Weak Gravity Conjecture
from
Amplitudes' Positivity

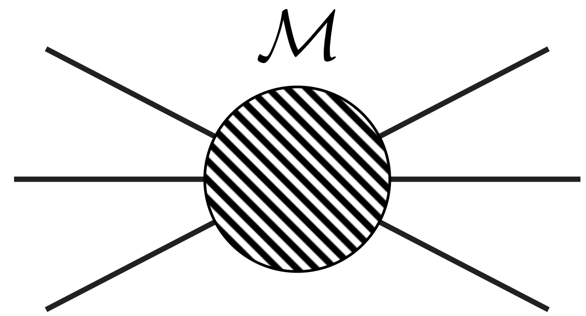
UC DAVIS
UNIVERSITY OF CALIFORNIA

Javi Serra

Technische Universität München

B.Bellazzini, M.Lewandowski and JS
arXiv:1902.03250

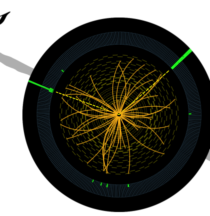
Program to constrain EFTs from consistency with basic physical principles:



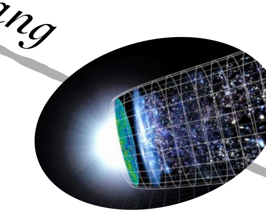
unitarity, locality and causality.

cosmology

colliders

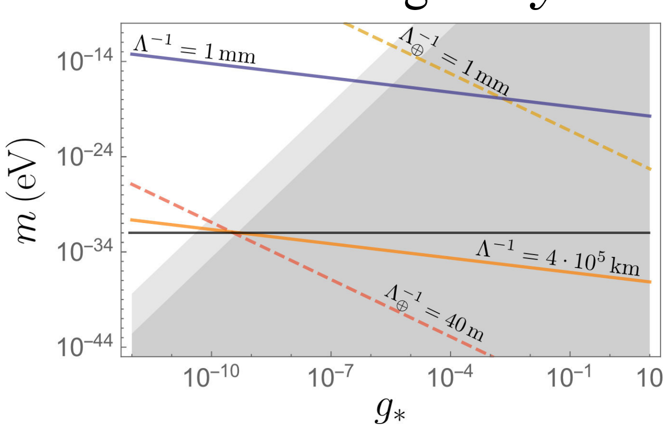


big bang



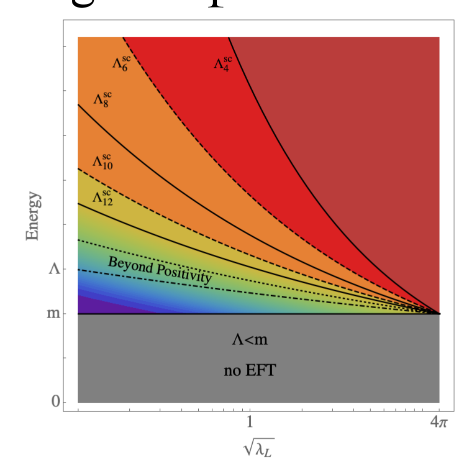
$E \sim \frac{1}{r}$

massive gravity



(Bellazzini, Riva, JS, Sgarlata '17)

higher-spin theories



(Bellazzini, Riva, JS, Sgarlata '19)

fermion compositeness

composites	goldstino $\sqrt{F}$ $[m_*]$ (TeV)	chiral f $[m_*]$ (TeV)
d_R	2.6 [9.4]	2.9 [36]
u_R	3.8 [13.5]	4.7 [59]
u_R, d_R	3.9 [13.7]	4.9 [62]
q_L	3.9 [13.7]	4.9 [62]
q_L, d_R	4.0 [14.2]	5.0 [63]
q_L, u_R	4.5 [16.1]	5.7 [72]
q_L, u_R, d_R	4.6 [16.2]	5.8 [73]

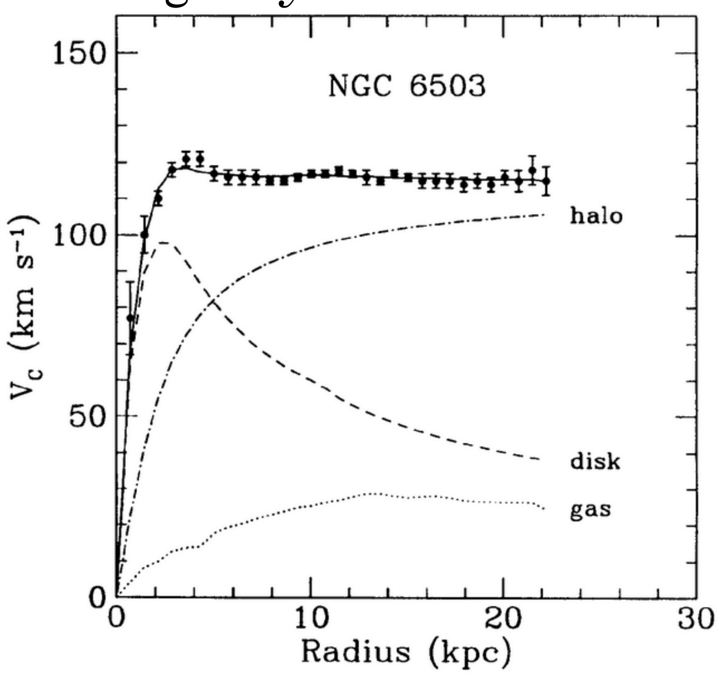
(Bellazzini, Riva, JS, Sgarlata '17)

Motivations

Gravity is related to many of the deepest puzzles in fundamental physics today.

Dark Matter

galaxy rotation curves



Coma cluster

gravitational lensing

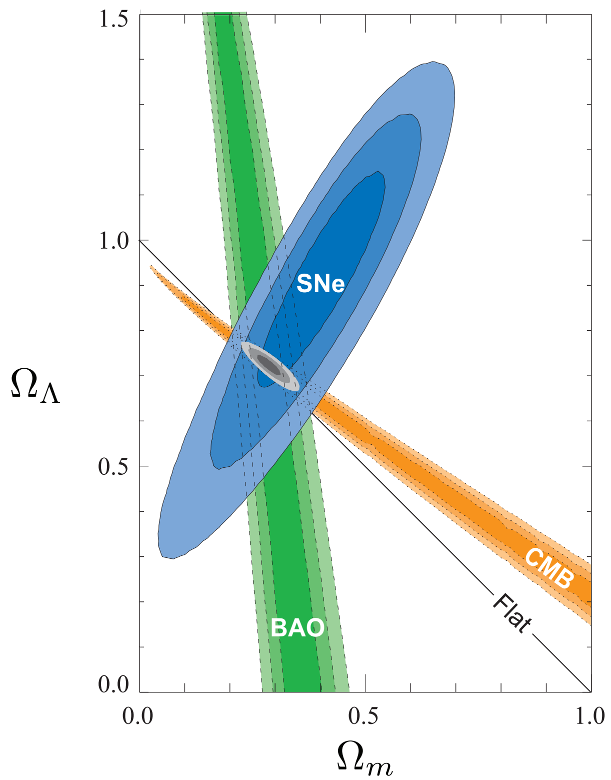
CMB, BAO, SNe, …

Dark matter has only been felt gravitationally.

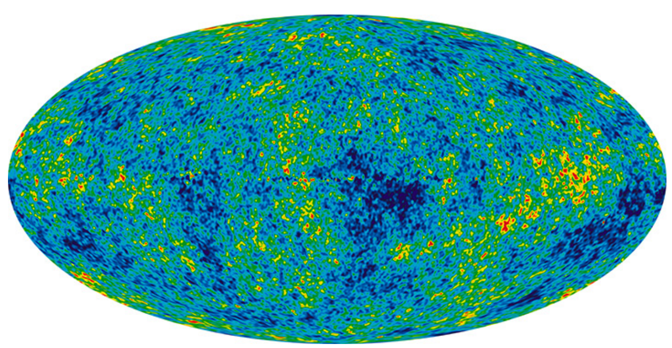
Gravity is related to many of the deepest puzzles in fundamental physics today.

Inflation

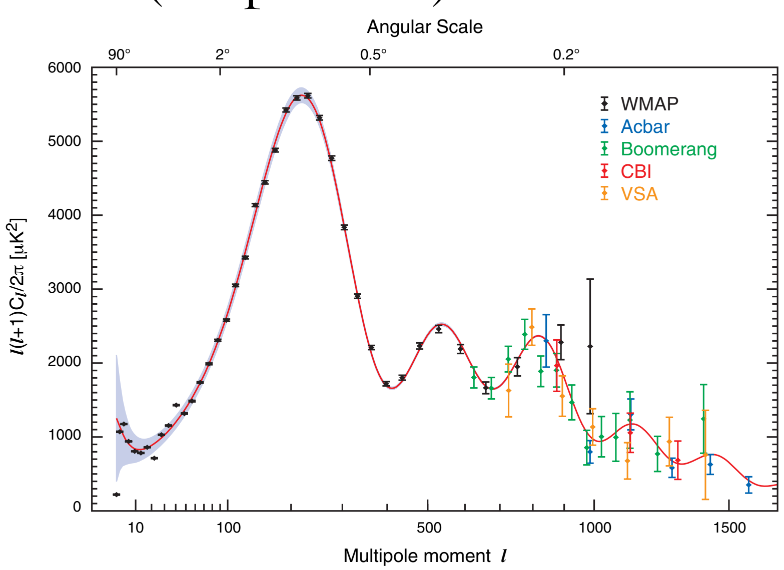
flatness



homogeneity



(temperature) fluctuations



Inflation resembles a gravitating, nearly constant, vacuum energy.

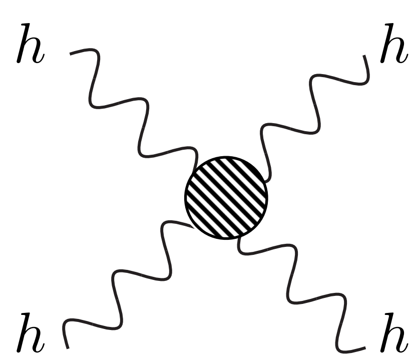
Gravity is related to many of the deepest puzzles in fundamental physics today.

Strong Quantum Gravity

$$\mathcal{S}_{\rm EH} = \frac{1}{2} M_{\rm Pl}^2 \int d^4x \sqrt{-g}\, R \;=\; O(\partial^2 h^n)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

graviton scattering



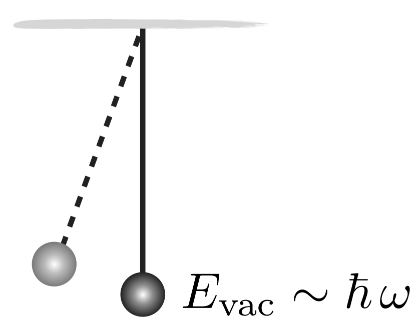
$$\mathcal{M} \sim \left(\frac{E}{M_{\rm Pl}}\right)^2 \xrightarrow{E \gg M_{\rm Pl}} P \sim |\mathcal{M}|^2 \gg 1$$

Gravity needs UV completion, e.g. string theory, and inevitably also the SM.

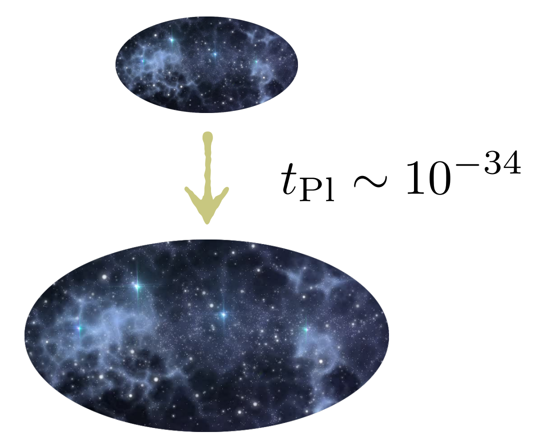
Gravity is related to many of the deepest puzzles in fundamental physics today.

Cosmological Constant

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \frac{1}{M_{\rm Pl}^2} g_{\mu\nu} \Lambda_{\rm CC} = 0$$



$$\frac{\Lambda_{\rm CC}}{M_{\rm Pl}^4} \sim 1$$



$$\frac{\Lambda_{\rm CC}}{M_{\rm Pl}^4} \sim 10^{-120}$$

Vacuum energy shows largest disagreement between natural expectation and experiment.

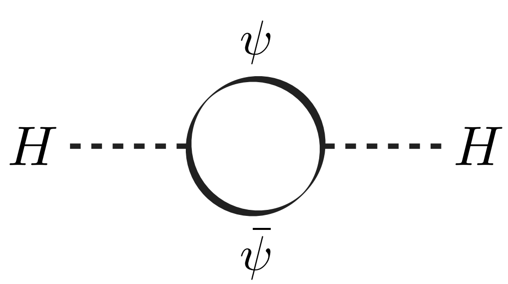
Gravity is related to many of the deepest puzzles in fundamental physics today.

Electroweak Scale

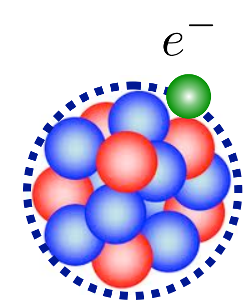
$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm kinetic} + m_H^2 |H|^2 - \lambda_H |H|^4 + y_\psi \bar{\psi}_L H \psi_R$$

$$\langle H \rangle = v\,, \quad m_\psi = y_\psi v$$

quantum corrections



$$m_H^2 \sim y_\psi^2 M_{\rm Pl}^2 \longrightarrow \frac{v^2}{M_{\rm Pl}^2} \sim 1$$



$$a_0 \sim \frac{\hbar}{c}\frac{1}{\alpha M_{\rm Pl}} \sim \ell_{\rm Pl}$$

$$\frac{v^2}{M_{\rm Pl}^2} \sim 10^{-30}$$

Higgs mass also shows large disagreement between natural expectation and experiment.

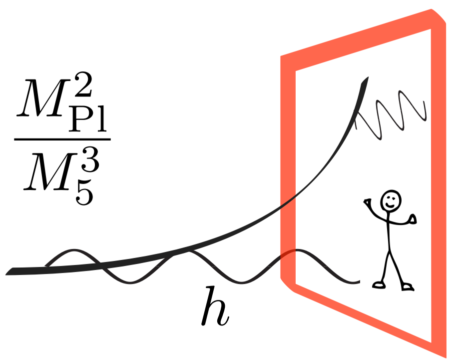
Tests of Gravity

experimental

modified gravity

DGP model

$$\ell_{\text{DGP}} = \frac{M_{\text{Pl}}^2}{M_5^3}$$



GRSMEFT

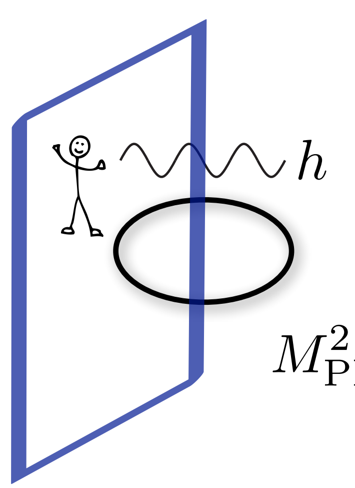
(Ruhdorfer, JS, Weiler '19)

$$\mathcal{L} = \frac{1}{2} M_{\text{Pl}}^2 R + \mathcal{L}_{\text{SM}} + \sum \mathcal{O}_{d>4}$$

e.g.

$$\mathcal{O} = F_{\mu\nu} F^{\rho\sigma} W^{\mu\nu\rho\sigma}$$

large extra-dimensions (ADD)

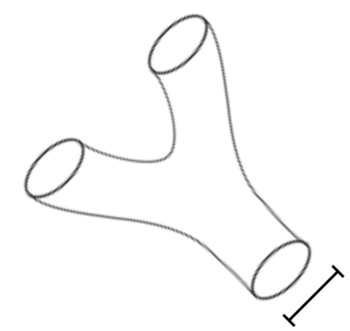


$$M_{\text{Pl}}^2 = M_*^{2+n} R^n$$

theoretical

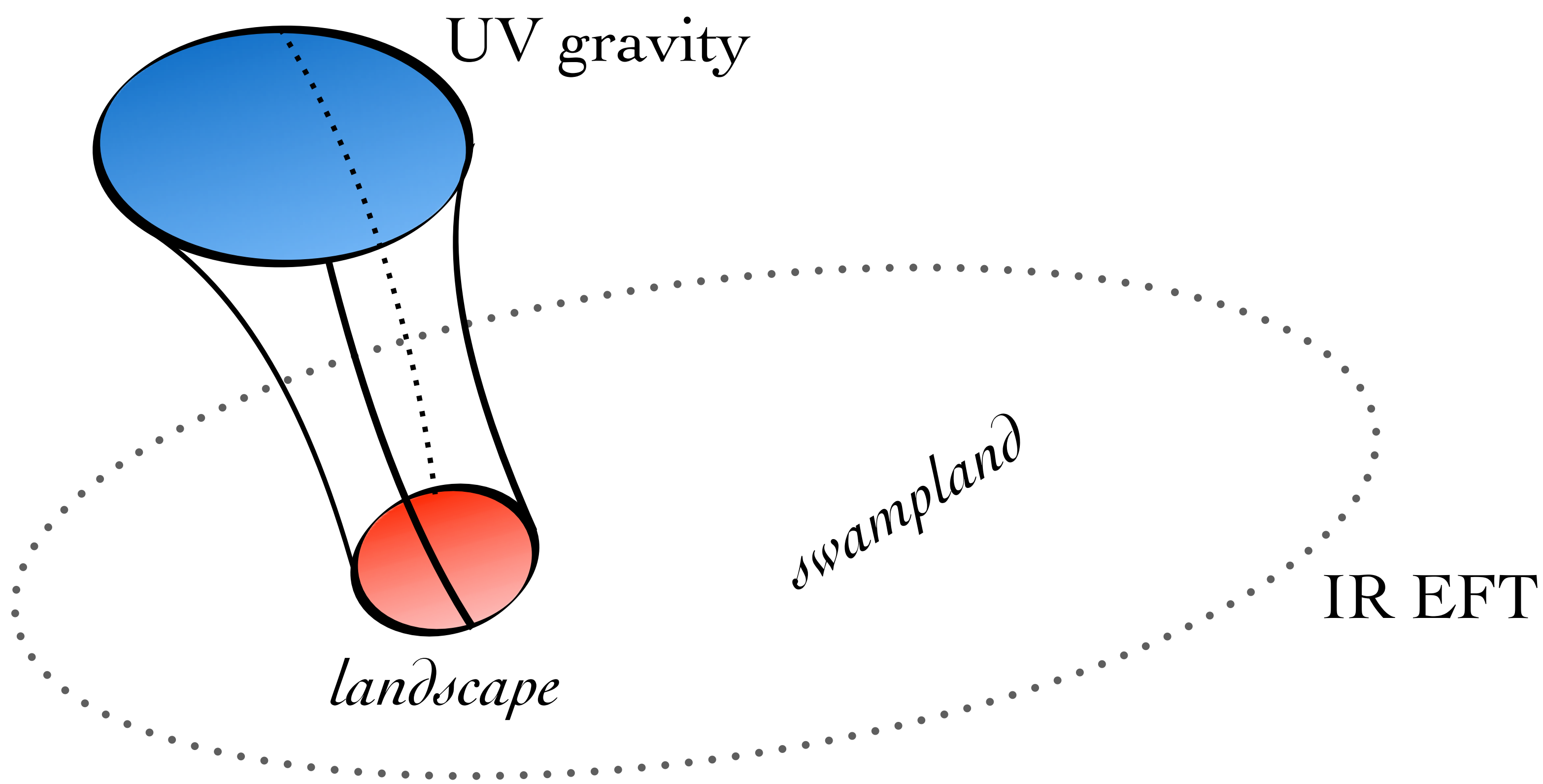
UV completion

string theory

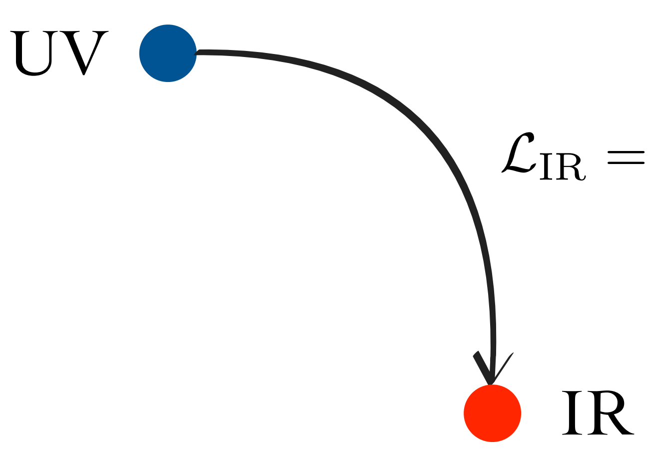


$$\ell_{\text{Pl}} \sim \frac{\hbar}{c} \frac{1}{M_{\text{Pl}}}$$

To charter the space of EFTs from gravity's UV completions is crucial.



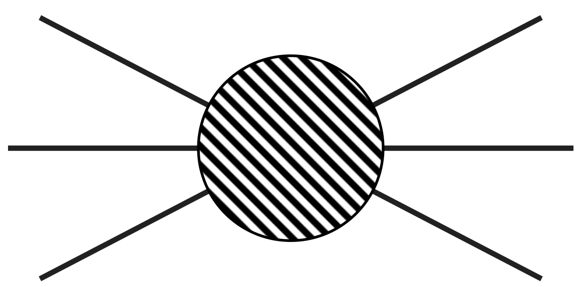
An even more general question: what is the space of consistent EFTs?



$$\mathcal{L}_{\rm IR} = \sum_i \frac{\mathcal{O}_i(x)}{\Lambda^{\Delta_i - 4}}$$

EFT = expansion in fields/derivatives consistent with symmetries of the system.

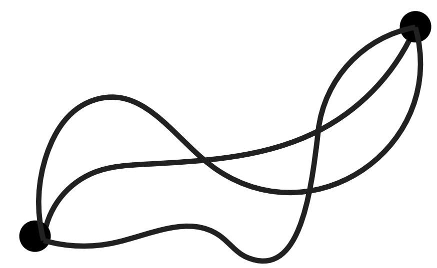
Amplitudes' positivity



Quantum Field Theory is the successful unification of Quantum Mechanics and Relativity.

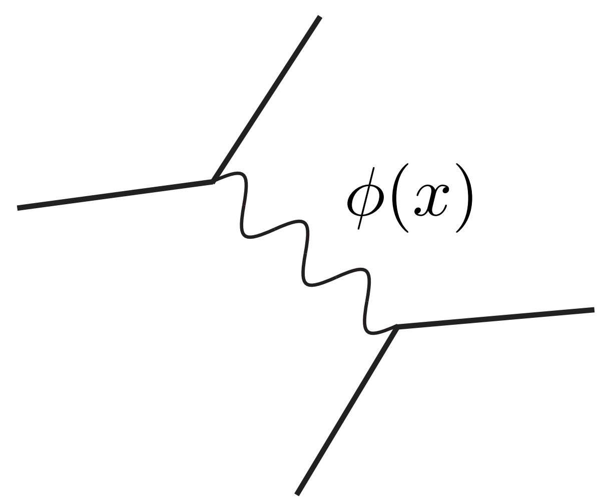
- Unitarity:

$$\sum P_i = 1$$

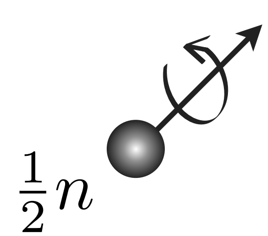


$$\sum_{\text{paths}} e^{i\mathcal{S}}$$

- (manifest) Locality:



- Poincaré invariance:



		little group
massive:	spin	$SU(2)$
massless:	helicity	$ISO(2)$

- Causality:

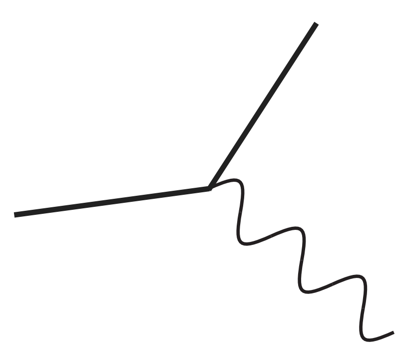
$$[\phi(x_A), \phi(x_B)] = 0 \qquad (x_A - x_B)^2 > 0$$

(spacelike)

- Gauge (redundancy) invariance:

  - h = ± 1 (2 d.o.f.), $A_\mu \to A_\mu + \partial_\mu \lambda$
  - h = ± 2 (2 d.o.f.), $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu - \partial_\mu \xi^\alpha \partial_\nu \xi_\alpha$

These principles alone make QFT a very powerful and constrained framework, e.g.

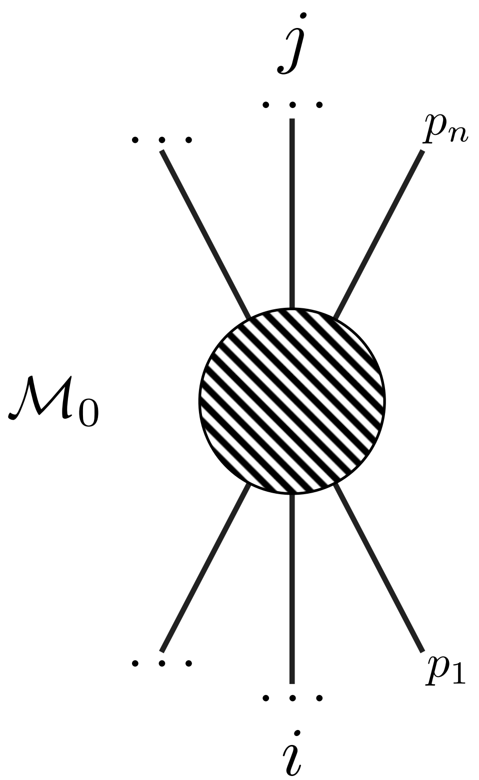


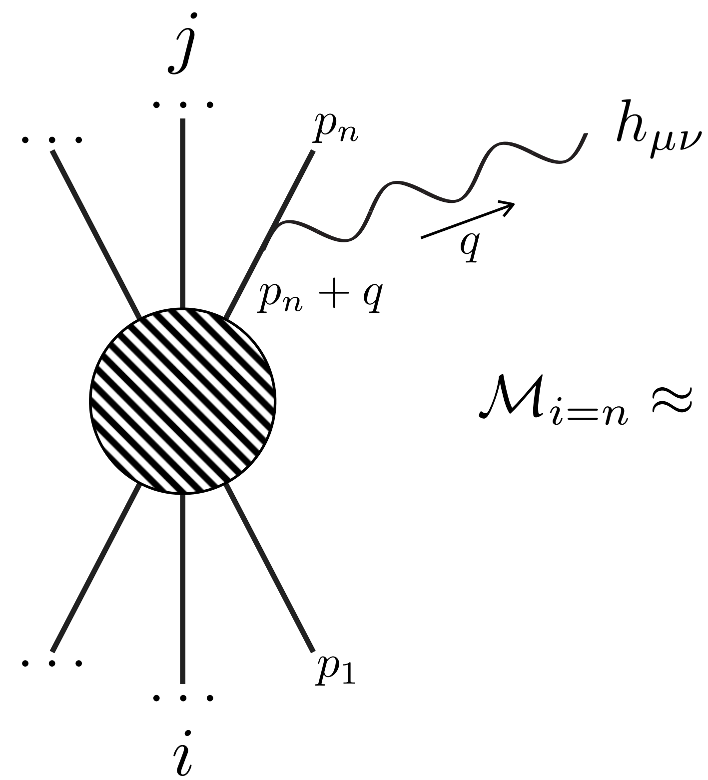
- h = ± 1: *charge conservation*
- h = ± 2: *equivalence principle*
- |h| ≥ 3: *no long-range forces*

Effective Field Theory

EFTs describe the relevant physics at low energies.

Weinberg soft theorem (gravity h = ± 2)





$$\mathcal{M}_{i=n} \approx -\kappa_n \frac{p_n^\mu \, p_n^\nu \, \epsilon_{\mu\nu}}{p_n \cdot q} \mathcal{M}_0$$

coupling at q = 0

$$\mathcal{M} \approx \mathcal{M}_0 \left[\sum_{\text{initial}} \kappa_i \frac{p_i^\mu \, p_i^\nu}{p_i \cdot q} - \sum_{\text{final}} \kappa_j \frac{p_j^\mu \, p_j^\nu}{p_j \cdot q} \right] \epsilon_{\mu\nu}$$

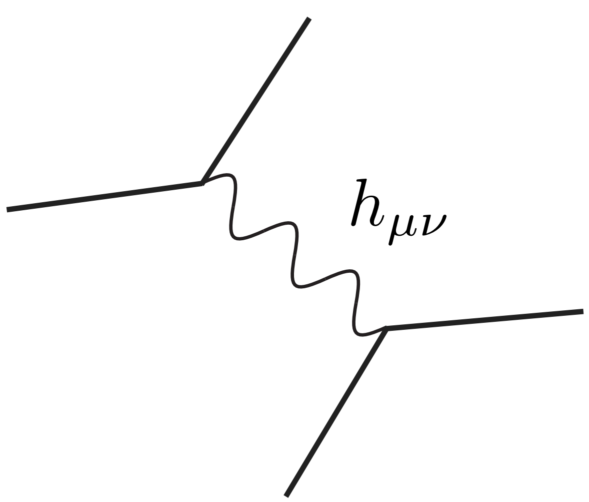
gauge invariance

$$\epsilon'_{\mu\nu} = \epsilon_{\mu\nu} + q_\mu \lambda_\nu + q_\nu \lambda_\mu$$

$$\mathcal{M}(\epsilon') = \mathcal{M}(\epsilon) \longrightarrow \sum_{\text{initial}} \kappa_i p_i^\nu = \sum_{\text{final}} \kappa_j p_j^\nu$$

Equivalence Principle

$$\kappa_{i,j} = \kappa = \frac{1}{M_{\rm Pl}} \;\forall\, i, j$$



$$\mathcal{L}_{\rm int} = h_{\mu\nu} T^{\mu\nu}\,, \quad \partial_\mu T^{\mu\nu} = 0$$

universal

General Relativity is the unique Lorentz invariant EFT for interacting massless spin-2 particles.

S-matrix Theory

$$S = \mathbb{I} + i(2\pi)^4 \delta(p) \mathcal{M}$$

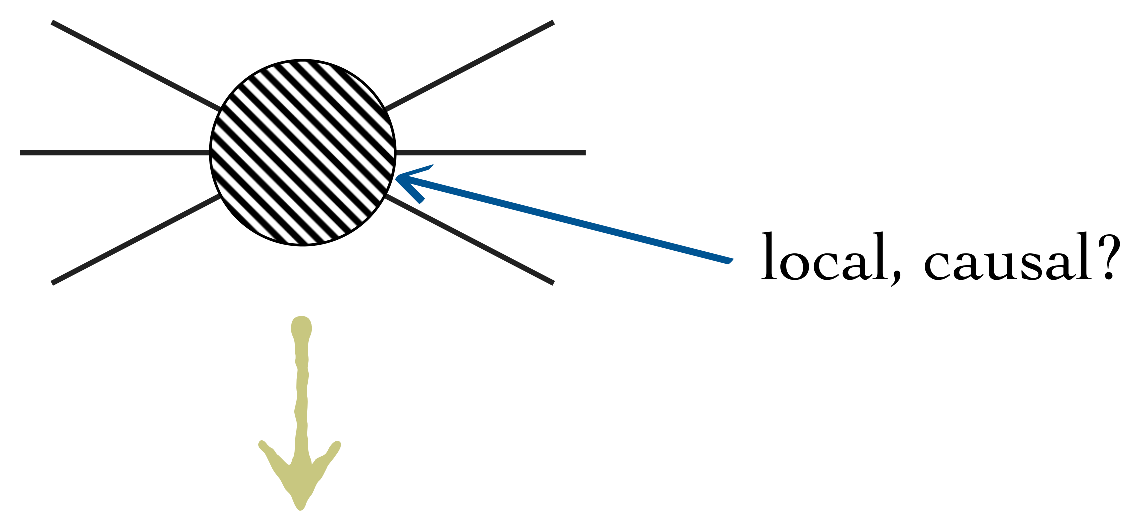
- Unitarity:

$$S^\dagger S = \mathbb{I} \qquad \text{(optical theorem)}$$

- Poincaré invariance:

  particle = (m, s)

  Little group covariant amplitudes.

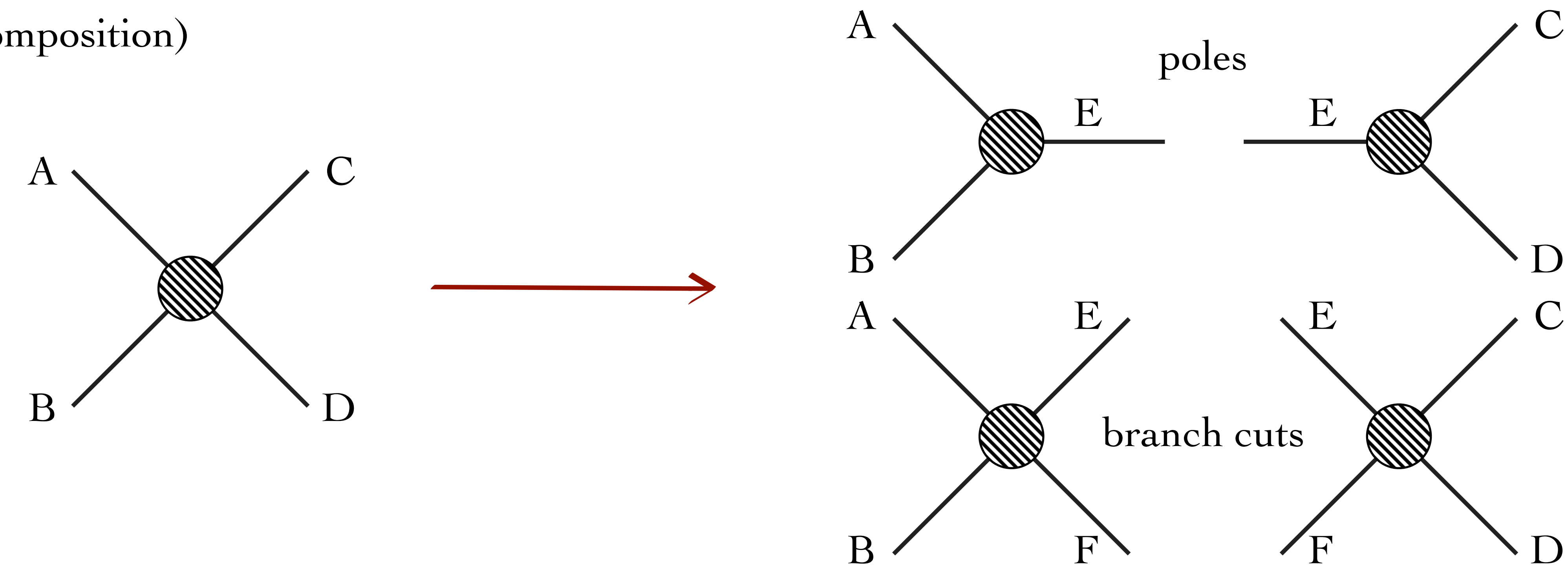
- 

Properties of scattering amplitudes in the complex plane of kinematical variables.

2 to 2 scattering Mandelstam variables: s, t, u

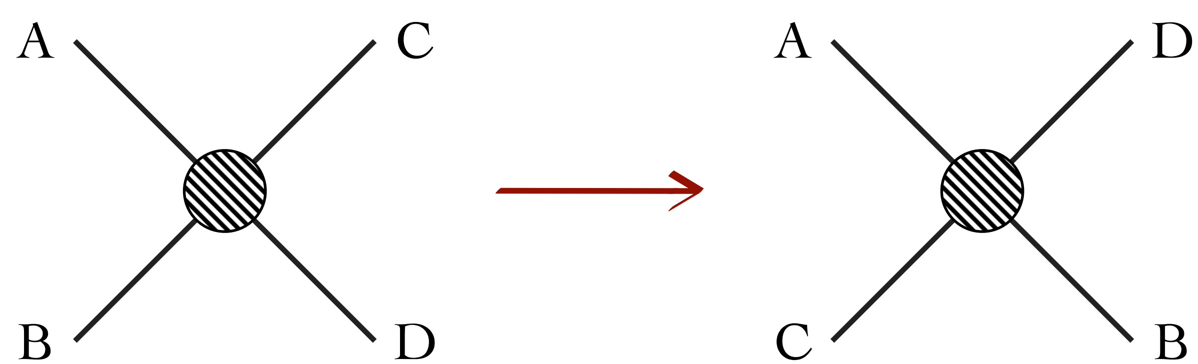
- Analyticity:
  (cluster decomposition)

$\mathcal{M}(s,t,u)$



- Crossing symmetry:

$\mathcal{M}(s,t,u)$



$\mathcal{M}(t,u,s)$

- Polynomial boundedness:
  (Froissart-Martin bound)

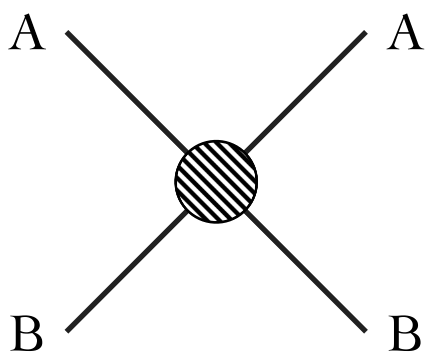
forward limit, $t \to 0$.

$$|\hat{\mathcal{M}}(s)| < c \cdot s \log^2 s$$

Satisfied by massive* QFT & perturbative (open) string theories.

(Gell-Mann, Goldberger, Thirring '54)

Dispersion Relation



$t \to 0$

$$\frac{1}{2\pi i}\oint_{\Gamma} \frac{\hat{\mathcal{M}}_{AB}(s)}{(s-\mu^2)^3}$$

forward elastic 2 to 2 scattering

Γ

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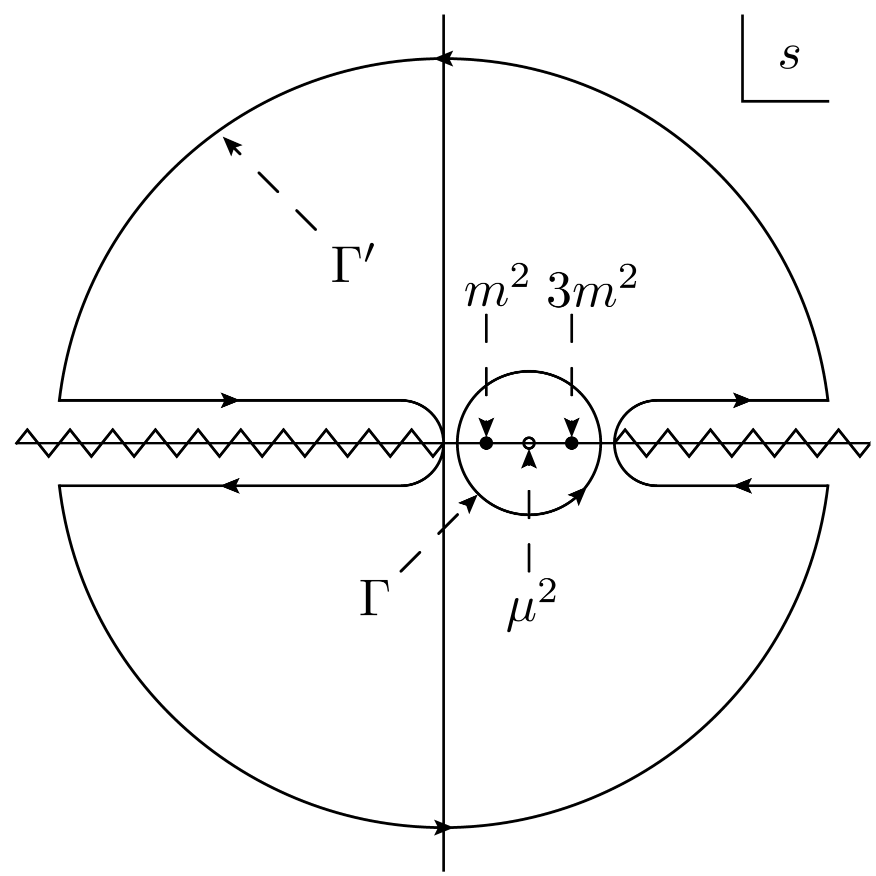


$\sum$ IR residues

$$\int_{4m^2}^{\infty} ds\,[\text{branch cuts}] + \text{boundary}$$

Froissart bound

crossing symmetry

optical theorem

tree-level EFT

$$\hat{\mathcal{M}}''_{AB}(s=\mu^2) = \frac{2}{\pi}\int^{\infty} \frac{ds}{s^2} \sum_X \sigma_{AB\to X}(s) > 0$$

Positivity constraint from a IR-UV connection.

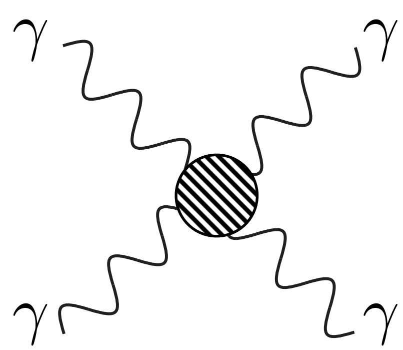
$$\times\left[1 + O\left(\frac{m^2}{\Lambda^2}, \frac{\mu^2}{\Lambda^2}\right)\right]$$

$$\hat{\mathcal{M}}''_{AB}(s=0) = \frac{2}{\pi}\int^{\infty} \frac{ds}{s^2} \sum_X \sigma_{AB\to X}(s) > 0$$

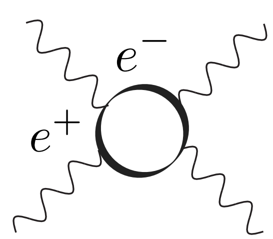
UV theories with canonical S-matrices give rise to EFTs satisfying positivity constraints.

- Theory of interacting photons: (Euler, Heisenberg '36)

$$\mathcal{L}_{\rm EH} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{\alpha_1}{4\Lambda^4}(F_{\mu\nu}F^{\mu\nu})^2 + \frac{\alpha_2}{4\Lambda^4}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2$$



$$\hat{\mathcal{M}}_{\uparrow\uparrow,\uparrow\downarrow} = \{|\#_1|\alpha_1, |\#_2|\alpha_2\}\frac{s^2}{\Lambda^4} \longrightarrow \alpha_{1,2} > 0$$



$$\frac{\alpha_1}{4\Lambda^4} = \frac{\alpha_{\rm em}^2}{90 m_e^4}\,, \qquad \frac{\alpha_2}{4\Lambda^4} = \frac{7\alpha_{\rm em}^2}{360 m_e^4}$$

Theories with wrong-sign coefficients live in the swampland of EFTs.

Many interesting applications:

a-theorem (4D)
Komargodski, Schwimmer '11
Luty, Polchinski, Rattazzi '12
...

Euler-Heisenberg, U(1) Goldstone
Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06
de Rham, Melville, Tolley, Zhou '18
...

Goldstini, R-axion
Dine, Festuccia, Komargodski '09
Bellazzini '16
Bellazzini, Mariotti, Redigolo, Sala, JS '17
...

compositeness, SMEFT
Distler, Grinstein, Porto, Rothstein '06
Vecchi '07
Low, Rattazzi, Vichi '09
Bellazzini, Riva, JS, Sgarlata '17
Bellazzini, Riva '18
Remmen, Rodd '19
...

massive gravity, Galileon
Cheung, Remmen '16
Bellazzini '16
de Rham, Melville, Tolley, Zhou '17
Bellazzini, Riva, JS, Sgarlata '17
...

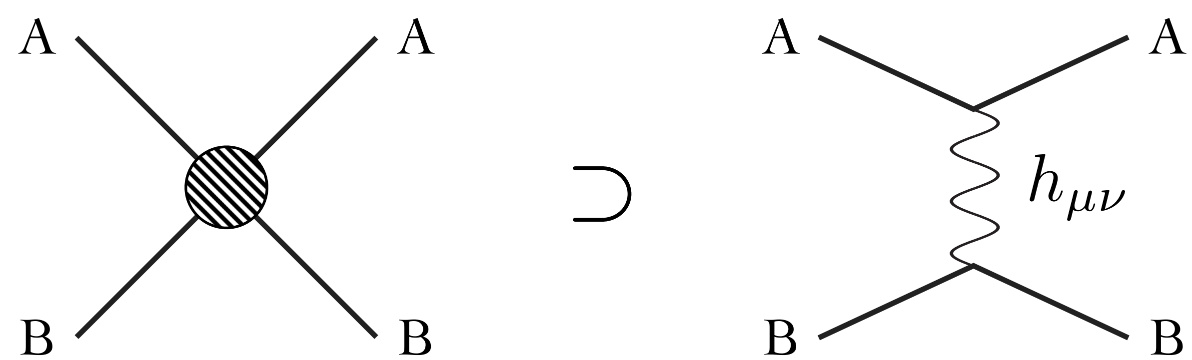
higher-spins
Bellazzini, Riva, JS, Sgarlata '19
...

quantum gravity
Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06
Bellazzini, Cheung, Remmen '15
Bellazzini, JS, Lewandowski '19
...

among others.

Gravitational Amplitudes

Turning on gravity gives rise to a universal forward (Coulomb) singularity.



$$\mathcal{M}(t \to 0) = -\frac{1}{M_{\mathrm{Pl}}^2}\frac{s^2}{t} + O(s)$$

$$\hat{\mathcal{M}}''_{AB}(s=0) = \frac{2}{\pi}\int^{\infty} \frac{ds}{s^2}\sum_X \sigma_{AB\to X}(s) > 0$$

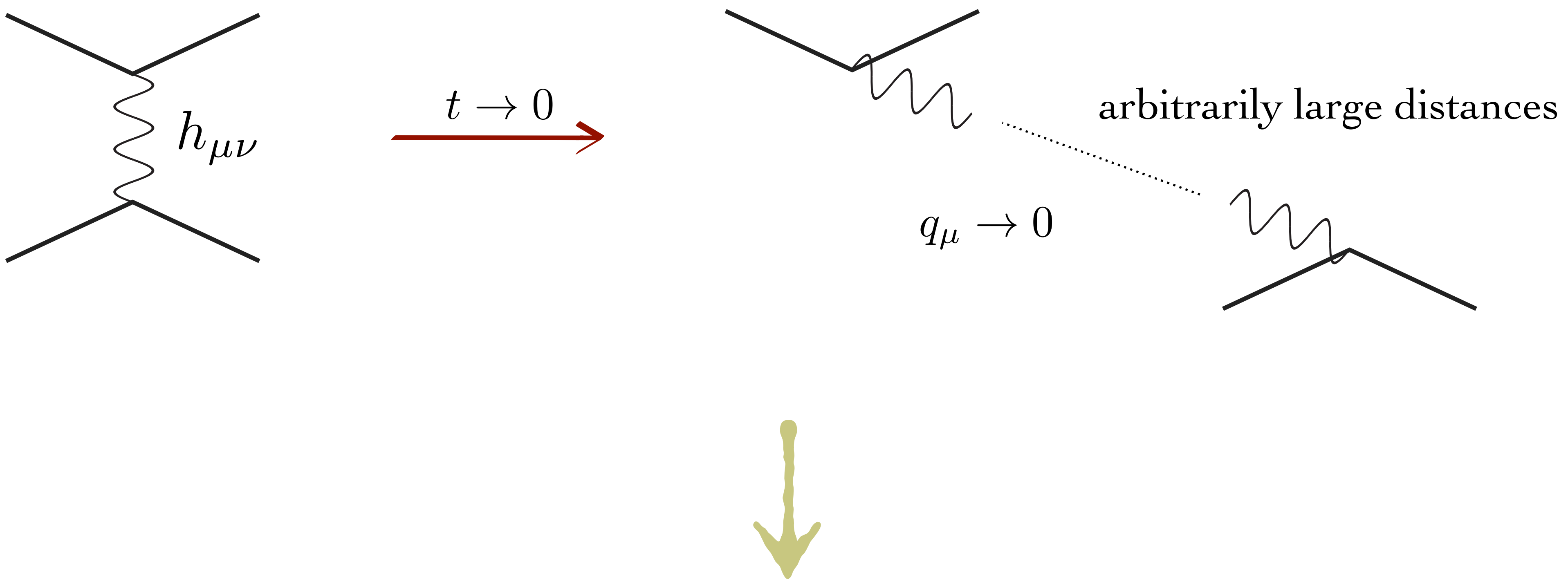
$$\infty = \infty$$

Amplitudes' positivity in gravitational theories useless unless we regulate the *t*-channel pole.

Graviton Singularity Regularization

The source of the problem: in the forward limit, soft graviton probes the infinite (flat) space.

$$\mathcal{M}(t \to 0) = -\tfrac{1}{M_{\rm Pl}^2}\frac{s^2}{t} + O(s)$$

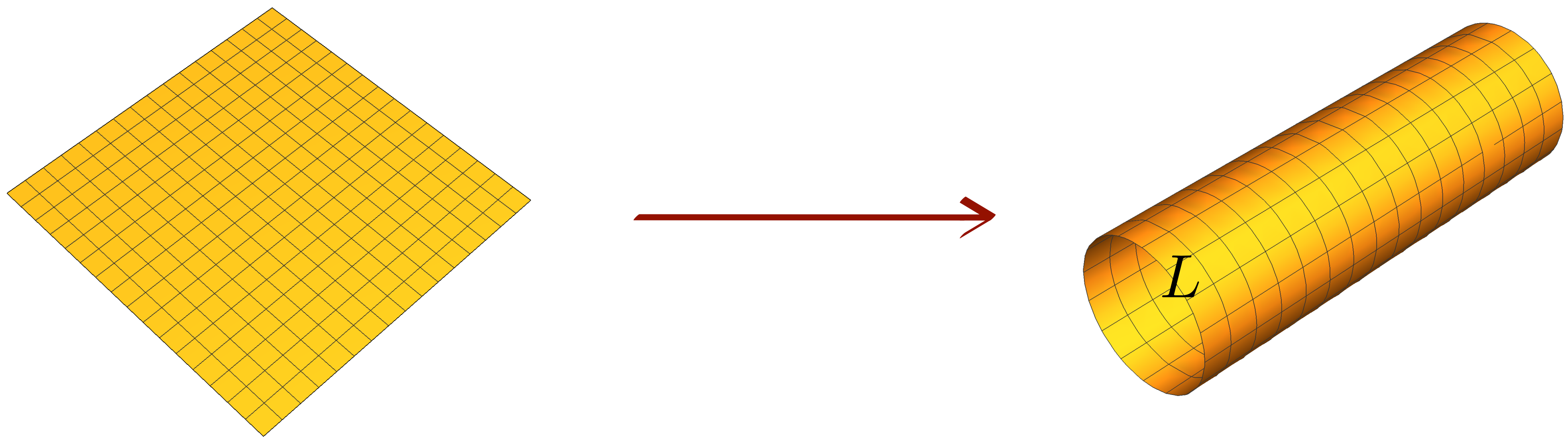


The idea of the solution: reduce available space.

(Bellazzini, Lewandowski, JS '19)

Compactification to 3D

(3+1)D flat space | (2+1)D flat space



$\hat{g}_{MN}$ → $g_{\mu\nu}$, σ , V_μ , KK-modes

- $g_{\mu\nu}$: non-dynamical metric
- σ: dilaton
- V_μ: graviphoton
- KK-modes: $m_n^2 \sim \frac{n^2}{L^2}$

$\hat{A}_M$ → A_μ , Φ , KK-modes

- A_μ: photon
- Φ: scalar-photon

$M, N = 0, 1, 2, 3$ | $\mu, \nu = 0, 1, 2$

$L \to \infty$ to recover 4D dynamics.

Several important comments on scattering in 3D.

- Little group:

		little group
massless:	helicity	Z_2
massive:	spin	$U(1)$

Any (even) massless field is dual to a scalar field.

e.g. 3D photon:

$$\Delta\mathcal{S} = \int d^3x\sqrt{-g}\ \tfrac{1}{2}F_{\mu\nu}\epsilon^{\mu\nu\rho}\partial_\rho\varphi$$

Several important comments on scattering in 3D.

- Gravitational scattering:

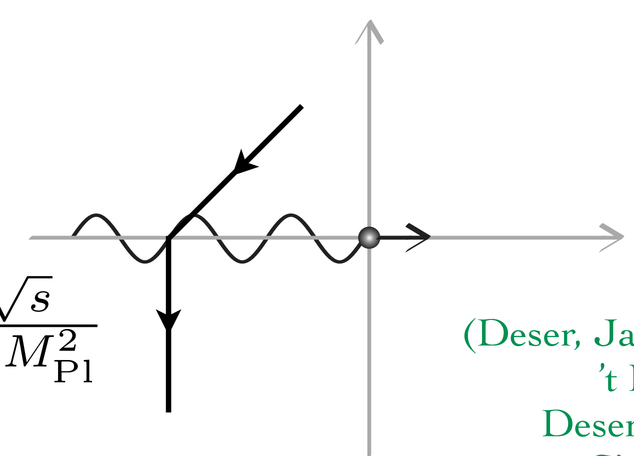
$$\mathcal{S} = L \int d^3x \sqrt{-g}\, \mathcal{L}_{\rm EM}\left[R^{\mu\nu},\, \sigma,\, V^{\mu\nu},\, \Phi,\, F^{\mu\nu}\right]$$

They mediate gravitational interactions.

Non-propagating, yet gravity is non-trivial: Aharonov-Bohm-like scattering

$$\mathcal{M}_{\rm EH} \simeq -\frac{1}{LM_{\rm Pl}^2}\frac{s^2}{t + \frac{s^2}{16(LM_{\rm Pl}^2)^2}}$$

$$\frac{\theta}{2} \simeq \frac{\sqrt{s}}{4LM_{\rm Pl}^2}$$



(Deser, Jackiw, 't Hooft '84
't Hooft '88
Deser, Jackiw '88
Ciafaloni '92
Zeni '93
Deser, McCarthy, Steif '94
…)

Forward limit is regular — leading gravitational amplitude can be explicitly subtracted.

Several important comments on scattering in 3D.

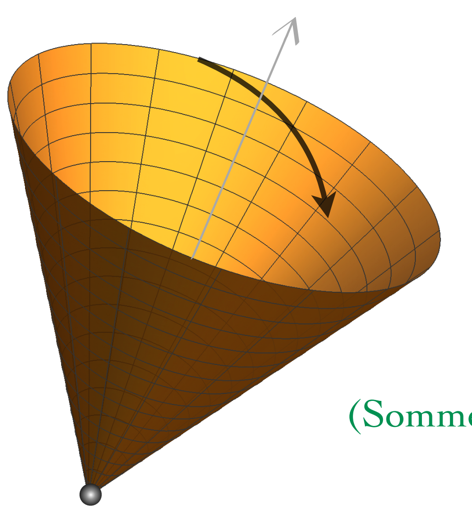
- Gravitational scattering:

$$\mathcal{S} = L \int d^3x \sqrt{-g}\, \mathcal{L}_{\rm EM} \left[R^{\mu\nu},\, \sigma,\, V^{\mu\nu},\, \Phi,\, F^{\mu\nu}\right]$$

They mediate gravitational interactions.

Non-propagating, yet gravity is non-trivial: scattering on a conical space

$$\mathcal{M}_{\rm EH} \simeq -\frac{1}{LM_{\rm Pl}^2} \frac{s^2}{t + \frac{s^2}{16(LM_{\rm Pl}^2)^2}}$$



(Sommerfeld, 19th century)

Forward limit is regular — leading gravitational amplitude can be explicitly subtracted.

Weak Gravity Conjecture

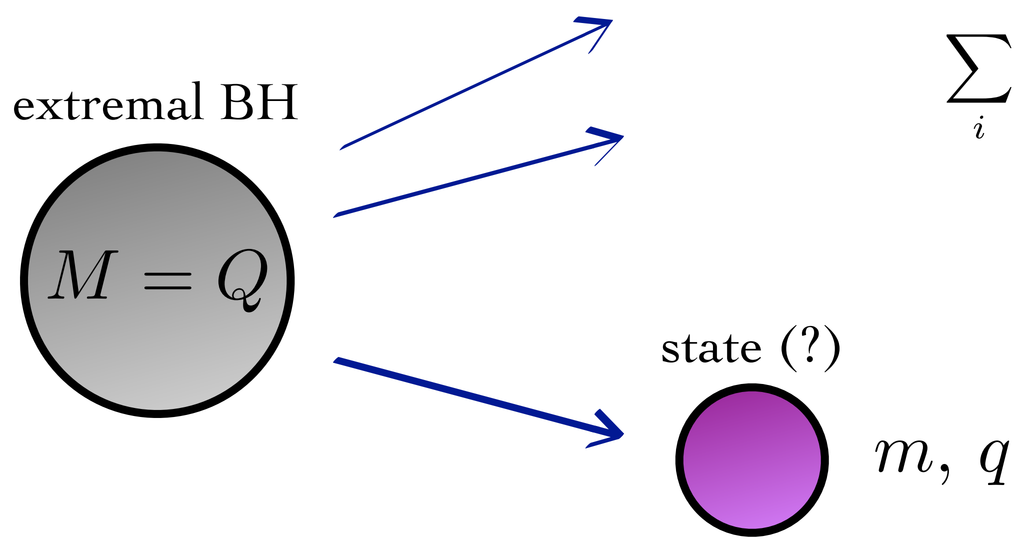
(Arkani-Hamed, Motl, Nicolis, Vafa '06)

Weak Gravity Conjecture

The (mild form of the) WGC is a statement about extremal Black Holes being able to decay.

$$\sum_i m_i = \sum_i \frac{m_i}{|q_i|}|q_i| > \sum_i |q_i| = M$$

$$\sqrt{2}\, g M_{\rm Pl} \equiv 1$$



$$\sqrt{2}\, g M_{\rm Pl} \left.\frac{q}{m}\right|_{\rm min} > 1$$

There must exist states for which gravity is the weakest force — seems statement beyond EFT.

(Arkani-Hamed, Motl, Nicolis, Vafa '06)

Einstein-Maxwell EFT

Effects of leading higher-dimensional operators on BH extremality condition.

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\tfrac{1}{2}M_{\rm Pl}^2 R - \tfrac{1}{4}F^{MN}F_{MN} + \alpha_1 \tfrac{1}{4M_{\rm Pl}^4}\left(F^{MN}F_{MN}\right)^2 + \alpha_2 \tfrac{1}{4M_{\rm Pl}^4}\left(\widetilde{F}^{MN}F_{MN}\right)^2 + \alpha_3 \tfrac{1}{2M_{\rm Pl}^2} F_{MN}F_{RS}W^{MNRS}\right]$$

Weyl tensor

↓

$$\sqrt{2}\, g M_{\rm Pl}\, \frac{Q}{M}\bigg|_{\rm extremal} = 1 + \tfrac{4}{5}(4\pi)^2 \frac{M_{\rm Pl}^2}{M^2}(2\alpha_1 - \alpha_3)$$

If $2\alpha_1 - \alpha_3 > 0$, extremal BHs are in fact the required states.

expansion parameter

$$\frac{M_{\rm Pl}^4}{\Lambda^2 M^2} = (\Lambda R_S)^{-2}$$

Proof of the WGC

Scattering photon 3D zero-modes.

$$\hat{\mathcal{M}}''_{AB}(s=0) = \frac{2}{\pi}\int^{\infty}\frac{ds}{s^2}\sum_X \sigma_{AB\to X}(s) > 0$$

$$\hat{\mathcal{M}}(\Phi\Phi \to \Phi\Phi) = \frac{2s^2}{M_{\rm Pl}^4 L}(2\alpha_1 - \alpha_3) > 0$$

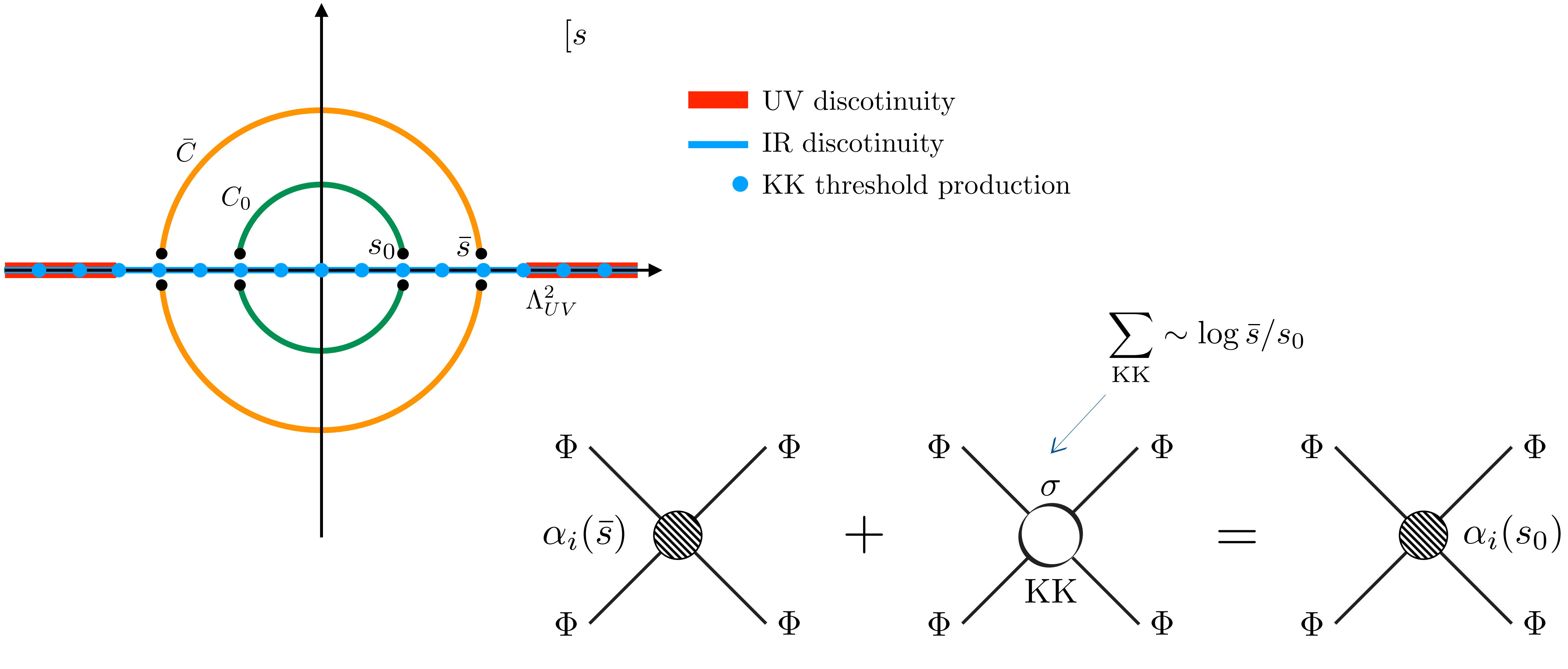
$$\hat{\mathcal{M}}(AA \to AA) = \frac{2s^2}{M_{\rm Pl}^4 L}(2\alpha_1 + \alpha_3) > 0$$

$$\hat{\mathcal{M}}(\Phi A \to \Phi A) = \frac{4s^2}{M_{\rm Pl}^4 L}\alpha_2 > 0$$

The mild form of the WGC follows from prime principles of the S-matrix.

Several important comments on the result are in order.

- Loop corrections:

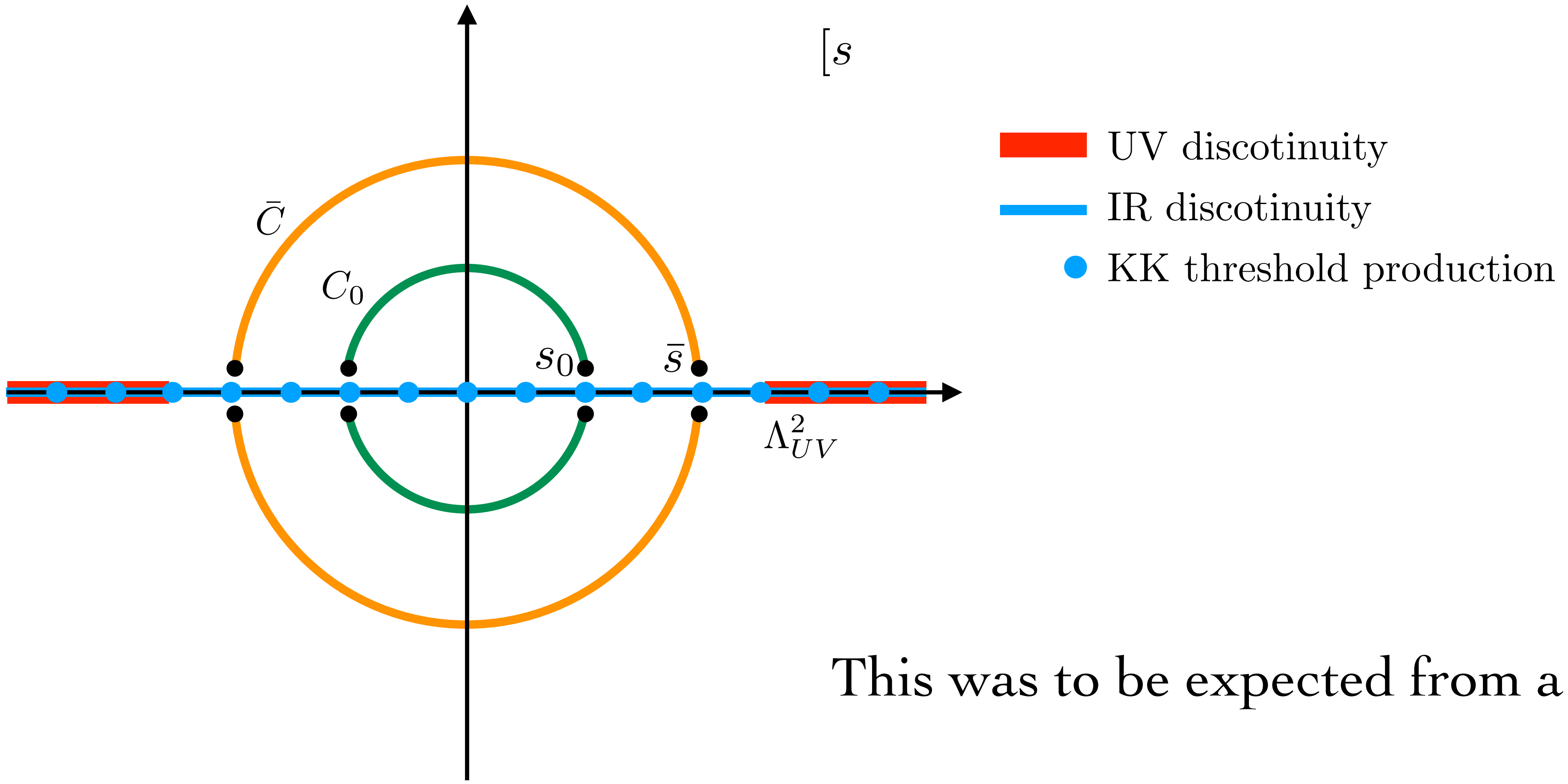


Positive contribution to the RG running of the higher-dimensional operators —can be subtracted.

$$\beta_{2\alpha_1 \pm \alpha_3} < 0\,,\ \ \beta_{\alpha_2} < 0$$

Several important comments on the result are in order.

- Loop corrections:



This was to be expected from a purely 4D calculation:

(Deser, Nieuwenhuizen '74)

$$\alpha_{1,2}(s_0) = \alpha_{1,2}(\bar{s}) + \frac{1}{8\pi^2}\frac{137}{60}\log\frac{\bar{s}}{s_0}$$

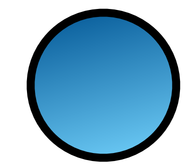
This implies that asymptotically large extremal BHs automatically satisfy the WGC.

Several important comments on the result are in order.

- Rest of positivity bounds:

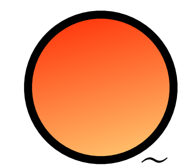
$$\hat{\mathcal{M}}(\Phi\Phi \to \Phi\Phi) = \frac{2s^2}{M_{\rm Pl}^4 L}\,(2\alpha_1 - \alpha_3) > 0$$

$$\hat{\mathcal{M}}(AA \to AA) = \frac{2s^2}{M_{\rm Pl}^4 L}\,(2\alpha_1 + \alpha_3) > 0$$ → magnetic extremal BH



$$\frac{\tilde{Q}}{M} > 1$$

$$\hat{\mathcal{M}}(\Phi A \to \Phi A) = \frac{4s^2}{M_{\rm Pl}^4 L}\alpha_2 > 0$$ → dyonic extremal BH



$Q = \tilde{Q}$

$$\frac{\sqrt{Q^2+\tilde{Q}^2}}{M} > 1$$

(e.g. Jones, McPeak '19)

↓

Continuous positivity bounds associated with arbitrary linear polarizations.

$$|c_{1,2}\rangle = c_{\theta_{1,2}}|\uparrow_{1,2}\rangle + s_{\theta_{1,2}}|\downarrow_{1,2}\rangle$$

$$\alpha_3(c_{2\theta_1} + c_{2\theta_2}) + 4\alpha_1 c^2_{\theta_1+\theta_2} + 4\alpha_2 s^2_{\theta_1+\theta_2} > 0$$

Match those derived from $\Delta S > 0$ for dyonic, non-extremal BHs.

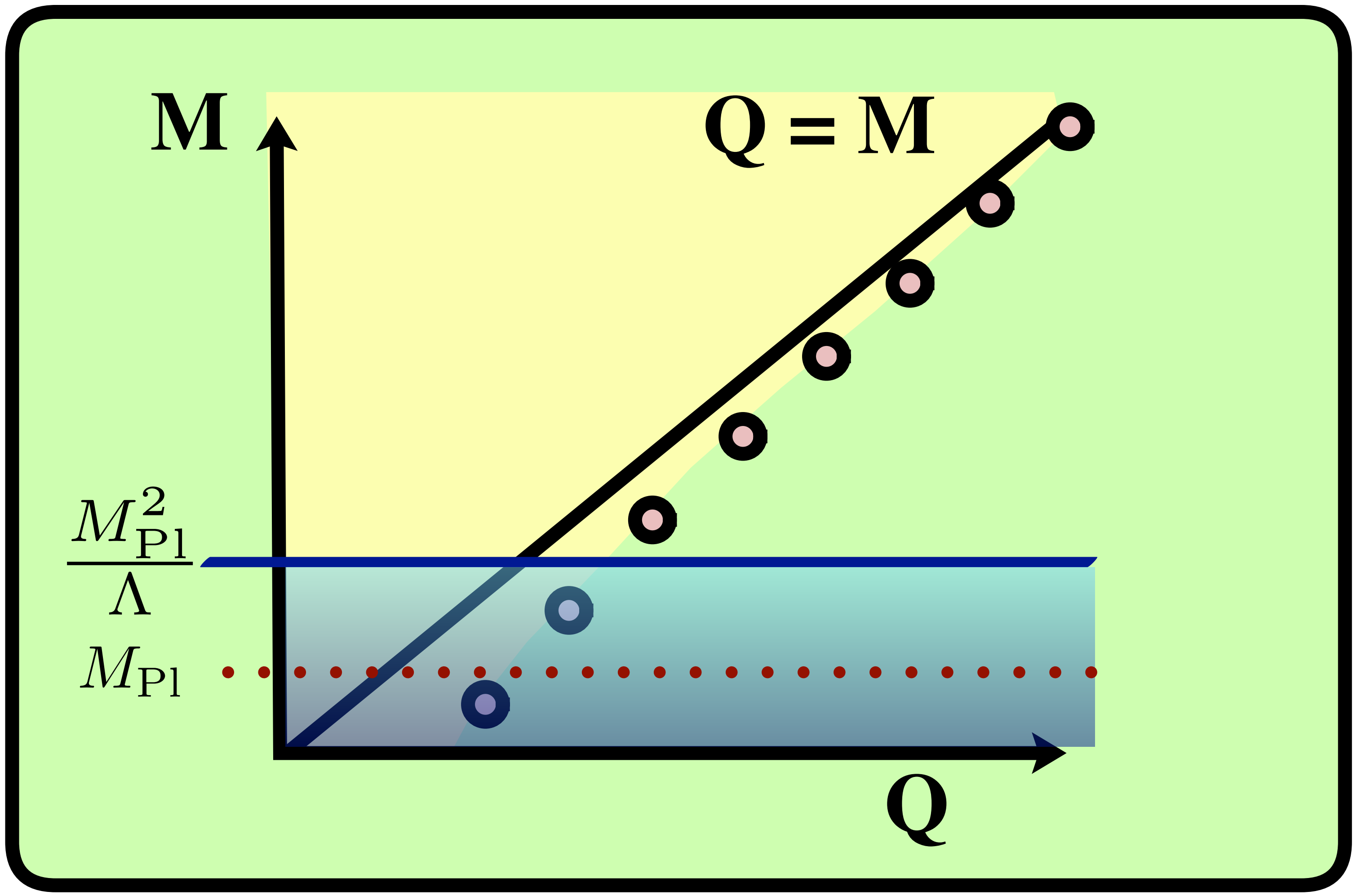
(Cheung, Liu, Remmen '19)

Several important comments on the result are in order.

- Beyond EFT leap:

Indication of super-extremal ($q > m$) sub-Planckian particle content,

(Arkani-Hamed, Motl, Nicolis, Vafa '06)



might follow from modular invariance in perturbative string theory.

(e.g. Heidenreich, Reece, Rudelius '17)

Several important comments on the result are in order.

- Relaxing Froissart bound:

Main UV input is polynomial bound — maximal consistent departure

$$\lim_{s\to\infty} |\hat{\mathcal{M}}(s)/s^2| \sim \frac{1}{\Lambda^2 M_{\rm Pl}^2 L}$$

Such behaviour is suggested by scattering of (3D) gravitational zero-modes.

$$\hat{\mathcal{M}}(\Phi\sigma \to \Phi\sigma) = -\hat{\mathcal{M}}(A\sigma \to A\sigma) = \frac{s^2}{M_{\rm Pl}^4 L}\alpha_3 \longrightarrow |\alpha_3| \lesssim \frac{M_{\rm Pl}^2}{\Lambda^2}$$

(Camanho, Edelstein, Maldacena, Zhiboedov '14)

Retaining Froissart bound for scattering of (3D) photon zero-modes

$$\alpha_{1,2} \sim g^2 \left(\frac{g}{4\pi}\right)^2 \frac{M_{\rm Pl}^4}{\Lambda^4} \qquad \alpha_3 \sim \left(\frac{g}{4\pi}\right)^2 \frac{M_{\rm Pl}^2}{\Lambda^2} \longrightarrow \Lambda \lesssim g M_{\rm Pl}$$

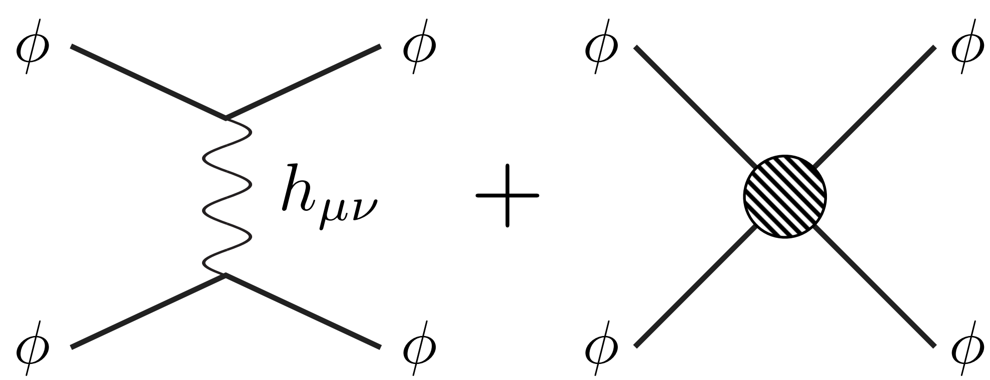
Modified gravity

Axion and P(X) theory

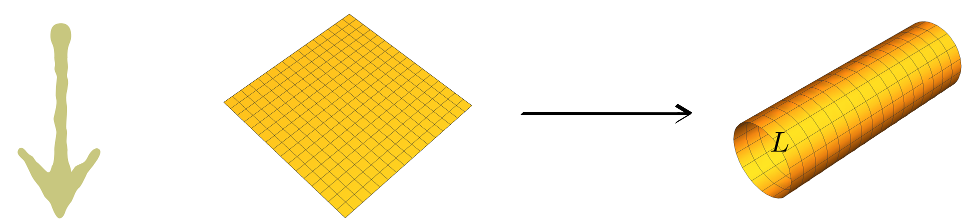
$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R - \frac{1}{2}(\partial\phi)^2 + \frac{a}{4f^4}(\partial\phi)^4\right]$$

Gravity turned off: $a > 0$

Late time cosmology: $f \sim \sqrt{M_{\rm Pl}H}$



$$\mathcal{M}(t \to 0) \sim -\frac{1}{M_{\rm Pl}^2}\frac{s^2}{H^2} + a\frac{s^2}{f^4}$$



$a > 0$

(Pirtskhalava, Santoni, Trincherini, Vernizzi '15)

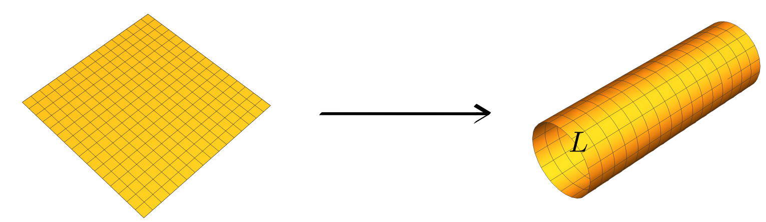
Weakly Broken Galileon

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\tfrac{1}{2} M_{\rm Pl}^2 R - \tfrac{1}{2}(\partial\pi)^2 - \frac{1}{2\Lambda_3^3}(\partial\pi)^2 \Box\pi + \frac{1}{4\Lambda_2^4}(\partial\pi)^4 + \ldots \right]$$

Admits stable Galileon-preserving vs -breaking hierarchy:

$$\Lambda_2^4 \simeq H^2 M_{\rm Pl}^2 \,, \;\; \Lambda_3^3 \simeq H^2 M_{\rm Pl}$$

$$\hat{\mathcal{M}}''_{AB}(s=0) = \frac{2}{\pi}\int^{\infty} \frac{ds}{s^2} \sum_X \sigma_{AB\to X}(s) > 0$$



$$\hat{\mathcal{M}}_{\pi\pi}(s=0) > \frac{2}{\pi}\int^{\Lambda^2} \frac{ds}{s^2} \sum_k \sigma_{\pi\pi\to\pi_k\pi_k}(s)$$

3D compactified beyond-positivity bounds.

(Bellazzini, Riva, JS, Sgarlata '17)

$$\Lambda < \left(H^3 M_{\rm Pl}\right)^{1/4} \left(\frac{16\pi^2}{c}\right)^{1/8} \sim \frac{1}{10^7\ \text{km}}$$

Regime of calculability limited — EFT needs UV completion at macroscopic distances.

Outlook

The WGC has many versions, ramifications, generalizations — checks.

- Multiple U(1)s.
- Supersymmetric extremal BHs.
- Axion decay constant.
- Einstein-Maxwell-dilaton.
- …

There are many potential applications of the gravitational positivity constraints.

- Gauss-Bonnet term.
- Inflationary models.
- General scalar-tensor theories.
- Scalar gravity.

(Ruhdorfer, JS, Weiler '19)

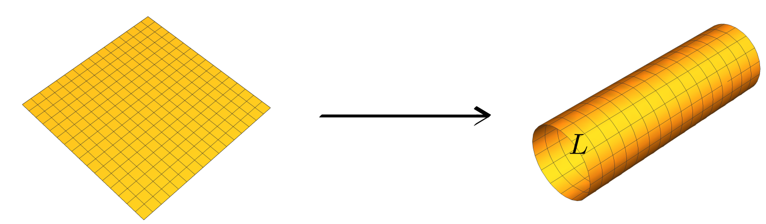
Positivity Constrains on the GRSMEFT

Most general non-redundant basis of operators for the SM + gravity EFT.

$$
\begin{aligned}
\mathcal{L}_6 = & \frac{c_1}{\Lambda^2} C_{\mu\nu}{}^{\rho\sigma} C^{\mu\nu\alpha\beta} C_{\alpha\beta\rho\sigma} + \frac{\tilde{c}_1}{\Lambda^2} C_{\mu\nu}{}^{\rho\sigma} C^{\mu\nu\alpha\beta} \tilde{C}_{\alpha\beta\rho\sigma} \\
& + \frac{c_2}{\Lambda^2} H^\dagger H C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \frac{\tilde{c}_2}{\Lambda^2} H^\dagger H C_{\mu\nu\rho\sigma} \tilde{C}^{\mu\nu\rho\sigma} \\
& + \frac{c_3}{\Lambda^2} B^{\mu\nu} B^{\rho\sigma} C_{\mu\nu\rho\sigma} + \frac{\tilde{c}_3}{\Lambda^2} B^{\mu\nu} B^{\rho\sigma} \tilde{C}_{\mu\nu\rho\sigma} + \frac{c_4}{\Lambda^2} G^{\mu\nu} G^{\rho\sigma} C_{\mu\nu\rho\sigma} + \frac{\tilde{c}_4}{\Lambda^2} G^{\mu\nu} G^{\rho\sigma} \tilde{C}_{\mu\nu\rho\sigma} \\
& + \frac{c_5}{\Lambda^2} W^{\mu\nu} W^{\rho\sigma} C_{\mu\nu\rho\sigma} + \frac{\tilde{c}_5}{\Lambda^2} W^{\mu\nu} W^{\rho\sigma} \tilde{C}_{\mu\nu\rho\sigma} \, .
\end{aligned}
$$

Hilbert series plus Weyl tensor as basic gravitational building block.

$$
\hat{\mathcal{M}}''_{AB}(s=0) = \frac{2}{\pi} \int^{\infty} \frac{ds}{s^2} \sum_X \sigma_{AB\to X}(s) > 0
$$



Constraints imposed by amplitudes' positivity.

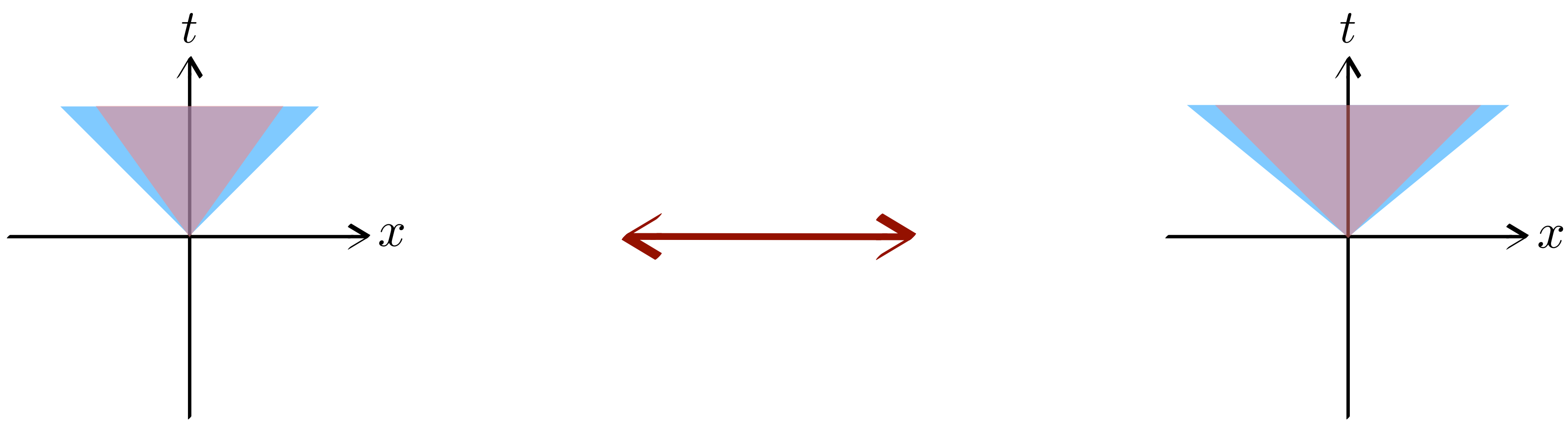
Possibility to learn about the connection of the SM with gravity's UV completion.

Fast Gravity Conjecture

Our positivity bounds have been found to imply gravity is the fastest force.
(de Rham, Tolley '19)

$$\frac{c_h}{c_\gamma} > 1$$

from leading R^2, C^2 operators — in FLRW and warped geometries.



Only relative speeds are physical.

Possibility this leads to a more general/deeper swampland criterion.

Conclusions

Amplitudes' positivity can be implemented in gravitational theories.

Obstruction due to graviton forward singularity avoided by 3D compactification.

The (mild) WGC is the result, from an EFT IR perspective,

of a UV theory of quantum gravity with a consistent S-matrix (unitary, local and causal).

Is string theory the only UV completion with such a canonical S-matrix?

Do all string theories in fact satisfy our assumptions?

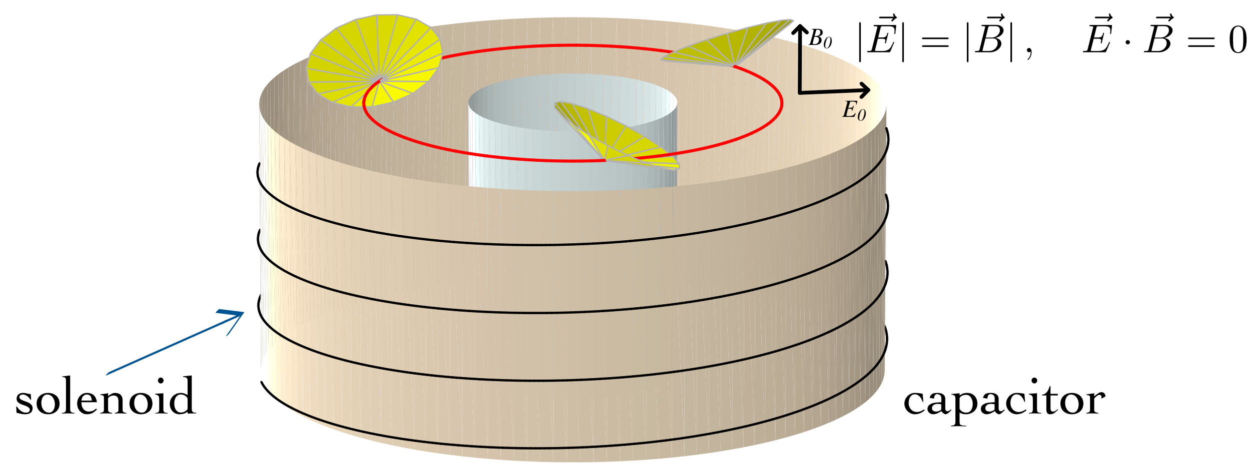
Modified gravity theories suffer from limited regime of predictivity.

Much remains to be explored within the EFT approach.

Thank you.

Superluminality

Theories with wrong-sign coefficients lead to violation of causality in non-trivial backgrounds.



$$v = \frac{1 - c\frac{32}{\Lambda^4}|\vec{E}|^2}{1 + c\frac{32}{\Lambda^4}|\vec{E}|^2}$$

Compactifying Einstein-Maxwell

$$ds_4^2[\hat{g}_{MN}] = e^{\sigma} ds_3^2[g_{\mu\nu}] + e^{-\sigma}(dz + V_\mu dx^\mu)$$

$$\hat{A}_M dx^M = A_\mu dx^\mu + \Phi dz$$

$$\Big\downarrow \quad \mathcal{S} = L \int d^3x \sqrt{-g} \times$$

$$R - \tfrac{1}{2}(\partial\sigma)^2 - \tfrac{1}{4} V^{\mu\nu} V_{\mu\nu}$$

$$-\tfrac{1}{4}(1-\sigma) F^{\mu\nu} F_{\mu\nu} - (1+\sigma)\tfrac{1}{2}(\partial\Phi)^2 - \tfrac{1}{2} F_{\mu\nu} V^{\mu\nu} \Phi$$

$$\alpha_1 \tfrac{1}{4M_{\rm Pl}^4} \left(F^{\mu\nu}F_{\mu\nu} + 2(\partial\Phi)^2\right)^2 + \alpha_2 \tfrac{1}{M_{\rm Pl}^4} \left(\epsilon^{\mu\nu\rho} F_{\mu\nu} \partial_\rho \Phi\right)^2$$

$$-\alpha_3 \tfrac{1}{M_{\rm Pl}^2} \left[\left(F_{\rho\mu} F^{\rho}{}_{\nu} - \partial_\mu \Phi \partial_\nu \Phi\right) \nabla^\mu \nabla^\nu \sigma + F_{\mu\nu} \partial_\rho \Phi \left(\nabla^\rho V^{\mu\nu} + g^{\mu\rho} \nabla_\alpha V^{\nu\alpha}\right)\right]$$

$$\alpha_3 \tfrac{1}{M_{\rm Pl}^4} \left[F_{\rho\mu} F^{\rho\nu} F^{\mu\sigma} F_{\nu\sigma} - \tfrac{1}{2} F^4 - (\partial\Phi)^4 + \tfrac{1}{2} F^2 (\partial\Phi)^2\right]$$

$+$ KK-modes