Studying challenging theories with the superconformal bootstrap

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connected to:

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with various subsets of:

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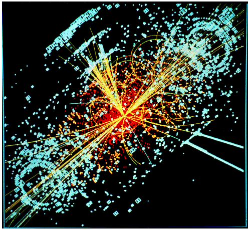
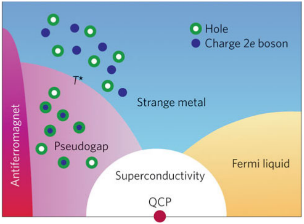
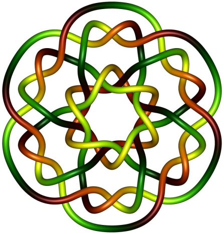
Quantum field theory

Quantum field theory is ubiquitous in modern theoretical physics (and mathematics).







However, quantum field theory is not a *technology* – can't just take it off the shelf and turn the crank.

This is more than a technical problem. There are hints that *we are missing something significant*.

Fields, Lagrangians, path integrals...

QFT is usually formulated as a theory of quantum fields:

$$\varphi(x)\ , \qquad \psi_\alpha(x)\ , \qquad A_\mu(x)\ , \qquad \dots\ ,$$

Write a Lagrangian (subject to some conditions), compute path integral:

$$\mathcal{L}[\varphi] = \partial_\mu\varphi\,\partial^\mu\varphi + m^2\varphi + g^2\varphi^4 + \dots$$

$$Z = \int [D\varphi] e^{\frac{i}{\hbar}S}\ , \qquad S = \int d^Dx\ \mathcal{L}[\varphi(x)]\ .$$

Many subtleties (regularization, renormalizability), but the story is basically established and useful.

Disclaimer: In this talk, QFT $\equiv$ Lorentz-invariant, unitary QFT

Fields, Lagrangians, path integrals...

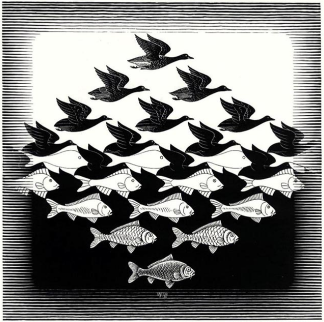
So $\{$quantum field theories$\} = \{$(UV complete) Lagrangians$\}$?

The FIELD Is Not Enough 007

Duality: Some theories admit multiple Lagrangian descriptions.

strongly coupled catfish

weakly coupled goose



weakly coupled catfish

strongly coupled goose

E.g., Electic-Magnetic duality in $\mathcal{N} = 4$ super Yang-Mills ($+$ many more...)

Duality connects to deep mathematics

Mirror symmetry (2d)
Geometric Langlands (4d) [Gukov; Kapustin; Witten]

Fields, Lagrangians, path integrals...

So $\{\text{quantum field theories}\} = \{\text{Lagrangians}\}/\text{Duality}$

$\ldots$

Lagrangians like coordinate charts?

The FIELD Is Not Enough 007

Non-Lagrangian theories: some theories seem to admit *no Lagrangian description*.

Existence deduced indirectly, often using *decoupling limits of string/M-theory*.

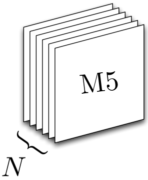
Such theories pose a serious conceptual challenge

In the rest of this talk, I'm going to describe a conservative approach to understanding a particularly interesting class of non-Lagrangian theories using algebraic methods.

$(2,0)$ theory in $d=6$ [Seiberg (1996)]

No (interacting) continuum Lagrangian QFTs in $d>4$ dimensions.

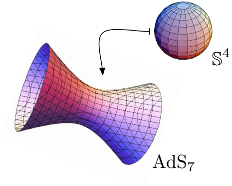
Nevertheless, *six-dimensional interacting QFTs exist*.



$\longleftarrow$

Conformally invariant: $SO(5,1)\to SO(6,2)$

Maximally supersymmetric: $SO(6,2)\to OSp(8|4)$



Holographic dual description for $N\to\infty$.
Can't compute $1/N$ corrections.

$(2,0)$ theory in $d = 6$ [Seiberg (1996)]

These theories appear to play be fairly important:

- No superconformal symmetry in $d > 6$ [Nahm (1978)].

  (Speculation: no interacting QFT in $d > 6$?).

- The "theory of M5 branes" (what is M-theory?)

- $d \leqslant 4$ landscape populated by compactifications. [Gaiotto (2008)]

  [Bah, Beem, Bobev, Wecht (2011, 2012);Beem, Dimofte, Pasquetti (2012)]

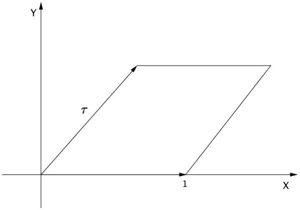
  [...]

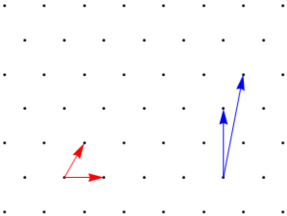
  Explains duality in lower dimensions.

“Explaining” duality in four dimensions [Witten (1995)]

$$(2,0)_N \text{ on } \mathbb{R}^4 \times T^2 \quad \xrightarrow{\text{IR}} \quad SU(N)\ \mathcal{N}=4 \text{ SYM on } \mathbb{R}^4$$

$$\text{Modular parameter of } T^2 \quad \longrightarrow \quad \tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}\ .$$





$$T^2_\tau \equiv T^2_{\tau'}\ , \qquad \tau' = \frac{a\tau + b}{c\tau + d} \quad \text{where} \quad ad - bc = 1\ .$$

Modular invariance $\longrightarrow$ S-duality

So what is the $(2,0)_N$ theory?

Liberal: Low energy limit of N coincident M5 branes.

Conservative: List of local operators with superconformally-covariant correlation functions.

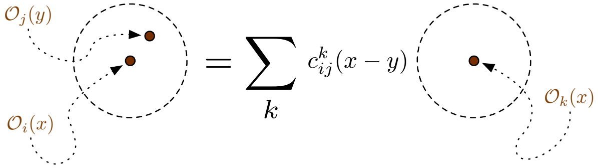
Moderate: Mostly conservative, but occasionally cross the aisle.

The conservative approach

Consequences of conformal symmetry

Operators in *conformal families*: $\{\mathcal{O}_{\Delta,\ell}, \partial\mathcal{O}_{\Delta,\ell}, \partial^2\mathcal{O}_{\Delta,\ell}\}$

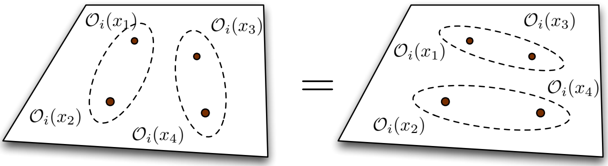
Algebraic structure: *convergent OPE*



Coefficients functions fixed by *three-point functions of primaries*.

n-point functions determined from spectrum and three-point functions (*CFT data*)

Consequences of conformal symmetry



$$u = \frac{x_{12}^2 x_{34}^2}{x_{14}^2 x_{23}^2}$$

$$v = \frac{x_{13}^2 x_{24}^2}{x_{14}^2 x_{23}^2}$$

CFT data is nontrivially constrained by *crossing symmetry:*

$$\sum_{\mathcal{O}_j} c_{iij}^2 G_{\Delta_j,\ell_j}(u,v) = \sum_{\mathcal{O}_j} c_{iij}^2 G_{\Delta_j,\ell_j}(v,u) \ .$$

One equation for each four-point function.

Conformal bootstrap: just solve these equations!

[Ferrara, Gatto, Grillo 1971-1975; Polyakov 1974]

[N.B. need *infinite number of conformal families*]

21st century bootstrap: convex optimization

In $d \geqslant 3$, no major progress until [Rattazzi, Rychkov, Tonni, Vichi (2008)] – numerical approach.

Roughly speaking, the technology is as follows:

- Rewrite crossing symmetry as *sum rule with positive coefficients:*

$$\sum_{\mathcal{O}_i} c_i^2 \left(G_{\Delta_i,\ell_i}(u,v) - G_{\Delta_i,\ell_i}(v,u) \right) = 1 \ .$$

- Make assumptions about spectrum – limits the basis of functions on LHS.

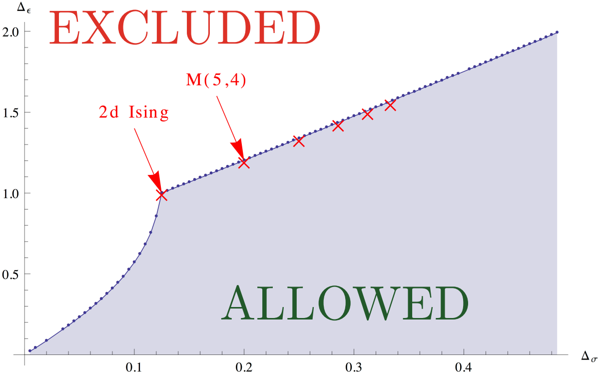
- Prove that no solution can exist:

$$\textit{i.e.,} \quad {f_i}''(0) > 0 \quad \Longrightarrow \quad \sum_i c_i^2 f_i(x) \neq 1 \ .$$

- Keyword: “convex optimization”

21st century bootstrap: convex optimization

In $d \geqslant 3$, no major progress until [Rattazzi, Rychkov, Tonni, Vichi (2008)] – numerical approach.



[El Showk, Paulos (2012)]

Theories on boundary can have CFT data *systematically reconstructed*.

Consequences of supersymmetry [Beem, et. al. (2013)]

Superconformal families: $\{\mathcal{O}, Q\mathcal{O}, \ldots, \mathcal{Q}^{16}\mathcal{O}, \text{descendants}\}$

Interesting supersymmetric operators: $Q^n\mathcal{O}_{BPS} = 0,\ n < 16.$

Expect infinitely many BPS operators

(*cp*. ordinary CFT: only conserved currents)

OPE algebra admits *truncation* involving *only BPS operators*

$$\text{“}\mathcal{O}^{(i)}_{BPS} \times \mathcal{O}^{(j)}_{BPS} = \sum_k \mathcal{O}^{(k)}_{BPS}\ .\text{”}$$

BPS algebra *much simpler* than full operator algebra.

Disclaimer: actual truncation very complicated

Consequences of supersymmetry [Beem, et. al. (2013)]

For $(2,0)$ theories, truncation requires operators lie in $\mathbb{C}^2_{[z,\bar{z}]} \subset \mathbb{R}^6$:

$$\mathcal{O}_i(z)\mathcal{O}_j(w) \sim \sum_k \frac{{c_{ij}}^k \mathcal{O}_k(w)}{(z-w)^{h_i+h_j-h_k}}$$

Known as *chiral algebras* (or *vertex algebras*) – appear in 2d CFT.

Crossing symmetry is nontrivial, but *tractable*.

analogy: complex analysis vs. real analysis

$(2,0)$ chiral algebra [Beem, Rastelli, van Rees (2014)]

Know enough about BPS spectrum to *solve chiral algebra bootstrap* completely.

BPS chiral algebra $=$ W_N algebra

First calculable correlation functions in the $(2,0)$ theory at finite N

At large N we can algebraically verify predictions from holography:

$$c_{k_1 k_2 k_3} = \frac{2^{2\alpha-2}}{(\pi N)^{3/2}} \Gamma\left(\frac{k_1+k_2+k_3}{2}\right)\left(\frac{\Gamma(\frac{k_{123}+1}{2})\Gamma(\frac{k_{231}+1}{2})\Gamma(\frac{k_{312}+1}{2})}{\sqrt{\Gamma(2k_1-1)\Gamma(2k_2-1)\Gamma(2k_3-1)}}\right).$$

(Research project): Finite N – *quantum gravity corrections* in M-theory.

Analytic $\Longrightarrow$ Numerical [Beem, Rastelli, van Rees (2013)]

BPS correlators alone are a big improvement, but can we do more?

Analytic results for BPS operators sets the stage for numerical analysis:

Solution to chiral algebra bootstrap

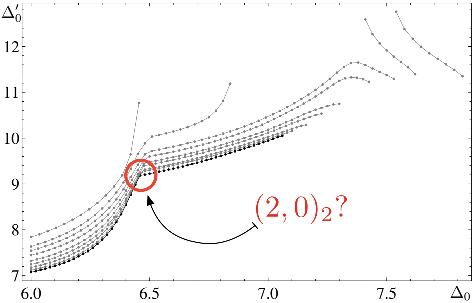
$\Downarrow$

CFT data for BPS operators

$\Downarrow$

Crossing symmetry for non-BPS operators

Cornering the $(2,0)_2$ theory [Beem, Lemos, Rastelli, van Rees (in progress)]



Numerical bootstrap results for $(2,0)$ theory.

"Interesting point" on the boundary seems to correspond to the $(2,0)_2$ theory.

$(2,0)$ theory = Ising model for the 21st century?

Where we are

- Rich algebraic structures connected to SCFTs.
  - $6d\ \mathcal{N} = (2,0) \Longrightarrow$ chiral algebra [Beem, Rastelli, van Rees (2014)]
  - $4d\ \mathcal{N} \geqslant 2 \Longrightarrow$ chiral algebra [Beem et. al. (2013)]
  - $3d\ \mathcal{N} = 4 \Longrightarrow$ deformation quantization [Beem, Peelaers, Rastelli (in progress)]

- Can compute BPS correlators in non-Lagrangian theories.

  [Beem, Rastelli, van Rees (2014); Beem, Peelaers, Rastelli, van Rees (2014)]

- Strong indications $(2,0)$ theory numerically accessible.

  [Beem, Lemos, Rastelli, van Rees (in progress)]

Future directions

- Explore "protected" algebraic structures –
  many connections to interesting mathematics.

- Are numerically accessible theories analytically special?

- Right mathematical framework for the bootstrap?

Thanks!