Positive Geometry of Scattering Amplitudes

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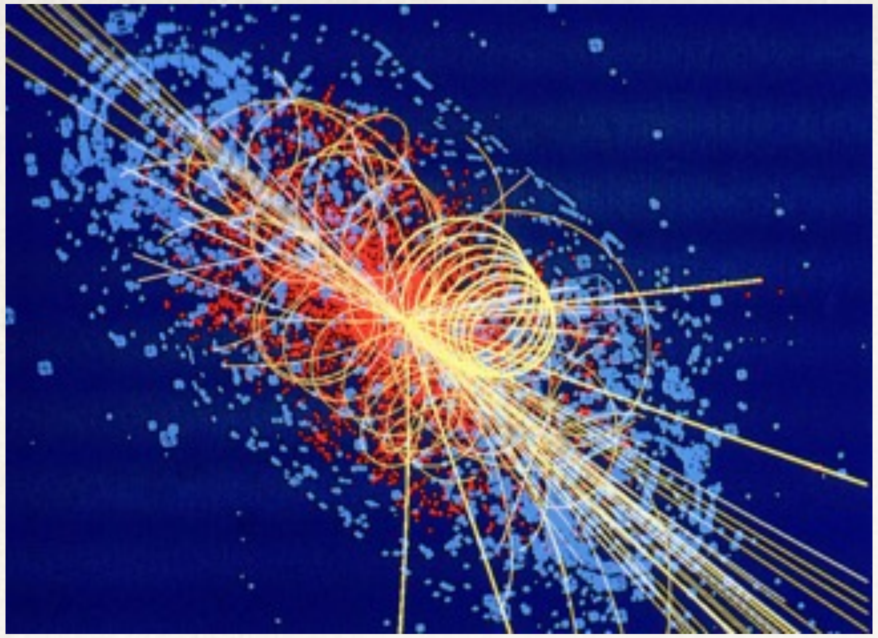
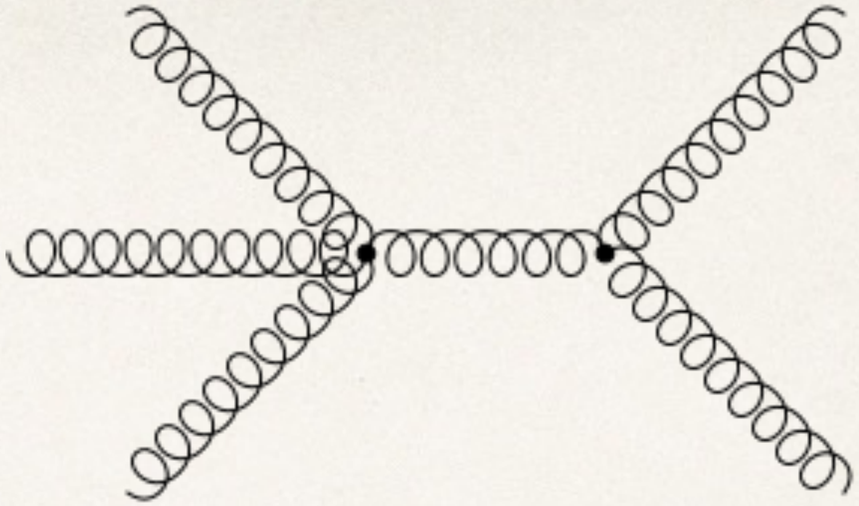
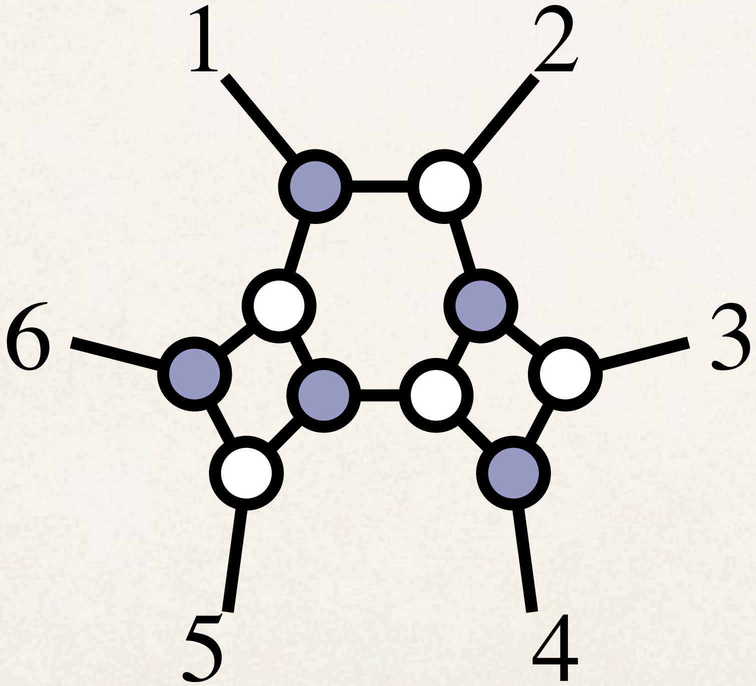
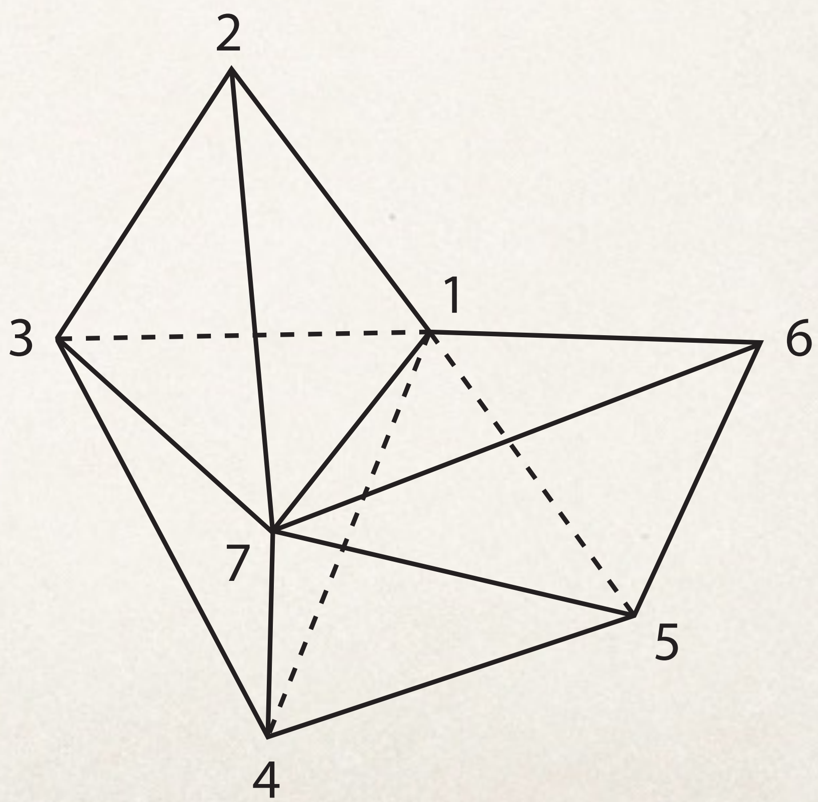
UC Davis, March 2, 2015





Scattering amplitudes



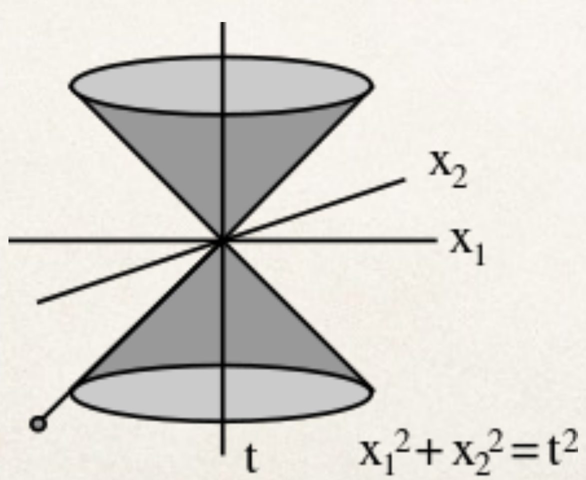


Quantum Field Theory (QFT)

- Our theoretical framework to describe Nature

- Compatible with two principles

Special relativity	Quantum mechanics



$$H(t)|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$$

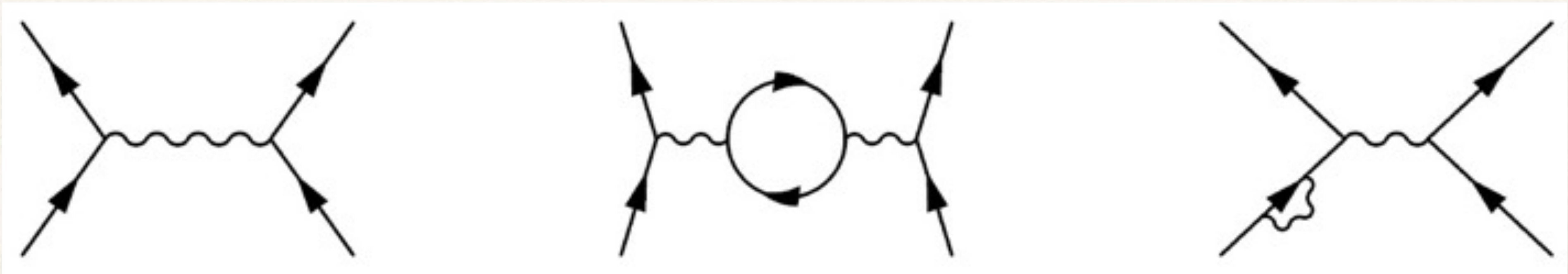
Standard formulation

(Dirac, Heisenberg, Pauli; Feynman, Dyson, Schwinger)

- Fields, Lagrangian, Path integral

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\overline{\psi} \not{D} \psi - m \overline{\psi} \psi \qquad \int \mathcal{D}A\, \mathcal{D}\psi\, \mathcal{D}\overline{\psi}\; e^{iS(A,\psi,\overline{\psi},J)}$$

- Feynman diagrams: pictures of particle interactions
  Perturbative expansion: trees, loops



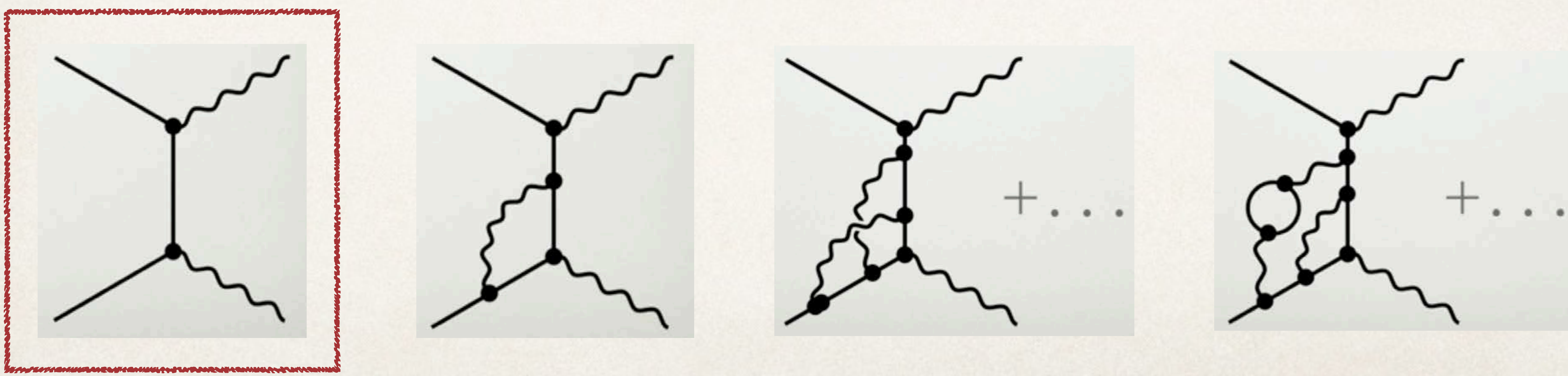
Great success of QFT

- QFT has passed countless tests in last 70 years
- Example: Magnetic dipole moment of electron

1928

Theory: $g_e = 2$

Experiment: $g_e \sim 2$



Great success of QFT

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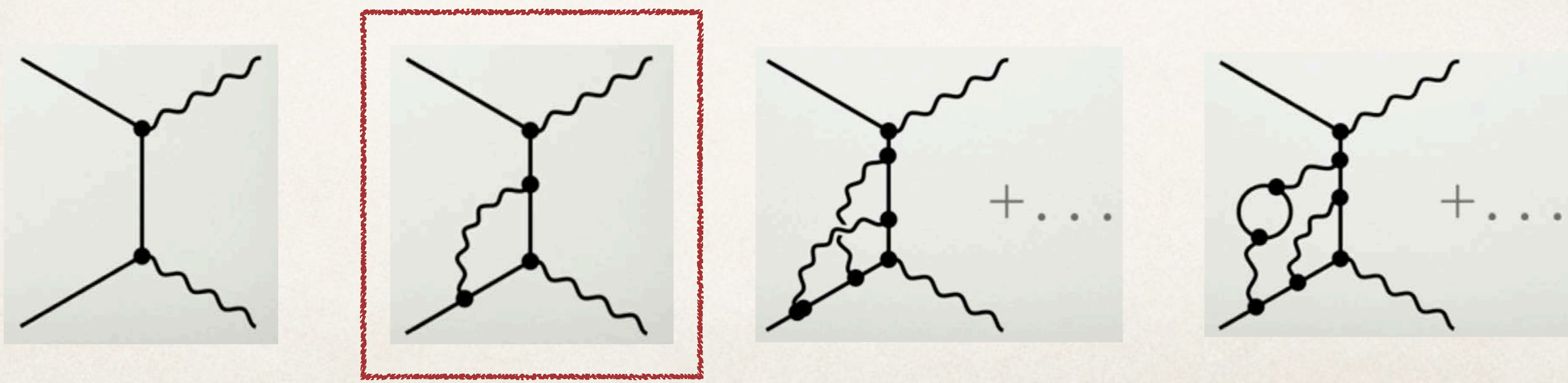
- Example: Magnetic dipole moment of electron

1947

Theory: $g_e = 2.00232$

Experiment: $g_e = 2.0023$



Great success of QFT

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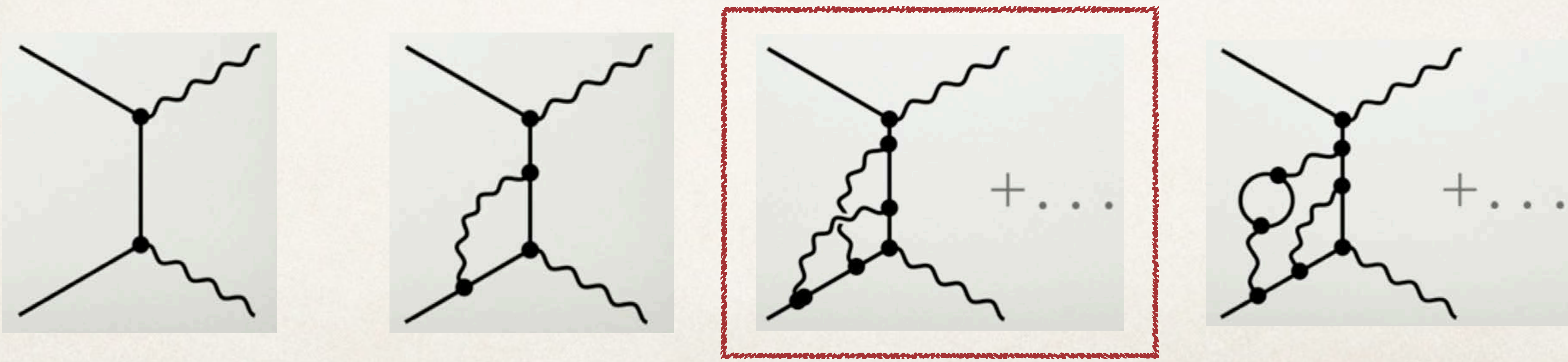
- Example: Magnetic dipole moment of electron

1957	Theory:	$g_e = 2.0023193$
1972	Experiment:	$g_e = 2.00231931$



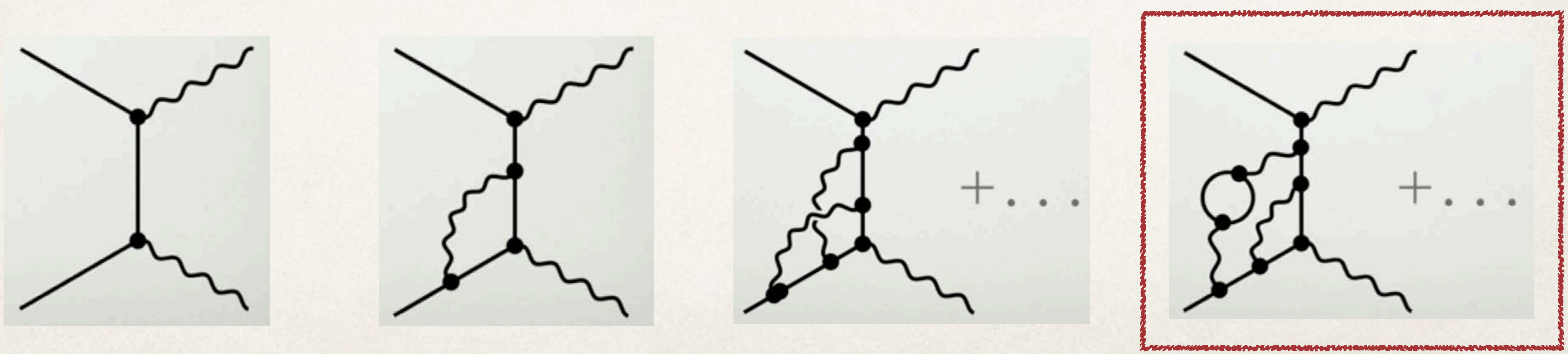
Great success of QFT

- QFT has passed countless tests in last 70 years
- Example: Magnetic dipole moment of electron

1990

Theory: $g_e = 2.0023193044$

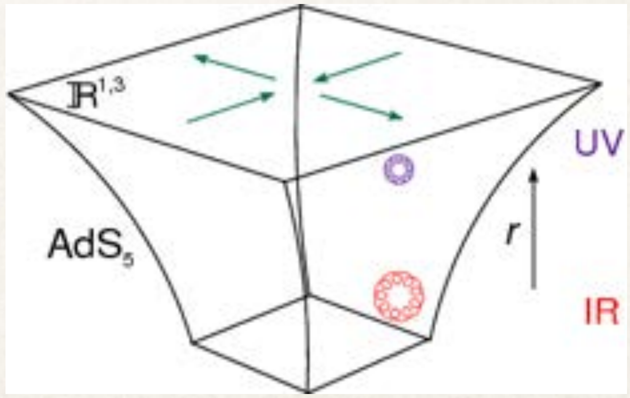
Experiment: $g_e = 2.00231930438$



Dualities

- At strong coupling: perturbative expansion breaks

- Surprises: dual to weakly coupled theory
  - Gauge-gauge dualities
    (Montonen-Olive 1977, Seiberg-Witten 1994)
  - Gauge-gravity duality
    (Maldacena 1997)



Motivation

- Our picture of QFT is incomplete
- Also, tension with gravity and cosmology

> If there is a new way of thinking about QFT,
> it must be seen even at weak coupling

- Explicit evidence: scattering amplitudes

Hidden simplicity
in scattering amplitudes

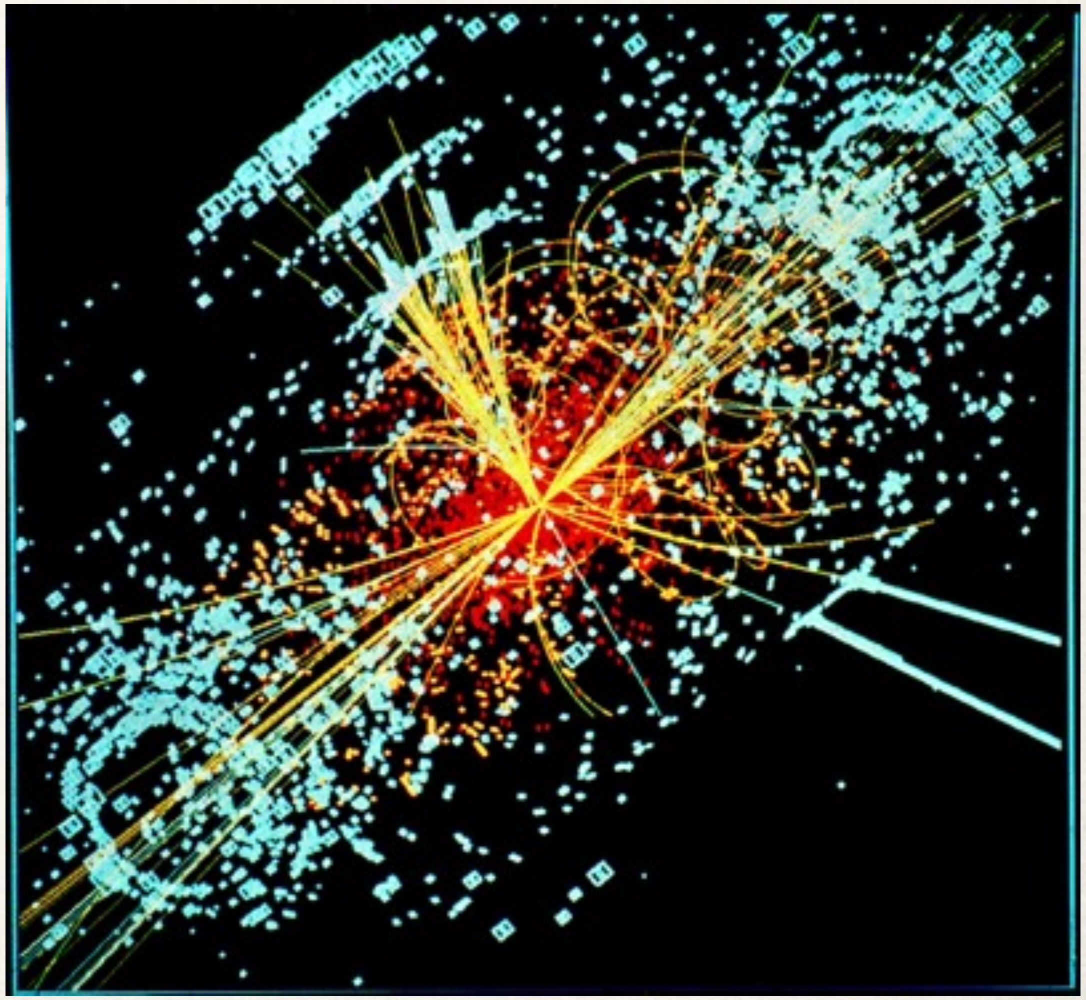
Scattering amplitudes

- Function of spin and external kinematics $\mathcal{M}(p, s, \ldots)$

- Probability of a given process during a particle collision

- Experimentalists measure cross-section

$$\sigma = \int d\Omega \, |\mathcal{M}|^2$$



Colliders at high energies

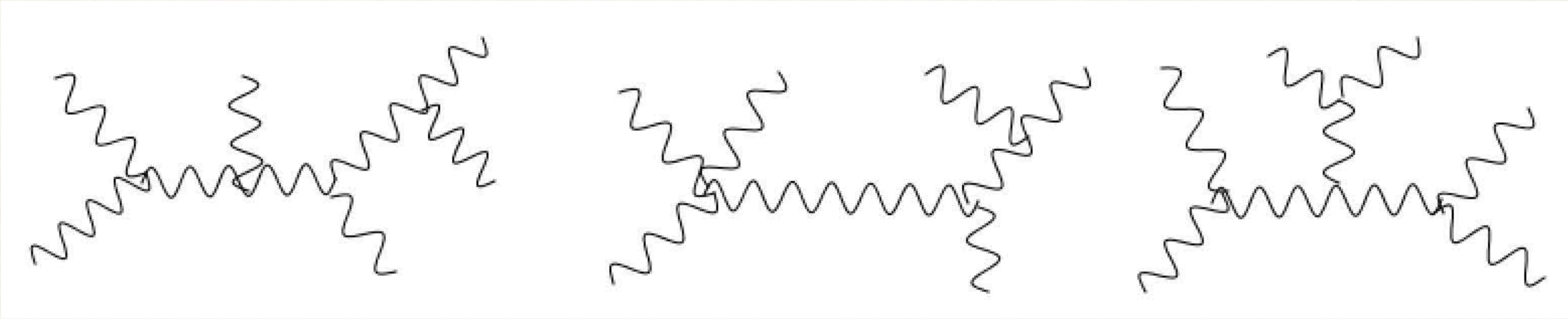
- Proton scattering at high energies

LHC - gluonic factory

- Needed: amplitudes of gluons for higher multiplicities

$$gg \to gg \ldots g$$

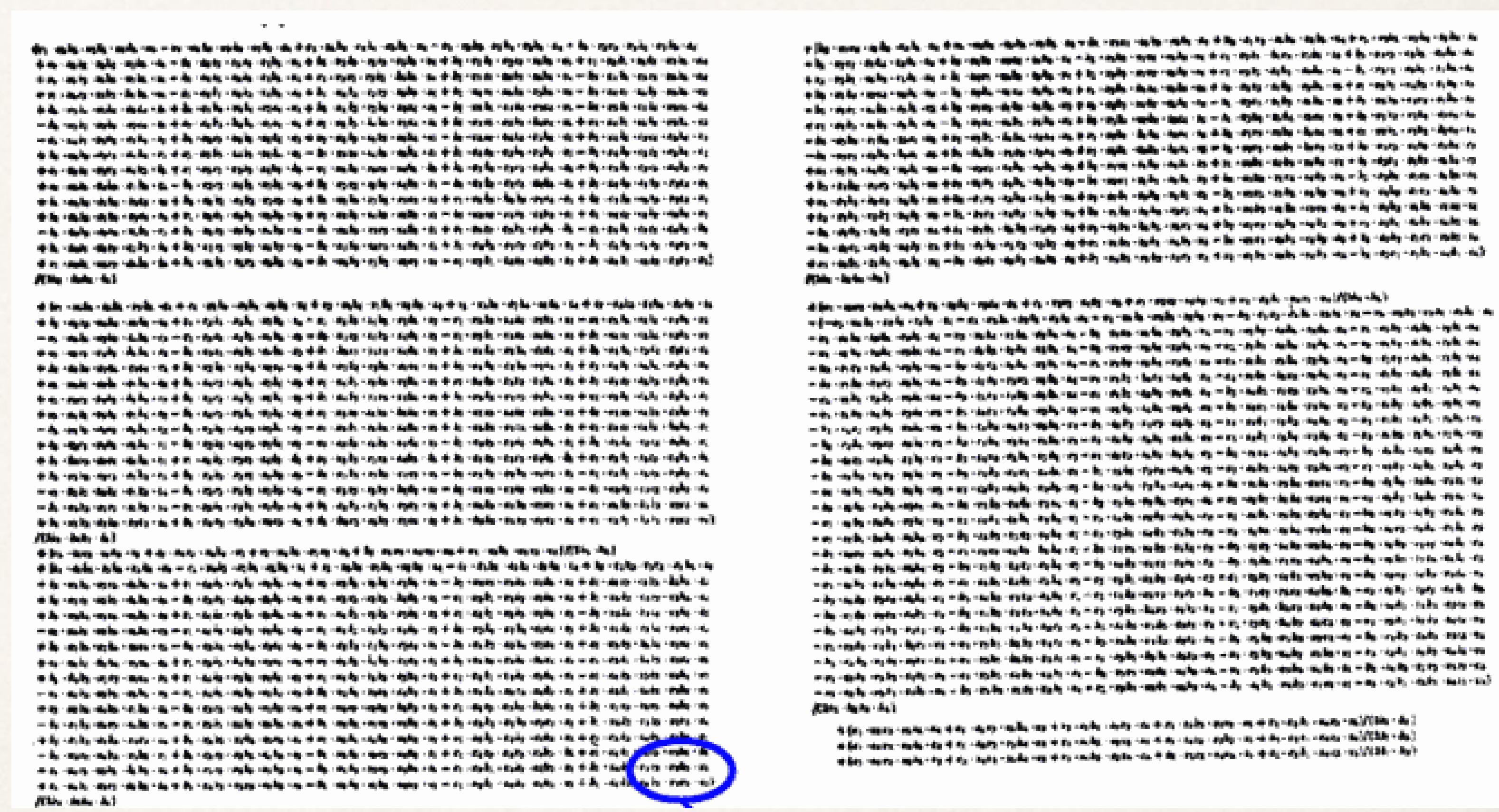
Two helicities: + -



Early 80s

- Status of the art: $gg \to ggg$

Brute force calculation
24 pages of result



$$(k_1 \cdot k_4)(\epsilon_2 \cdot k_1)(\epsilon_1 \cdot \epsilon_3)(\epsilon_4 \cdot \epsilon_5)$$

New collider

- 1983: Superconducting Super Collider approved
- Energy 40 TeV: many gluons!

- Demand for calculations, next on the list: $gg \to gggg$

Parke-Taylor formula

- Process $gg \to gggg$
- 220 Feynman diagrams, $\sim$100 pages of calculations
- 1985: Paper with 14 pages of result

GLUONIC TWO GOES TO FOUR

Stephen J. Parke and T.R. Taylor
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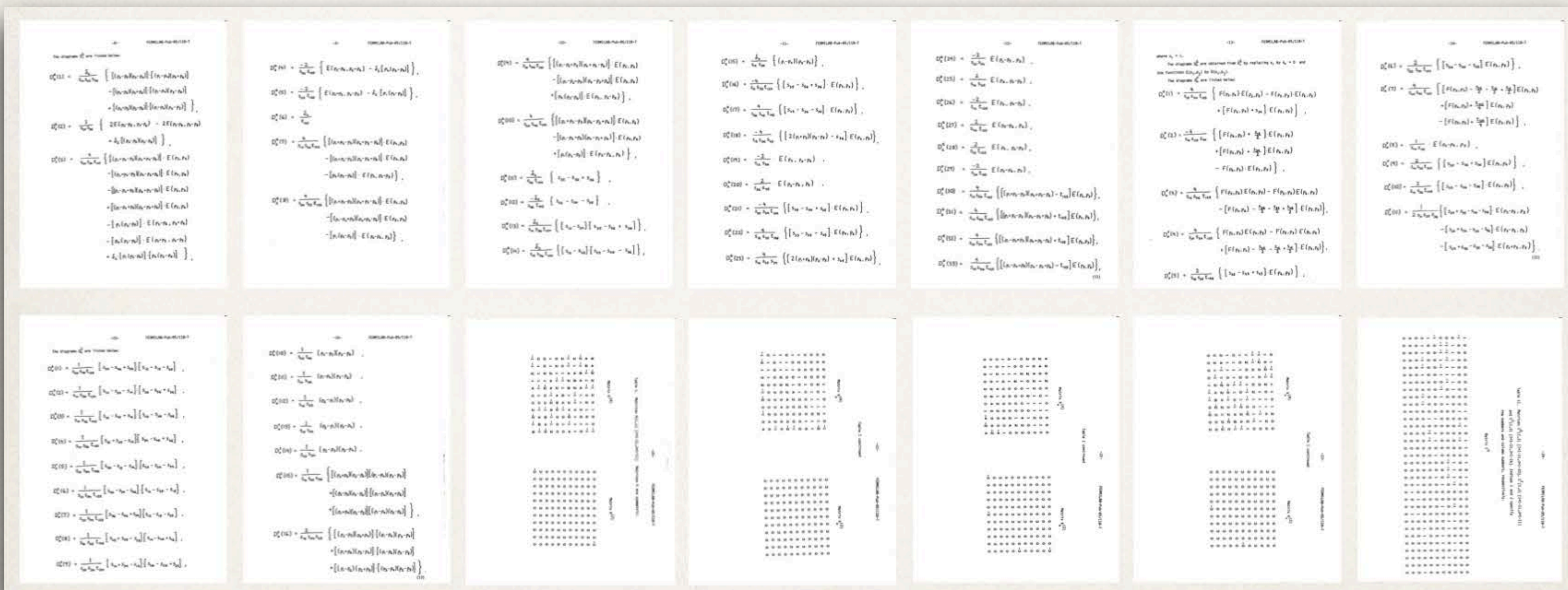
P.O. Box 500, Batavia, IL 60510
U.S.A.

ABSTRACT

The cross section for two gluon to four gluon scattering is given in a form suitable for fast numerical calculations.

Parke-Taylor formula

- Process $gg \to gggg$
- 220 Feynman diagrams, $\sim$100 pages of calculations



Parke-Taylor formula

Our result has succesfully passed both these numerical checks.

Details of the calculation, together with a full exposition of our techniques, will be given in a forthcoming article. Furthermore, we hope to obtain a simple analytic form for the answer, making our result not only an experimentalist's, but also a theorist's delight.

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- Within a year they realized

$$\mathcal{M}_6 = \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 61 \rangle}$$

Spinor-helicity variables

$$p^\mu = \sigma^\mu_{a\dot{a}} \lambda_a \tilde{\lambda}_{\dot{a}}$$
$$\langle 12 \rangle = \epsilon_{ab} \lambda_a^{(1)} \lambda_b^{(2)}$$
$$[12] = \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}_{\dot{a}}^{(1)} \tilde{\lambda}_{\dot{b}}^{(2)}$$

(Mangano, Parke, Xu 1987)

Parke-Taylor formula

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- Within a year they realized

$$\mathcal{M}_n = \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 34\rangle\langle 45\rangle\ldots\langle n1\rangle}$$

AN AMPLITUDE FOR n GLUON SCATTERING

STEPHEN J. PARKE and T. R. TAYLOR

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, IL 60510.

Gauge redundancy

- Where is the problem? Massless particles
- Particles with spin: gauge redundancy $\epsilon^\mu \to \epsilon^\mu + \alpha p^\mu$
- Individual Feynman diagrams not gauge invariant
  Huge cancellations among diagrams

Locality and unitarity

- Redundancy: local interaction picture, off-shell particles

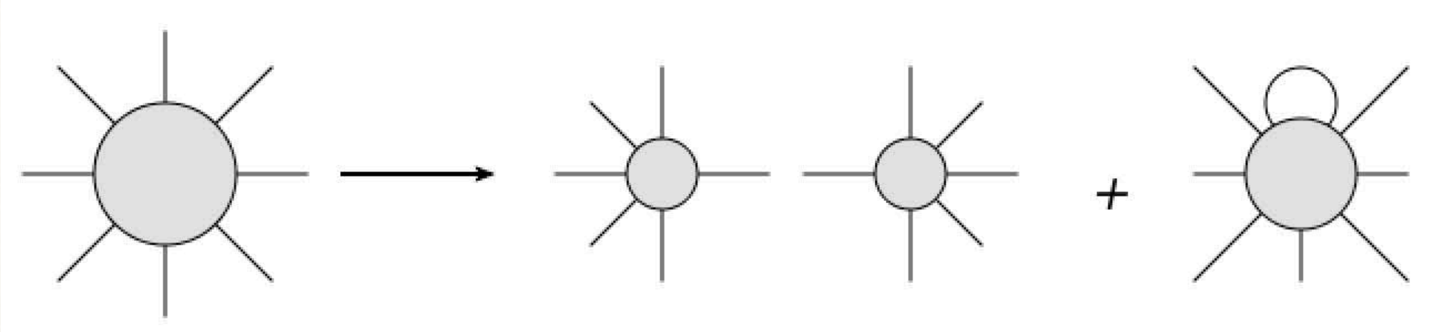
- Two principles manifest:

I) Locality: particles interact point-like

Amplitude: only poles $\quad \frac{1}{P^2} \to \infty \qquad P = \sum_{i \in \sigma} p_i$

II) Unitarity: sum of probabilities is 1

Amplitude: factorization



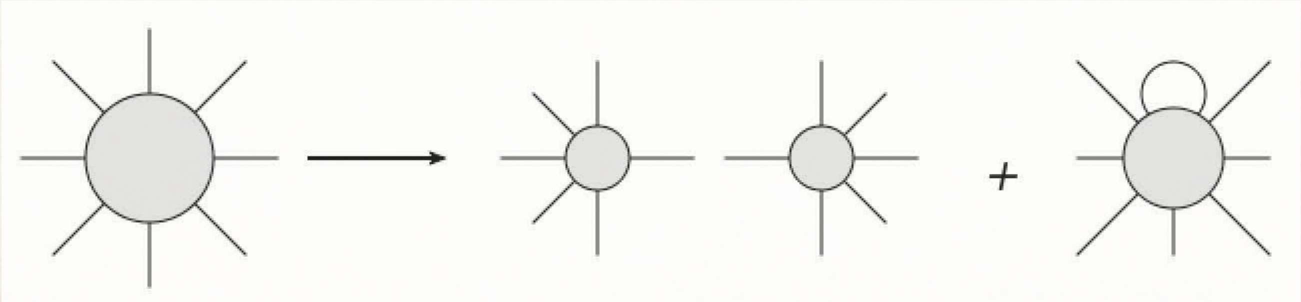
Modern methods for amplitudes

- Lessons from Parke-Taylor calculation:

Gauge invariance + physical states

- No fields, Lagrangians or path integrals

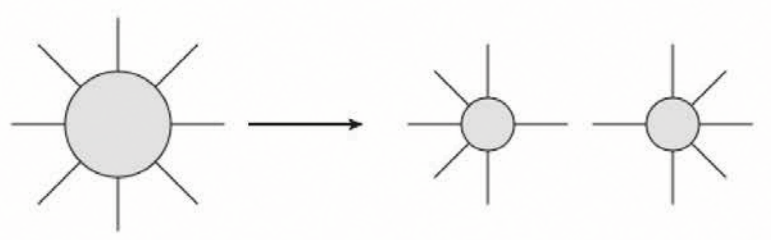
- Exploit locality and unitarity: fix the amplitude



Recursion relations

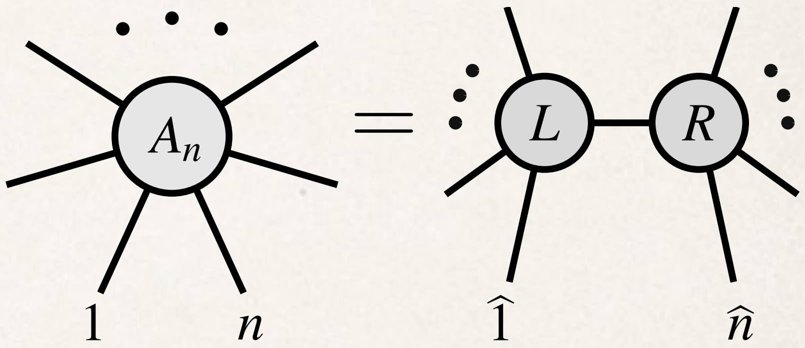
(Britto-Cachazo-Feng-Witten 2005)

- Large class of theories at tree-level
- Tree-level unitarity



- Shift momenta + Cauchy formula

$$p_1 \to p_1 + zq$$
$$p_2 \to p_2 - zq$$



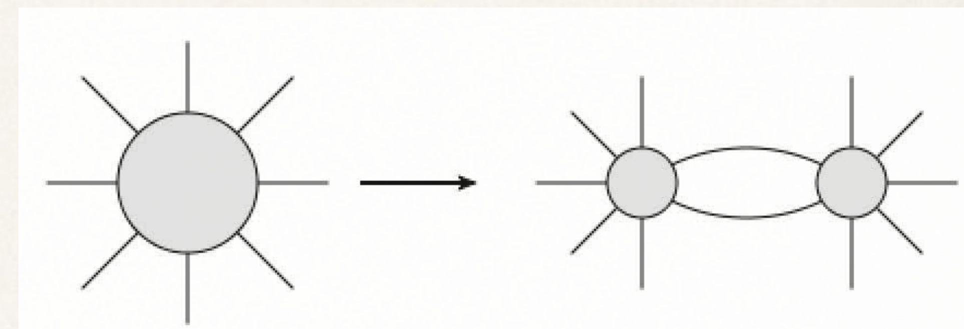
- Very efficient method:

	$gg \to 4g$	$gg \to 5g$	$gg \to 6g$
Feynman diagrams	220	2485	34300
Recursion relations	3	6	20

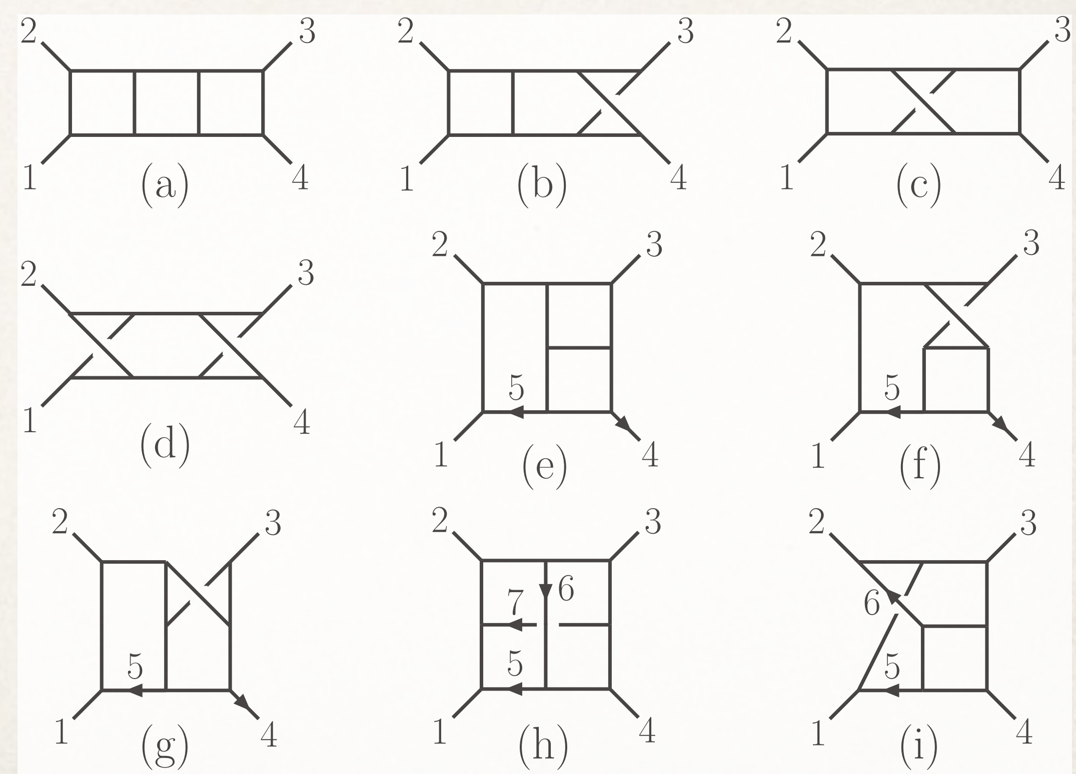
Unitarity methods

(Bern-Dixon-Kosower)

- Iterative use of the unitary cut



- Generate basis of integrals, fixing coefficients from cuts
- Tremendous success
  in calculations in 1990-today



Example: Four point 3-loop amplitudes
in supersymmetric Yang-Mills theory and gravity

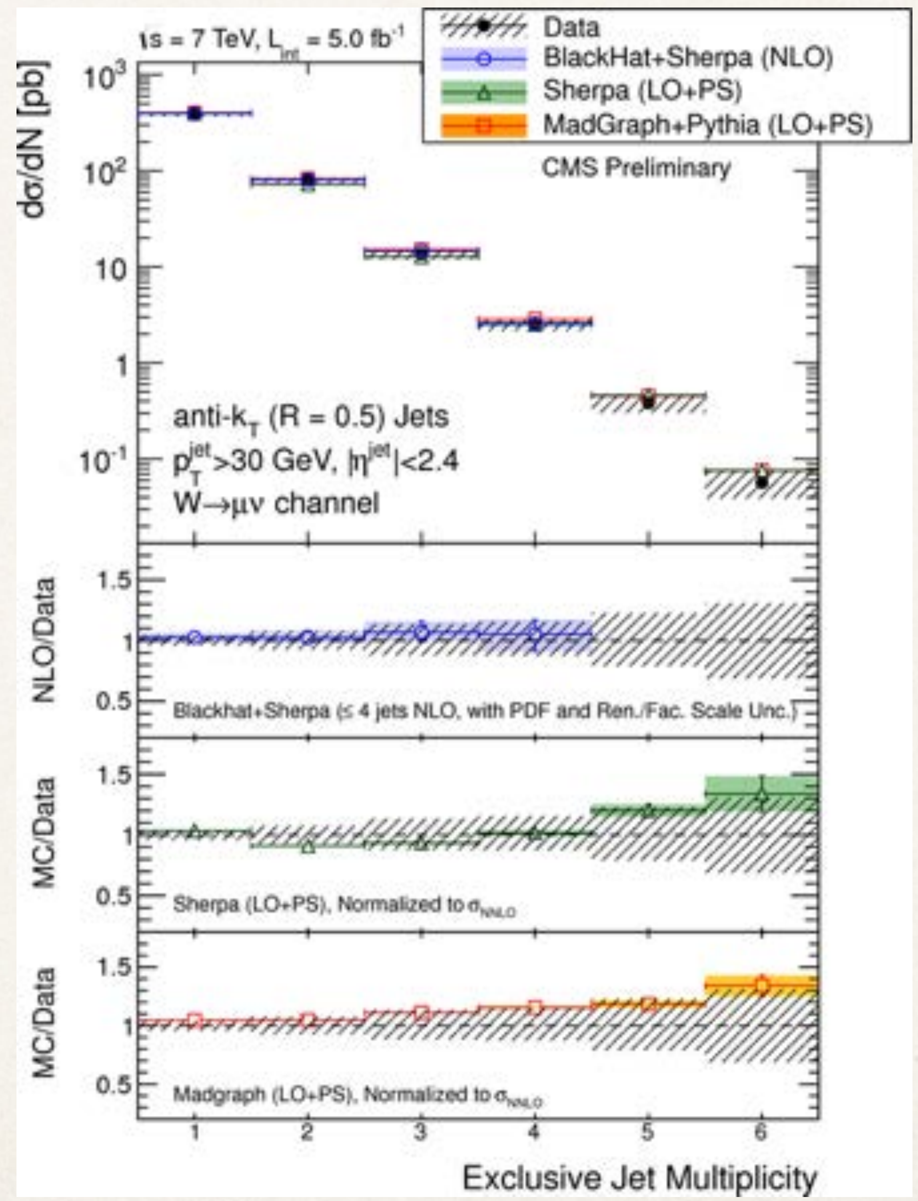
Unitarity methods



(Bern-Dixon-Kosower)

- QCD background at LHC
- BlackHat collaboration

- Huge efficiency in NLO calculations



Used by CMS in comparison to data, March 2014

Toy model

- This is a great success; is there a deeper structure?
- Time-proven method: study a toy model first

  Wish list:

  - Four-dimensional interacting theory
  - Close to the real world (QCD) as much as possible
  - Ability to generate plenty of explicit results

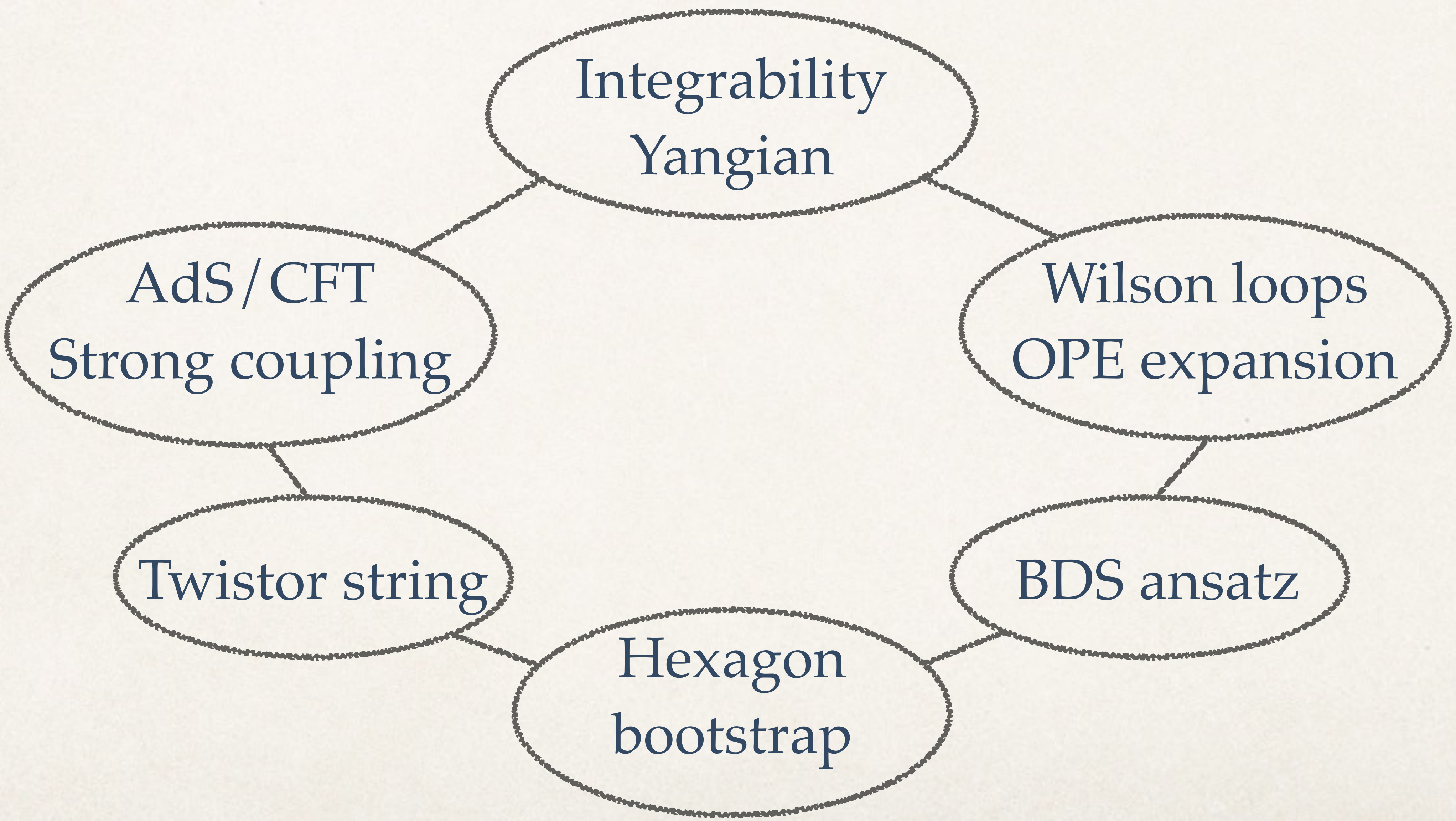
Maximally supersymmetric Yang-Mills theory in planar limit

(Brink-Scherk-Schwarz 1977)

- Conformal, convergent series
- Great toy model for QCD
  - Tree-level amplitudes identical
  - Loop amplitudes simpler, structures similar
  - But, no confinement :(
- Past: new methods for amplitudes originated here

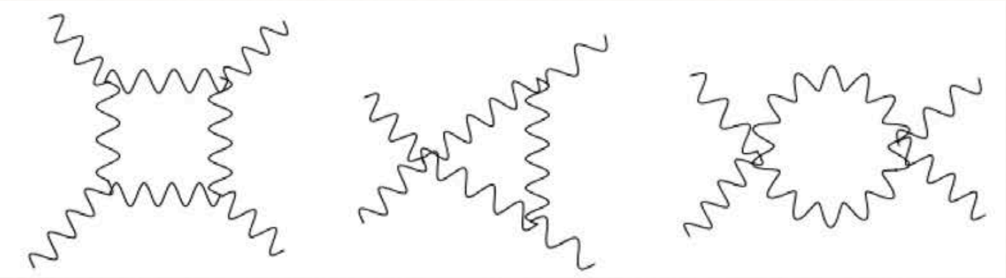
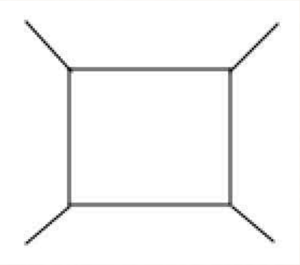
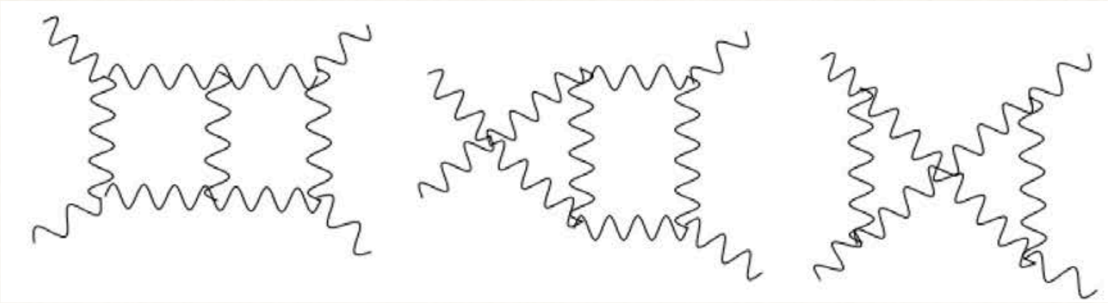
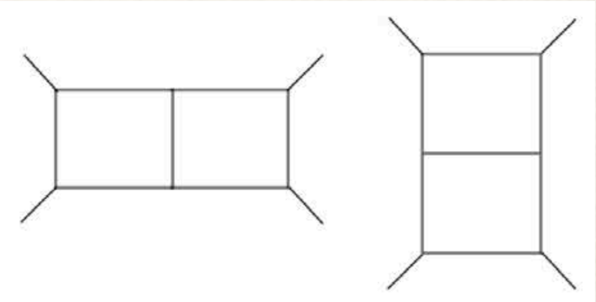
Many faces of the theory

- Useful playground for many theoretical ideas



Simple amplitudes

- Comparison: Feynman diagrams vs unitary methods

$gg \to gg$	Number of graphs	
	87 vs 1	
	$\sim$1000 vs 2	

What is the amplitude?

New definition of the amplitude

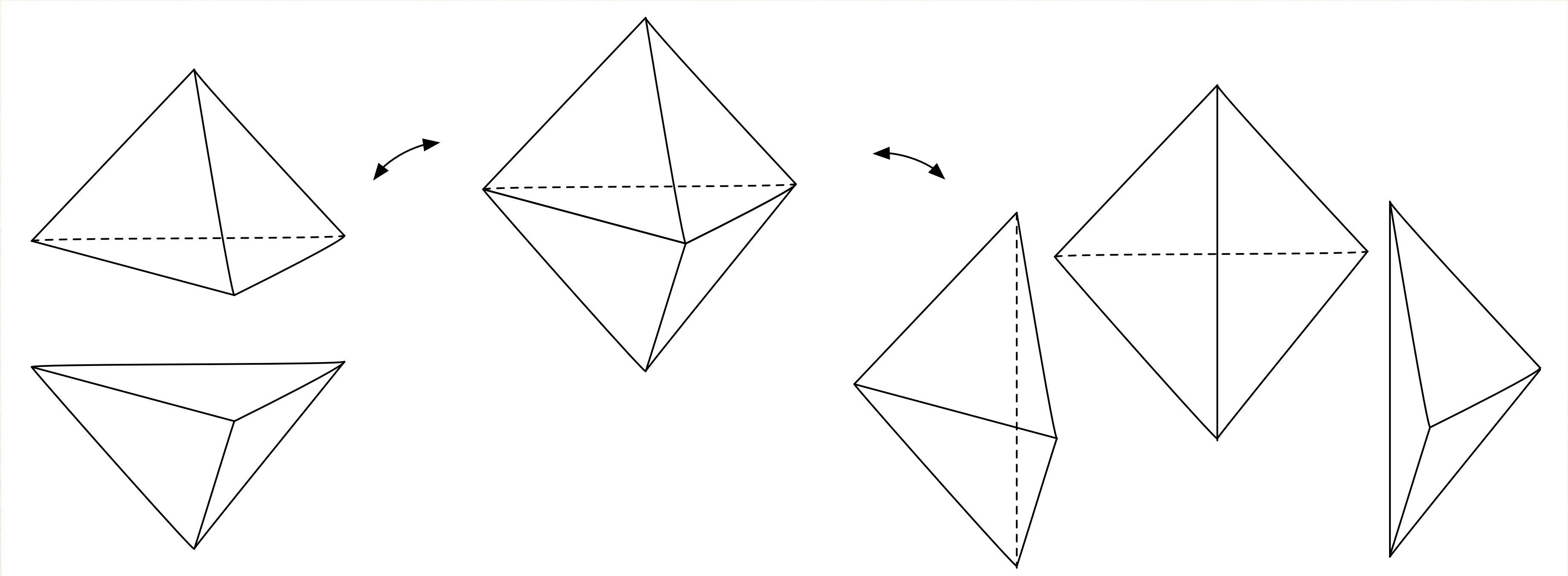
- Standard: Function consistent with locality and unitarity
- Our goal: Different definition
    - No fields, Lagrangians, path integrals
    - Unitarity, locality emergent from other principles
    - Powerful method for calculations

Prelude

Volume of polyhedron

(Hodges 2009)

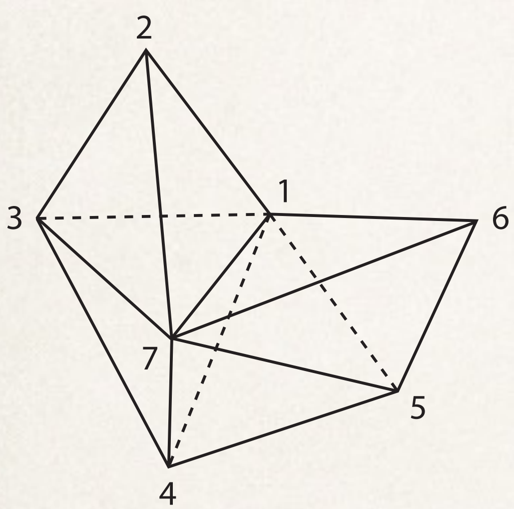
- New kinematical variables — momentum twistors $Z \in \mathbb{C}^3$
- Tree-level process: $gg \to 5g$
- Comparison of two calculations of recursion relations



(Picture by Stavros Garoufalidis)

Evidence for a new structure

Volume of polyhedron



$gg \to gg \ldots g$

at tree-level

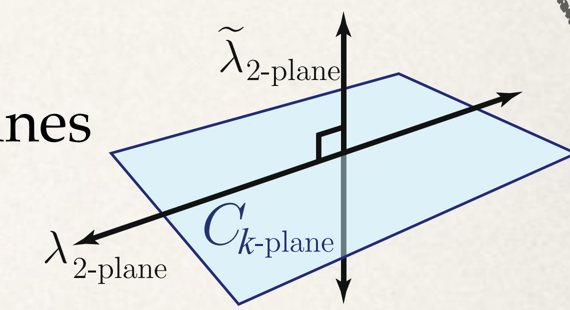
$$\int_{\widetilde{P}_n} \frac{D^4\mathcal{W}}{(\mathcal{Z}_0 \cdot \mathcal{W})^5}$$

Amplitude = volume

(Arkani-Hamed, Bourjaily, Cachazo, Hodges, JT 2010)

Grassmannian

Configurations of k-planes in n dimensions



$$\oint_{C\subset\Gamma_\sigma} \frac{d^{k\times n}C}{\text{vol}(GL(k))}\, \frac{\delta^{k\times 4}(C\cdot\widetilde{\eta})}{(1\cdots k)\cdots(n\cdots k\text{–}1)}\delta^{k\times 2}(C\cdot\widetilde{\lambda})\delta^{2\times(n-k)}(\lambda\cdot C^{\perp})$$

All-loop order information

(Arkani-Hamed, Cachazo, Cheung, Kaplan 2009)

"Conjecture"

Amplitudes are volumes
of *some regions* in *some space*

The Amplituhedron

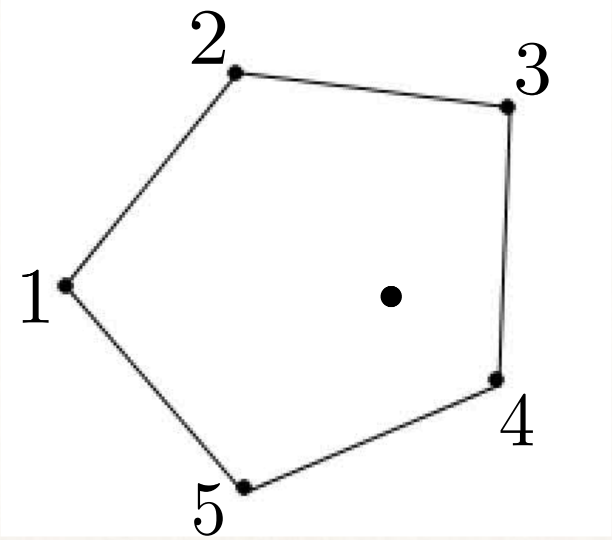
(Arkani-Hamed, JT 2013)

Strategy

- Simple intuitive geometric ideas: use equations
- Generalization:
  - More complicated geometry
  - Higher dimensions
- Same equations persist

Road to Amplituhedron

<u>Start:</u>
Point inside a convex polygon



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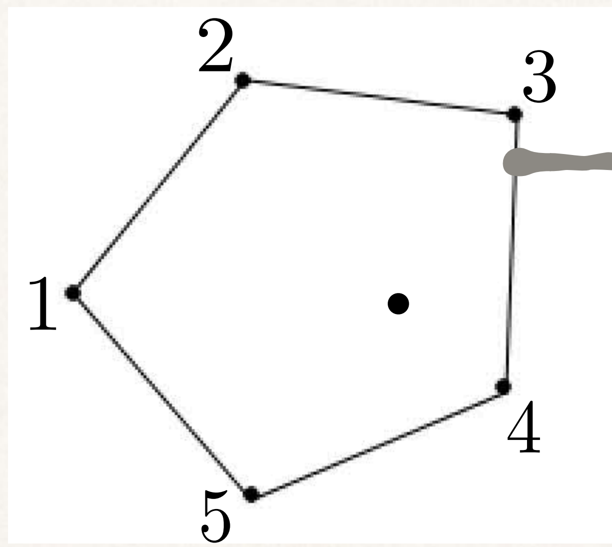

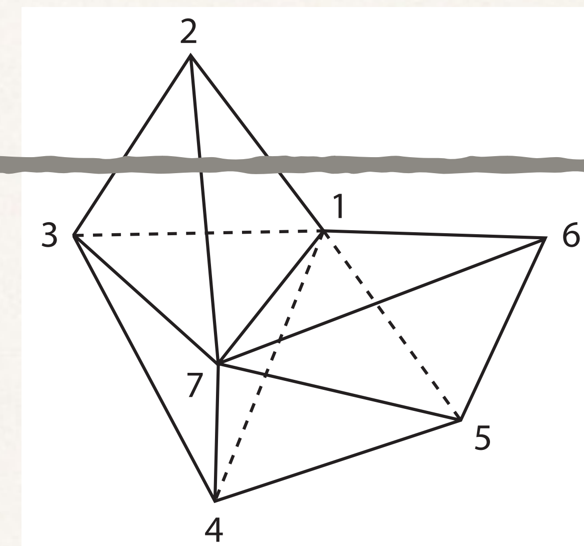

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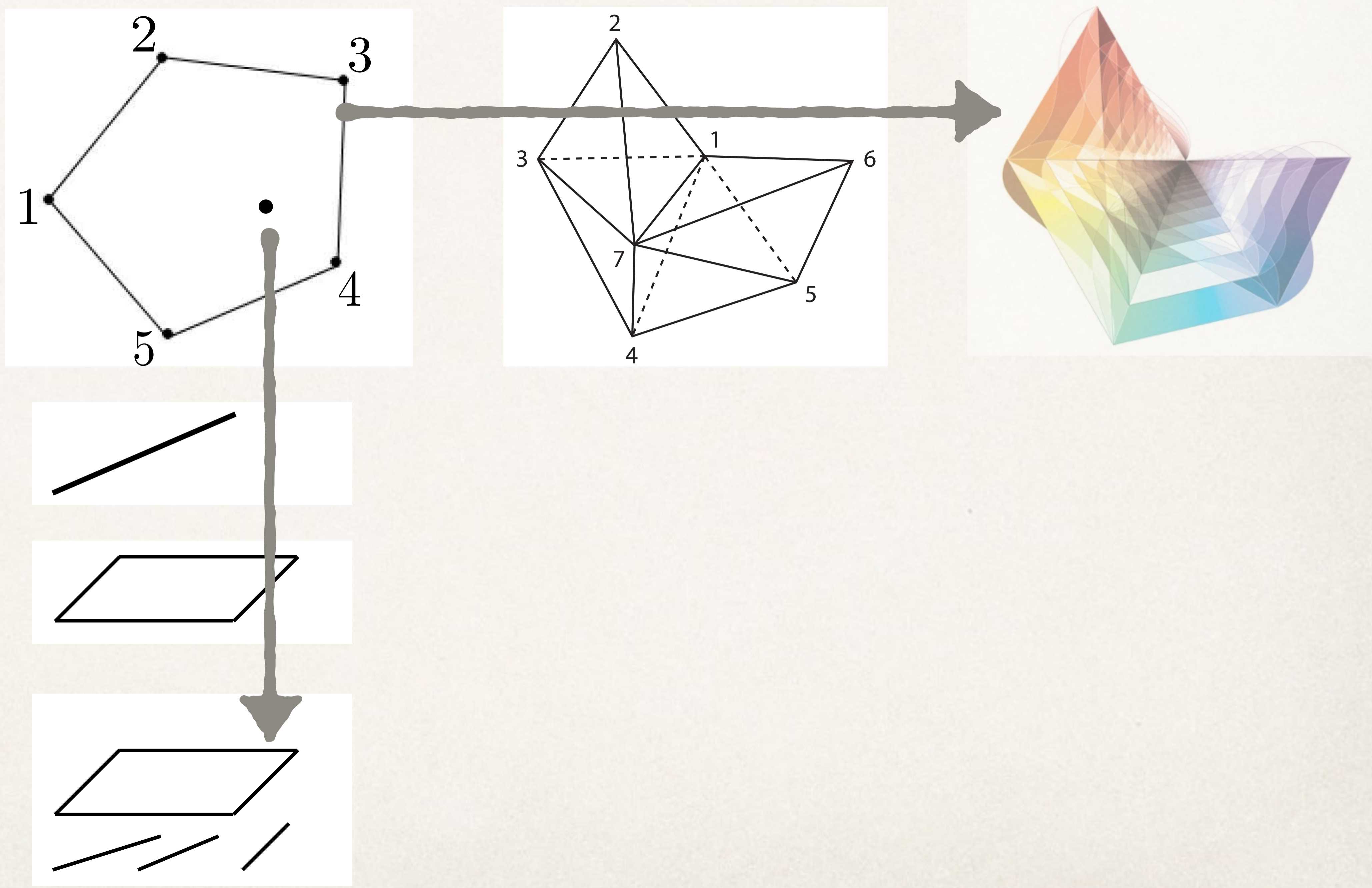

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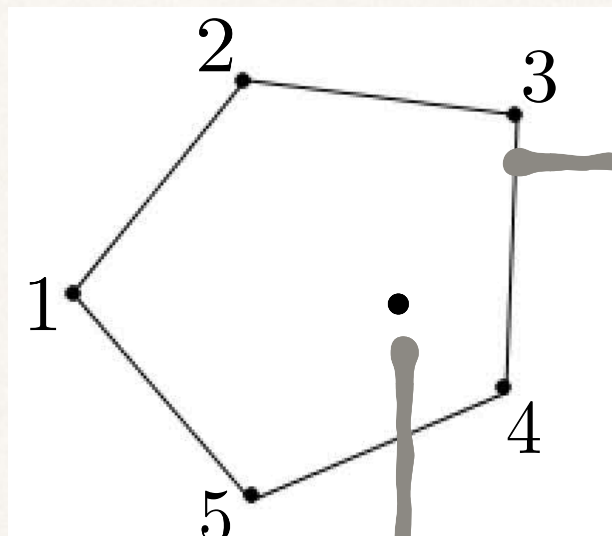
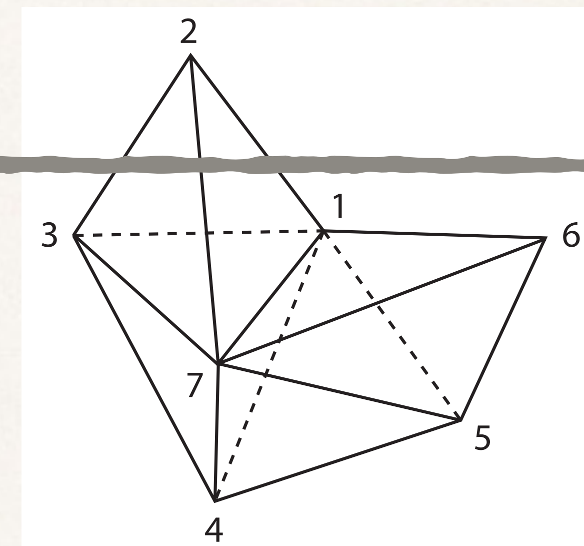
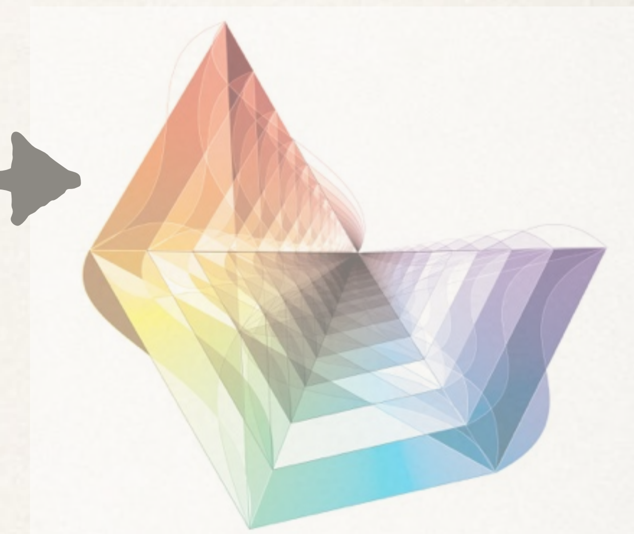
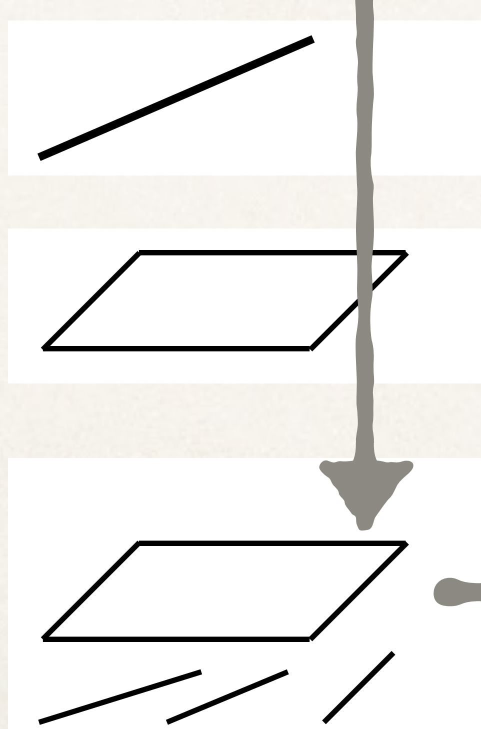






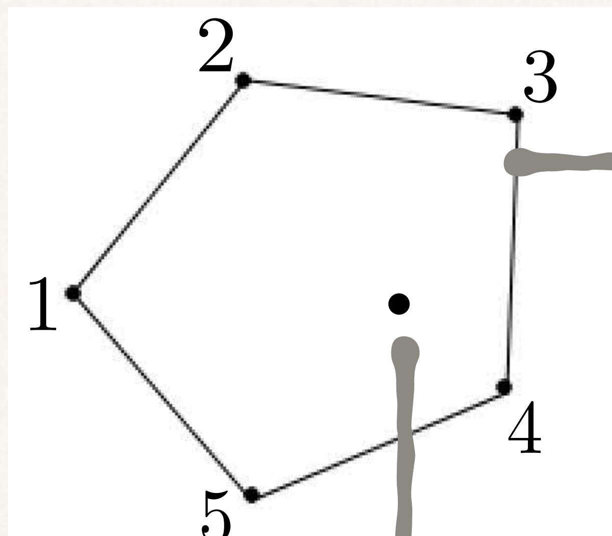


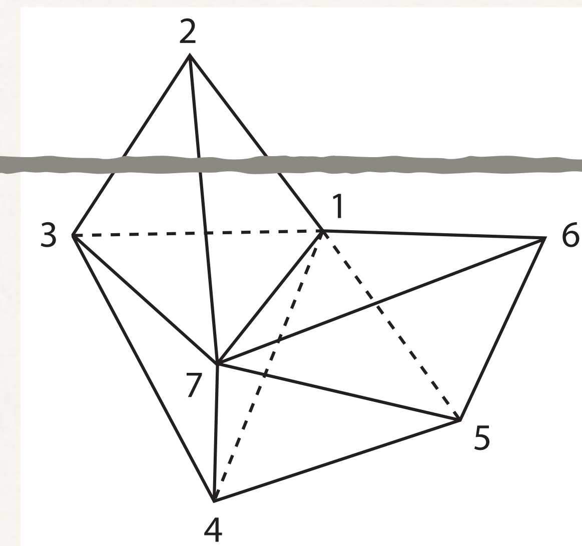
<u>Amplituhedron $\mathcal{A}_{n,k,\ell}$</u>

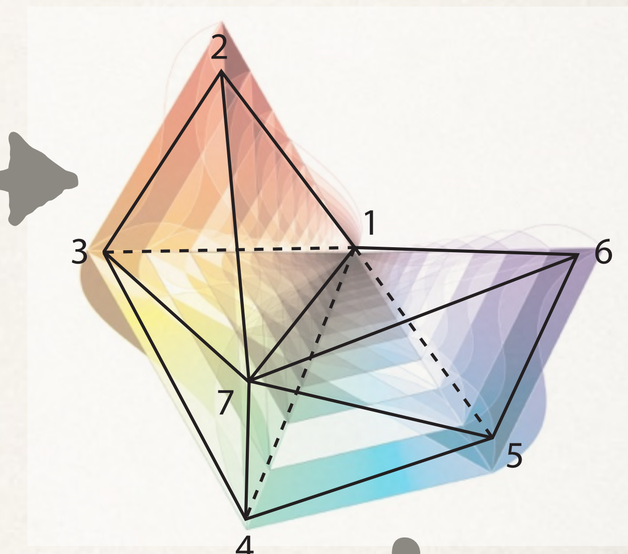
A k-dim plane and ℓ lines inside a $(k+4)$-dim convex space defined by n vertices

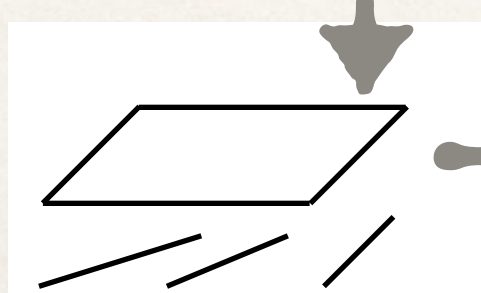
Road to Amplituhedron

<u>Start:</u>
Point inside a convex polygon









<u>Amplituhedron $\mathcal{A}_{n,k,\ell}$</u>

A k-dim plane and ℓ lines inside a $(k+4)$-dim convex space defined by n vertices

Amplituhedron conjecture

- Volume of $\mathcal{A}_{n,k,\ell}$:

Amplitudes in maximally supersymmetric Yang-Mills theory

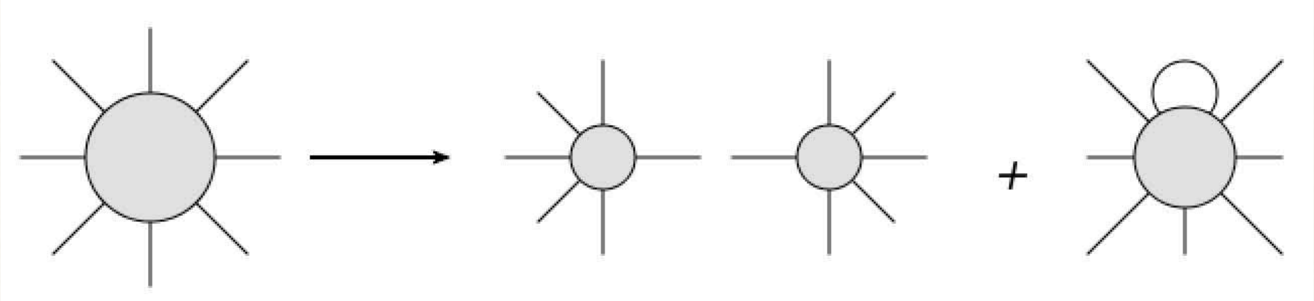
$\ell = 0$: Amplitudes of gluons in QCD

n
number of particles

k
helicity information

ℓ
number of loops

- Consistency check: Locality and Unitarity



- Explicit checks against reference theoretical data

Volume of the space

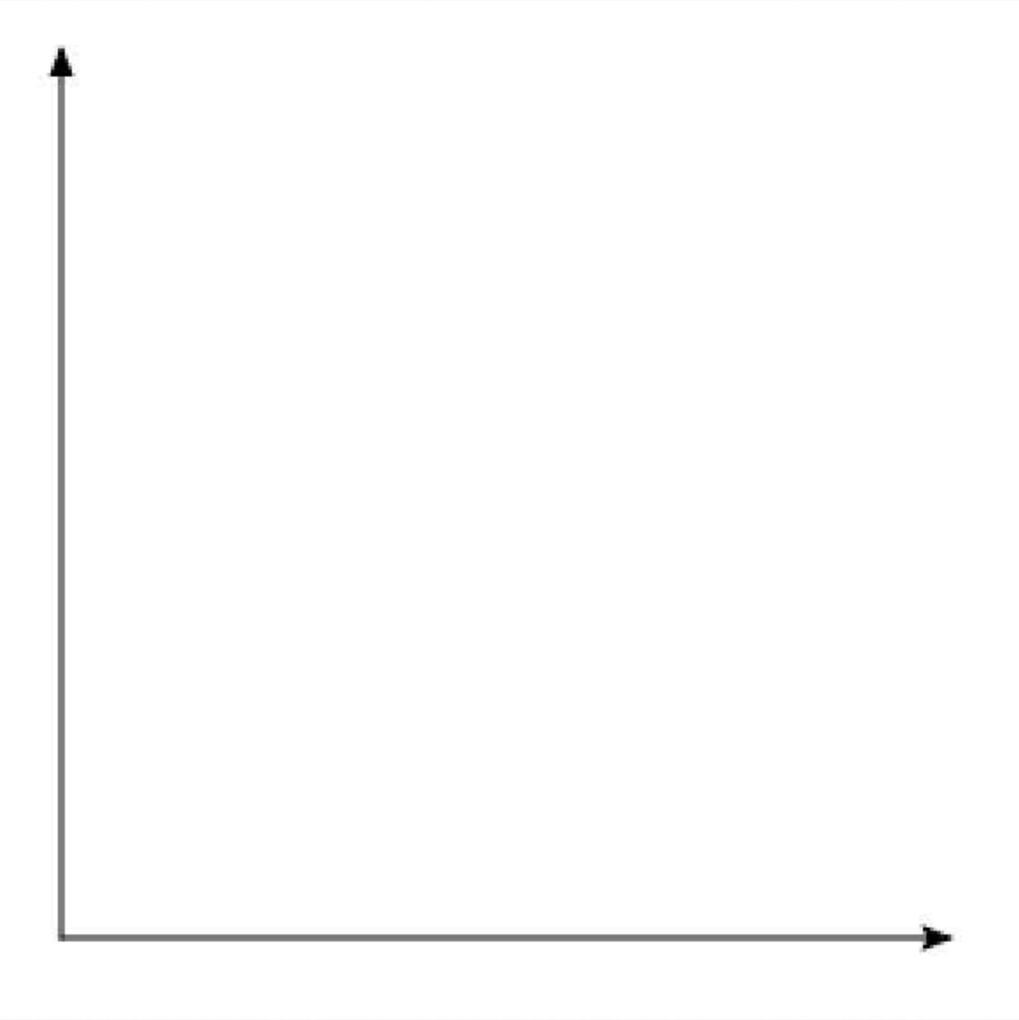
- Set of inequalities: Volume = differential form
- Simple examples: $x > 0: \quad \text{Vol} = \frac{dx}{x}$

$$y > 0,\, x > 0: \quad \text{Vol} = \frac{dx}{x}\frac{dy}{y} \qquad\qquad y > x > 0: \quad \text{Vol} = \frac{dx}{x}\frac{dy}{y-x}$$

- Amplituhedron for amplitude $gg \to gg$
  - Nice interpretation: Configuration of vectors on a plane
  - Easy to state, hard to solve — "High school problem"

High school problem $gg \to gg$

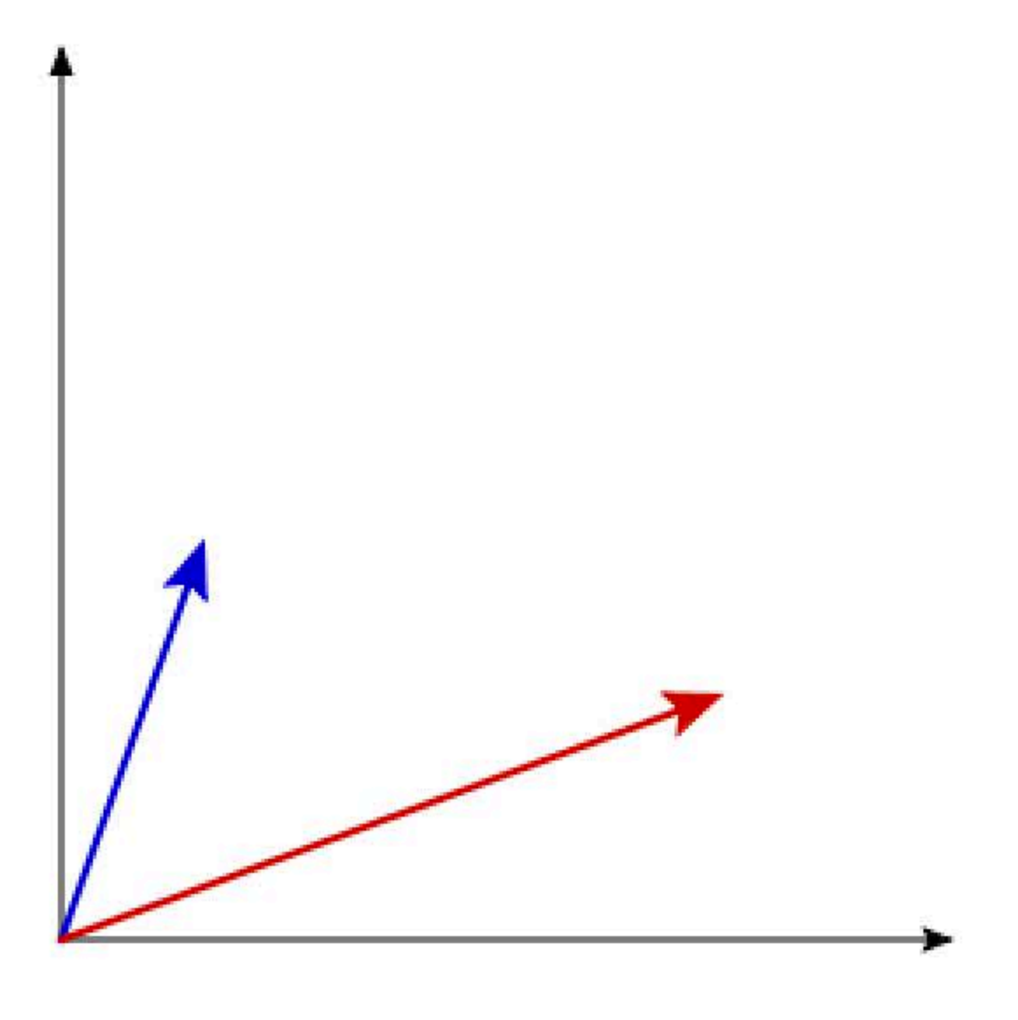
- Positive quadrant



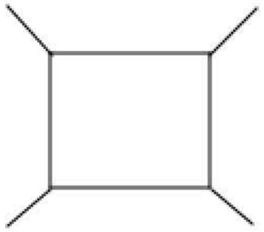
High school problem $gg \to gg$

- Positive quadrant
- Vectors

$$\vec{a}_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \quad \vec{b}_1 = \begin{pmatrix} z_1 \\ w_1 \end{pmatrix}$$



$$\text{Vol}\,(1) = \frac{dx_1}{x_1}\frac{dy_1}{y_1}\frac{dz_1}{z_1}\frac{dw_1}{w_1} =$$



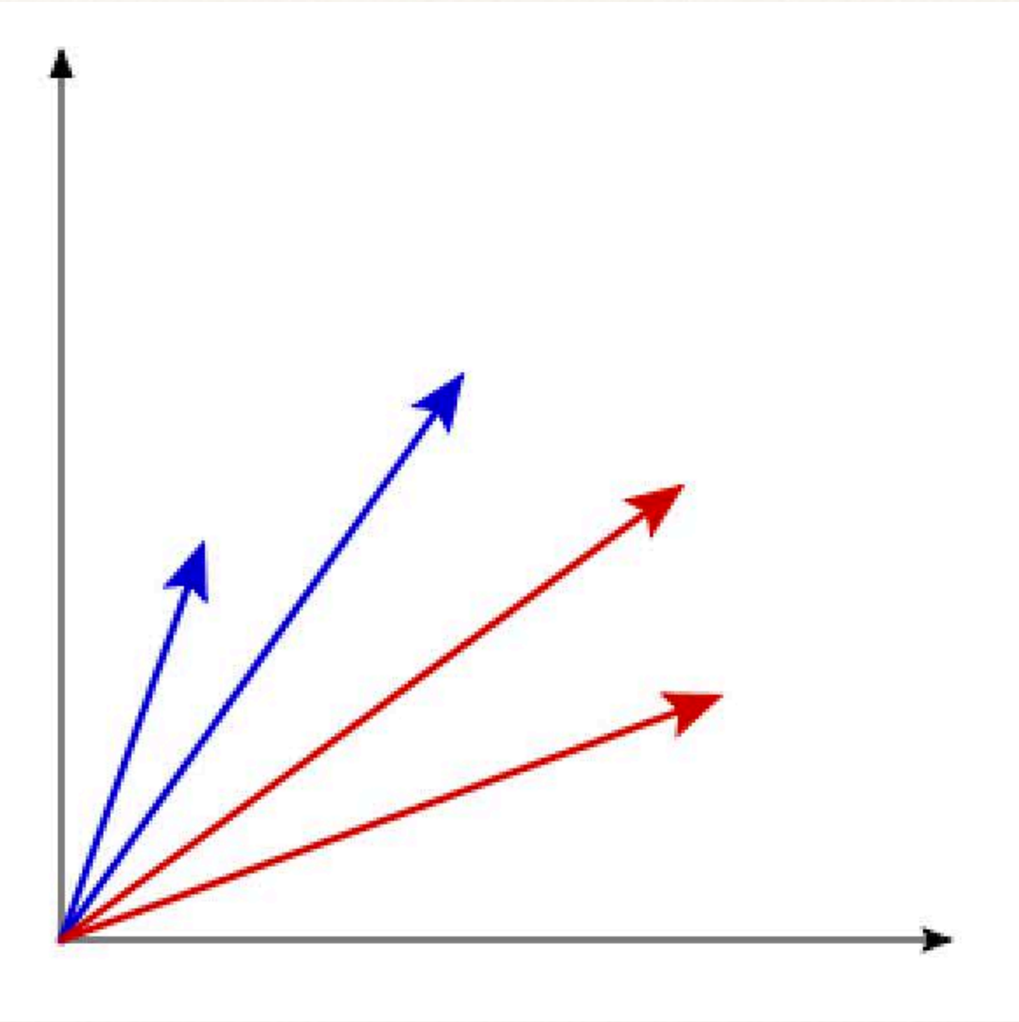
High school problem $gg \to gg$

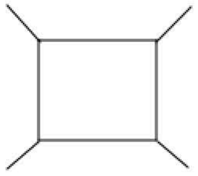
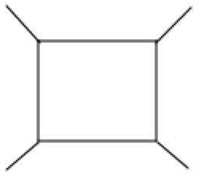
- Positive quadrant

- Vectors

$$\vec{a}_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \quad \vec{b}_1 = \begin{pmatrix} z_1 \\ w_1 \end{pmatrix}$$

$$\vec{a}_2 = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \quad \vec{b}_2 = \begin{pmatrix} z_2 \\ w_2 \end{pmatrix}$$



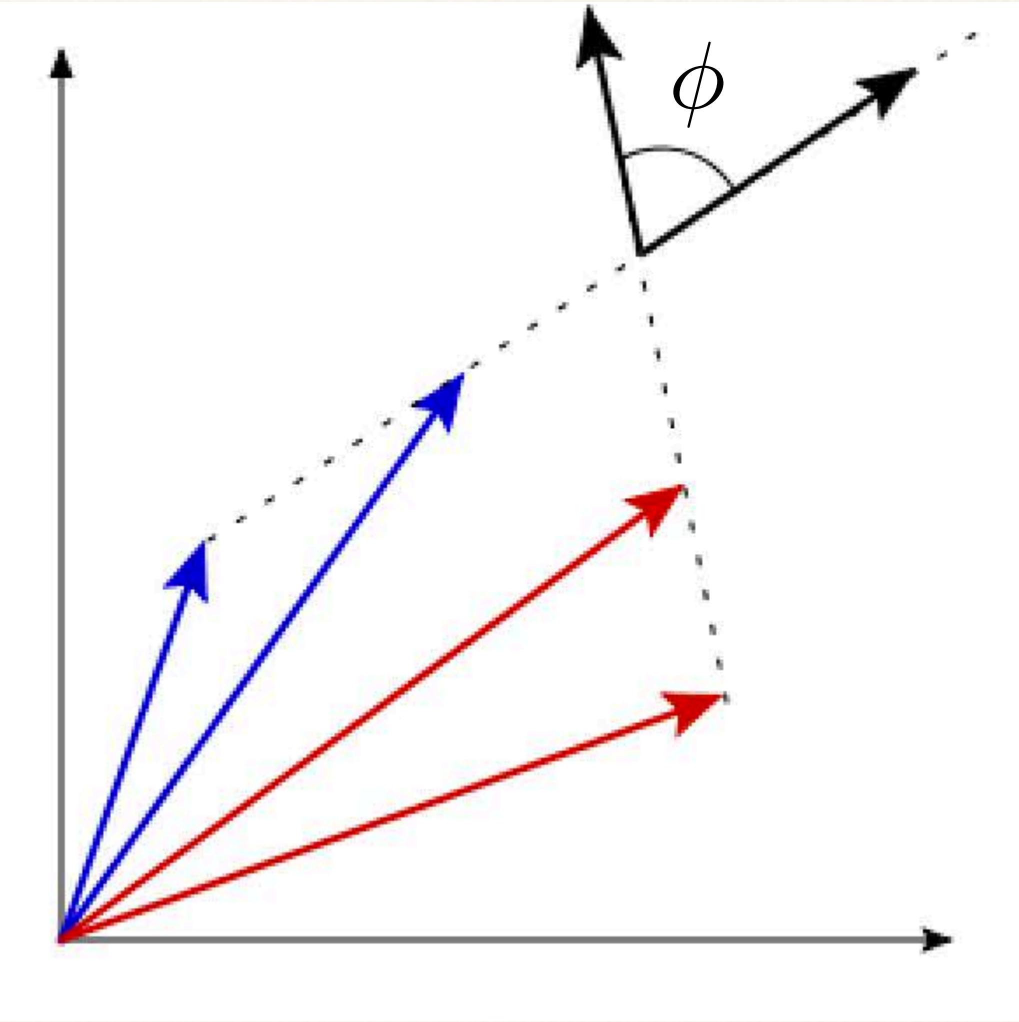
$$[\mathrm{Vol}\,(1)]^2 = \frac{dx_1}{x_1}\frac{dy_1}{y_1}\frac{dz_1}{z_1}\frac{dw_1}{w_1}\frac{dx_2}{x_2}\frac{dy_2}{y_2}\frac{dz_2}{z_2}\frac{dw_2}{w_2} =$$  $\times$ 

High school problem $gg \to gg$

- Positive quadrant
- Vectors

$$\vec{a}_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \quad \vec{b}_1 = \begin{pmatrix} z_1 \\ w_1 \end{pmatrix}$$

$$\vec{a}_2 = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \quad \vec{b}_2 = \begin{pmatrix} z_2 \\ w_2 \end{pmatrix}$$



- Impose: $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_2 - \vec{b}_1) \leq 0 \qquad \phi > 90^o$

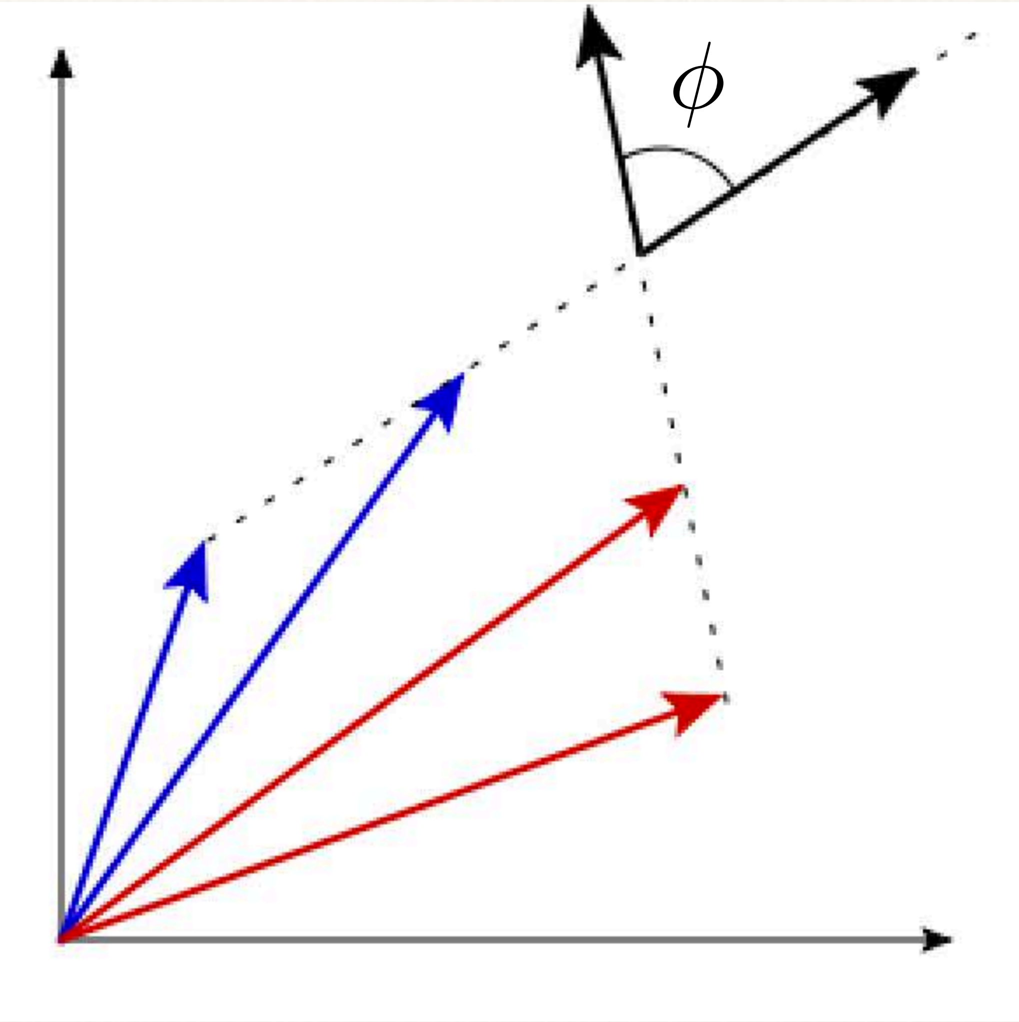
Subset of configurations allowed

High school problem $gg \to gg$

- Positive quadrant
- Vectors

$$\vec{a}_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \quad \vec{b}_1 = \begin{pmatrix} z_1 \\ w_1 \end{pmatrix}$$

$$\vec{a}_2 = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \quad \vec{b}_2 = \begin{pmatrix} z_2 \\ w_2 \end{pmatrix}$$



$$\mathrm{Vol}\,(2) = \frac{dx_1}{x_1}\frac{dy_1}{y_1}\frac{dz_1}{z_1}\frac{dw_1}{w_1}\frac{dx_2}{x_2}\frac{dy_2}{y_2}\frac{dz_2}{z_2}\frac{dw_2}{w_2}\left[\frac{\vec{a}_1\cdot\vec{b}_2+\vec{a}_2\cdot\vec{b}_1}{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_2-\vec{b}_1)}\right]$$

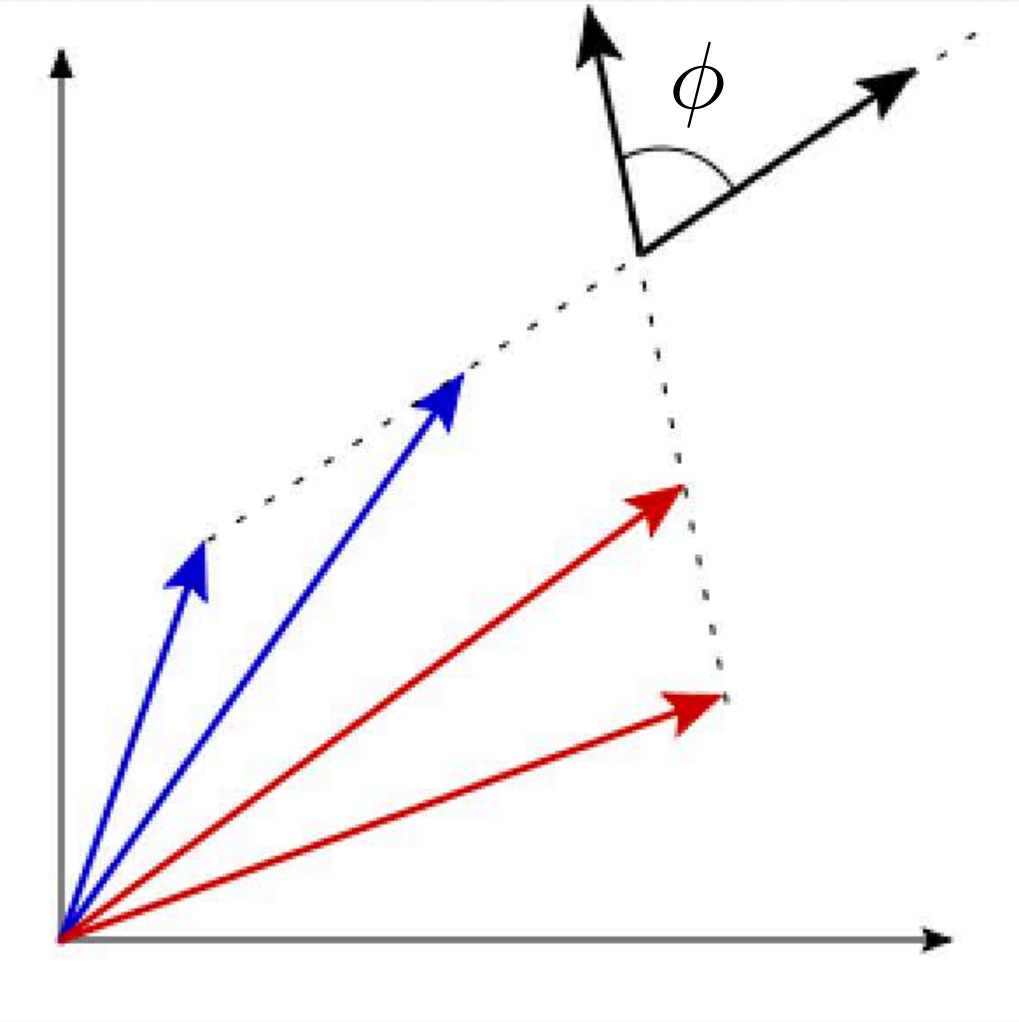
High school problem $gg \to gg$

- Positive quadrant

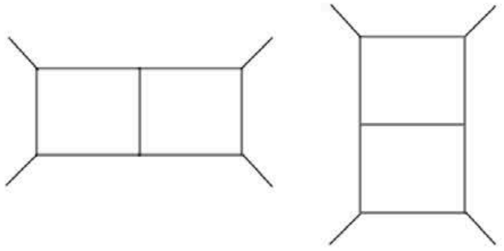
- Vectors

$$\vec{a}_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \quad \vec{b}_1 = \begin{pmatrix} z_1 \\ w_1 \end{pmatrix}$$

$$\vec{a}_2 = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \quad \vec{b}_2 = \begin{pmatrix} z_2 \\ w_2 \end{pmatrix}$$



$\mathrm{Vol}\,(2) =$



High school problem $gg \to gg$

- Positive quadrant
- Vectors

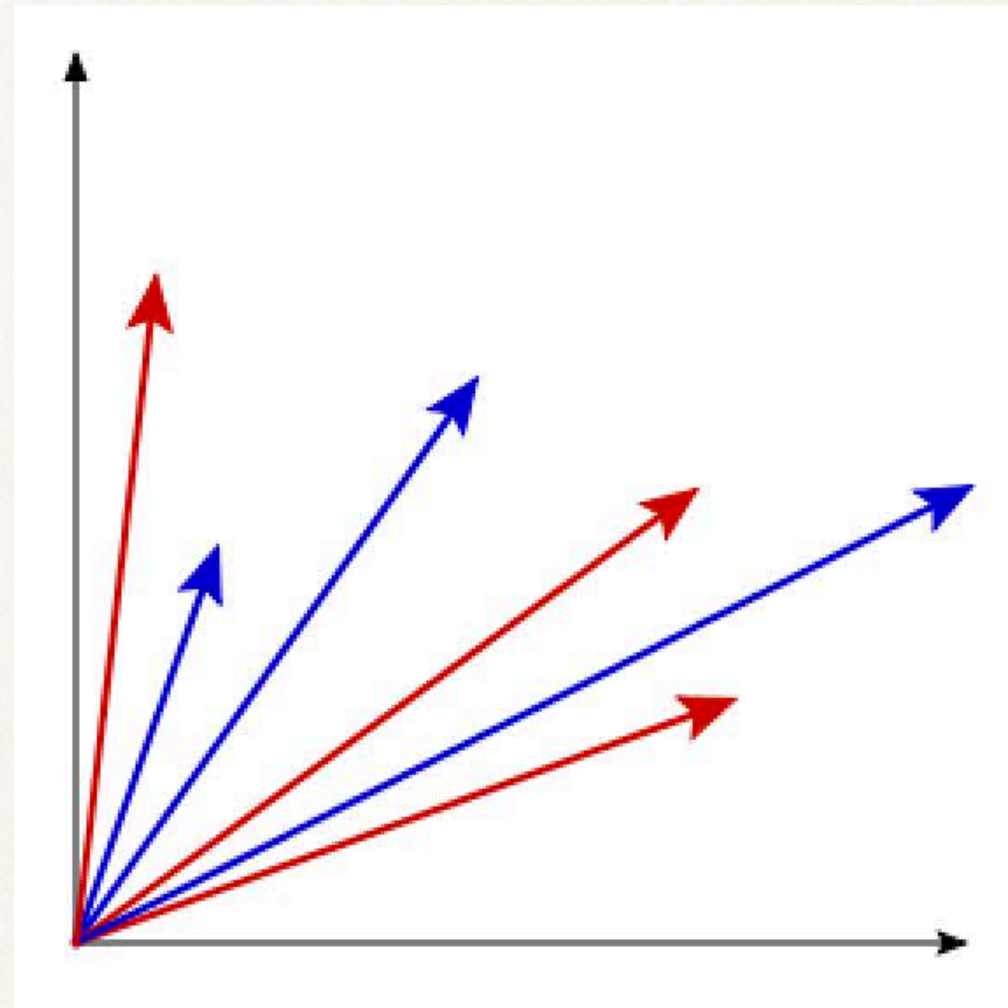
  $\vec{a}_1, \vec{a}_2, \vec{a}_3$ $\qquad$ $\vec{b}_1, \vec{b}_2, \vec{b}_3$

- Conditions

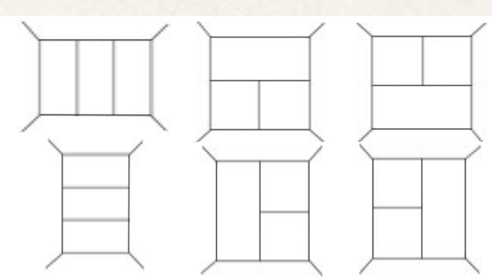
$$(\vec{a}_1 - \vec{a}_2) \cdot (\vec{b}_1 - \vec{b}_2) \leq 0$$

$$(\vec{a}_1 - \vec{a}_3) \cdot (\vec{b}_1 - \vec{b}_3) \leq 0$$

$$(\vec{a}_2 - \vec{a}_3) \cdot (\vec{b}_2 - \vec{b}_3) \leq 0$$



$\mathrm{Vol}\,(3) =$



High school problem $gg \to gg$

- Positive quadrant
- Vectors

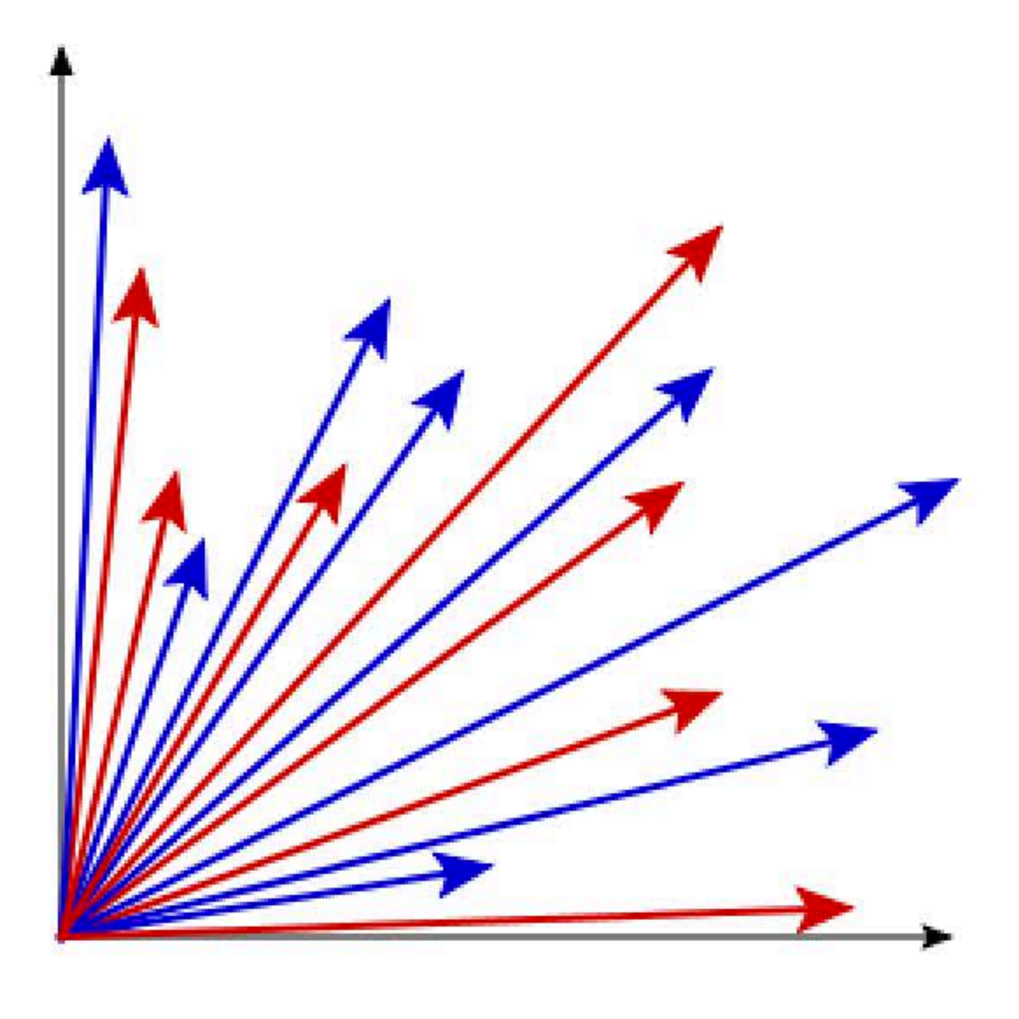
$$\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_\ell \quad \vec{b}_1, \vec{b}_2, \ldots, \vec{b}_\ell$$

- Conditions

$$(\vec{a}_i - \vec{a}_j) \cdot (\vec{b}_i - \vec{b}_j) \leq 0$$

for all pairs i, j

Let me know if you solve it!



$$\text{Vol}\,(\ell) = \ldots\ldots$$

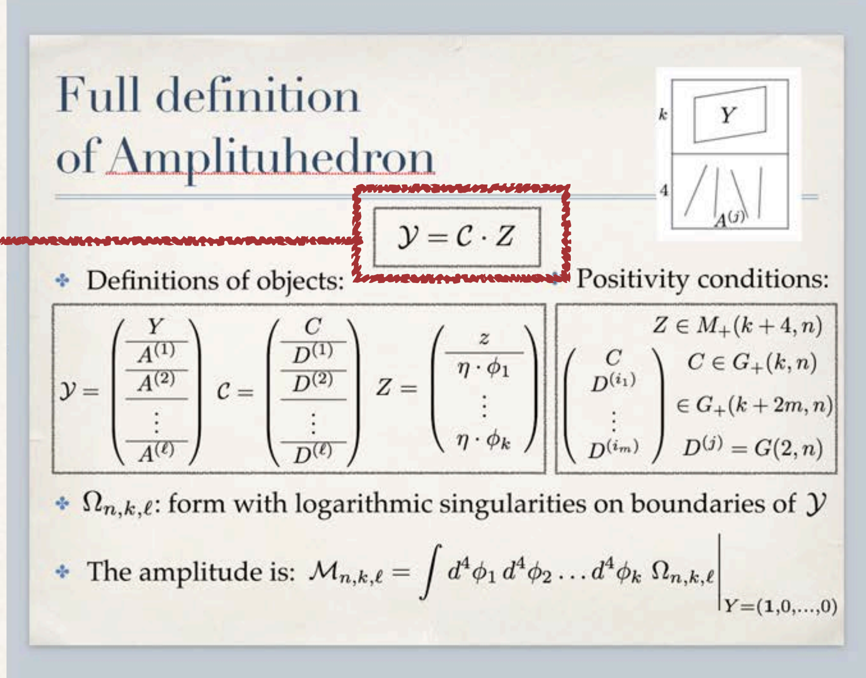
Positivity

- In the definition of Amplituhedron

$$\mathcal{Y} = \mathcal{C} \cdot Z$$

Amplituhedron

Positive matrices: Minors are positive $\begin{vmatrix} * & * \\ * & * \end{vmatrix} > 0$



- Positivity: crucial property of geometry
  - Locality, unitarity, even planarity derived from it
  - Hidden symmetry of this theory (Yangian) manifest

Question for mathematicians

How to make big positive matrices?

For the case $\ell = 0$ solved by Alexander Postnikov in 2006

Positive Grassmannian $G_+(k,n)$

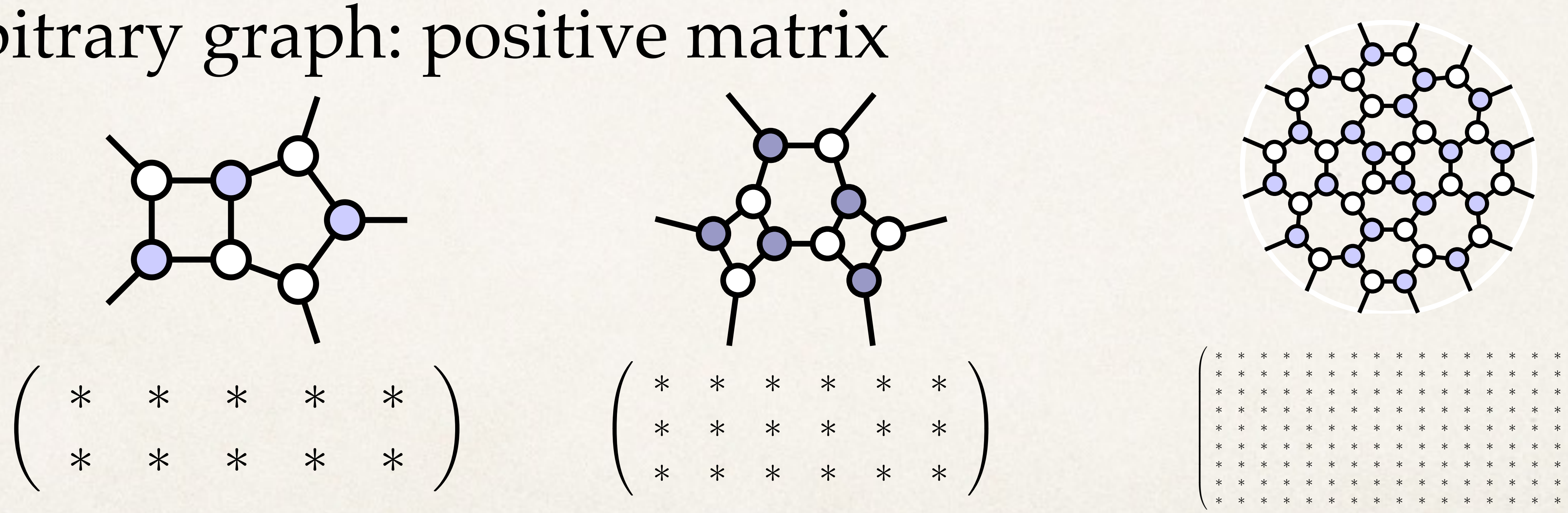
$$\begin{pmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{pmatrix} \qquad \begin{vmatrix} * & * & * \\ * & * & * \\ * & * & * \end{vmatrix} > 0$$

Gluing procedure

- Construct big positive matrix from small ones

Gluing preserves positivity of minors

- Arbitrary graph: positive matrix

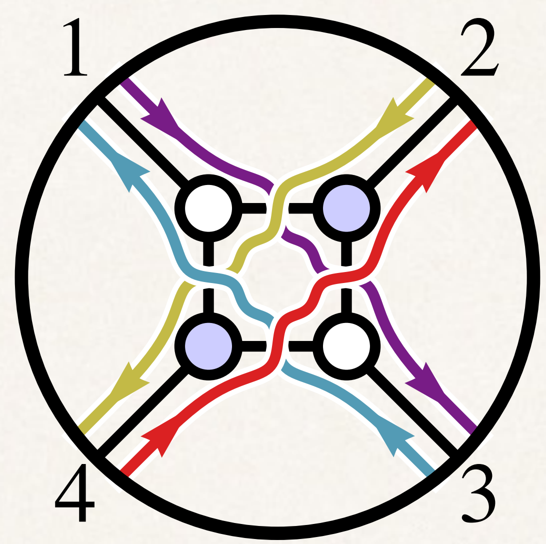


- Triangulation of Amplituhedron: set of these diagrams

Permutations

- "Basis" of these matrices: labeled by permutations

$$\begin{pmatrix} * & * & * & * \\ * & * & * & * \end{pmatrix} \leftrightarrow$$



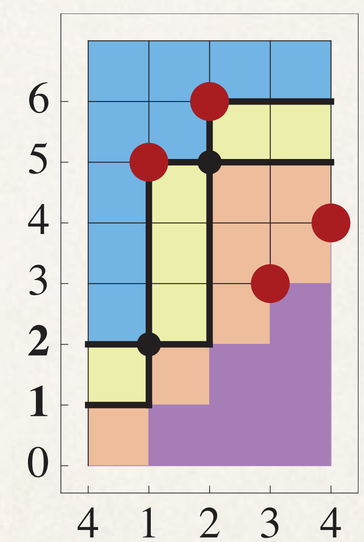
$$\leftrightarrow \quad (3, 4, 1, 2)$$

- Juggling patterns

Allen Knutson (Cornell U.)
1990-1995 world record in juggling (12 balls)

POSITROID VARIETIES I: JUGGLING AND GEOMETRY

ALLEN KNUTSON, THOMAS LAM, AND DAVID E SPEYER

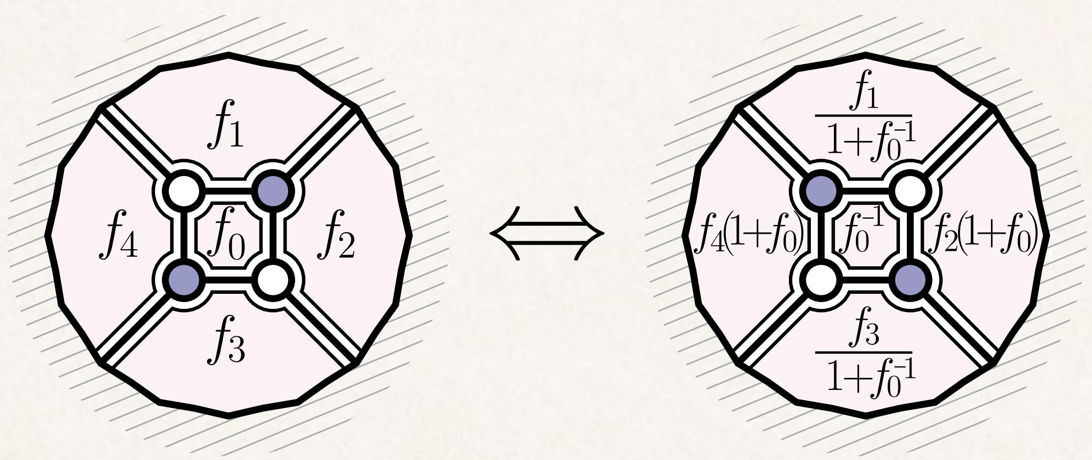


Deligne table

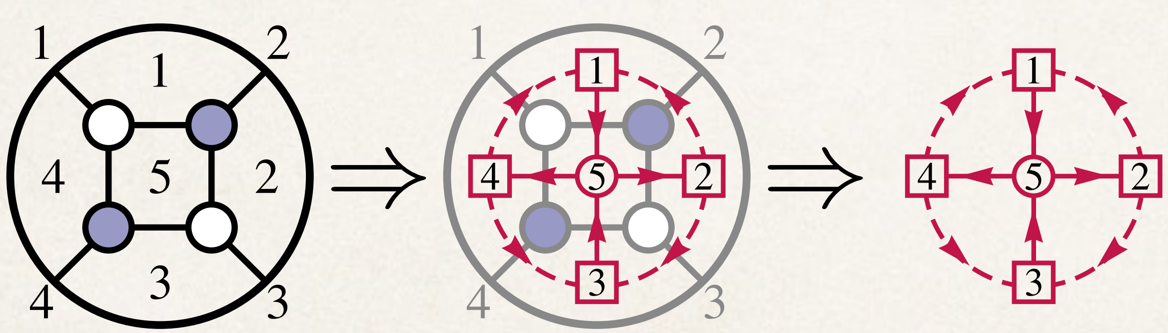
Other appearances of graphs

- Cluster variables associated with each graph

(Fock-Goncharov 2003)



- Dual graphs: quivers, Seiberg duality, shalow water waves,…

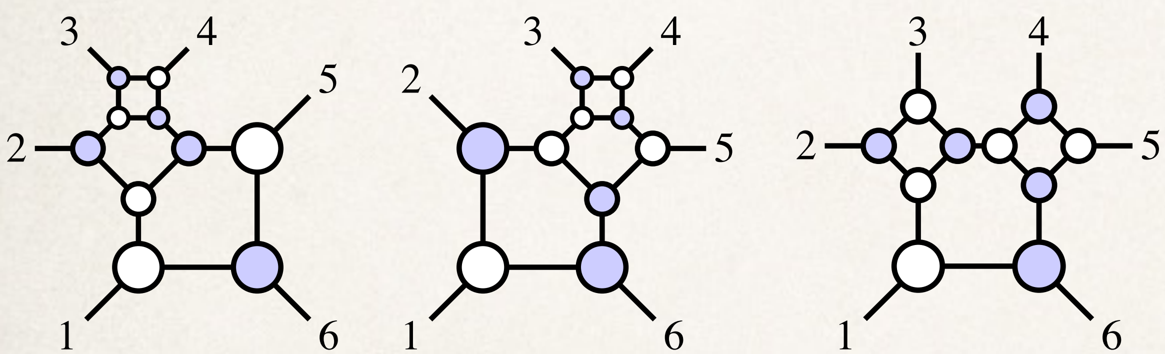


(Kodama-Williams 2011)

On-shell diagrams

- Same diagrams: radically different interpretation

- Physical on-shell processes: product of 3pt amplitudes



$$= \sum \mathcal{M}_3 \cdot \overline{\mathcal{M}}_3 \cdot \mathcal{M}_3 \ldots \overline{\mathcal{M}}_3 \cdot \mathcal{M}_3$$

$$= \text{Volume}\,(\mathcal{A})$$

Tree-level
$gg \to gggg$

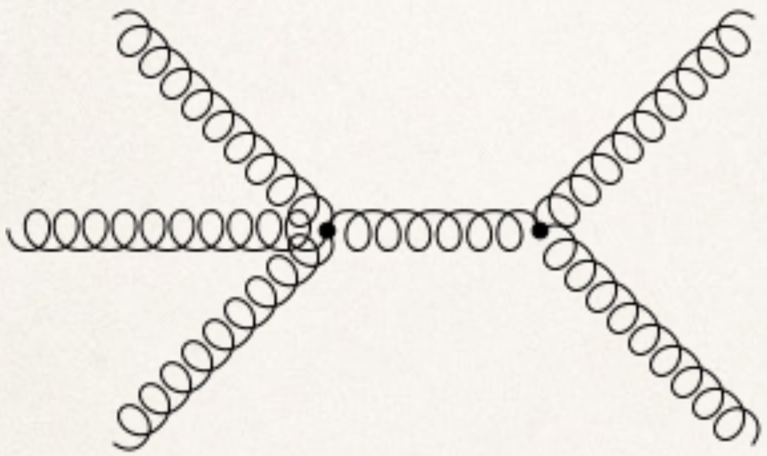
- Detailed study of this connection

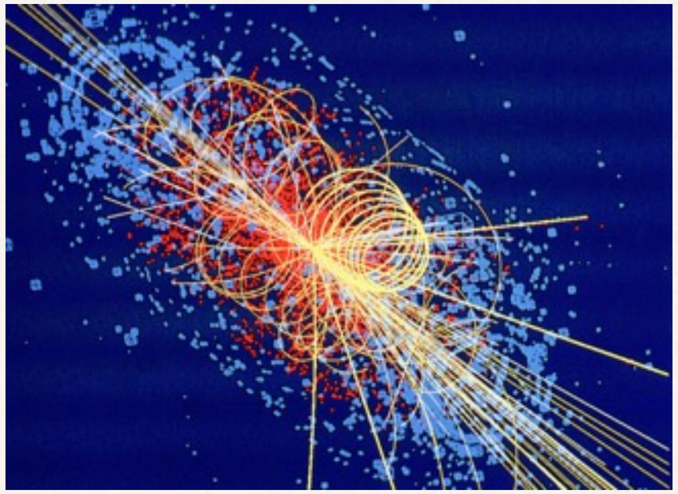
arXiv:1212.5605 [pdf, other]
Scattering Amplitudes and the Positive Grassmannian
Nima Arkani-Hamed, Jacob L. Bourjaily, Freddy Cachazo, Alexander B. Goncharov, Alexander Postnikov, Jaroslav Trnka
Comments: 158 pages, 264 figures
Subjects: **High Energy Physics - Theory (hep-th)**; Algebraic Geometry (math.AG); Combinatorics (math.CO)

Record on arxiv?

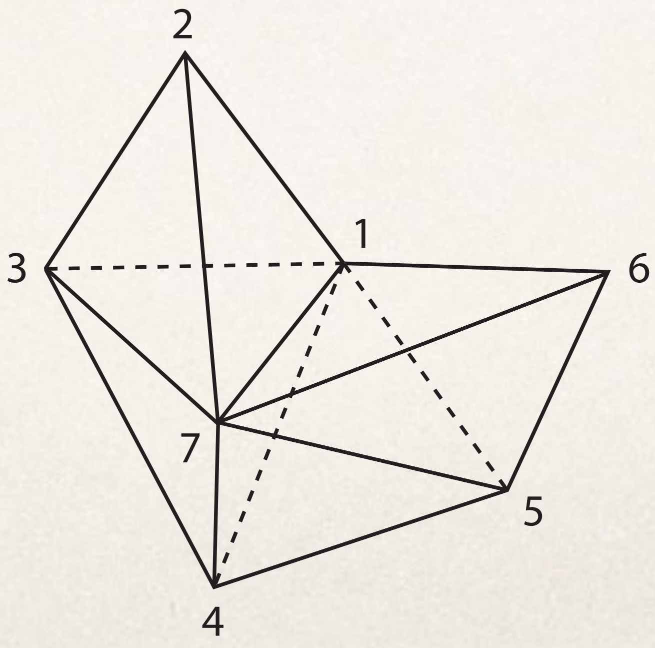
Physics vs geometry

- Dynamical particle interactions in 4-dimensions





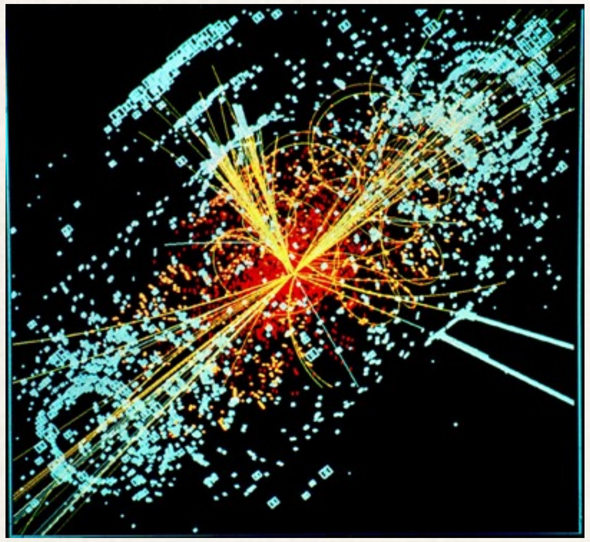
- Static geometry in high dimensional space



Real process at LHC

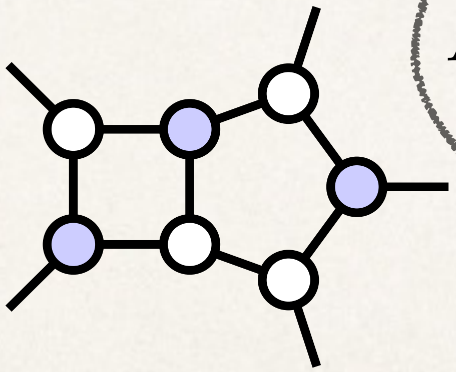
At the intersection

- Fascinating connection between fields which have never interacted so far



Predictions of particle collisions for experiments

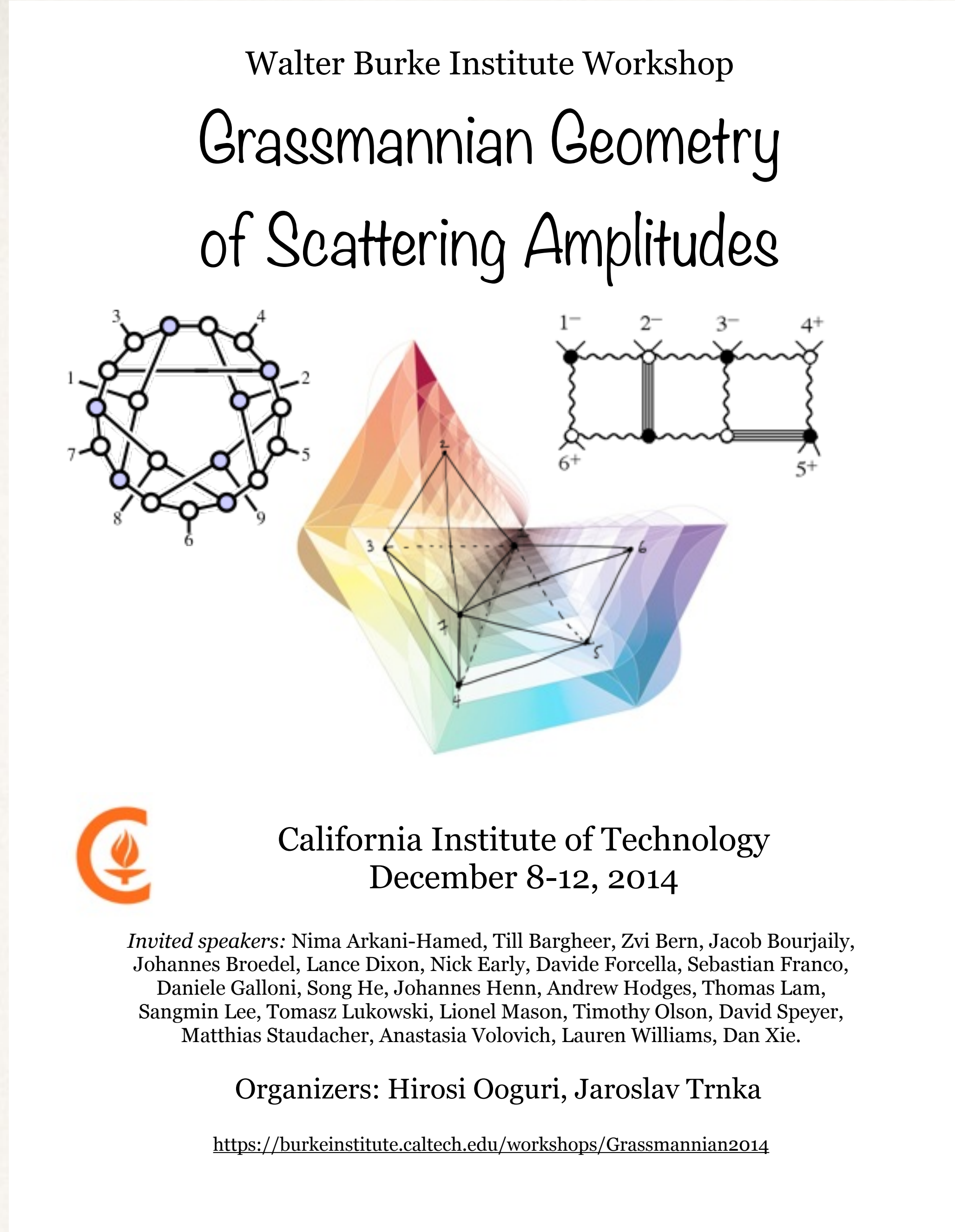
Combinatorics
Algebraic geometry
Cluster algebras

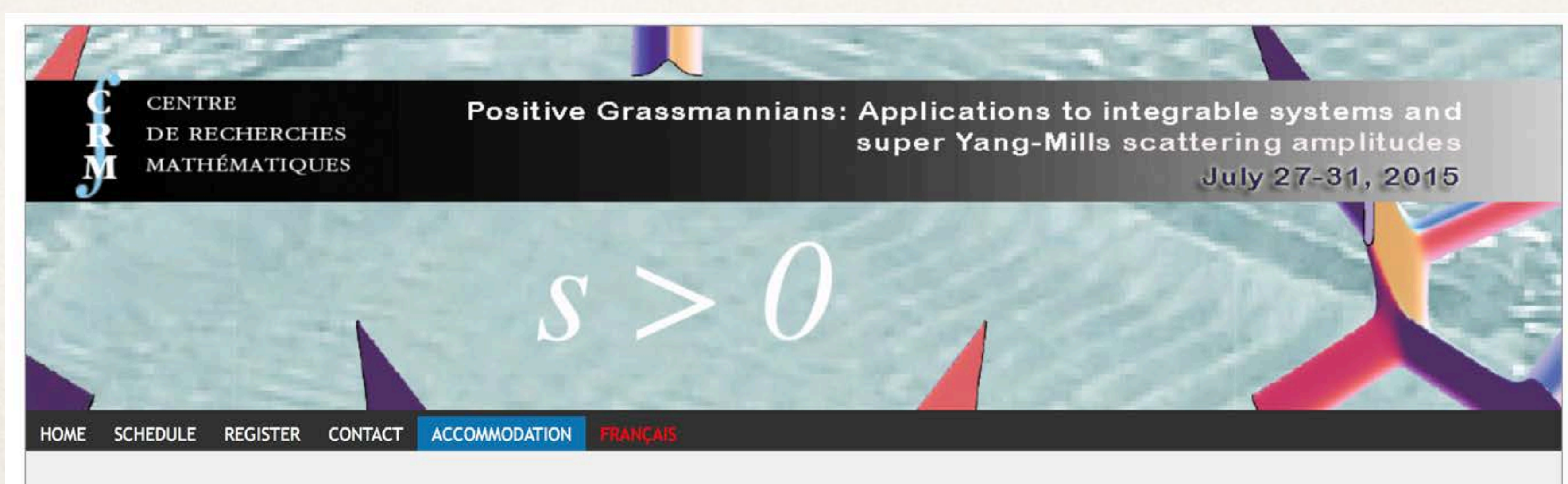


$(3,4,5,1,2)$

$$\begin{pmatrix} * & * & * & * & * \\ * & * & * & * & * \end{pmatrix} \in G_+(2,5)$$

$$A_{n,k} : G_+(k,n) \underset{Z}{\longrightarrow} G(k,k+4)$$


Walter Burke Institute Workshop
Grassmannian Geometry
of Scattering Amplitudes
California Institute of Technology
December 8-12, 2014
Invited speakers: Nima Arkani-Hamed, Till Bargheer, Zvi Bern, Jacob Bourjaily, Johannes Broedel, Lance Dixon, Nick Early, Davide Forcella, Sebastian Franco, Daniele Galloni, Song He, Johannes Henn, Andrew Hodges, Thomas Lam, Sangmin Lee, Tomasz Lukowski, Lionel Mason, Timothy Olson, David Speyer, Matthias Staudacher, Anastasia Volovich, Lauren Williams, Dan Xie.
Organizers: Hirosi Ooguri, Jaroslav Trnka
https://burkeinstitute.caltech.edu/workshops/Grassmannian2014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Back to Parke-Taylor formula

- Scattering $gg \to ggg \dots gg$
  in our toy model

$$\mathcal{M}_n^{tree} = \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \dots \langle n1 \rangle}$$

Back to Parke-Taylor formula

- Scattering $gg \to ggg \dots gg$
  in our toy model

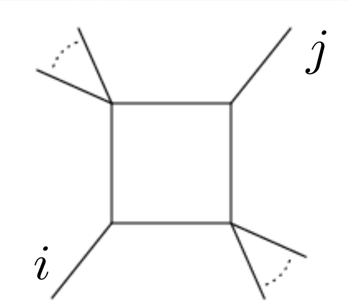
$$\mathcal{M}_n = \mathcal{M}_n^{tree} \left\{ \boxed{1} + \right.$$

Tree-level
(1985)

Back to Parke-Taylor formula

- Scattering $gg \to ggg\dots gg$
  in our toy model

$$\mathcal{M}_n = \mathcal{M}_n^{tree} \left\{ 1 + \sum_{ij} \right.$$



$+$

Tree-level (1985)

One-loop (1994)

Back to Parke-Taylor formula

- Scattering $gg \to ggg \dots gg$

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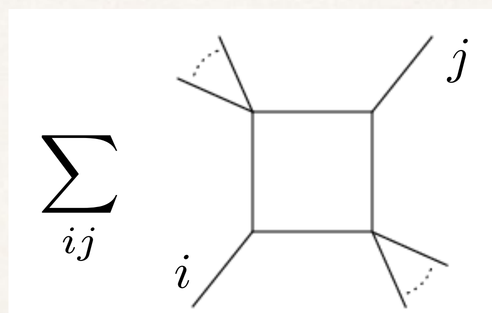
$$\mathcal{M}_n = \mathcal{M}_n^{tree} \left\{ 1 + \sum_{ij} \right.$$



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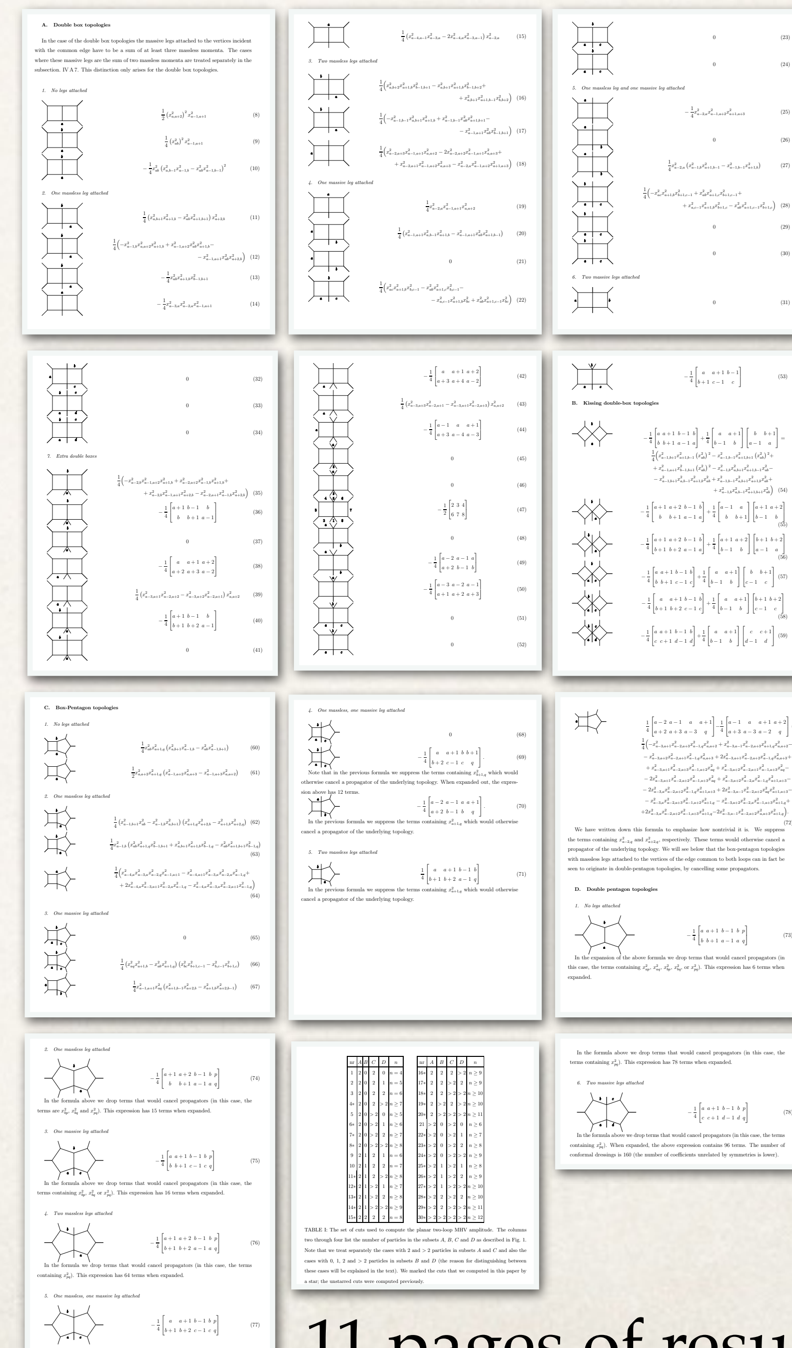


Tree-level (1985)

One-loop (1994)

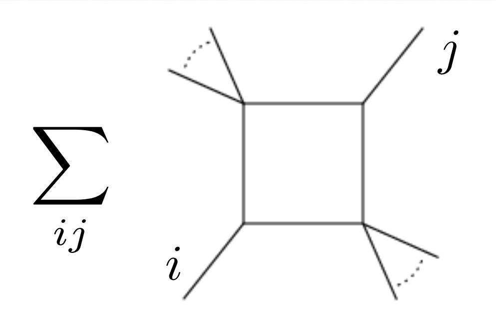
Two-loop (2009)

11 pages of result

Back to Parke-Taylor formula

- Scattering $gg \to ggg\ldots gg$
  in our toy model

$$\mathcal{M}_n = \mathcal{M}_n^{tree}\left\{ 1 + \sum_{ij} \right.$$

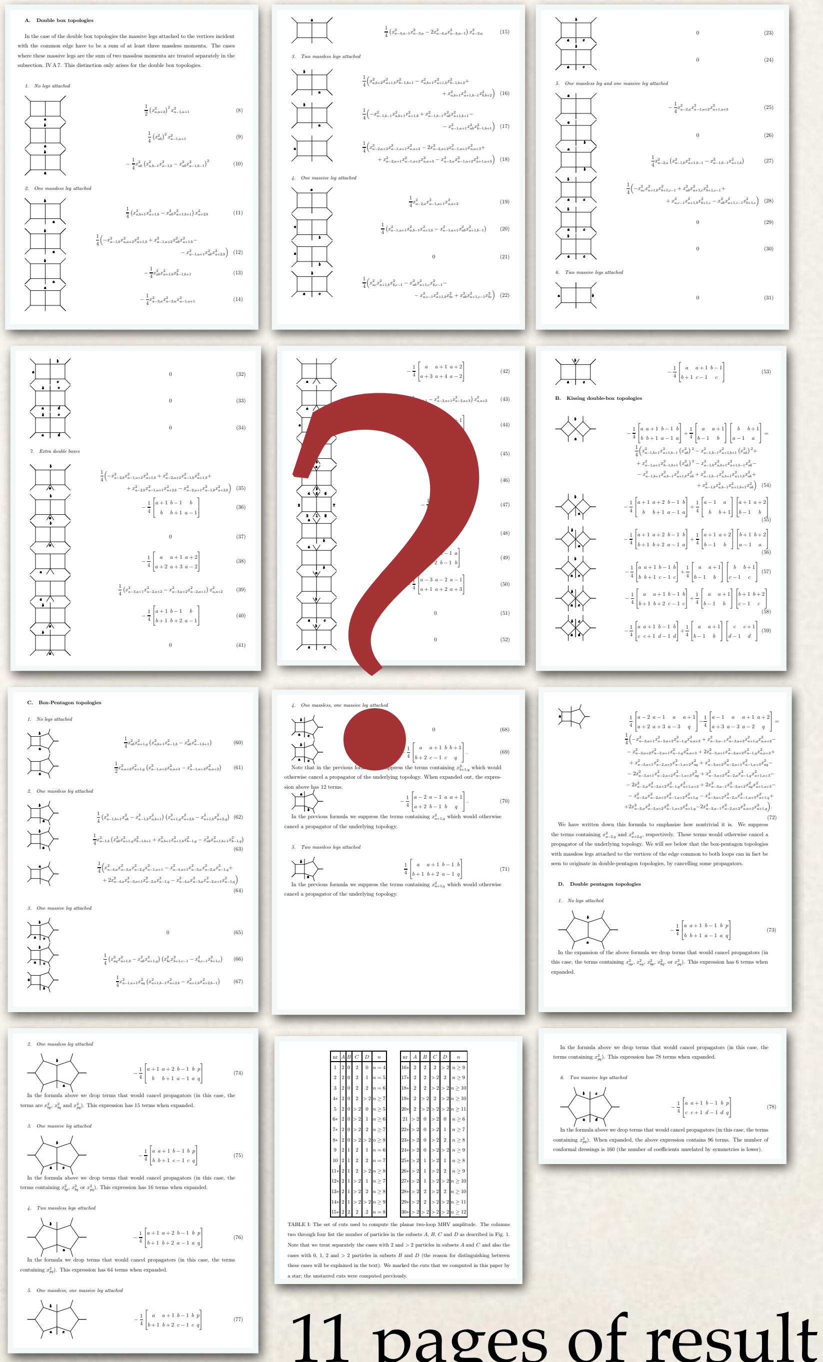


$+$

Tree-level (1985)

One-loop (1994)

Two-loop (2009)



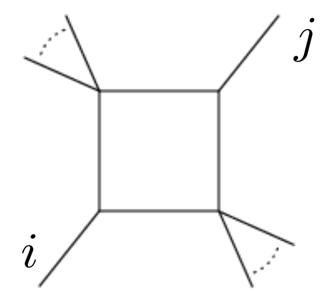
11 pages of result

Back to Parke-Taylor formula

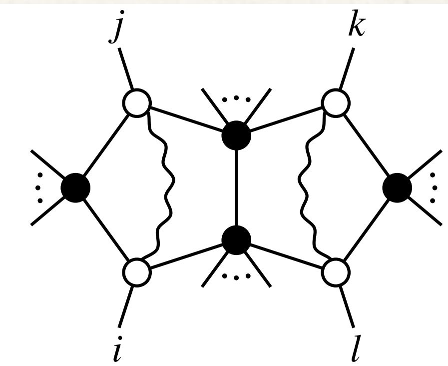
- Scattering $gg \to ggg\ldots gg$

  in our toy model

$$\mathcal{M}_n = \mathcal{M}_n^{tree}\left\{ 1 + \sum_{ij} \right.$$



$$\left. + \frac{1}{2}\sum_{i<j<k<l<i} \right.$$



(Arkani-Hamed, Bourjaily, Cachazo, JT 2010)

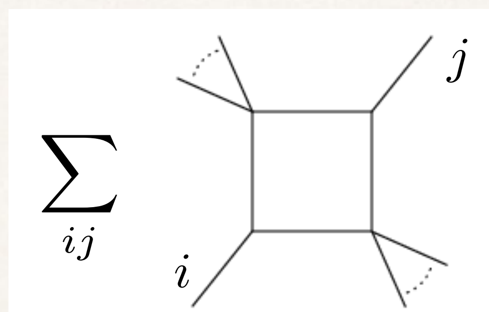
Tree-level
(1985)

One-loop
(1994)

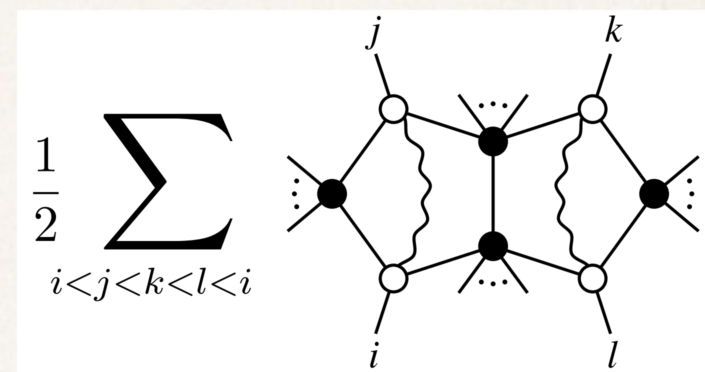
Back to Parke-Taylor formula

- Scattering $gg \to ggg \ldots gg$ in our toy model

$$\mathcal{M}_n = \mathcal{M}_n^{tree} \Big\{ \boxed{1} + $$

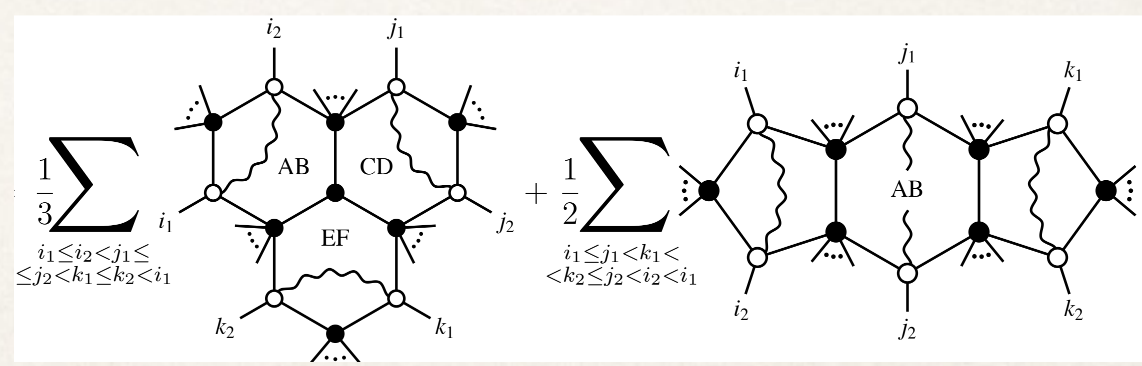


$$+$$



(Arkani-Hamed, Bourjaily, Cachazo, JT 2010)

$$+$$



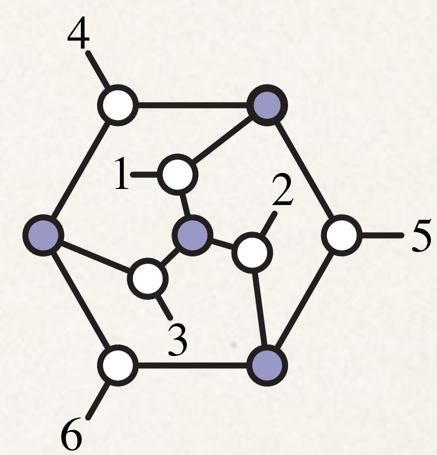
$$+ \ldots \Big\}$$

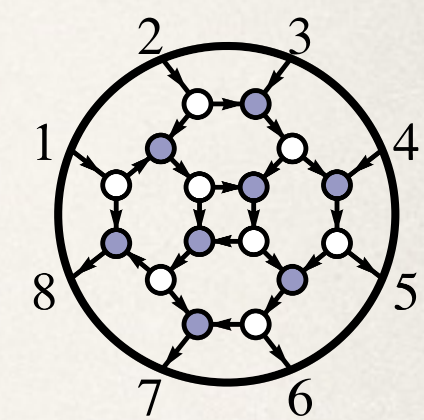
Tree-level (1985)

One-loop (1994)

Outlook: Beyond the toy model

- Amplituhedron: Geometric picture for amplitudes
- Next step: non-planar, gravity, string amplitudes, <u>QCD</u>
- Evidence beyond the toy model





  - On-shell diagram: non-planar, no susy
    (Arkani-Hamed, Bourjaily, Cachazo, Postnikov, JT 2014)
  - Amplituhedron-type construction beyond the planar limit
    (Arkani-Hamed, Bourjaily, Cachazo, JT 2014) (Bern, Hermann, Litsey, Stankowicz, JT 2014)
  - Connection to EFTs (NLσM, DBI, Galileon) via soft limits
    (Cheung, Kampf, Novotny, JT 2014)

Thank you for your attention