Collider Physics

in The LHC Era And Beyond

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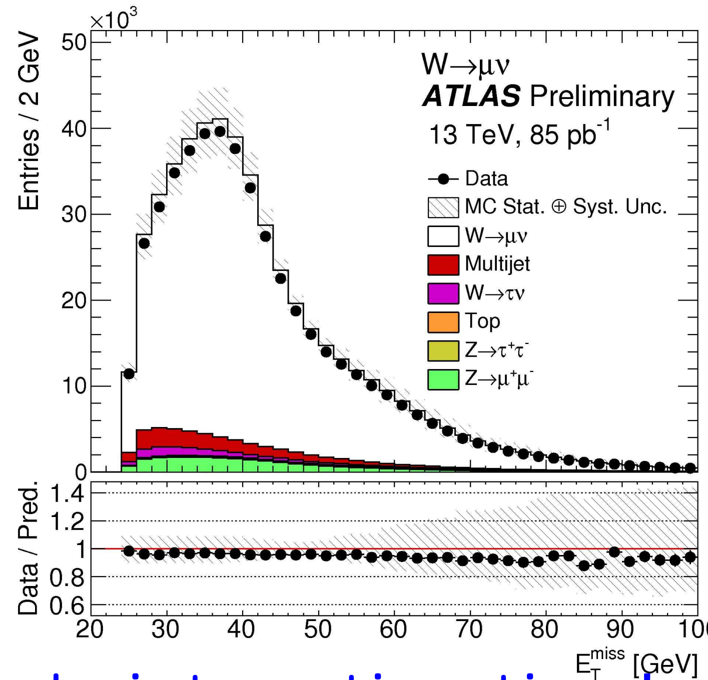
Contents:

Prelude: LHC Run-II is in mission!

June 3, 2015: Run-II started at
$E_{cm} = 6.5 \oplus 6.5 = 13$ TeV.
New era in science has begun!



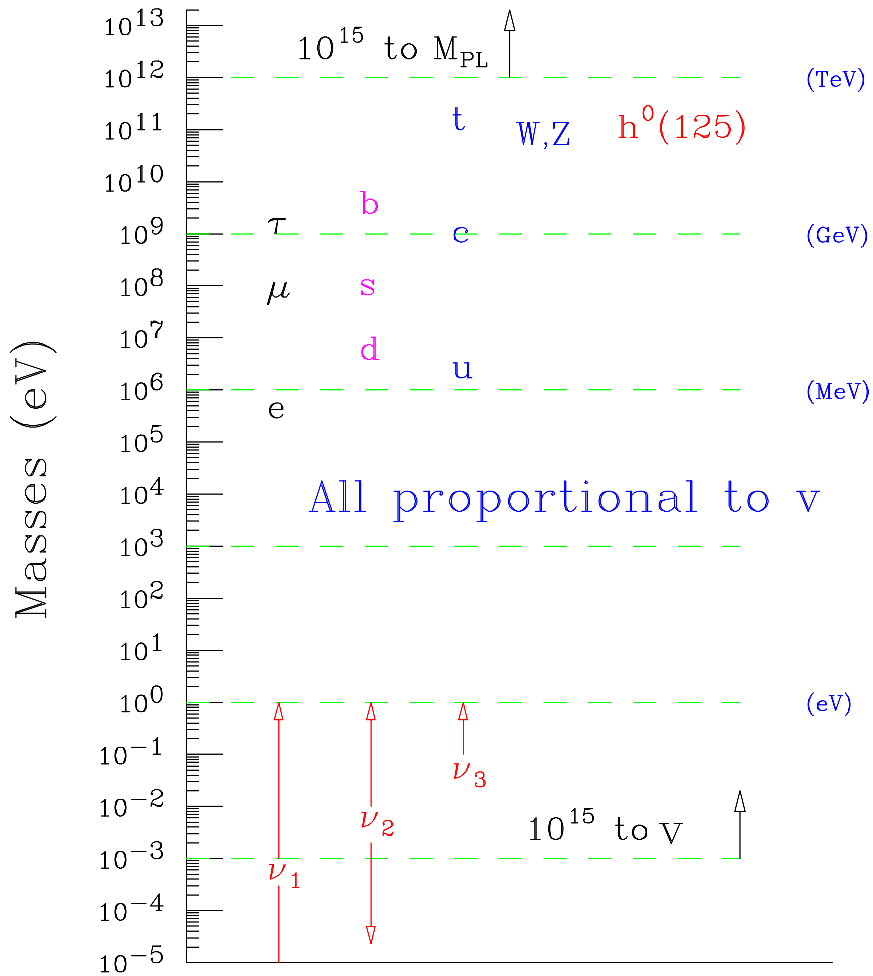
High Energy Physics IS at an extremely interesting time!

The completion of the Standard Model: With the discovery of the Higgs boson, for the first time ever, we have a consistent relativistic quantum-mechanical theory, weakly coupled, unitary, renormalizable, vacuum (quasi?) stable, valid up to an exponentially high scale!

Question: Where IS the next scale?

$\mathcal{O}(1$ TeV)? M_{GUT}? M_{Planck}?

Large spread of masses for elementary particles:

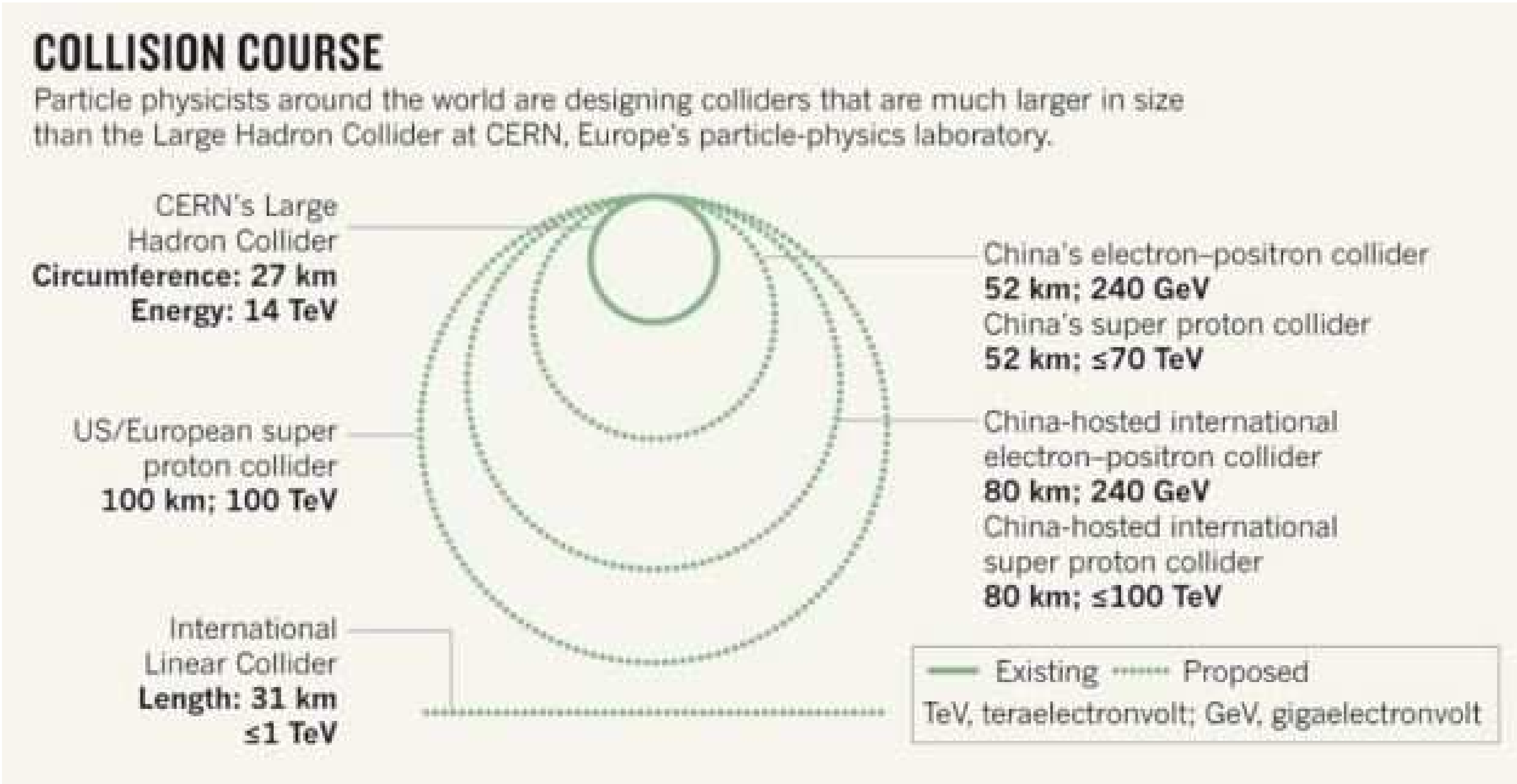


Large hierarchy: Electroweak scale $\Leftrightarrow M_{Planck}$? Conceptual.

Little hierarchy: Electroweak scale $\Leftrightarrow$ Next scale at TeV? Observational.

Consult with the other excellent lectures.

That motivates us to the new energy frontier! *



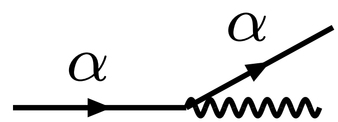
- LHC (300 fb^{-1}), HL-LHC (3 ab^{-1}) lead to way: 2015–2030
- ILC as a Higgs factory (250 GeV) and beyond: 2020–2030 (250/500/1000 GeV, 250/500/1000 fb^{-1}).
- FCC_{ee} (4×2.5 ab^{-1})/CEPC as a Higgs factory: 2028–2035
- FCC_{hh}/SPPC/VLHC (100 TeV, 3 ab^{-1}) to the energy frontier: 2040–

*Nature News (July, 2014)

I-A. Colliders and Detectors

(0). A Historical Count:

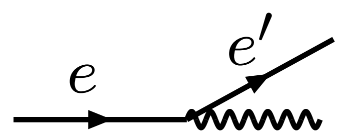
Rutherford's experiments were the first

to study matter structure: 

Gold foil target

discover the point-like nucleus:

$$\frac{d\sigma}{d\Omega} = \frac{(\alpha Z_1 Z_2)^2}{4E^2 \sin^4 \theta/2}$$

SLAC-MIT DIS experiments 

Proton target

discover the point-like structure of the proton:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \left(\frac{F_1(x,Q^2)}{m_p} \sin^2 \frac{\theta}{2} + \frac{F_2(x,Q^2)}{E-E'} \cos^2 \frac{\theta}{2} \right)$$

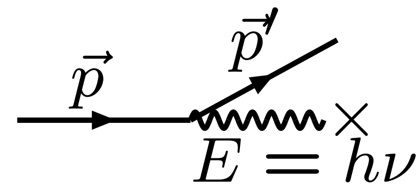
QCD parton model $\Rightarrow 2xF_1(x,Q^2) = F_2(x,Q^2) = \sum_i x f_i(x) e_i^2.$

Rutherford's legendary method continues to date!

(A). High-energy Colliders:

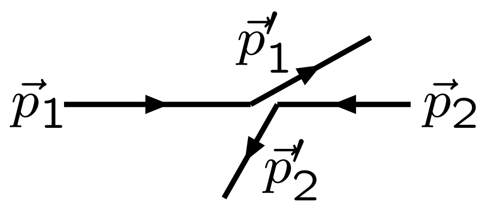
To study the deepest layers of matter,

we need the probes with highest energies.



Two parameters of importance:

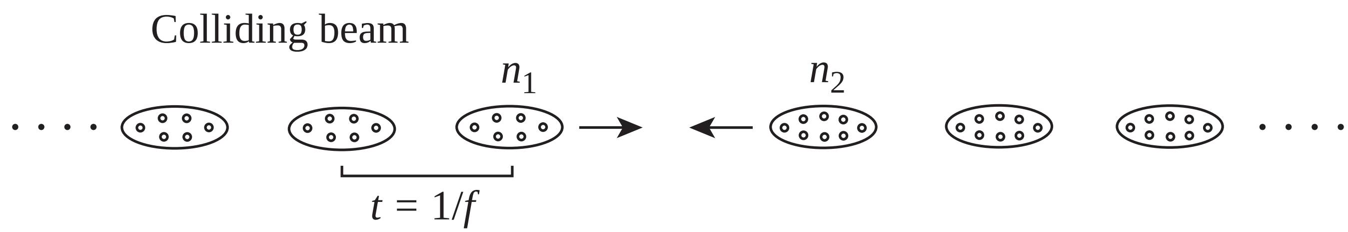
1. The energy:



$$s \equiv (p_1 + p_2)^2 = \begin{cases} (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2, \\ m_1^2 + m_2^2 + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2). \end{cases}$$

$$E_{cm} \equiv \sqrt{s} \approx \begin{cases} 2E_1 \approx 2E_2 & \text{in the c.m. frame } \vec{p}_1 + \vec{p}_2 = 0, \\ \sqrt{2E_1 m_2} & \text{in the fixed target frame } \vec{p}_2 = 0. \end{cases}$$

2. The luminosity:



$$\mathcal{L} \propto f n_1 n_2 / a,$$

(a some beam transverse profile) in units of #particles/cm^2/s

$\Rightarrow$ 10^{33} cm^{-2}s^{-1} = 1 nb^{-1} s^{-1} $\approx$ 10 fb^{-1}/year.

Current and future high-energy colliders:

Hadron Colliders	$\sqrt{s}$ (TeV)	$\mathcal{L}$ (cm^{-2}s^{-1})	$\delta E/E$	f (MHz)	#/bunch (10^{10})	L (km)
LHC Run (I) II	(7,8) 13	(10^{32}) 10^{33}	0.01%	40	10.5	26.66
HL-LHC	14	7×10^{34}	0.013%	40	22	26.66
FCC$_{hh}$ (SppC)	100	1.2×10^{35}	0.01%	40	10	100

e^+e^- Colliders	$\sqrt{s}$ (TeV)	$\mathcal{L}$ (cm^{-2}s^{-1})	$\delta E/E$	f (MHz)	polar.	L (km)
ILC	0.5–1	2.5×10^{34}	0.1%	3	80, 60%	14 – 33
FCC$_{ee}$/CEPC	0.25–0.35	$4 \cdot 10^{35}/2 \cdot 10^{34}$	0.13%			50-100
CLIC	3–5	$\sim 10^{35}$	0.35%	1500	80, 60%	33 – 53

(B). e^+e^- Colliders

The collisions between e^- and e^+ have major advantages:

- The system of an electron and a positron has zero charge, zero lepton number etc.,

$\Longrightarrow$ it is suitable to create new particles after e^+e^- annihilation.

- With symmetric beams between the electrons and positrons, the laboratory frame is the same as the c.m. frame,

$\Longrightarrow$ the total c.m. energy is fully exploited to reach the highest possible physics threshold.

- With well-understood beam properties,

$\Longrightarrow$ the scattering kinematics is well-constrained.

- Backgrounds low and well-undercontrol:

For $\sigma \approx 10$ pb $\Rightarrow 0.1$ Hz at 10^{34} cm^{-2}s^{-1}.

- Linear Collider: possible to achieve high degrees of beam polarizations,

$\Longrightarrow$ chiral couplings and other asymmetries can be effectively explored.

Disadvantages

- Large synchrotron radiation due to acceleration,

$$\Delta E \sim \frac{1}{R} \left(\frac{E}{m_e}\right)^4 .$$

Thus, a multi-hundred GeV e^+e^- collider will have to be made a linear accelerator.

- This becomes a major challenge for achieving a high luminosity when a storage ring is not utilized; beamsstrahlung severe.

CEPC/FCC$_{ee}$ Higgs Factory

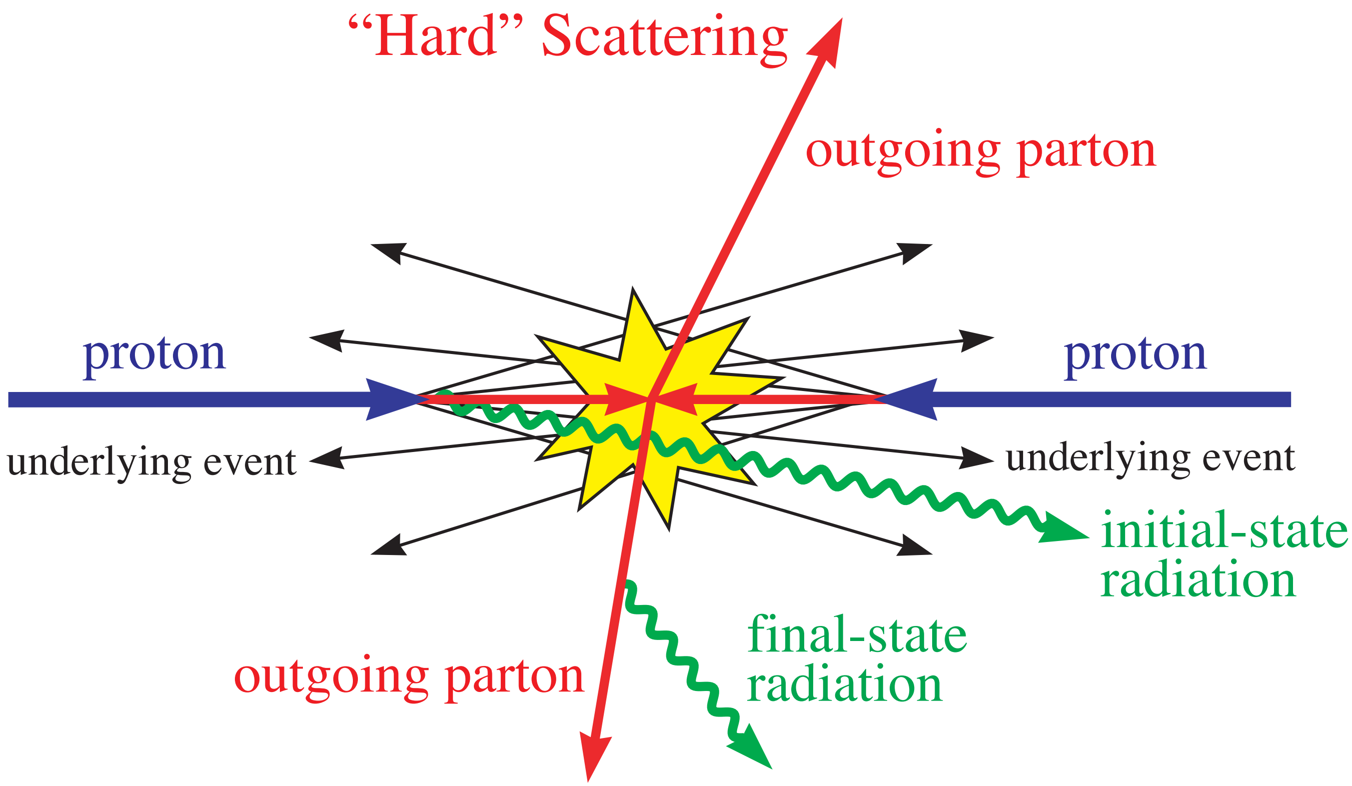
It has been discussed to build a **circular** e^+e^- collider

$$50 - 100 \text{ km}, \quad E_{cm} = 245 \text{ GeV}–350 \text{ GeV}$$

with multiple interaction points for very high luminosities.

(C). Hadron Colliders

LHC: the new high-energy frontier



<u>Advantages</u>

• Higher c.m. energy, thus higher energy threshold:

$\sqrt{S} = 14$ TeV: $\quad M_{new}^2 \sim s = x_1 x_2 S \quad \Rightarrow \quad M_{new} \sim 0.3\sqrt{S} \sim 4$ TeV.

- Higher luminosity: 10^{34}/cm^2/s $\Rightarrow$ 100 fb^{-1}/yr.
  Annual yield: 1B $W^\pm$; 100M $t\bar{t}$; 10M W^+W^-; 1M H^0...
- Multiple (strong, electroweak) channels:
  $q\bar{q}'$, gg, qg, $b\bar{b} \to$ colored; $Q = 0, \pm 1$; $J = 0, 1, 2$ states;
  WW, WZ, ZZ, $\gamma\gamma \to I_W = 0, 1, 2$; $Q = 0, \pm 1, \pm 2$; $J = 0, 1, 2$ states.

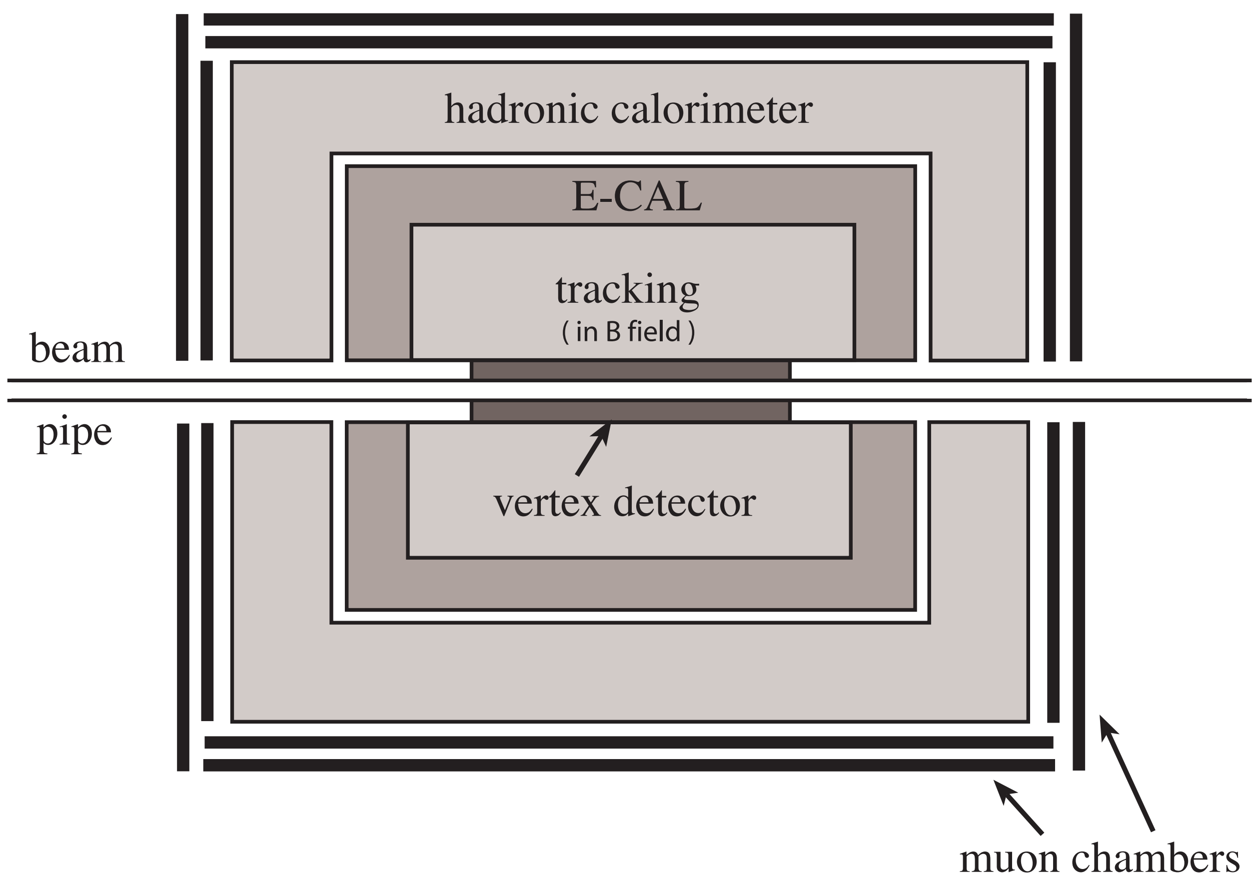
<u>Disadvantages</u>

- Initial state unknown:
  colliding partons unknown on event-by-event basis;
  parton c.m. energy unknown: $E^2_{cm} \equiv s = x_1 x_2 S$;
  parton c.m. frame unknown.
  $\Rightarrow$ largely rely on final state reconstruction.
- The large rate turns to a hostile environment:
  $\Rightarrow$ Severe backgrounds!

Our primary job !

(D). Particle Detection:

The detector complex:
Utilize the strong and electromagnetic interactions between detector materials and produced particles.



What we "see" as particles in the detector: (a few meters)

For a relativistic particle, the travel distance:

$$d = (\beta c\ \tau)\gamma \approx (300\ \mu m)(\frac{\tau}{10^{-12}\ s})\ \gamma$$

- stable particles directly "seen":

$$p,\ \bar{p},\ e^{\pm},\ \gamma$$

- quasi-stable particles of a life-time $\tau \geq 10^{-10}$ s also directly "seen":

$$n, \Lambda, K_L^0, ...,\ \mu^{\pm},\ \pi^{\pm}, K^{\pm}...$$

- a life-time $\tau \sim 10^{-12}$ s may display a secondary decay vertex, "vertex-tagged particles":

$$B^{0,\pm},\ D^{0,\pm},\ \tau^{\pm}...$$

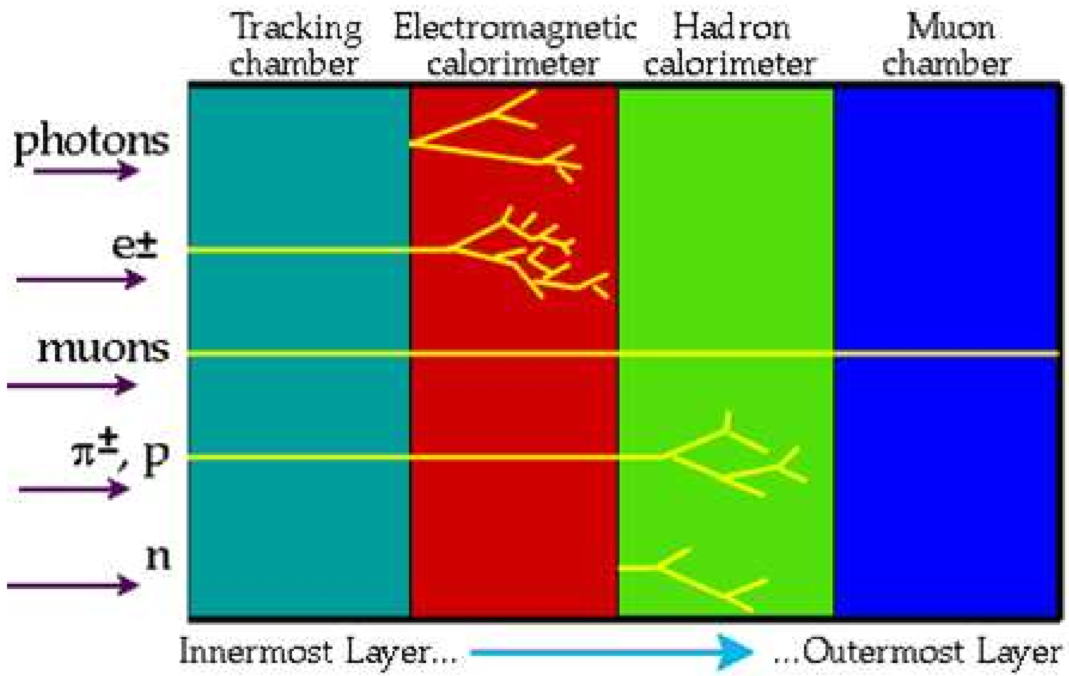
- short-lived not "directly seen", but "reconstructable":

$$\pi^0,\ \rho^{0,\pm}...\ ,\quad Z, W^{\pm}, t, H...$$

- missing particles are weakly-interacting and neutral:

$$\nu,\ \tilde{\chi}^0, G_{KK}...$$

† For stable and quasi-stable particles of a life-time $\tau \geq 10^{-10} - 10^{-12}$ s, they show up as

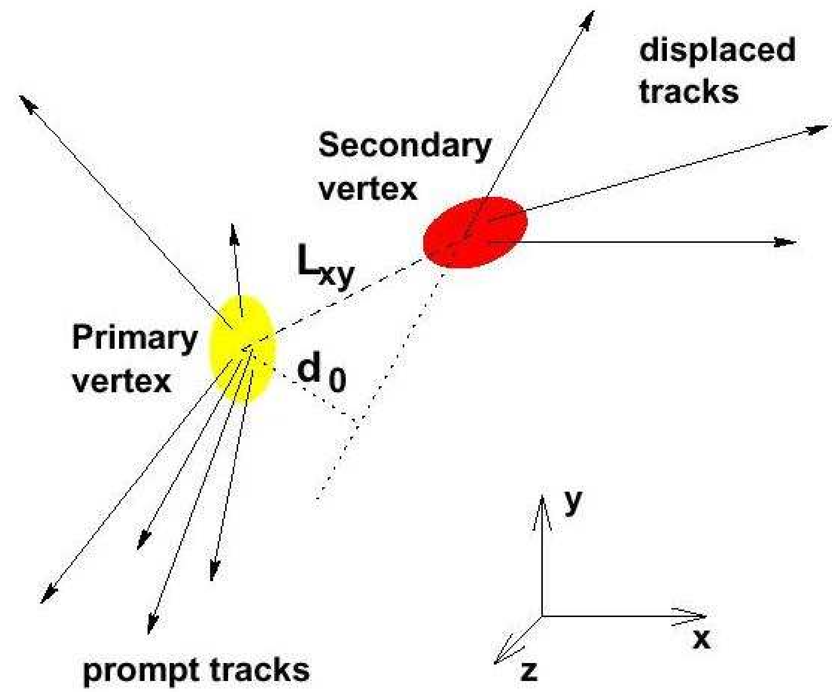


Theorists should know:

$$\begin{aligned} \text{For charged tracks:} \quad \Delta p/p &\propto p, \\ \text{typical resolution:} \quad &\sim p/(10^4\ \text{GeV}). \\ \text{For calorimetry:} \quad \Delta E/E &\propto \frac{1}{\sqrt{E}}, \\ \text{typical resolution:} \quad &\sim (10\%_{ecal},\ 50\%_{hcal})/\sqrt{E/\text{GeV}} \end{aligned}$$

† For vertex-tagged particles $\tau \approx 10^{-12}$ s,

heavy flavor tagging: the secondary vertex:



Typical resolution: $d_0 \sim 30 - 50\ \mu$m or so

⇒ Better have two (non-collinear) charged tracks for a secondary vertex;

Or use the "impact parameter" w.r.t. the primary vertex.

For theorists: just multiply a "tagging efficiency":

$$\epsilon_b \sim 70\%; \quad \epsilon_c \sim 40\%; \quad \epsilon_\tau \sim 40\%.$$

† For short-lived particles $(Z, W^{\pm}, t, H...)$: $\tau < 10^{-12}$ s or so,
make use of final state kinematics to reconstruct the resonance.

† For missing particles:
make use of energy-momentum conservation to deduce their existence.

$$p_1^i + p_2^i = \sum_f^{obs.} p_f \; + \; p_{miss}.$$

But in hadron collisions, the longitudinal momenta unknown,
thus transverse direction only:

$$0 = \sum_f^{obs.} \vec{p}_{f\ T} \; + \; \vec{p}_{miss\ T}.$$

often called "missing p_T" ($\not{p}_T$) or (conventionally) "missing E_T" ($\not{E}_T$).

Note: "missing E_T" (MET) is *conceptually* ill-defined!
It is only sensible for massless particles: $\not{E}_T = \sqrt{\vec{p}^{\,2}_{miss\ T} + m^2}$.

What we "see" for the SM particles (no universality!)

Leptons	Vetexing	Tracking	ECAL	HCAL	Muon Cham.
$e^\pm$	$\times$	$\vec{p}$	E	$\times$	$\times$
$\mu^\pm$	$\times$	$\vec{p}$	$\surd$	$\surd$	$\vec{p}$
$\tau^\pm$	$\surd\times$	$\surd$	$e^\pm$	$h^\pm$; $3h^\pm$	$\mu^\pm$
ν_e, ν_μ, ν_τ	$\times$	$\times$	$\times$	$\times$	$\times$
Quarks					
u, d, s	$\times$	$\surd$	$\surd$	$\surd$	$\times$
$c \to D$	$\surd$	$\surd$	$e^\pm$	h's	$\mu^\pm$
$b \to B$	$\surd$	$\surd$	$e^\pm$	h's	$\mu^\pm$
$t \to bW^\pm$	b	$\surd$	$e^\pm$	$b+$ 2 jets	$\mu^\pm$
Gauge bosons					
γ	$\times$	$\times$	E	$\times$	$\times$
g	$\times$	$\surd$	$\surd$	$\surd$	$\times$
$W^\pm \to \ell^\pm\nu$	$\times$	$\vec{p}$	$e^\pm$	$\times$	$\mu^\pm$
$\to q\bar{q}'$	$\times$	$\surd$	$\surd$	2 jets	$\times$
$Z^0 \to \ell^+\ell^-$	$\times$	$\vec{p}$	$e^\pm$	$\times$	$\mu^\pm$
$\to q\bar{q}$	$(b\bar{b})$	$\surd$	$\surd$	2 jets	$\times$
the Higgs boson					
$h^0 \to b\bar{b}$	$\surd$	$\surd$	$e^\pm$	h's	$\mu^\pm$
$\to ZZ^*$	$\times$	$\vec{p}$	$e^\pm$	$\surd$	$\mu^\pm$
$\to WW^*$	$\times$	$\vec{p}$	$e^\pm$	$\surd$	$\mu^\pm$

Homework:

Exercise 1.1: For a π^0, μ^-, or a τ^- respectively, calculate its decay length for $E = 10$ GeV.

Exercise 1.2: An event was identified to have a $\mu^+\mu^-$ pair, along with some missing energy. What can you say about the kinematics of the system of the missing particles? Consider both an e^+e^- and a hadron collider.

Exercise 1.3: Electron and muon measurements: Estimate the relative errors of energy-momentum measurements for an electron by an electromagnetic calorimetry ($\Delta E/E$) and for a muon by tracking ($\Delta p/p$) at energies of $E = 50$ GeV and 500 GeV, respectively.

Exercise 1.4: A 125 GeV Higgs boson will have a production cross section of 20 pb at the 14 TeV LHC. How many events per year do you expect to produce for the Higgs boson with an instantaneous luminosity $10^{33}/\text{cm}^2/\text{s}$? Do you expect it to be easy to observe and why?

I-B. Basic Techniques and Tools for Collider Physics

(A). Scattering cross section

For a $2 \to n$ scattering process:

$$\sigma(ab \to 1+2+...n) = \frac{1}{2s} \overline{\sum} |\mathcal{M}|^2 \, dPS_n,$$

$$dPS_n \equiv (2\pi)^4 \; \delta^4 \left(P - \sum_{i=1}^{n} p_i \right) \Pi_{i=1}^{n} \frac{1}{(2\pi)^3} \frac{d^3\vec{p}_i}{2E_i},$$

$$s = (p_a + p_b)^2 \equiv P^2 = \left(\sum_{i=1}^{n} p_i \right)^2,$$

where $\overline{\sum}|\mathcal{M}|^2$: dynamics (dimension $4-2n$);
dPS_n: kinematics (Lorentz invariant, dimension $2n-4$.)

For a $1 \to n$ decay process, the partial width in the rest frame:

$$\Gamma(a \to 1+2+...n) = \frac{1}{2M_a} \overline{\sum} |\mathcal{M}|^2 \, dPS_n.$$

$$\tau = \Gamma_{tot}^{-1} = (\sum_f \Gamma_f)^{-1}.$$

(B). Phase space and kinematics *

One-particle Final State $a+b\to 1$:

$$
\begin{aligned}
dPS_1 &\equiv (2\pi)\,\frac{d^3\vec{p}_1}{2E_1}\delta^4(P-p_1)\\
&\doteq \pi|\vec{p}_1|d\Omega_1\delta^3(\vec{P}-\vec{p}_1)\\
&\doteq 2\pi\ \delta(s-m_1^2).
\end{aligned}
$$

where the first and second equal signs made use of the identities:

$$
|\vec{p}|d|\vec{p}| = EdE, \qquad \frac{d^3\vec{p}}{2E} = \int d^4p\ \delta(p^2-m^2).
$$

Kinematical relations:

$$
\begin{aligned}
\vec{P} &\equiv \vec{p}_a+\vec{p}_b=\vec{p}_1, \quad E_1^{cm}=\sqrt{s}\ \text{in the c.m. frame},\\
s &= (p_a+p_b)^2 = m_1^2.
\end{aligned}
$$

The "dimensinless phase-space volume" is $s(dPS_1) = 2\pi$.

*E.Byckling, K. Kajantie: Particle Kinemaitcs (1973).

Two-particle Final State $a+b\to 1+2$:

$$
\begin{aligned}
dPS_2 &\equiv \frac{1}{(2\pi)^2}\,\delta^4(P-p_1-p_2)\,\frac{d^3\vec{p}_1}{2E_1}\frac{d^3\vec{p}_2}{2E_2} \\
&\doteq \frac{1}{(4\pi)^2}\,\frac{|\vec{p}_1^{\,cm}|}{\sqrt{s}}\,d\Omega_1 = \frac{1}{(4\pi)^2}\,\frac{|\vec{p}_1^{\,cm}|}{\sqrt{s}}\,d\cos\theta_1 d\phi_1 \\
&= \frac{1}{4\pi}\frac{1}{2}\,\lambda^{1/2}\left(1,\frac{m_1^2}{s},\frac{m_2^2}{s}\right)dx_1dx_2, \\
d\cos\theta_1 &= 2dx_1, \quad d\phi_1 = 2\pi dx_2, \quad 0\le x_{1,2}\le 1,
\end{aligned}
$$

The magnitudes of the energy-momentum of the two particles are fully determined by the four-momentum conservation:

$$
|\vec{p}_1^{\,cm}| = |\vec{p}_2^{\,cm}| = \frac{\lambda^{1/2}(s,m_1^2,m_2^2)}{2\sqrt{s}},\;\; E_1^{cm} = \frac{s+m_1^2-m_2^2}{2\sqrt{s}},\;\; E_2^{cm} = \frac{s+m_2^2-m_1^2}{2\sqrt{s}},
$$

$$
\lambda(x,y,z) = (x-y-z)^2-4yz = x^2+y^2+z^2-2xy-2xz-2yz.
$$

The phase-space volume of the two-body is scaled down with respect to that of the one-particle by a factor

$$
\frac{dPS_2}{s\;dPS_1} \approx \frac{1}{(4\pi)^2}.
$$

just like a “loop factor”.

Exercise 2.1: Assume that $m_a = m_1$ and $m_b = m_2$. Show that

$$
\begin{aligned}
t &= -2p_{cm}^2(1 - \cos\theta^*_{a1}), \\
u &= -2p_{cm}^2(1 + \cos\theta^*_{a1}) + \frac{(m_1^2 - m_2^2)^2}{s},
\end{aligned}
$$

$p_{cm} = \lambda^{1/2}(s, m_1^2, m_2^2)/2\sqrt{s}$ is the momentum magnitude in the c.m. frame.
Note: t is negative-definite; $t \to 0$ in the collinear limit.

Exercise 2.2: A particle of mass M decays to two particles isotropically in its rest frame. What does the momentum distribution look like in a frame in which the particle is moving with a speed β_z? Compare the result with your expectation for the shape change for a basket ball.

Three-particle Final State $a+b\to 1+2+3$:

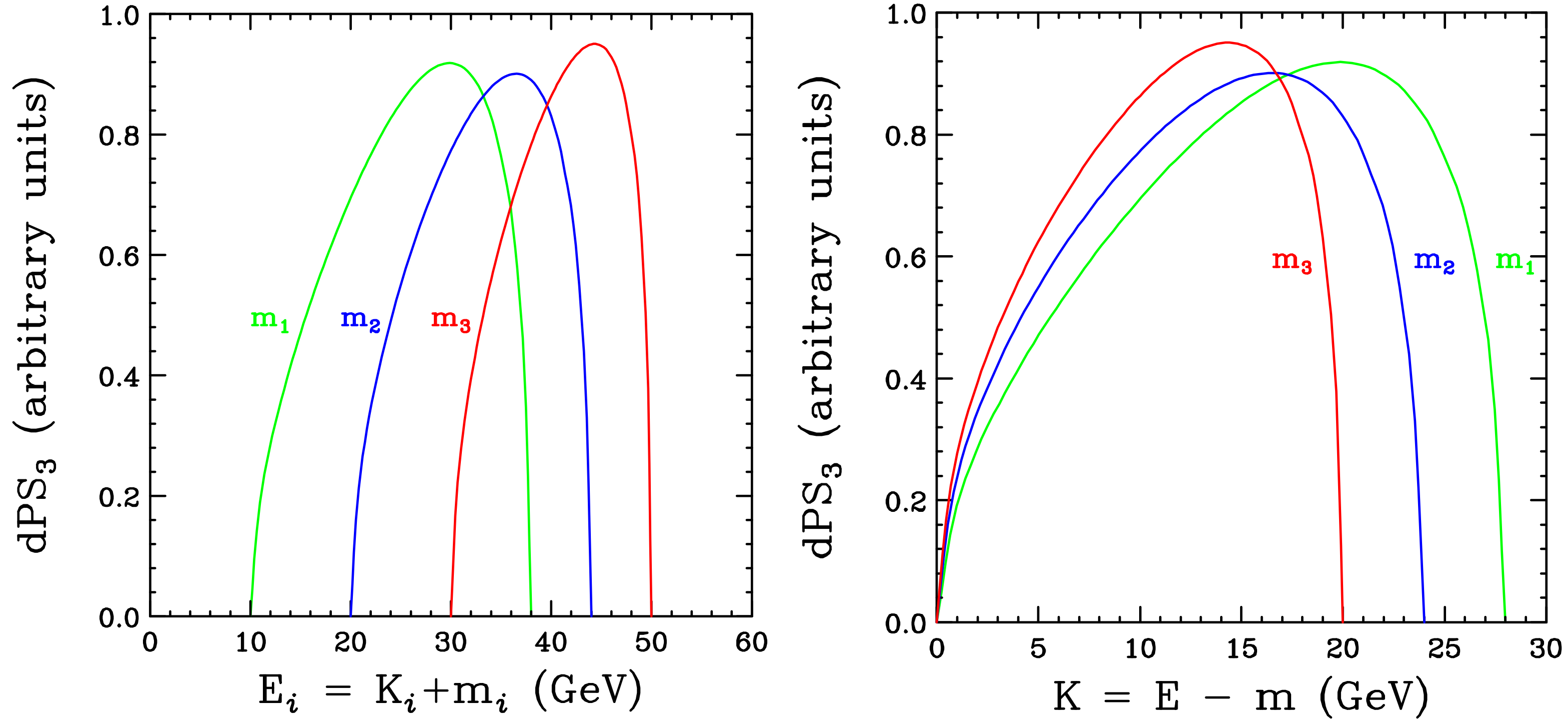
$$\begin{aligned}
dPS_3 &\equiv \frac{1}{(2\pi)^5}\,\delta^4(P-p_1-p_2-p_3)\,\frac{d^3\vec{p}_1}{2E_1}\frac{d^3\vec{p}_2}{2E_2}\frac{d^3\vec{p}_3}{2E_3}\\
&\doteq \frac{|\vec{p}_1|^2\ d|\vec{p}_1|\ d\Omega_1}{(2\pi)^3\ 2E_1}\ \frac{1}{(4\pi)^2}\ \frac{|\vec{p}_2^{(23)}|}{m_{23}}\ d\Omega_2\\
&= \frac{1}{(4\pi)^3}\ \lambda^{1/2}\left(1,\frac{m_2^2}{m_{23}^2},\frac{m_3^2}{m_{23}^2}\right)\ 2|\vec{p}_1|\ dE_1\ dx_2dx_3dx_4dx_5.
\end{aligned}$$

$$\begin{aligned}
d\cos\theta_{1,2} &= 2dx_{2,4},\quad d\phi_{1,2}=2\pi dx_{3,5},\quad 0\le x_{2,3,4,5}\le 1,\\
&|\vec{p}_1^{cm}|^2 = |\vec{p}_2^{cm}+\vec{p}_3^{cm}|^2=(E_1^{cm})^2-m_1^2,\\
&m_{23}^2 = s-2\sqrt{s}E_1^{cm}+m_1^2,\quad |\vec{p}_2^{23}|=|\vec{p}_3^{23}|=\frac{\lambda^{1/2}(m_{23}^2,m_2^2,m_3^2)}{2m_{23}},
\end{aligned}$$

The particle energy spectrum is not monochromatic.
The maximum value (the end-point) for particle 1 in c.m. frame is

$$\begin{aligned}
E_1^{max} &= \frac{s+m_1^2-(m_2+m_3)^2}{2\sqrt{s}},\quad m_1\le E_1\le E_1^{max},\\
|\vec{p}_1^{max}| &= \frac{\lambda^{1/2}(s,m_1^2,(m_2+m_3)^2)}{2\sqrt{s}},\quad 0\le p_1\le p_1^{max}.
\end{aligned}$$

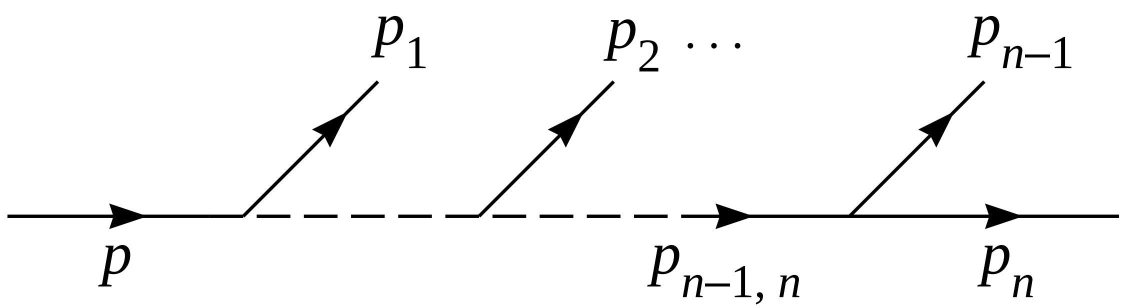
With $m_i = 10,\ 20,\ 30,\ \sqrt{s} = 100$ GeV.



More intuitive to work out the end-point for the kinetic energy,
– recall the direct neutrino mass bound in β-decay:

$$K_1^{max} = E_1^{max} - m_1 = \frac{(\sqrt{s} - m_1 - m_2 - m_3)(\sqrt{s} - m_1 + m_2 + m_3)}{2\sqrt{s}}.$$

Recursion relation $P \rightarrow 1 + 2 + 3... + n$:



$$dPS_n(P; p_1, ..., p_n) = dPS_{n-1}(P; p_1, ..., p_{n-1,n})$$
$$dPS_2(p_{n-1,n}; p_{n-1}, p_n)\frac{dm^2_{n-1,n}}{2\pi}.$$

For instance,

$$dPS_3 = dPS_2(i)\ \frac{dm^2_{prop}}{2\pi}\ dPS_2(f).$$

This is generically true, but particularly useful when the diagram has an s-channel particle propagation.

Breit-Wigner Resonance, the Narrow Width Approximation

An unstable particle of mass M and total width Γ_V, the propagator is

$$R(s) = \frac{1}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2}.$$

Consider an intermediate state V^*

$$a \longrightarrow bV^* \longrightarrow b\ p_1 p_2.$$

By the reduction formula, the resonant integral reads

$$\int_{(m_*^{min})^2=(m_1+m_2)^2}^{(m_*^{max})^2=(m_a-m_b)^2} dm_*^2.$$

Variable change

$$\tan\theta = \frac{m_*^2 - M_V^2}{\Gamma_V M_V},$$

resulting in a flat integrand over θ

$$\int_{(m_*^{min})^2}^{(m_*^{max})^2} \frac{dm_*^2}{(m_*^2 - M_V^2)^2 + \Gamma_V^2 M_V^2} = \int_{\theta^{min}}^{\theta^{max}} \frac{d\theta}{\Gamma_V M_V}.$$

In the limit

$$(m_1+m_2)+\Gamma_V \ll M_V \ll m_a-m_b-\Gamma_V,$$
$$\theta^{min} = \tan^{-1}\frac{(m_1+m_2)^2-M_V^2}{\Gamma_V M_V} \to -\pi,$$
$$\theta^{max} = \tan^{-1}\frac{(m_a-m_b)^2-M_V^2}{\Gamma_V M_V} \to 0,$$

then the Narrow Width Approximation

$$\frac{1}{(m_*^2-M_V^2)^2+\Gamma_V^2 M_V^2} \approx \frac{\pi}{\Gamma_V M_V}\ \delta(m_*^2-M_V^2).$$

Exercise 2.4: Consider a three-body decay of a top quark, $t \to bW^* \to b\ e\nu$. Making use of the phase space recursion relation and the narrow width approximation for the intermediate W boson, show that the partial decay width of the top quark can be expressed as

$$\Gamma(t \to bW^* \to b\ e\nu) \approx \Gamma(t \to bW) \cdot BR(W \to e\nu).$$

(C). Matrix element: The dynamics

Properties of scattering amplitudes $T(s,t,u)$

• Analyticity: A scattering amplitude is analytical except:
simple poles (corresponding to single particle states, bound states etc.);
branch cuts (corresponding to thresholds).

• Crossing symmetry: A scattering amplitude for a $2 \to 2$ process is symmetric among the s-, t-, u-channels.

• Unitarity:
S-matrix unitarity leads to :

$$-i(T - T^\dagger) = TT^\dagger$$

Partial wave expansion for $a + b \to 1 + 2$:

$$\mathcal{M}(s,t) = 16\pi \sum_{J=M}^{\infty} (2J+1) a_J(s) d^J_{\mu\mu'}(\cos\theta)$$
$$a_J(s) = \frac{1}{32\pi}\int_{-1}^{1} \mathcal{M}(s,t)\ d^J_{\mu\mu'}(\cos\theta) d\cos\theta.$$

where $\mu = s_a - s_b,\ \mu' = s_1 - s_2,\ M = \max(|\mu|, |\mu'|)$.

By Optical Theorem: $\sigma = \frac{1}{s}\mathrm{Im}\mathcal{M}(\theta=0) = \frac{16\pi}{s}\sum_{J=M}^{\infty}(2J+1)|a_J(s)|^2$.

The partial wave amplitude have the properties:

(a). partial wave unitarity: $\mathrm{Im}(a_J) \geq |a_J|^2$, or $|\mathrm{Re}(a_J)| \leq 1/2$,

(b). kinematical thresholds: $a_J(s) \propto \beta_i^{l_i}\ \beta_f^{l_f}\quad (J = L + S)$.

$\Rightarrow$ well-known behavior: $\sigma \propto \beta_f^{2l_f+1}$.

Exercise 2.5: Appreciate the properties (a) and (b) by explicitly calculating the helicity amplitudes for

$$e^-_L e^+_R \to \gamma^* \to H^- H^+, \quad e^-_L e^+_{L,R} \to \gamma^* \to \mu^-_L \mu^+_R, \quad H^- H^+ \to G^* \to H^- H^+.$$

(D). Calculational Tools

Traditional "Trace" Techniques in QFT:

Good for simple processes

Helicity Techniques:

technical simplification, necessary for multiple particles;

conceptual advancements.
(Henriette's lectures)

`Exercise 2.6: Calculate the squared matrix element for` $\overline{\sum}|\mathcal{M}(f\bar{f} \to ZZ)|^2$, `in terms of` s, t, u, `in whatever technique you like.`

Calculational packages:

- Monte Carlo packages for phase space integration:

VEGAS by P. LePage: adaptive important-sampling MC
  http://en.wikipedia.org/wiki/Monte-Carlo_integration

- Automated evaluation of cross sections:

(1) MadGraph/MadEvent and MadSUSY:
Generate Fortran codes on-line! http://madgraph.hep.uiuc.edu
(Now allows you to input new models.)

(2) CompHEP/CalHEP: computer program for calculation of elementary particle processes in Standard Model and beyond. CompHEP has a built-in numeric interpreter. So this version permits to make numeric calculation without additional Fortran/C compiler. It is convenient for more or less simple calculations.

— It allows your own construction of a Lagrangian model!

http://theory.npi.msu.su/~kryukov
(Now allows you to input new models.)

(3) SHERPA (F. Krauss et al.): (Gaining popularity)
Generate Fortran codes on-line! Merging with MC generators (see next).
http://www.sherpa-mc.de/

- Cross sections at NLO packages: (Gaining popularity)

(1) MC(at)NLO (B. Webber et al.):
http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO/
Combining a MC event generator with NLO calculations for QCD processes.

(2) MCFM (K. Ellis et al.):
http://mcfm.fnal.gov/
Parton-level, NLO processes for hadronic collisions.

(3) BlackHat (Z.Bern, L.Dixon, D.Kosover et al.):
http://blackhat.hepforge.org/
Parton-level, NLO processes to combine with Sherpa

• Numerical simulation packages: Monte Carlo Event Generators

Reading: http://www.sherpa-mc.de/

(1) PYTHIA:

PYTHIA is a Monte Carlo program for the generation of high-energy physics events, i.e. for the description of collisions at high energies between e^+, e^-, p and $\bar{p}$ in various combinations.

They contain theory and models for a number of physics aspects, including hard and soft interactions, parton distributions, initial and final state parton showers, multiple interactions, fragmentation and decay.

— It can be combined with MadGraph and detector simulations.

http://www.thep.lu.se/ torbjorn/Pythia.html

Already made crucial contributions to Tevatron/LHC.

(2) HERWIG

HERWIG is a Monte Carlo program which simulates $pp, p\bar{p}$ interactions at high energies. It has the most sophisticated perturbative treatments, and possible NLO QCD matrix elements in parton showing.

http://hepwww.rl.ac.uk/theory/seymour/herwig/

• Detector Simulations "Pretty Good Simulation" (PGS):

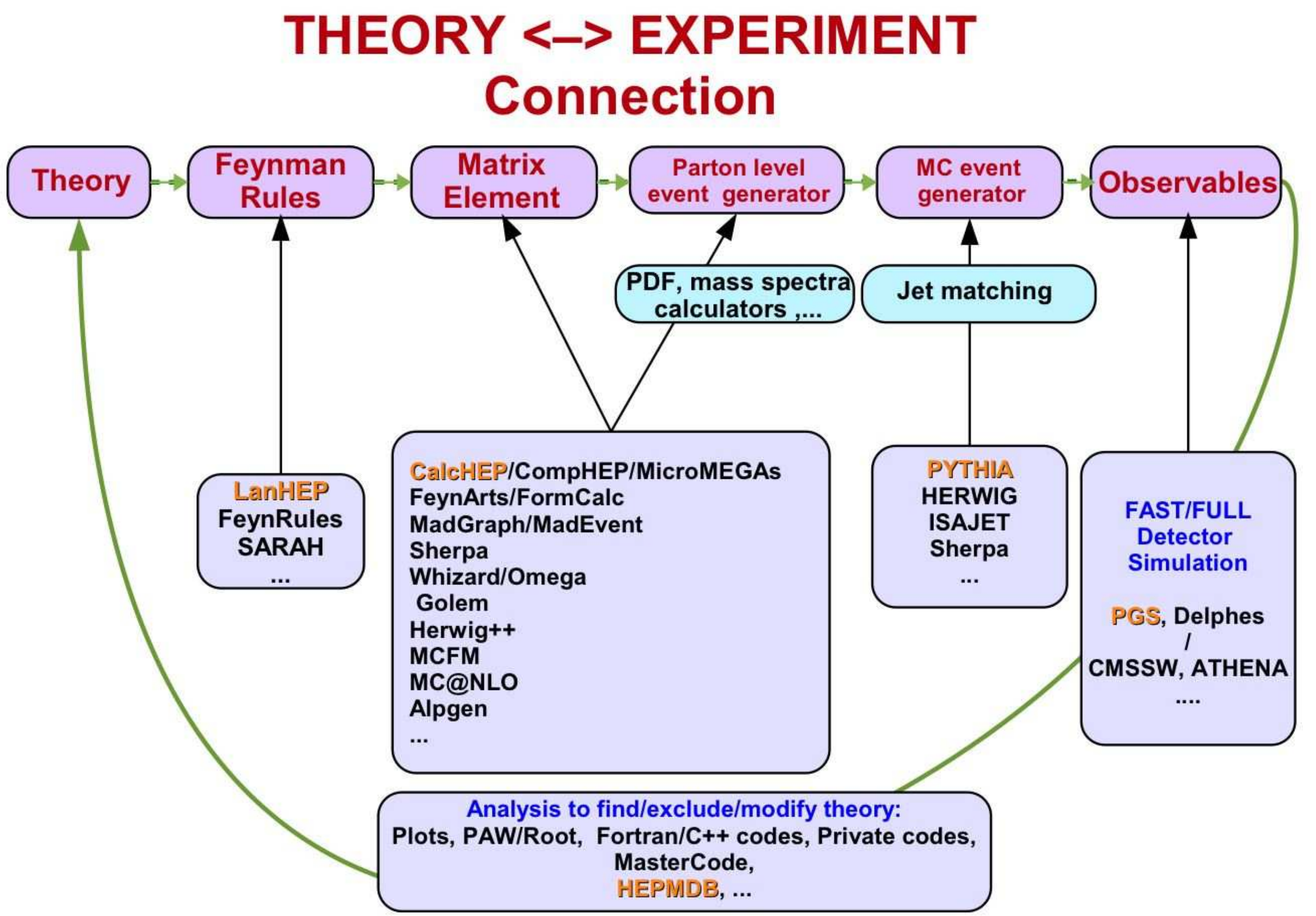
By John Conway: A simplified detector simulation,

mainly for theorists to estimate the detector effects.
http://www.physics.ucdavis.edu/ conway/research/software/pgs/pgs.html

PGS has been adopted for running with PYTHIA and MadGraph.
(but just a "toy".)

DELPHES: A modular framework for fast simulation of a generic collider experiment.
http://arxiv.org/abs/1307.6346

Over all:


THEORY <–> EXPERIMENT
Connection
Theory
Feynman Rules
Matrix Element
Parton level event generator
MC event generator
Observables
PDF, mass spectra calculators ,...
Jet matching
LanHEP
FeynRules
SARAH
...
CalcHEP/CompHEP/MicroMEGAs
FeynArts/FormCalc
MadGraph/MadEvent
Sherpa
Whizard/Omega
Golem
Herwig++
MCFM
MC@NLO
Alpgen
...
PYTHIA
HERWIG
ISAJET
Sherpa
...
FAST/FULL Detector Simulation
PGS, Delphes
/
CMSSW, ATHENA
....
Analysis to find/exclude/modify theory:
Plots, PAW/Root, Fortran/C++ codes, Private codes,
MasterCode,
HEPMDB, ...

I-C. Physics at an e^+e^- Collider

(A.) Simple Formalism

Event rate of a reaction:

$$R(s) = \sigma(s)\mathcal{L}, \quad \text{for constant } \mathcal{L}$$
$$= \mathcal{L}\int d\tau \frac{dL(s,\tau)}{d\tau}\,\sigma(\hat{s}), \quad \tau = \frac{\hat{s}}{s}.$$

As for the differential production cross section of two-particle a, b,

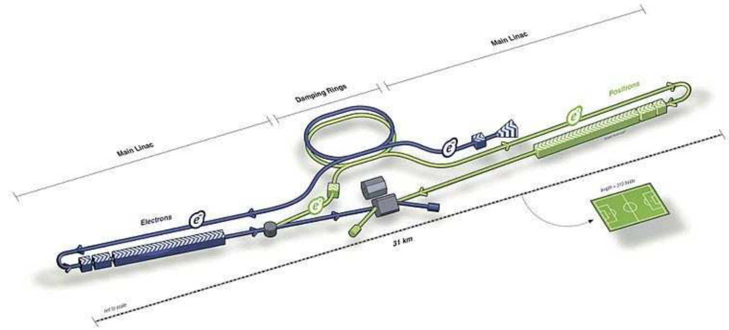
$$\frac{d\sigma(e^+e^- \to ab)}{d\cos\theta} = \frac{\beta}{32\pi s}\overline{\sum}|\mathcal{M}|^2$$

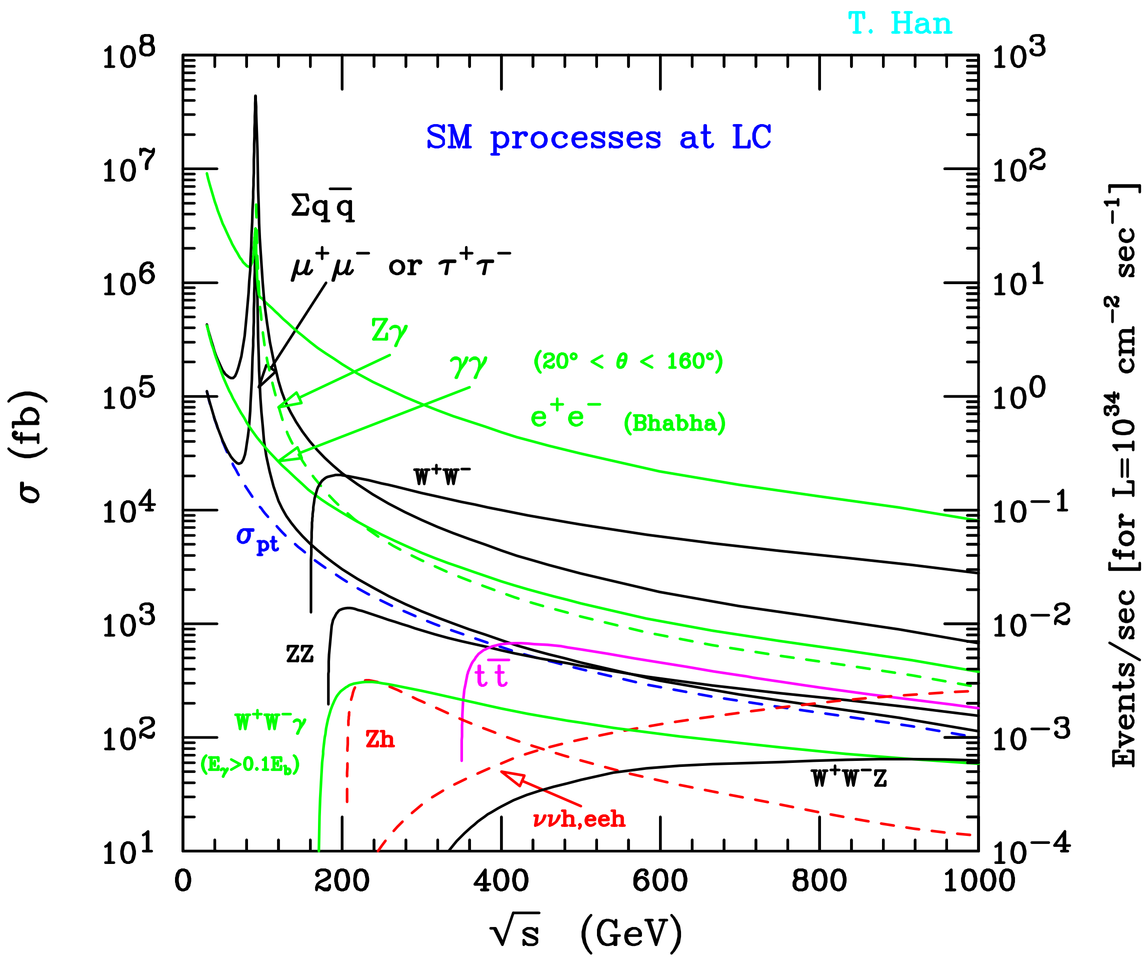
where

• $\beta = \lambda^{1/2}(1, m_a^2/s, m_b^2/s)$, is the speed factor for the out-going particles in the c.m. frame, and $p_{cm} = \beta\sqrt{s}/2$,

• $\overline{\sum|\mathcal{M}|^2}$ the squared matrix element, summed and averaged over quantum numbers (like color and spins etc.)

• unpolarized beams so that the azimuthal angle trivially integrated out, Total cross sections and event rates for SM processes:





(B). Resonant production: Breit-Wigner formula

$$\frac{1}{(s-M_V^2)^2+\Gamma_V^2 M_V^2}$$

If the energy spread $\delta\sqrt{s} \ll \Gamma_V$, the line-shape mapped out:

$$\sigma(e^+e^- \to V^* \to X) = \frac{4\pi(2j+1)\Gamma(V\to e^+e^-)\Gamma(V\to X)}{(s-M_V^2)^2+\Gamma_V^2 M_V^2}\ \frac{s}{M_V^2},$$

If $\delta\sqrt{s} \gg \Gamma_V$, the narrow-width approximation:

$$\frac{1}{(s-M_V^2)^2+\Gamma_V^2 M_V^2} \to \frac{\pi}{M_V\Gamma_V}\ \delta(s-M_V^2),$$

$$\sigma(e^+e^- \to V^* \to X) = \frac{2\pi^2(2j+1)\Gamma(V\to e^+e^-)BF(V\to X)}{M_V^2}\frac{dL(\hat{s}=M_V^2)}{d\sqrt{\hat{s}}}$$

`Exercise 3.1: sketch the derivation of these two formulas, assuming a Gaussian distribution for`

$$\frac{dL}{d\sqrt{\hat{s}}} = \frac{1}{\sqrt{2\pi}\ \Delta}\exp[\frac{-(\sqrt{\hat{s}}-\sqrt{s})^2}{2\Delta^2}].$$

Note: Away from resonance

For an s-channel or a finite-angle scattering:

$$\sigma \sim \frac{1}{s}.$$

For forward (co-linear) scattering:

$$\sigma \sim \frac{1}{M_V^2} \ln^2 \frac{s}{M_V^2}.$$

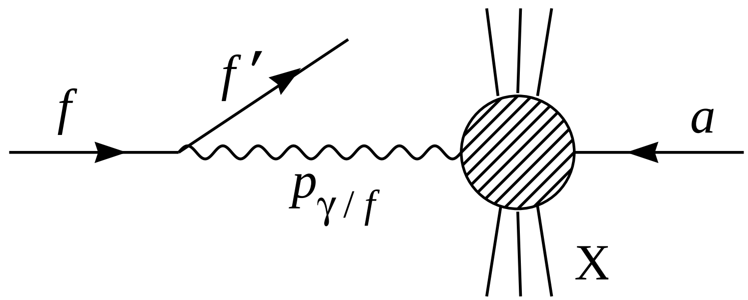
- The simplest reaction

$$\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-) \equiv \sigma_{pt} = \frac{4\pi\alpha^2}{3s}.$$

In fact, $\sigma_{pt} \approx 100\ \text{fb}/(\sqrt{s}/\text{TeV})^2$ has become standard units to measure the size of cross sections.

(C). Gauge boson radiation:

A qualitatively different process is initiated from gauge boson radiation, typically off fermions:



The simplest case is the photon radiation off an electron, like:

$$e^+e^- \to e^+,\ \gamma^* e^- \to e^+e^-.$$

The dominant features are due to the result of a t-channel singularity, induced by the collinear photon splitting:

$$\sigma(e^- a \to e^- X) \approx \int dx\ P_{\gamma/e}(x)\ \sigma(\gamma a \to X).$$

The so called the effective photon approximation.

For an electron of energy E, the probability of finding a collinear photon of energy xE is given by

$$P_{\gamma/e}(x) = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\ln\frac{E^2}{m_e^2},$$

known as the Weizsäcker-Williams spectrum.

Exercise 3.3: Try to derive this splitting function.

We see that:

- m_e enters the log to regularize the collinear singularity;
- $1/x$ leads to the infrared behavior of the photon;
- This picture of the photon probability distribution is also valid for other photon spectrum:

Based on the back-scattering laser technique, it has been proposed to produce much harder photon spectrum, to construct a "photon collider"...

(massive) Gauge boson radiation:

A similar picture may be envisioned for the electroweak massive gauge bosons, $V = W^\pm, Z$.

Consider a fermion f of energy E, the probability of finding a (nearly) collinear gauge boson V of energy xE and transverse momentum p_T (with respect to $\vec{p}_f$) is approximated by

$$P^T_{V/f}(x, p_T^2) = \frac{g_V^2 + g_A^2}{8\pi^2} \frac{1 + (1-x)^2}{x} \frac{p_T^2}{(p_T^2 + (1-x)M_V^2)^2},$$

$$P^L_{V/f}(x, p_T^2) = \frac{g_V^2 + g_A^2}{4\pi^2} \frac{1-x}{x} \frac{(1-x)M_V^2}{(p_T^2 + (1-x)M_V^2)^2}.$$

Although the collinear scattering would not be a good approximation until reaching very high energies $\sqrt{s} \gg M_V$, it is instructive to consider the qualitative features.

(D). Recoil mass technique:

One of the most important techniques, that distinguishes an e^+e^- collisions from hadronic collisions.

Consider a process:

$$e^+ + e^- \to V + X,$$

where V: a (bunch of) visible particle(s); X: unspecified.

Then:

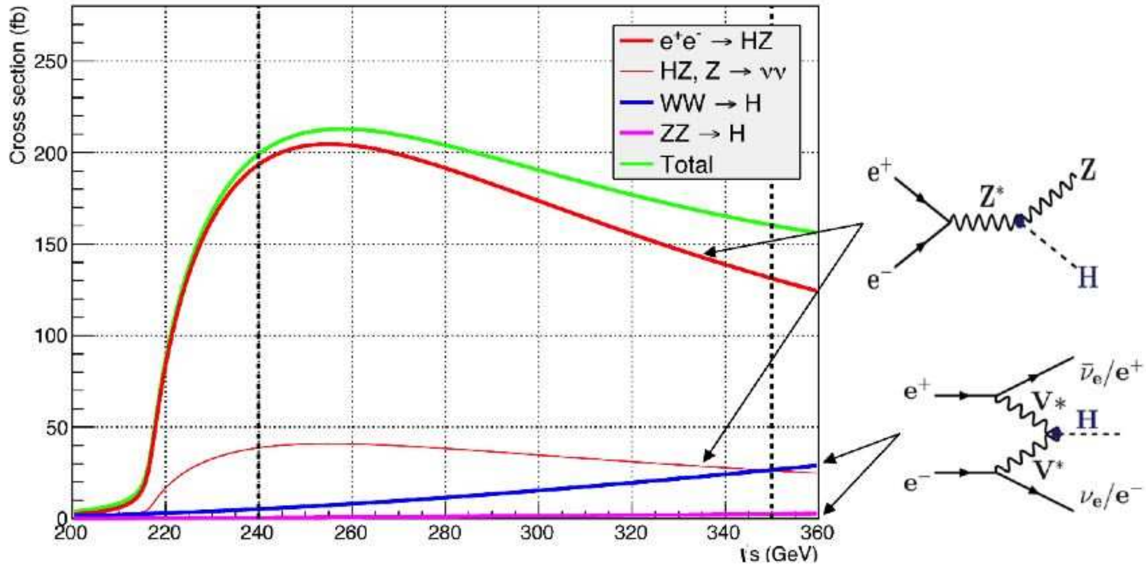
$$p_{e^+} + p_{e^-} = p_V + p_X, \ (p_{e^+} + p_{e^-} - p_V)^2 = p_X^2,$$
$$M_X^2 = (p_{e^+} + p_{e^-} - p_V)^2 = s + M_V^2 - 2\sqrt{s}E_V.$$

One thus obtain the "model-independent" inclusive measurements

a. mass of X by the recoil mass peak

b. coupling of X by simple event-count at the peak

(E). Physics at a Higgs Factory:



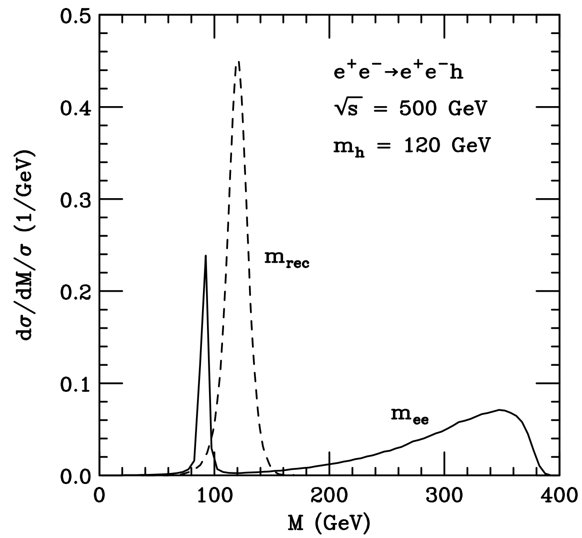
At peak cross section $\approx$ 200 fb with 5 ab^{-1} $\Rightarrow$ 1M h^0!

The key point for a Higgs factory:

Model-independent measurements on the ZZh coupling in a clean experimental environment.

Consider: $e^+ + e^- \to f\bar{f} + h.$

$$M_h^2 = (p_{e^+} + p_{e^-} - p_f - p_{\bar{f}})^2 = s + M_V^2 - 2\sqrt{s}E_{f\bar{f}}.$$



Kinematical selection of “inclusive” signal events!

Marching to higher energies: 500 GeV−1 TeV: