Bootstrapping Conformal Field Theories

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IAS

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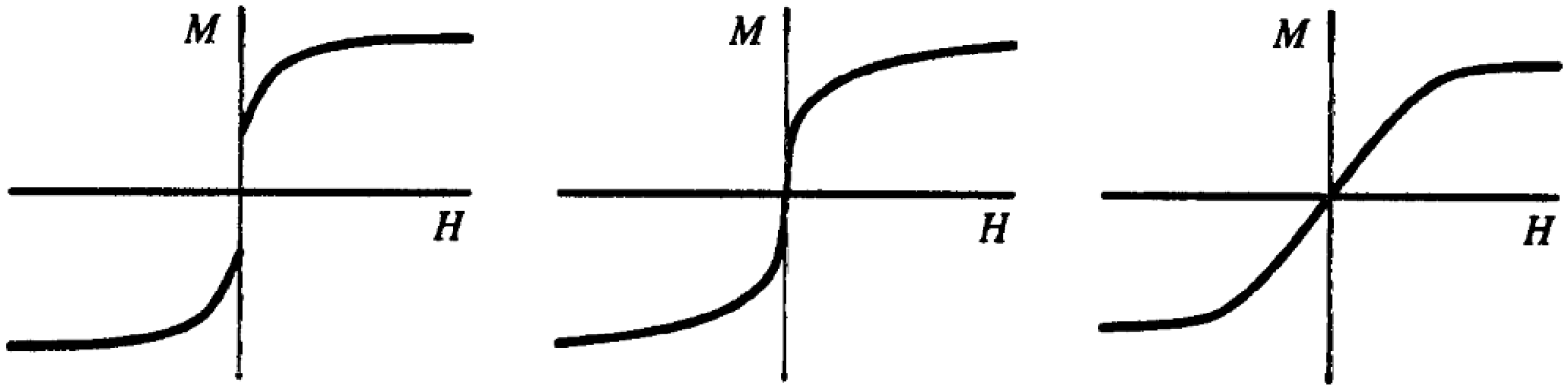
Outline

Critical Point of a Ferromagnet

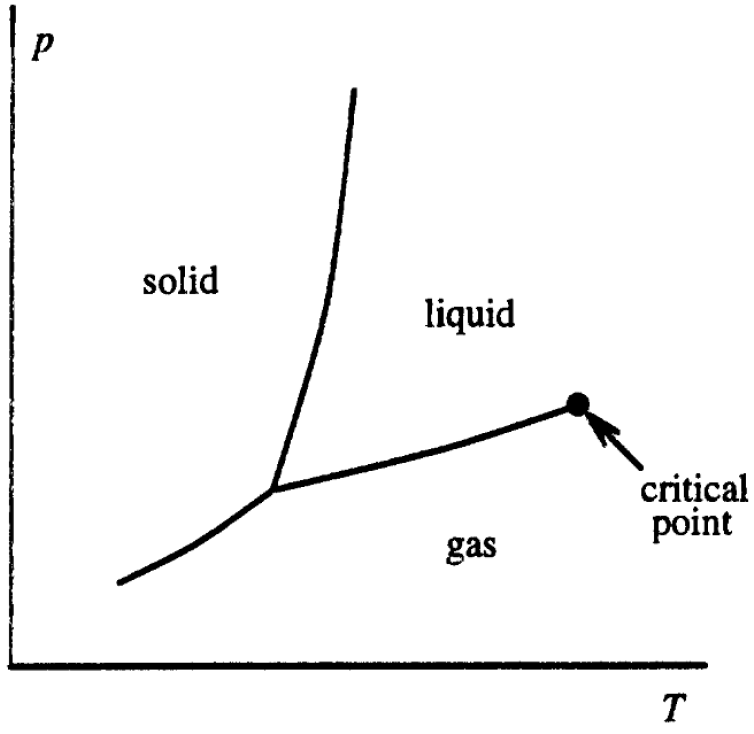


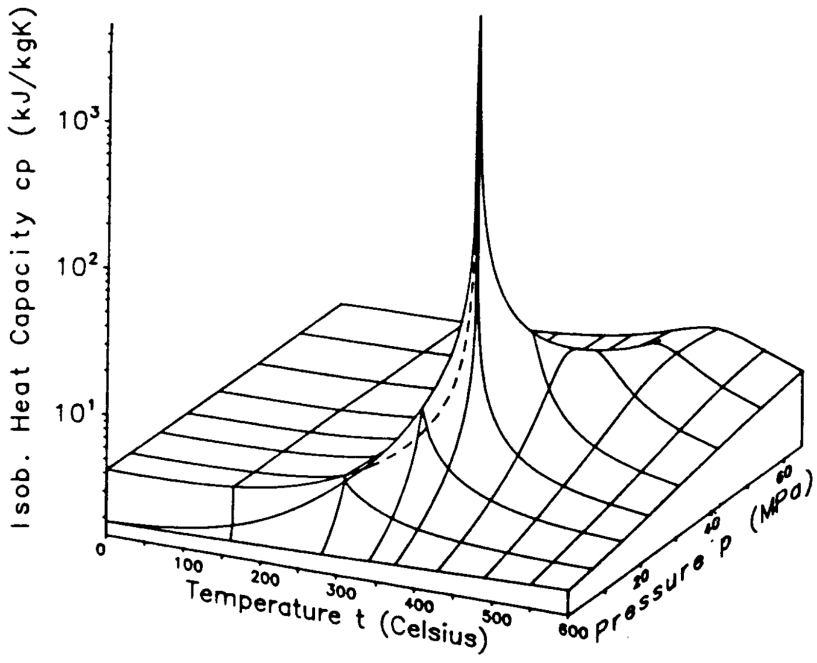
(a) $T < T_c$ (b) $T = T_c$ (c) $T > T_c$

Critical exponents:

- Specific heat $C \propto |T - T_c|^{-\alpha}$
- Susceptibility $\chi \propto |T - T_c|^{-\gamma}$

Critical Point of Water





Critical exponents:

- Specific heat $C \propto |T - T_c|^{-\alpha}$
- Compressibility $\chi_T \propto |T - T_c|^{-\gamma}$

Critical Universality

- magnet vs. liquid
- $\{T, H\}$ vs. $\{T, p\}$
- **Same** critical exponents!

$$\begin{aligned} \alpha_{\text{Magnet}} &= \alpha_{\text{Water}} = 0.110\ldots \\ \gamma_{\text{Magnet}} &= \gamma_{\text{Water}} = 1.237\ldots \\ &\vdots \end{aligned}$$

Universality class: 3d Ising model

$$H = -J \sum_{\langle ij \rangle} s_i s_j$$

Critical Universality

- Why?
- What are $\alpha, \gamma, \ldots$?
- How do we calculate them?

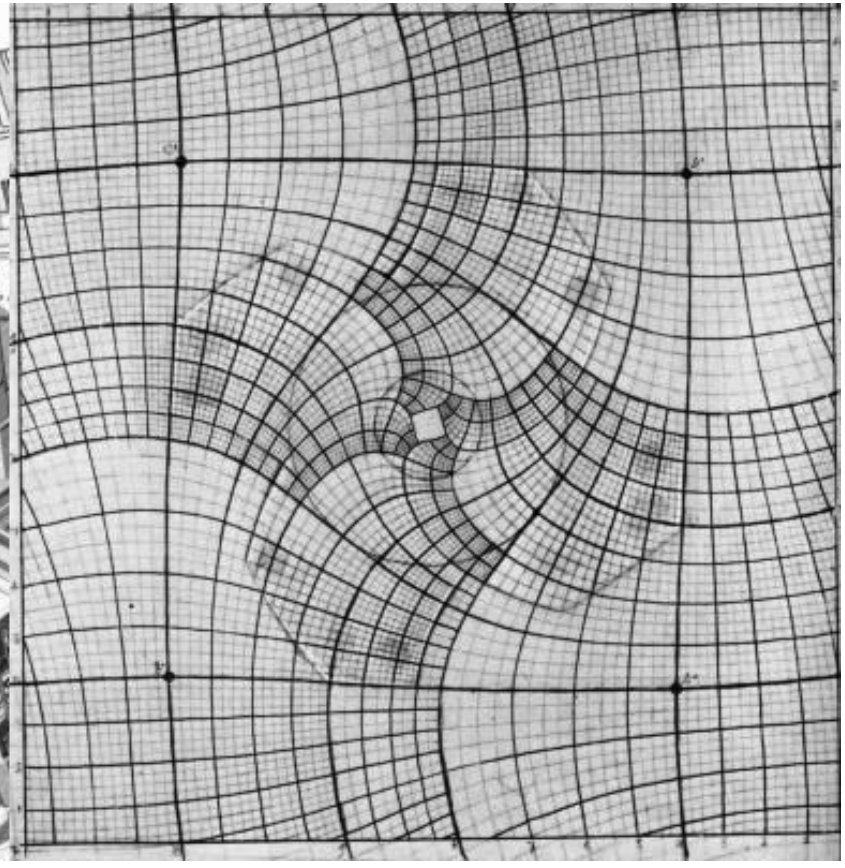
Beautiful story, spanning decades.

Today: a new chapter.

Outline

Scale Invariance

Scale $\Longrightarrow$ Conformal



conformal transformation $=$ rescaling + rotation near each point

Conformal Field Theory (CFT)

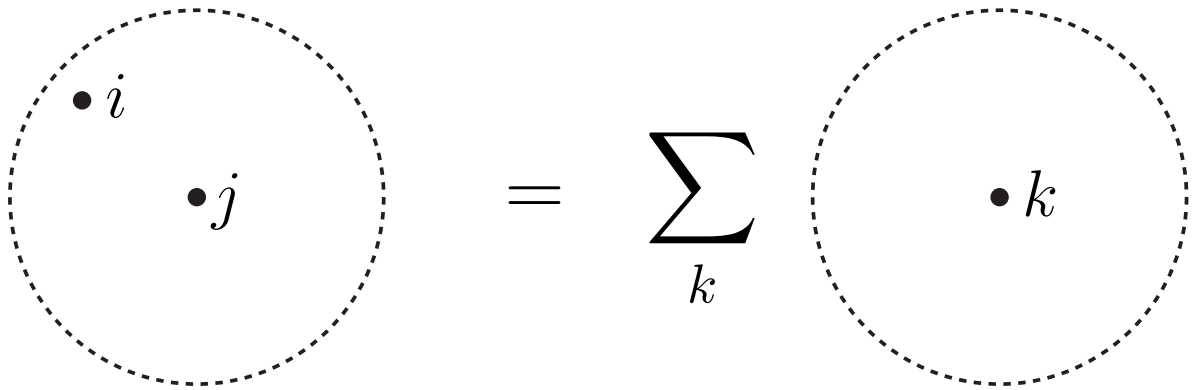
- Local operators $\mathcal{O}_1(x)$, $\mathcal{O}_2(x)$, ...
  - Magnet: magnetization, energy, ...
  - Liquid: density, energy, ...
- Scaling dimensions $\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = |x-y|^{-2\Delta_\mathcal{O}}$

$$\alpha = \frac{3-2\Delta_\epsilon}{3-\Delta_\epsilon}$$
$$\gamma = \frac{3-2\Delta_\sigma}{3-\Delta_\epsilon}$$

where σ, ϵ are operators.

Operator Product Expansion

$$\mathcal{O}_i(x)\mathcal{O}_j(0) \;=\; \sum_k f_{ijk}\, x^{\Delta_k-\Delta_i-\Delta_j}\,(\mathcal{O}_k(0)+\ldots)$$



- "..." determined by conformal symmetry
- Associativity $(\mathcal{O}_1\mathcal{O}_2)\mathcal{O}_3 = \mathcal{O}_1(\mathcal{O}_2\mathcal{O}_3)$

Using the OPE

- Reduce n-pt correlator to $(n-1)$-pt correlator

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\ldots\mathcal{O}_l(x_n)\rangle$$
$$= \sum_k f_{ijk} x_{12}^{\Delta_k-\Delta_i-\Delta_j} \langle \mathcal{O}_k(x_2)\ldots\mathcal{O}_l(x_n)\rangle + \ldots$$

- 1-pt correlators are simple:

$$\begin{aligned} \langle \mathbf{1} \rangle &= 1 \\ \langle \mathcal{O}_i(x) \rangle &= 0 \qquad \text{(otherwise)} \end{aligned}$$

The Conformal Bootstrap

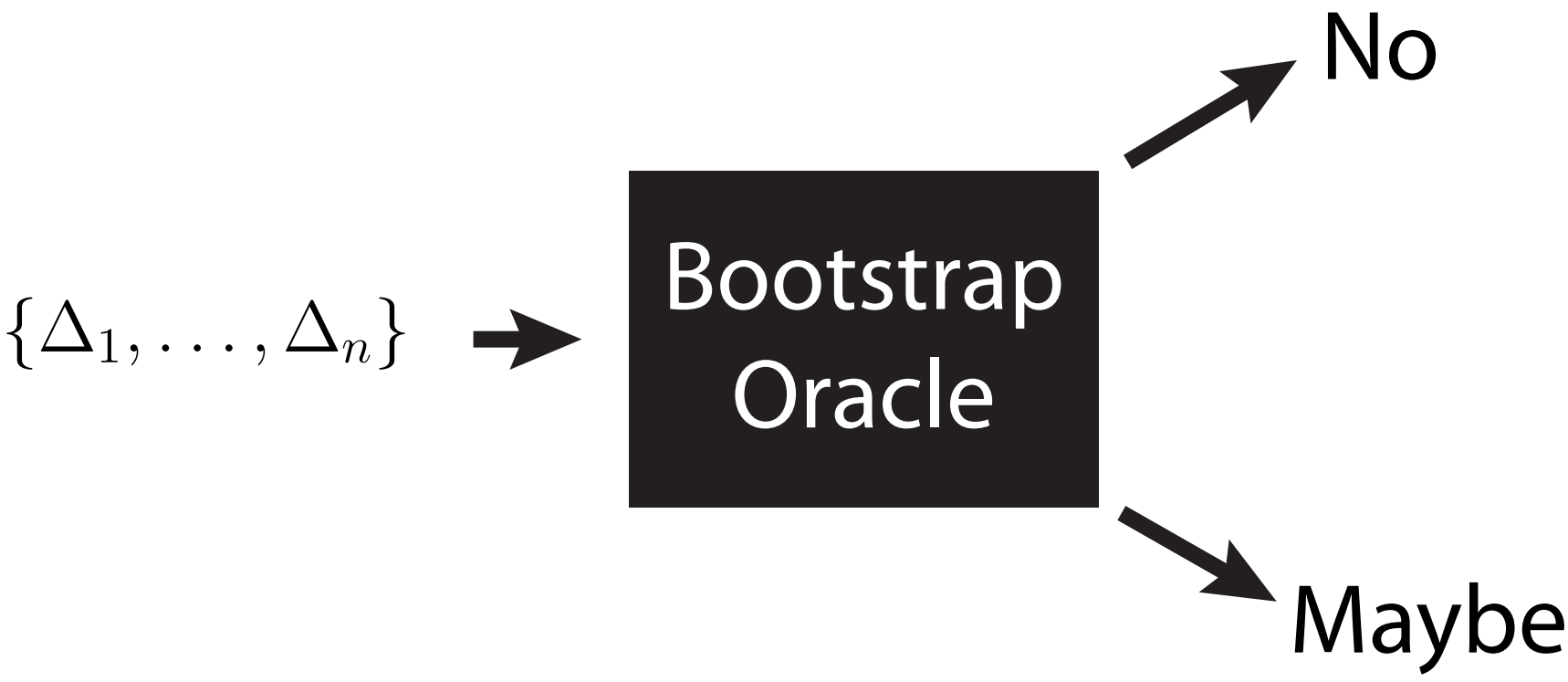
Solve CFTs by classifying OPE algebras [Polyakov '74]

- Progress in $d = 2$ throughout 80's and 90's.
  - ∞-dim conformal group [BPZ '83]
  - 2d Ising CFT exactly solved
  - Partial classification of 2d CFTs
- Huge revival for $d > 2$ a few years ago...

Outline

Bootstrap Revival [Rattazzi, Rychkov, Tonni, Vichi '08]

Is $\{\Delta_1, \ldots, \Delta_n\}$ part of some CFT spectrum?



An Oracle

- Scalar $\phi(x)$
- OPE

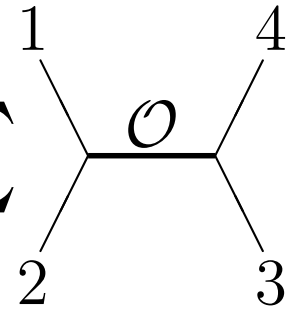
$$\phi(x)\phi(0) \;=\; \sum_{\mathcal{O}} f_{\phi\phi\mathcal{O}}\, x^{\Delta_{\mathcal{O}}-2\Delta_\phi}\,(\mathcal{O}(0)+\ldots)$$

- Unitarity: $f_{\phi\phi\mathcal{O}} \in \mathbb{R}$

Idea: Study constraints of associativity + unitarity on $\langle\phi\phi\phi\phi\rangle$

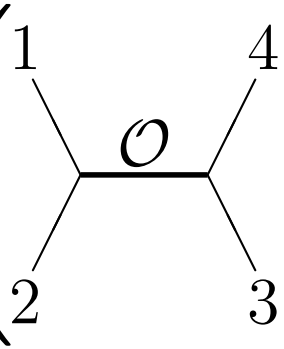
Conformal Blocks & Crossing Symmetry

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \sum_{\mathcal{O}}$$



Crossing Symmetry

$$\sum_{\mathcal{O}} \left(\right. $$



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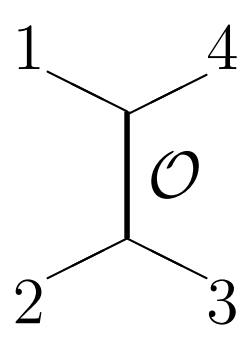


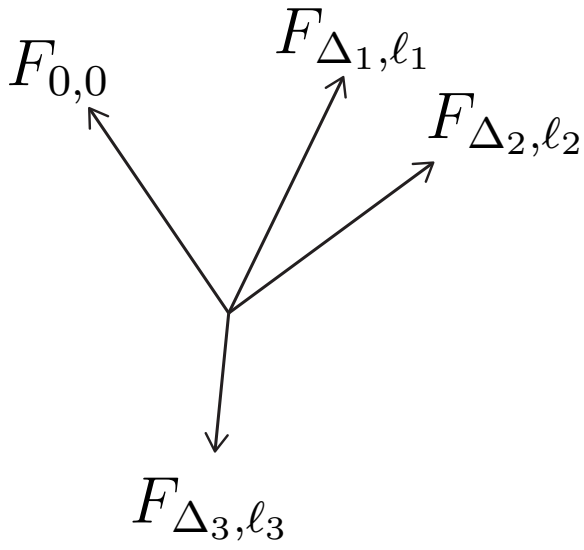
$$\left. \right) = 0$$

$$\sum_{\mathcal{O}} f^2_{\phi\phi\mathcal{O}} F_{\Delta_{\mathcal{O}},\ell_{\mathcal{O}}}(u,v) = 0$$

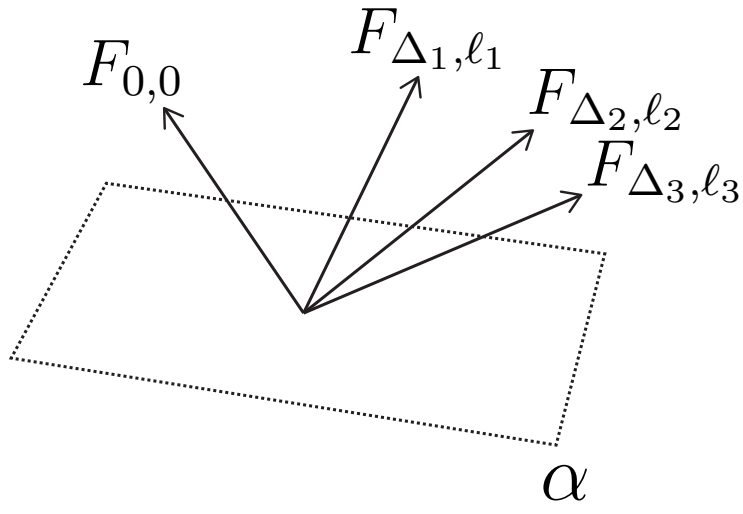
Crossing Symmetry vs. Unitarity

$$\sum_{\mathcal{O}} f^2_{\phi\phi\mathcal{O}} F_{\Delta_{\mathcal{O}},\ell_{\mathcal{O}}}(u,v) \;=\; 0$$

Maybe

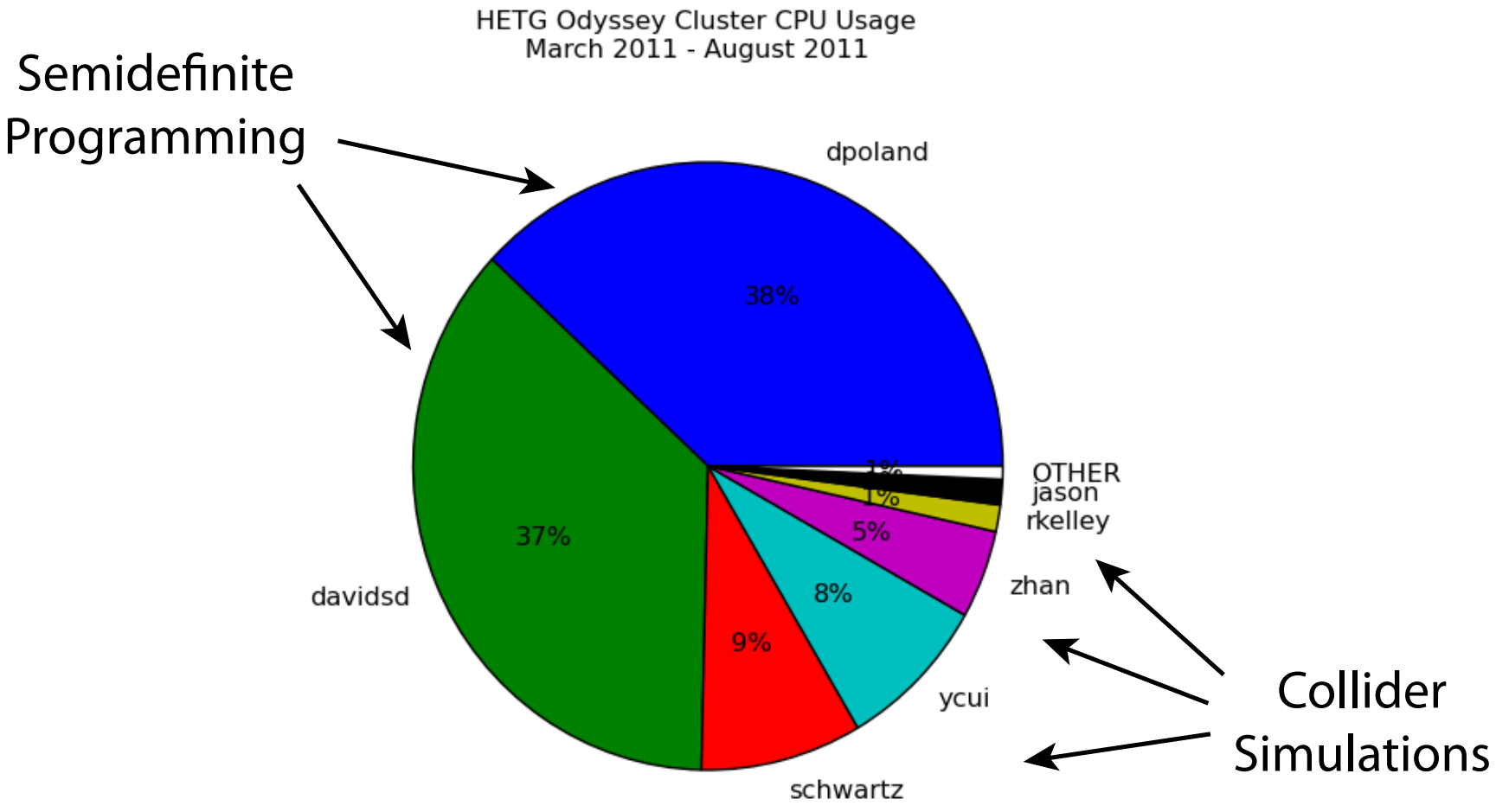


No



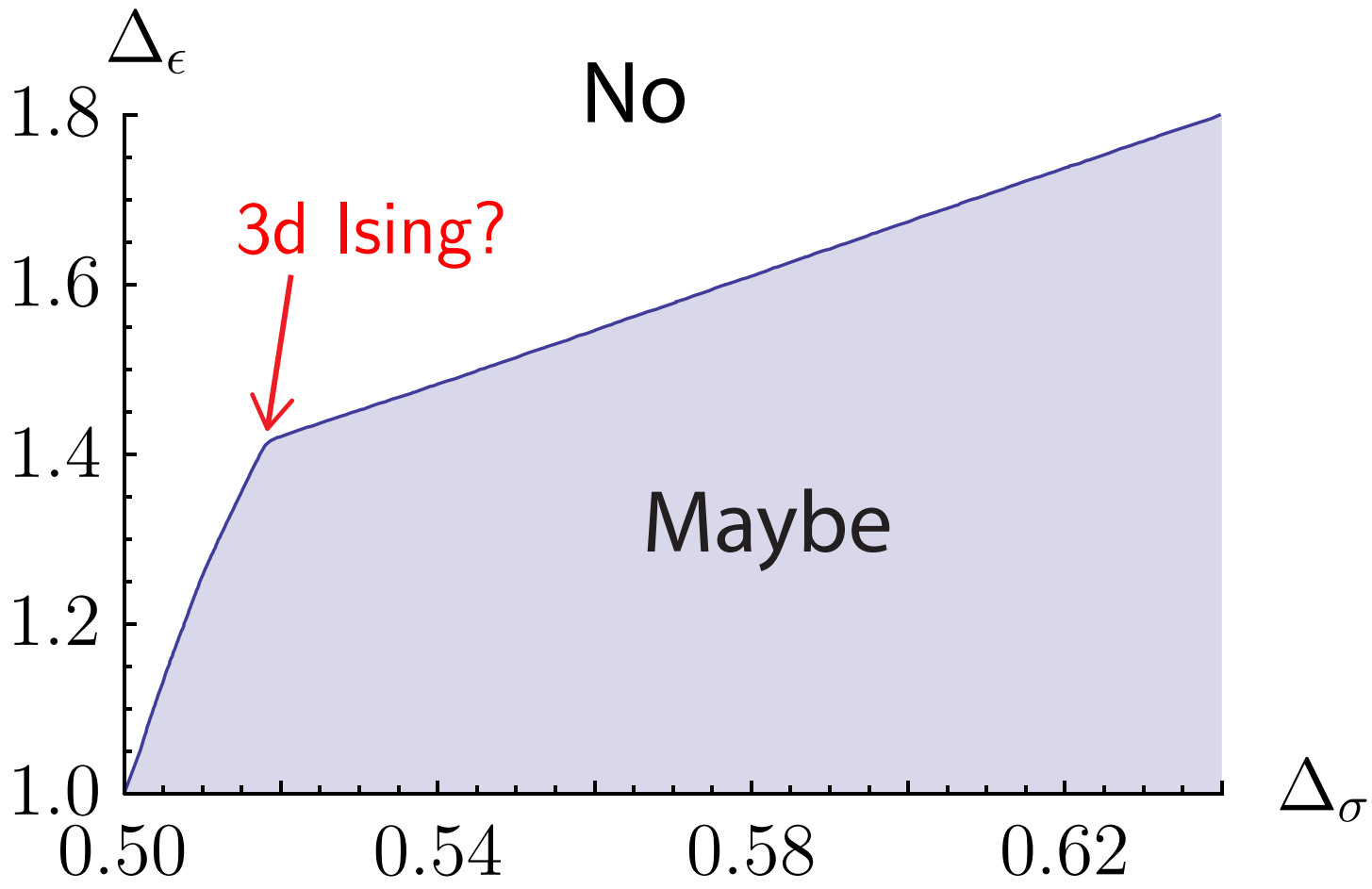
Finding $\alpha \implies$ Linear/Semidefinite Programming

Harvard Linux Cluster Usage, Spring 2011



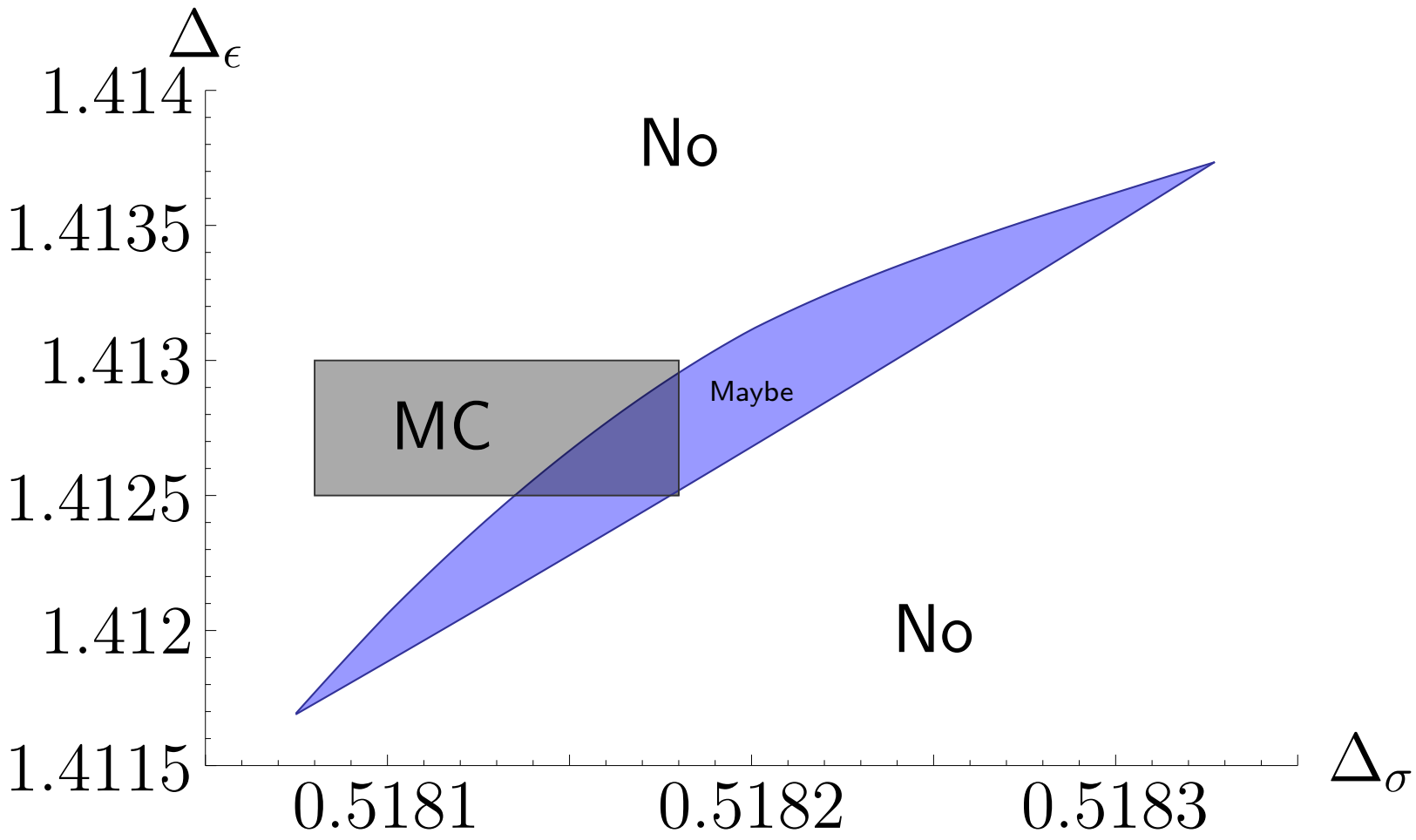
Outline

A Surprise in 3d [El-Showk, Paulos, Poland, Rychkov, DSD, Vichi '12]



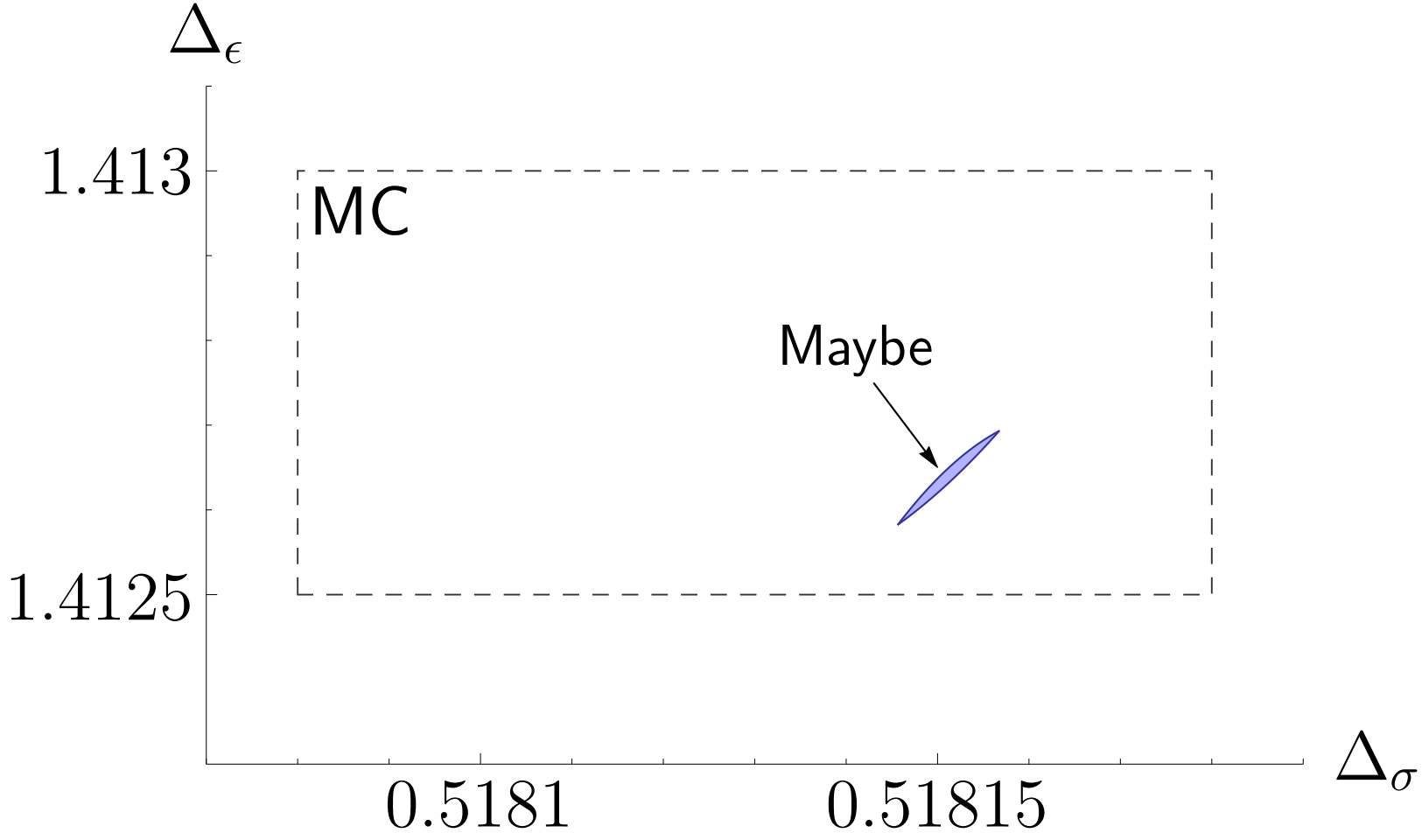
- Unitary 3d CFT with $\epsilon \sim \sigma^2$
- From studying $\langle \sigma\sigma\sigma\sigma \rangle$

Multiple Correlators [Kos, Poland, DSD '14]



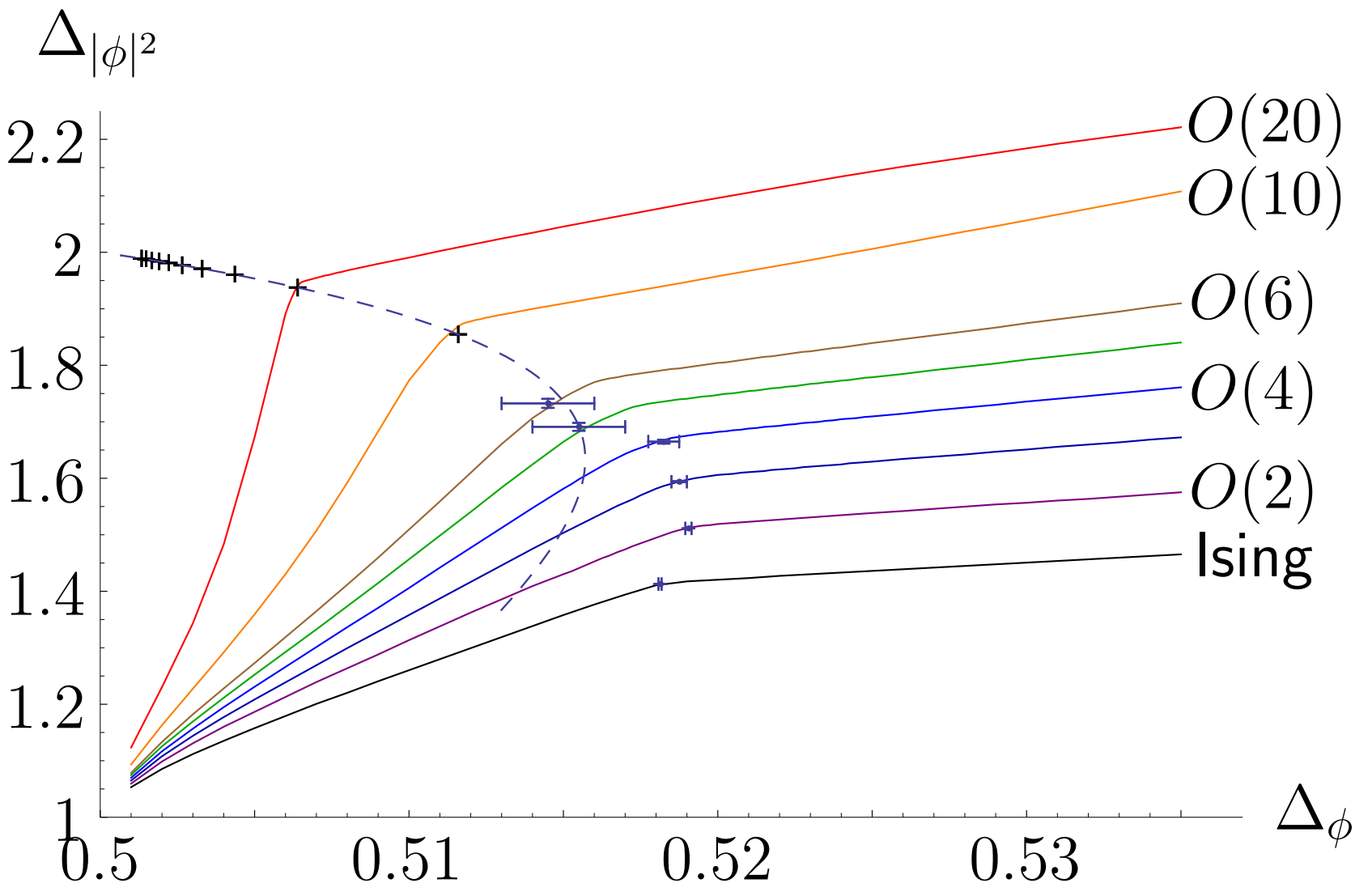
- From studying $\langle\sigma\sigma\sigma\sigma\rangle, \langle\sigma\sigma\epsilon\epsilon\rangle, \langle\epsilon\epsilon\epsilon\epsilon\rangle$
- Assuming σ, ϵ are only relevant scalars

New Oracle [DSD to appear]

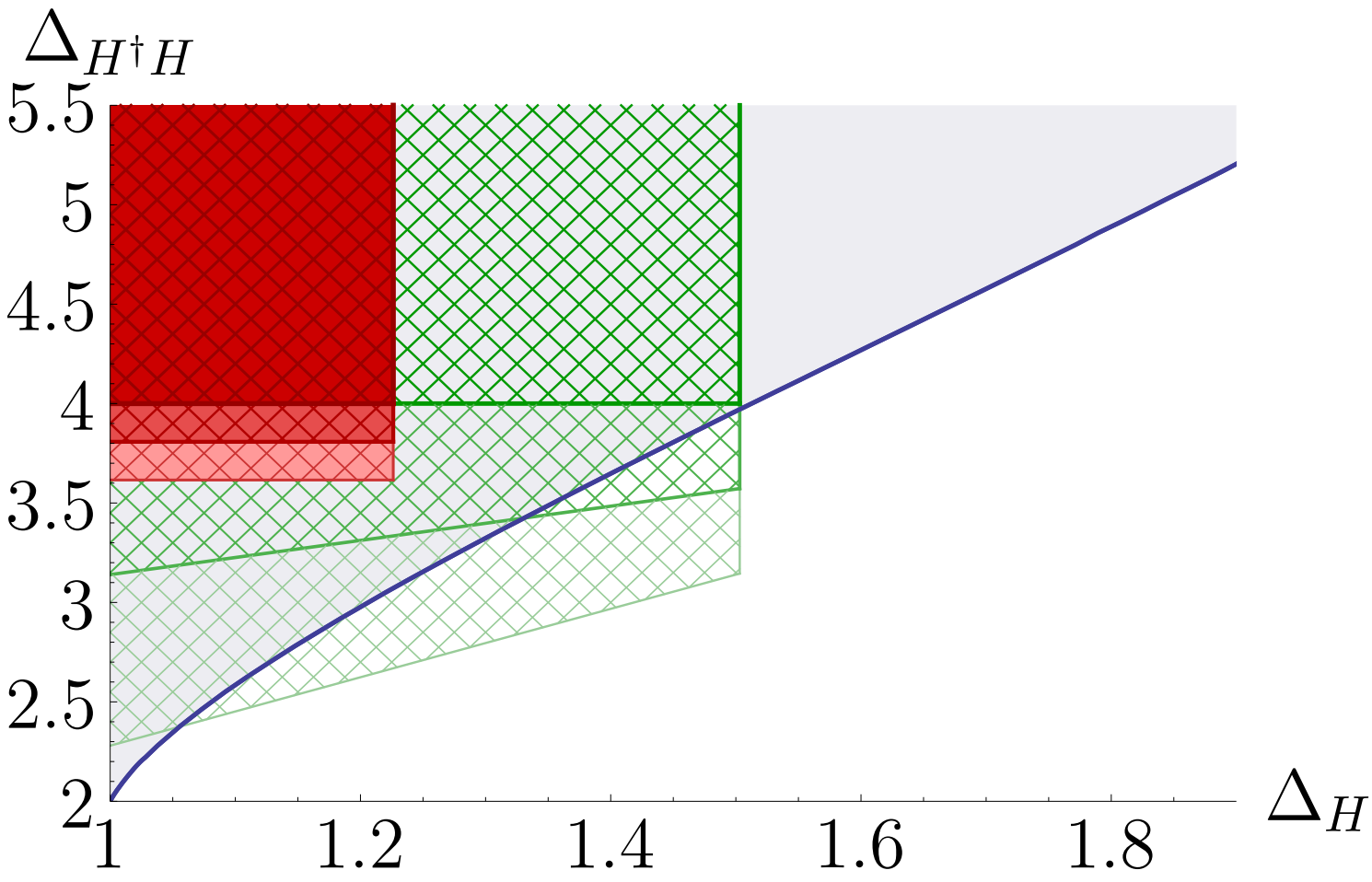


- Same setup as before
- $\Delta_\sigma = 0.518151(5)$, $\Delta_\epsilon = 1.41263(5)$.

3d $O(N)$ Vector Models [Kos, Poland, DSD '13]



Viable Regions for Conformal Technicolor



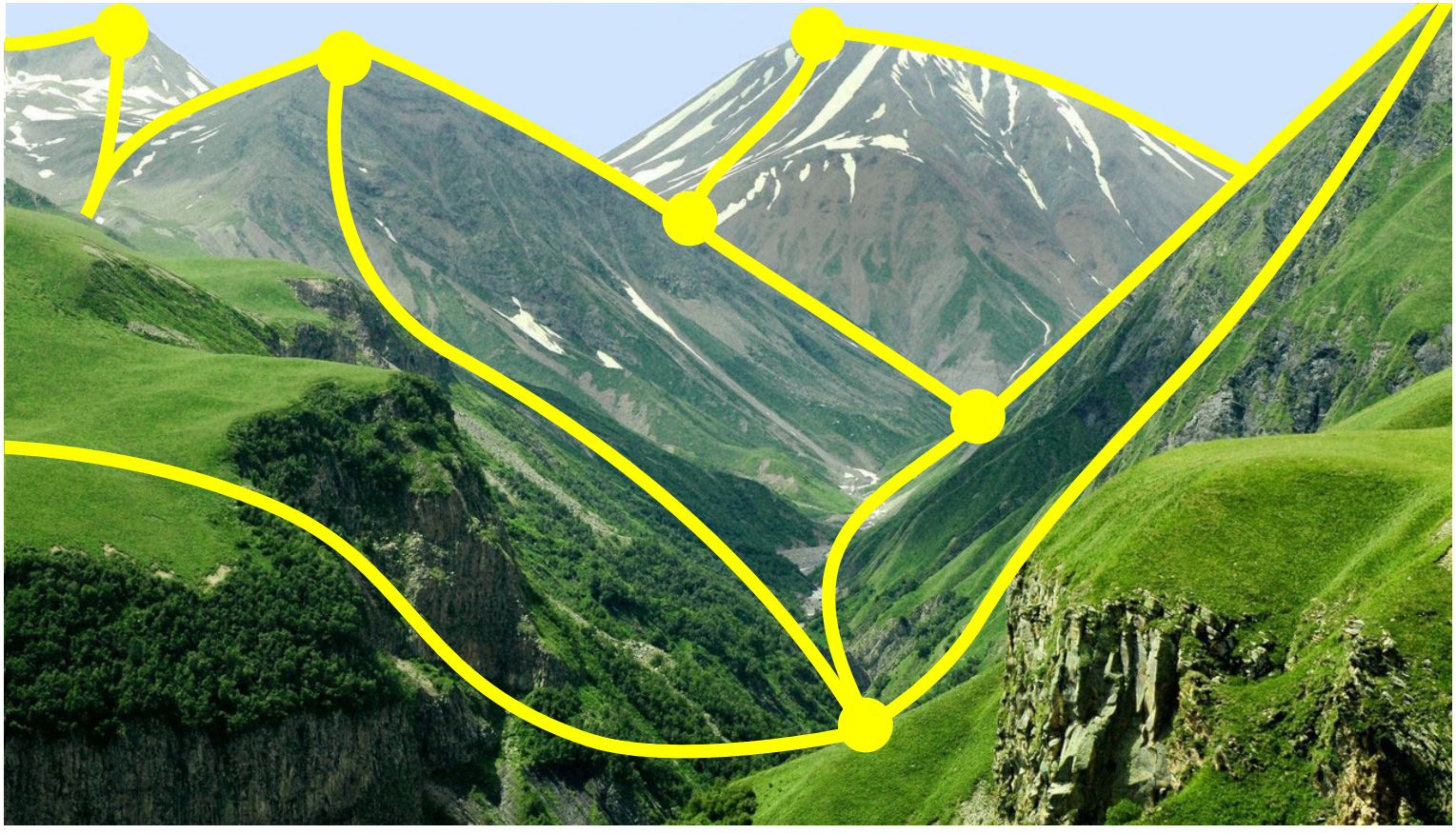
- Flavor generic
- Flavor optimistic

Outline

CFTs are Ubiquitous

- They describe 2nd order phase transitions in condensed matter systems
- They may appear in Beyond the Standard Model physics
- They encode theories of quantum gravity via AdS/CFT

Landmarks in the Space of Physical Theories



Future Directions

- Improve precision/compute more quantities in 3d Ising,
- Isolate other 3d theories ($O(N)$ models, SUSY Ising, ...),
- Conserved currents J_μ and stress tensors $T_{\mu\nu}$,
- Other dimensions, different amounts of SUSY,
- Analytic results, new consistency conditions,
- Classify 3d critical points with small number of relevant operators,
- Study strongly-coupled 4d gauge theories (perhaps find the conformal window of QCD)...