Spacetime Locality and Quantum Information

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March 11, 2015

Quantum Gravity in Perspective

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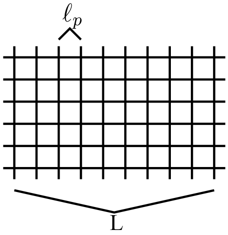
- Creating a theory of quantum gravity has been a long-standing goal of theoretical physics for at least 60 years, going back to Feynman, DeWitt, Wheeler, etc.
- By far the biggest obstacle is the universal nature of the gravitational force; everything couples to gravity.
- This makes it quite difficult to locally probe gravitational physics at the Planck scale; a microscopic array of Planck-sized "rods and clocks" must be heavy enough to be localized despite the uncertainty principle, but this causes it to immediately collapse into a black hole.

Say we want to use a network of rods to define the locations of points at distances of order ℓ_p:



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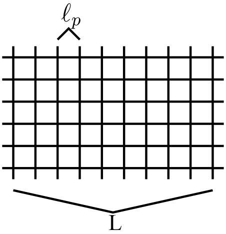

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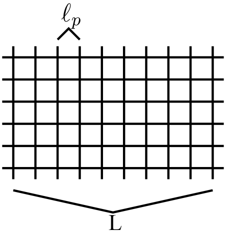

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Rods localized:

$$\ell_p \gg \Delta x > \frac{1}{m_{rod}\Delta v} \gg \frac{1}{m_{rod}} \implies m_{rod} \gg m_p. \tag{2}$$

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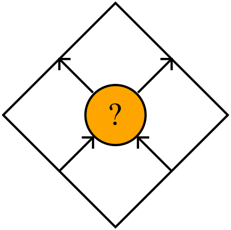
- There is also the *practical* problem that $\ell_{planck} \ll \ell_{people}$, which makes it difficult to come up with feasible experiments! I'll touch on this briefly soon, but my primary focus today is the theoretical problem.

Experiments in Asymptopia

In fact there *have* been consistent theories of quantum gravity that have been discovered in the last ~ 20 years, but the way this problem manifests itself is that we always find that we are only able to easily describe scattering experiments:



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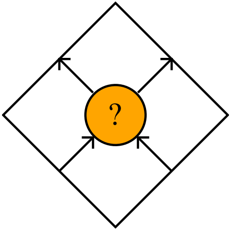

We send in well-isolated particles from asymptotically far away, which allows us to carefully prepare and measure them without disrupting whatever is happening in region where strong gravitational interactions are important.

Cosmology

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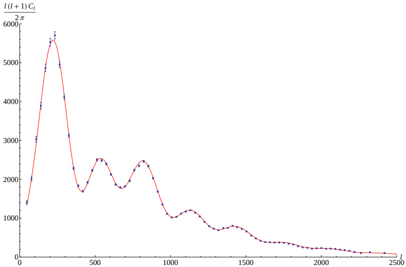
The first reason is that the evidence is now quite compelling that our universe underwent a period of rapid exponential expansion in the distant past (inflation). Moreover we seem to have recently entered another period of exponential expansion (the cosmological constant), albeit at a more stately pace:



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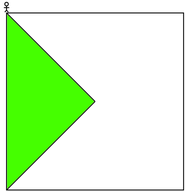

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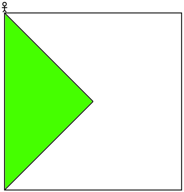

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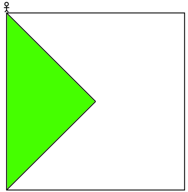
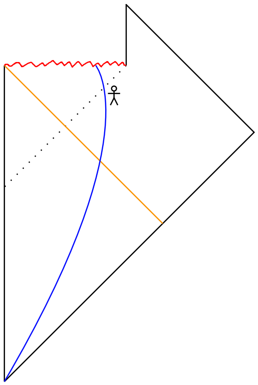


- Everything we will ever see is confined within a finite volume ($\sim 10GPa$ across); anything outside of this is beyond the horizon.
- In this type of cosmological setting, we have no choice but to attempt to make sense of experiments done by observers who are fundamentally limited in how well they can separate themselves from their experiments.

Black Holes

The second empirical reason we need a theory of quantum gravity away from asymptopia is that black holes exist in nature:

Consider an observer (Alice) who jumps into a black hole:



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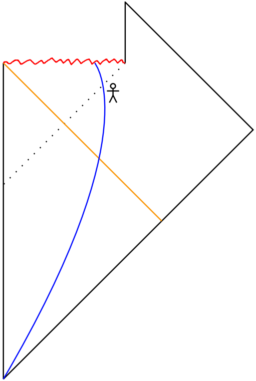

Once she crosses the horizon, Alice has only a limited amount of time and space available to make measurements before she dies horribly at the singularity. She has no asymptopia to escape to. But what does she see?

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It seems likely (to me at least, and to others) that learning how to describe Alice's experiences in a theory of quantum gravity will teach us important lessons for cosmology.

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- In fact in the remainder of this talk, I will not discuss cosmology further. Black holes will make an appearance near the end, but even they will be set aside for a while.
- The question I will focus on is the following: in a certain type of quantum gravity theory that we understand fairly well, which does have a nice asymptotic region where scattering-like experiments can be done, how is the physics directly in the middle of the space encoded in the fundamental degrees of freedom?

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- In the last 17 years, Maldacena's paper has become the most cited paper in the history of theoretical physics ($> 12,000$ times).

Let's first understand what is meant by "Asymptotically-AdS" gravity. Anti de Sitter space is the maximally symmetric solution of Einstein's equation with negative cosmological constant:

$$ds^2 = -\left(1 + \left(\frac{r}{\ell}\right)^2\right) dt^2 + \frac{dr^2}{\left(1 + \left(\frac{r}{\ell}\right)^2\right)} + r^2 d\Omega_{d-1}^2. \tag{3}$$

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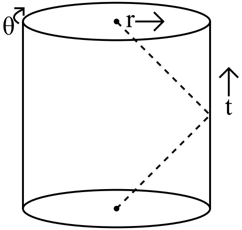
Diagrammatically it can be represented as a solid cylinder:



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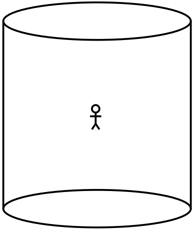

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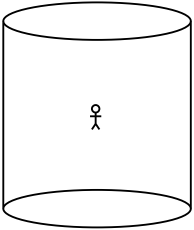

An essential point here is that as $r \to \infty$, in any such geometry the induced metric at fixed r approaches:

$$ds^2 \to r^2 \left(-dt^2/\ell^2 + d\Omega_{d-1}^2\right); \tag{4}$$

this is a round sphere times time ($\mathbb{S}^{d-1} \times \mathbb{R}$).

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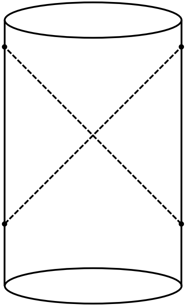
- The Hilbert spaces are equivalent; any state in the CFT has a "bulk" interpretation, and vice versa.
- The Hamiltonians are equivalent, as are the other generators of the AdS symmetries.
- For any bulk field $\phi(x)$, as we pull it to the boundary it becomes a CFT local operator:

$$\lim_{r\to\infty} \phi(t, r, \Omega) r^{\Delta} = \mathcal{O}(t, \Omega). \tag{5}$$

This is sometimes called the "extrapolate dictionary".

As promised earlier, the extrapolate dictionary means that "scattering experiments" in AdS are easy to describe in the CFT:



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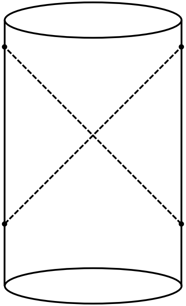

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Note that locality of bulk operators that are near the boundary reduces to CFT locality.

A Paradox

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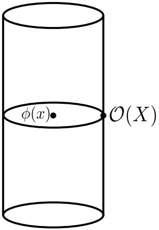

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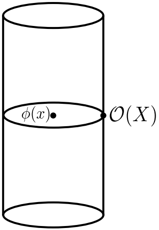

Do we have

$$[\phi(x), \mathcal{O}(X)] = 0? \tag{7}$$

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But this is impossible! In a quantum field theory, any operator that commutes with all local operators at a fixed time must be proportional to the identity.
Intuitively, this is because this set of local operators acts irreducibly on the Hilbert space:

$$|\phi(x) + \alpha(x)\rangle = e^{i\int dx \alpha(x) p(x)} |\phi(x)\rangle. \tag{9}$$

What are we to make of this?

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- I will then argue that the formalism of "quantum error correction", first developed as part of the quest to build a quantum computer, provides a natural resolution of both problems.
- Finally I will return to the problem with we started with; how does the possible formation of a black hole disrupt the emergence of spacetime locality in quantum gravity?

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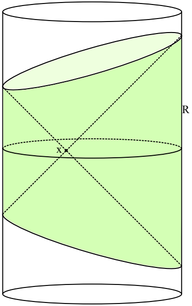

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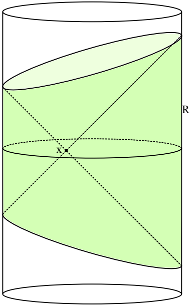

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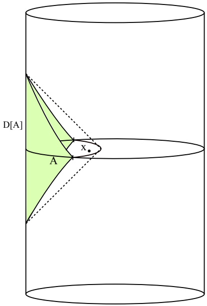
This is called *global reconstruction*.

There is also a more subtle reconstruction on subregions, called *AdS-Rindler reconstruction*:



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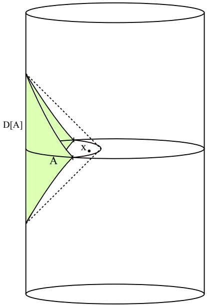

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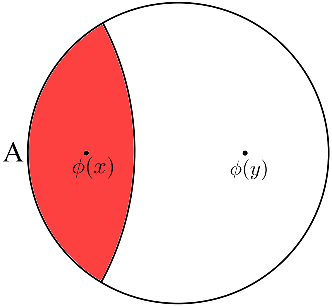
In the bulk these two reconstructions produce the same operator, and are related by a Bogoliubov transformation.

For example, consider:



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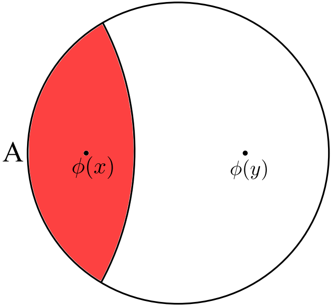

The operator $\phi(x)$ can be represented on A, but the operator $\phi(y)$ cannot.

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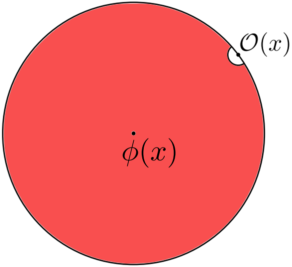
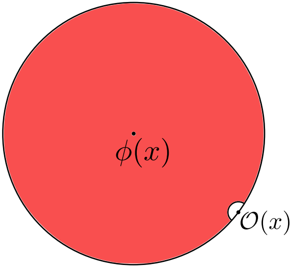


or



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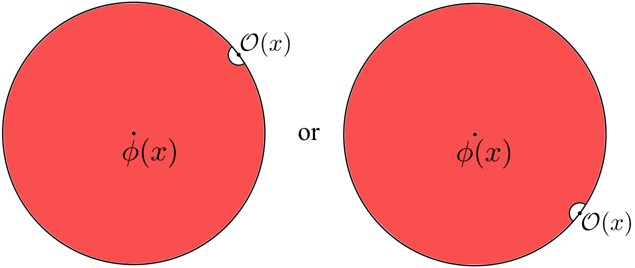

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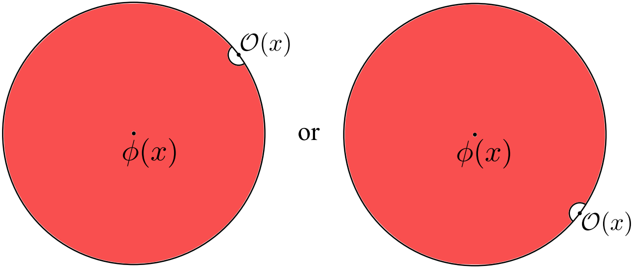

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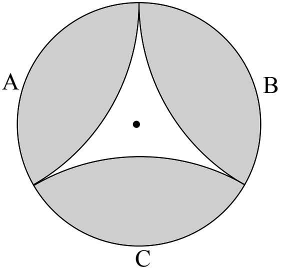
This can only be possible if the different representations aren't actually equal as operators!

Another illustration:



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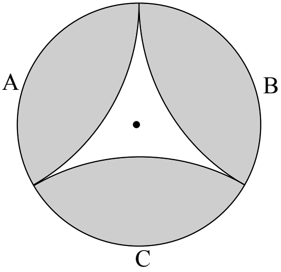

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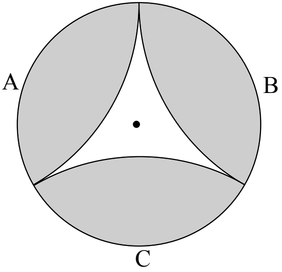

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Something interesting is going on here, but what is it?

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- Nonetheless, there is a way of a encoding the state which protects it against postal corruption - quantum error correction.
- QEC was first developed as a necessary part of building a quantum computer: decoherence of your memory is almost inevitable, so you need a way to fix it!

An Example

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The idea is to instead send you three qutrits in the state

$$|\widetilde{\psi}\rangle = \sum_{i=0}^{2} C_i |\widetilde{i}\rangle, \tag{13}$$

where $|\widetilde{i}\rangle$ is a basis for a special subspace of the full 27-dimensional Hilbert space, which is called the *code subspace*.

Explicitly, we take

$$
\begin{aligned}
|\widetilde{0}\rangle &= \frac{1}{\sqrt{3}}\left(|000\rangle + |111\rangle + |222\rangle\right) \\
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- This leads to the remarkable fact that we can *completely recover* the quantum state from any two of the qutrits!

To see this explicitly, we can define a two-qutrit unitary operation U_{12} that acts as

$$\begin{array}{lll} |00\rangle \to |00\rangle & |11\rangle \to |01\rangle & |22\rangle \to |02\rangle \\ |01\rangle \to |12\rangle & |12\rangle \to |10\rangle & |20\rangle \to |11\rangle \\ |02\rangle \to |21\rangle & |10\rangle \to |22\rangle & |21\rangle \to |20\rangle \end{array} . \tag{15}$$

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- By symmetry there must also exist U_{13} and U_{23}.

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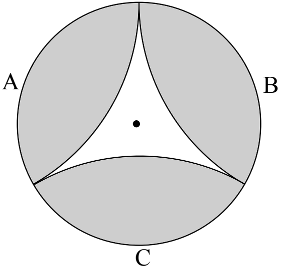
Generically this operator will have nontrival support on all three qutrits, but using our U_{12} we can define

$$O_{12} \equiv U_{12}^{\dagger} O_1 U_{12}, \tag{20}$$

which acts nontrivially only on the first two but still implements O on the code subspace.

The point now is that we can interpret O_{12}, O_{13}, and O_{23} as being analogous to the representations of $\phi(0)$ on AB, AC, and BC in this example:



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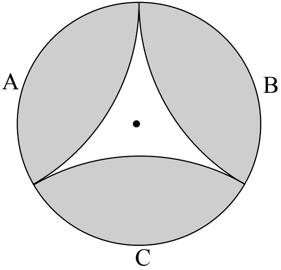

By using the entanglement of the code subspace, we can replicate the paradoxical properties of the AdS-Rindler reconstruction.

We can also make contact with the commutator puzzle: let's compute

$$\langle\widetilde{\psi}|[\widetilde{O}, X_3]|\widetilde{\phi}\rangle, \tag{21}$$

where X_3 is some operator on the third qutrit and $|\widetilde{\phi}\rangle$, $|\widetilde{\psi}\rangle$ are arbitrary states in the code subspace.

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This is the lesson to learn for AdS/CFT; the bulk algebra of operators *holds only on a subspace of states*!

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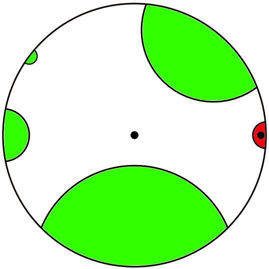
- First of all the code has the property that quantum information located further from the boundary is better protected in the CFT:



- We can also consider the question of how large the code subspace can be; on how many states can we realize the bulk algebra of operators? It turns out that the answer, which is given by a general theorem in quantum error correction, can be matched directly onto the bulk regime where we make a black hole:

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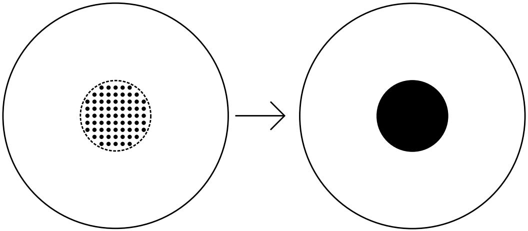

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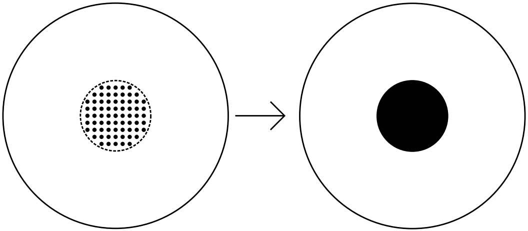

We thus come back to my original motivation: any attempt to probe the locality of spacetime precisely *does* eventually break down due to black hole formation! The full theory of quantum gravity, given by the CFT in this case, simply has no notion of locality beyond this point.

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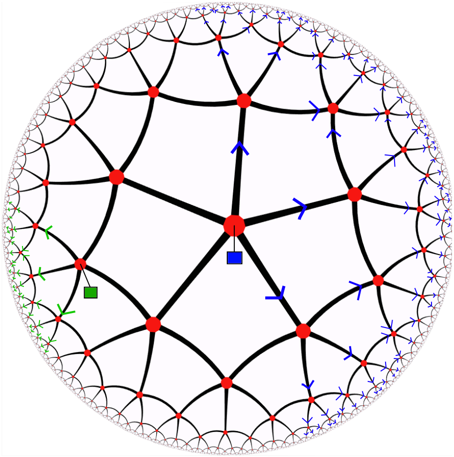
- These codes are a generalization of those currently used in designing quantum computer algorithms, and they may be superior. An engineering application for quantum gravity?

Here is a picture:



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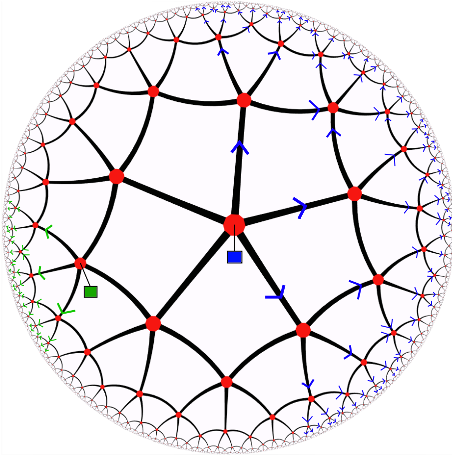

Thanks!