Composite Higgs


[Reviews: Contino 1005.4296; Belleazzini, Csehi, Serra: 1401.2457; Panico, Walzer 1506.01961]

EWSB

Higgs potential: \[ V(H) = -m_H^2 |H|^2 + \lambda_H |H|^4 \] where \( H = \frac{1}{\sqrt{2}} (\psi_{+H}) \)

Higgs discovery: \[ m_h^2 = 2 \lambda_H v^2 \approx (125 \text{ GeV})^2 \quad \text{and} \quad v^2 = \frac{m_h^2}{\lambda_H} = (246 \text{ GeV})^2 \]

\[ m_h^2 = (89 \text{ GeV})^2 \quad \text{and} \quad \lambda_H \approx 0.13 \]

Why is \( m_h \ll M_{\text{Pl}} \)? Possible answer: strong dynamics!

Planck scale = \( 10^{16} \text{ GeV} \)

New strong force with coupling \( g_s \): \[ \frac{d}{dx} \left( \frac{1}{g_s^2} \right) = -\frac{b_s}{16\pi^2} \]

\[ \Rightarrow \Lambda_5 = M_{\text{Pl}} \approx \frac{g_5^2(M_{\text{Pl}}) b_s}{16\pi^2} \quad (b_s < 0) \]

\[ \Rightarrow m_h^2 \sim \Lambda_5^2 \ll M_{\text{Pl}}^2 \]

Similar to QCD! \( \Lambda_{\text{QCD}} \sim 250 \text{ MeV} \ll M_{\text{Pl}} \)

Idea: Higgs boson \( \Leftrightarrow \) bound state of new strong dynamics

But strong dynamics will also produce other bound states.

Question: Can Higgs “bound state” be naturally lighter?

Yes! Higgs can be a pseudo Nambu-Goldstone boson

[Georgi, Kaplan 1984]
**Analogy:** Pions in QCD

In limit $m_u, m_d \to 0$

\[
\begin{align*}
&(u_L \to g_L(u_L), d_L \to g_L(d_L)) \\
&\text{global symmetry: } G = SU(2)_L \times SU(2)_R \\
&\text{"chiral" symmetry } g_{L,R} \in SU(2)_{L,R} \\
&\text{Spontaneously broken } \iff \langle \bar{q}_L q_R \rangle \neq 0 \\
&\text{QCD interactions cause quark-antiquark pairs to condense} \\
&\text{unbroken global } SU(2)_V \text{ symmetry preserved}
\end{align*}
\]

**Goldshtein's theorem:** For every spontaneously broken continuous symmetry, there is a massless particle (Nambu-Goldstone boson).

If global symmetry $G$ broken to $H$, then the number of NG bosons = dimension of coset space $G/H$. i.e. $\dim G/H = \dim G - \dim H$

Consider $g, g_1 \in G$ then $g \sim g_1$ if $\exists h \in H$ s.t. $g_1 = gh$

QCD $G = SU(2)_L \times SU(2)_R \to H = SU(2)_V \implies \dim G/H = \dim G - \dim H = 6 - 3 = 3$

$\implies 3$ Nambu-Goldstone bosons (pions - $\pi^+$)

**Isospin triplet**
Strong dynamics can be described by "linear $\sigma$-model" (mimics effect of $\bar{q}q$ condensate)

$$\mathcal{L} = \frac{1}{2} \partial_{\mu} \Sigma \partial_{\nu} \Sigma + m^2 |\Sigma|^2 - \frac{\lambda}{4} |\Sigma|^4$$

where $\Sigma = \text{complex scalar}$, $\Sigma \rightarrow g_L Z g_R^\dagger$ $g_{\mu\nu} \in SU(2)_L \times SU(2)_R$

Potential minimum: $\langle \Sigma \rangle = \frac{V}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ where $V^2 = \frac{2 m^2}{\sqrt{\lambda}}$

Fluctuations: $\Sigma(x) = \frac{1}{\sqrt{V}} (V + \sigma(x)) \exp \left[ -\frac{i}{\sqrt{V}} \frac{\hat{r} \cdot \hat{F}}{2 m} \right]$ where $\hat{F} = \nabla \Sigma - \Sigma \nabla$

Note: Symmetry is not really "broken", instead is realized nonlinearly

$$\Sigma \rightarrow g_L Z g_R^\dagger \quad (g_{\mu\nu} = e^{i \theta_\mu x^\mu}) \Rightarrow \pi_a \rightarrow \pi_a + \frac{e}{2 \sqrt{\lambda}} \Theta_{\mu \nu} \epsilon_{\mu \nu a} \cdots \text{"shift symmetry"}$$

$$\Rightarrow \text{Pion potential: } V(\pi) = 0 \quad (m_\pi = 0) \quad \text{at quantum level}$$

Shift symmetry forbids a pion mass term!

However: Quark masses explicitly break chiral symmetry by small amount

$$\mathcal{L}_m = \bar{q} M q$$ where $M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$

Treat $M$ as a spurion where $M \rightarrow g_L M g_R^\dagger \Rightarrow \mathcal{L} = \frac{V^3}{2} \text{Tr}(M U + M U^\dagger)$

$$\Rightarrow m_\pi \neq 0 \quad \text{but } m_\pi \ll m_q \quad \text{since } m_u \text{ and } m_d \text{ are small}$$

$$\Rightarrow \text{pion = pseudo Nambu-Goldstone boson}$$

Substitute $\Sigma$ into $\mathcal{L} \Rightarrow \text{chiral Lagrangian}$

$\mathcal{L}_3$ describes pion interactions
Do something similar for the Higgs boson!

Consider global symmetry groups with coset $G/H \supset$ Higgs doublet!

Generate Higgs potential? Break EW symmetry?

Note Strong dynamics does not break EW symmetry

$\rightarrow$ differs from technicolor (scaled-up version of QCD)!

Suppose new strong dynamics spontaneously breaks global symmetry $G$ at scale $f$:

$$G \xrightarrow{f} \text{unbroken global group}$$

Require:

- $H \supset SU(2)_L \times U(1)_Y =$ SM electroweak gauge group
- Coset $G/H$ must contain a Higgs doublet (4 real scalar fields)

Note The underlying strong dynamics is not specified and not needed to determine Higgs properties.

**SO(5)/SO(4) model** [Agashe, Contino, Pomarol 2004]

Consider $SO(5) \xrightarrow{f} SO(4)$

Recall $SO(n) =$ group of $n \times n$ orthogonal matrices with unit determinant

- Lie algebra = $n \times n$, traceless, imaginary Hermitian matrices $\Rightarrow \dim = \frac{n(n-1)}{2}$

  - $SO(5)_c =$ custodial symmetry (required to ensure $F = \frac{m^2}{m^2_{\text{W}} \cos^2 \theta_W} = 1$)

- $SO(4) \cong SU(2)_L \times SU(2)_R \supset SU(2)_L \times U(1)_Y$

- number NG bosons $= \dim SO(5)/SO(4) = \dim SO(5) - \dim SO(4)$

  $= [10 - 6] = 4$

  $\Rightarrow$ Higgs doublet

  $\Rightarrow$ $SO(5)/SO(4) =$ minimal model
Model $SO(5) \rightarrow SO(4)$ breaking with scalar fields $\vec{\Phi} = E$ of $SO(5)$ and effective Lagrangian: “linear $\sigma$-model”

$$
\mathcal{L} = \frac{1}{2} \partial_{\mu} \vec{\Phi} \partial^{\mu} \vec{\Phi} - \frac{g_{5}}{8} (\vec{\Phi} \cdot \vec{\Phi} - f^{2})^{2} \quad \text{where} \quad g_{5} = \text{composite sector coupling} \quad (1 \leq g_{5} < 4\pi)
$$

Fluctuations about potential minimum:

$$
\vec{\Phi} = e^{i \frac{\vec{t}}{f}} \vec{\Pi}(\omega) \vec{\tau} \left[ \begin{array}{c}
0 \\
0 \\
0 \\
\cos \frac{\vec{t}}{f} \\
\sin \frac{\vec{t}}{f}
\end{array} \right] = (f + \sigma) \left[ \begin{array}{c}
\sin \frac{\vec{t}}{f} \\
\cos \frac{\vec{t}}{f}
\end{array} \right]
$$

where $\vec{t}$ = broken $SO(5)/SO(4)$ generators ($i = 1, ..., 4$)

$$
\vec{\Pi} = \left( \begin{array}{c}
\eta_{1} \\
\eta_{2} \\
\eta_{3} \\
\eta_{4}
\end{array} \right) \quad \vec{\Pi} = \sqrt{\vec{t}^{2} \cdot \vec{t}} \quad \text{with} \quad \text{Higgs doublet} \quad H = \frac{1}{f} \left( \begin{array}{c}
\eta_{2} + i \eta_{1} \\
\eta_{4} - i \eta_{3}
\end{array} \right)
$$

$SO(4)$ fourplet

Substituting $\vec{\Phi}$ into $\mathcal{L}$ gives:

$$
\mathcal{L} = \frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma - \frac{1}{2} (g_{5} f)^{2} \partial_{\mu} \sigma \partial^{\mu} \sigma - \frac{1}{6} g_{5}^{2} \sigma^{3} - \frac{1}{8} g_{5}^{2} \sigma^{4}
$$

$$
+ \frac{1}{4} \left( f + \sigma \right)^{2} \left[ \frac{\varepsilon}{4 \chi^{2}} \sin^{2} \frac{\chi}{f} \partial_{\mu} H \partial^{\mu} H + \frac{\varepsilon}{4 \chi^{4}} \left( \frac{2 \chi^{2}}{f} - \sin^{2} \frac{\chi}{f} \right) \partial_{\mu} H \partial^{\mu} H + \frac{\varepsilon}{2 \chi^{4}} \partial_{\mu} H \partial^{\mu} H \right]
$$

Note

1. $\sigma$ is a massive resonance $m_{\sigma} = g_{5} f$
2. $H^{i}$ fields are massless (since $V(H) = 0$) and derivatively coupled (like QCD chiral Lagrangian) i.e. $H$'s are NG bosons
(3) Electroweak gauge-Higgs interactions obtained by:

\[ D_H \rightarrow \bar{D}_H = (\frac{g}{v} W^{\mu} H + \frac{g'}{v} B_{\mu} \frac{1}{2}) H \]

\[ SM: SU(2)_L \times U(1)_Y \subset SU(2)_L \times SU(2)_R \cong SO(4) \]

Assume \[ H = \frac{1}{i} \begin{pmatrix} 0 \\ v \end{pmatrix} \] (unitary gauge) then obtain:

\[ \mathcal{L} \supset \frac{1}{2} (\frac{g}{v})^2 + \frac{g^2 f^2}{4} \sin^2 \frac{v}{f} \left( |W|^2 + \frac{1}{2 \cos^2 \theta_w} Z^2 \right) + \ldots \]

\[ \Rightarrow m_W = \cos \theta_w m_Z = \frac{1}{2} g f \sin \frac{v}{f} \equiv \frac{1}{2} g f \sin \frac{v}{f} \]

\[ \Rightarrow \tilde{f} \sin \frac{v}{f} = \frac{v}{f} \quad \text{where } \tilde{f} = \text{electroweak VEV} = 246 \text{ GeV} \]

Higgs couplings to gauge bosons:

\[ \mathcal{L} \supset \frac{g^2}{4} \left( |W|^2 + \frac{1}{2 \cos^2 \theta_w} Z^2 \right) \left[ \frac{1}{2} - \frac{v^2}{2 f^2} + (1 - 2 \delta) \frac{v^2}{2 f^2} + \ldots \right] \quad \text{where } \delta = \frac{v^2}{f^2} \]

\[ \Rightarrow \frac{g_{HWW}}{g_{HHV}} = \sqrt{1 - \delta} \quad \text{and} \quad \frac{g_{HVV}}{g_{HHV}} = 1 - 2 \delta \]

Due to compact group \( g \)

Expansion of \( \mathcal{L} \) for large \( f \):

\[ \mathcal{L} \supset \frac{2}{3 f^2} |H|^2 D_H^\dagger D_H^\dagger H + \frac{1}{6 f^2} \partial \mu (H^\dagger H) \partial^\mu (H^\dagger H) + \ldots \]

Note 1: \( \mathcal{L} \) is absent! \( \Rightarrow \)

\[ \mathcal{L} = \frac{m^2}{(\cos \theta_w m_Z)^2} = 1 \]

As expected since \( \tilde{f} = \text{so(4)} \subset SU(2)_L \times SU(2)_R \rightarrow \text{custodial symmetry} \)

(2) Linear \( \sigma \)-model is an effective description of the (unknown) strong dynamics

More general formalism based on non-linearly realized symmetry

Callan, Coleman, Weiss, Zumino (CCWZ)

(see Pomico, Wulzer review)
How to generate $V(H) \neq 0$?

$\rightarrow$ need an explicit violation of global symmetry $G$

Early attempts introduced new interactions [see Georgi-Kaplan 1984, Banks 1984]

Instead, assume that global symmetry of composite sector is broken by mixing with an elementary sector: [Agashe, Contino, Pomarol 2004]

$$\mathcal{L}_{\text{mix}} = \lambda_{ij} \bar{\Psi}_i \gamma^\mu D^\mu_{ij} \Phi_j + \tilde{g}_i A_{\mu}^i \tilde{J}^\mu_{\text{vector}}$$

Elementary fields are not complete $G$ multiplets $\Rightarrow$ explicitly breaks $G$ parametrised by SM couplings $g_{\text{SM}}$

**Note**

1. This is motivated by 5D models and AdS/CFT correspondence

2. Explicit breaking could also be due to constituent masses (like quark mass in QCD) $\rightarrow$ assume strong sector does not have this contribution.

$\mathcal{L}_{\text{mix}} \Rightarrow$ mass eigenstates are mixtures of elementary and composite states

$$|\text{phys.}\rangle = \cos \theta_i |\text{elem.}\rangle + \sin \theta_i |\text{comp.}\rangle$$

**Known as partial compositeness**

**Note**

Similar to $\gamma-\rho$ mixing in QCD:

- explains $\gamma \rightarrow e^+ e^-$

$$\mathcal{L}_{\text{mix}} = \lambda_{ij} \bar{\Psi}_i \gamma^\mu D^\mu_{ij} \Phi_j + \tilde{g}_i A_{\mu}^i \tilde{J}^\mu_{\text{vector}}$$
Explains fermion mass hierarchy: [Kaplan 1991; TG, Pomarol 2000]

Consider: \( L = \left( \begin{array}{c} \lambda^L_{\psi L} \bar{\psi}_L \psi_R + \lambda^R_{\psi R} \bar{\psi}_R \psi_R \\ \lambda^L_{\psi L} \bar{\psi}_L \psi_R + \lambda^R_{\psi R} \bar{\psi}_R \psi_L \end{array} \right) \) - fermionic resonances \( F_{l, r}^T, F_{l, r}^c \)

\( \Omega_{l, r} = \) fermionic op. \( \lambda_{l, r} = O(1) \) constants \( \psi_{l, r} = \) elementary fermions

Now \( \langle 0 | \Omega \hat{c}_{l, r} | F_{l, r}^c \rangle \neq 0 \)

\( \Rightarrow \text{excites single-particle state from vacuum} \)

\( \Rightarrow \mathcal{L}_{\text{mass}} = -(\bar{\psi}_{l, r} F_{l, r}^c \psi_{l, r}) \left( \begin{array}{c} 0 \ 0 \ \lambda_{l, r} \\ 0 \ \lambda_{l, r} \ g_f \\ g_f \ 0 \end{array} \right) \left( \begin{array}{c} \psi_{l, r} \\ F_{l, r}^c \\ \psi_{l, r} \end{array} \right) \)

Mass eigenstates:
\( |\text{phys.}> = \cos \theta_i |\text{elem.}> + \sin \theta_i |\text{comp.}> \)

where \( \sin \theta_{l, r} = \frac{\lambda_{l, r}}{\sqrt{g^2_f + \lambda^2_{l, r}}} \Rightarrow \frac{\lambda_{l, r}}{g^2_f} \) assuming \( \lambda_{l, r} \ll g_f, \quad (1 \lesssim g_f < \Phi) \)

Yukawa interaction:

\( \Rightarrow \lambda_f = g_f \sin \theta_L \sin \theta_R \approx \frac{\lambda_f(\mu)\lambda_f(\mu)}{g_f} \)

Light fermions: \( \lambda_f \ll 1 \Rightarrow d_{l, r} > \frac{\pi}{2} \Rightarrow \text{mostly elementary!} \)

Top quark: \( \lambda_f \sim 1 \Rightarrow d_{l, r} \sim \frac{\pi}{2} \Rightarrow \text{mostly composite!} \)

Note: Requiring \( \delta g_b \sim \frac{(\lambda_{l, r})^2}{g_f^2} \) to satisfy expl. constraints (\( \delta g_b \leq 10^{-3} \)) \( \Rightarrow \{ \lambda_{l, r} \ll 1, \lambda_{l, r} \approx g_f \Rightarrow \text{composite!} \}, \lambda_{l, r} \ll \lambda_{l, r} = \sqrt{\lambda_f g_f} \)
Fermion embeddings

Recall: \( \mathcal{L} = \lambda_L f_L \mathcal{O}_L + \lambda_R f_R \mathcal{O}_R \) What are possible reps of \( \mathcal{O}_{L,R} \)?

Convenient to embed \( f_{L,R} \) into reps. of \( SO(5) \):

\[ \mathcal{L} = \lambda_L (\mathcal{O}_L)^{I_L} + \lambda_R (\mathcal{O}_R)^{I_R} = (\Psi_L)^{I_L} = (\Psi_R)^{I_R} \]

\( \alpha = \text{SM group index} \)
\( I_{L,R} = \text{G group index} \)

SM fermions: \( 2_\frac{1}{2}, 1_0, 1_{-1} \) cannot be embedded into \( SO(5) \)!

Instead, consider \( SO(5) \times U(1)_x \rightarrow SO(4) \times U(1)_x \)

\[ \Rightarrow \text{hypercharge} \ Y = T_{3R} + X \]

\( SO(5) \) fermion representations

Consider fundamental \( 5 \) rep. with \( U(1)_x \) charge \( \pm \frac{2}{3} \) then

\[ 5_{\pm \frac{2}{3}} \rightarrow 4_{\pm \frac{2}{3}} \oplus 1_{\pm \frac{2}{3}} \rightarrow 2_{\frac{1}{3}} \oplus 2_{\frac{1}{3}} \oplus 1_{\frac{2}{3}} \]

Thus have the following embeddings:

\[ Q_L = \begin{pmatrix} \frac{\lambda_L}{\sqrt{2}} \\ -i \frac{\lambda_L}{\sqrt{2}} \\ -i \frac{\lambda_L}{\sqrt{2}} \\ \frac{\lambda_L}{\sqrt{2}} \\ 0 \end{pmatrix}, \quad T_R = \begin{pmatrix} \frac{0}{0} \\ \frac{0}{0} \\ \frac{0}{0} \\ \frac{0}{0} \\ \frac{0}{0} \end{pmatrix} \]

Partial compositeness \( \Rightarrow \mathcal{L} = \lambda_L (\mathcal{T}_L)^{I_L} (\mathcal{O}_L)^{I_L} + \lambda_R (\mathcal{T}_R)^{I_R} (\mathcal{O}_R)^{I_R} + h.c \)

Integrate out strong dynamics

\( \Rightarrow \mathcal{L}_{\text{eff}}[Q_L, T_R] \) must be \( G \) invariant

Given \( U(\Gamma) = e^{i\frac{\Gamma_{ij}}{2}} \) transformations under \( G \)

(Implicitly defined by \( U(\Gamma^3) = g \cdot U(\Gamma) \cdot h^{-1} [\Gamma, g] \) where \( g, \Gamma, h \in G \))

\( \Rightarrow \) Yukawa interactions are fixed by symmetry!
Higgs couplings to fermions

With \( q_L < 5 = q_L, \ t_R < 5 = T_R \) where \( U(N) = \left[ \begin{array}{c} \frac{(1 - \cos \beta)}{\sin \beta} \\sin \frac{\beta}{\sin \frac{\beta}{\sin \frac{\beta}{\sin \beta}} \\ \frac{\sin \beta}{\sin \frac{\beta}{\sin \frac{\beta}{\sin \beta}}} \end{array} \right] \)

obtain:

\[
L_L = -m \bar{e} \gamma \cdot \gamma_5 E - \frac{k}{\sqrt{2}} \frac{m}{\bar{y}^2} \left( \begin{array}{c} \frac{1 - 2\xi}{\sin \beta} \\ \frac{1 - 2\xi}{\sin \beta} \end{array} \right) \left( \begin{array}{c} \frac{1 - 2\xi}{\sin \beta} \\ \frac{1 - 2\xi}{\sin \beta} \end{array} \right) \left( \begin{array}{c} \frac{1 - 2\xi}{\sin \beta} \\ \frac{1 - 2\xi}{\sin \beta} \end{array} \right) \]

new dim 5 interaction!

\[
k^4 = \frac{c^4}{\sqrt{1 - \xi}} \quad \frac{\xi}{\sqrt{1 - \xi}} \quad \text{MCHM}_5 \quad (5+5 \text{ model})
\]

Note: As \( \xi \to 0 (f \to 0) \) obtain \( k_5 \to 1, \ c_5 \to 0 \) (SM limit)

Other possibilities

1. Spinor rep 4:

\[
4_{\psi} \rightarrow 2_{\bar{\psi}} \oplus 1_{\psi} \oplus 1_{\bar{\psi}}
\]

\[
q_L < 4; \ t_R < 4 \quad \Rightarrow \quad k^4 = \frac{c^4}{\sqrt{1 - \xi}} \quad \frac{\xi}{\sqrt{1 - \xi}} \quad \text{MCHM}_4 \quad (4+4 \text{ model})
\]

Note: This model is ruled out because \( 2 T_{\bar{L}L} \) coupling correction is too large

2. Symmetric traceless rep 14:

\[
14_{Y} \rightarrow 3_{\bar{Y}} \oplus 3_{Y} \oplus 3_{Y} \oplus 3_{Y} \oplus 3_{Y} \oplus 1_{Y} \oplus 1_{Y} \oplus 1_{Y} \oplus 1_{Y}
\]

(i) 14+1 model:

\[
q_L < 14_{\bar{Y}}; \ t_R < 14_{\bar{Y}} \quad \Rightarrow \quad k^4 = \frac{C(1 - 8^2 + 8^4)}{2 \xi (1 - 8)^2 \left[ 5 (2 - 1) + 2 (4 - 5) \right]} \quad \textbf{Y}_{\bar{Y}} \quad \text{Yukawa couplings}
\]

(ii) 14+1 model:

\[
q_L < 14_{Y}; \ t_R < 14_{Y} \quad \Rightarrow \quad \text{same as MCHM}_5 \quad \text{allows for the possibility of fully composite} t_R
\]
LHC limits on Higgs couplings

Note: Coupling corrections < 1 (due to compact global group $G$)

$LHC (7+8 TeV)$

$K_{V, F} = \frac{g_{V, F}}{g_{SM}}$

$F = \frac{g^2}{\lambda^2}$ values
**Higgs potential**

\[ \mathcal{L} = \lambda_L q_L c_L + \lambda_H t_R c_L + g A_L J^a \]

*Examples:*

\[ \begin{array}{c}
\text{gauge boson contribution:} \\
\begin{array}{c}
\text{Fermion contribution:}
\end{array}
\end{array} \]

\[ V_{\text{gauge}} = \frac{A}{16\pi^2} (3g^2 + g'^2) \sin^2 \frac{H}{f} \]  
*Assumes SO(4) enlarged to O(4) to suppress corrections to ZEW coupling*

\[ V_{\text{Fermion}} = \frac{\lambda}{2} (c_1 \lambda^2 + c_2 \lambda^2 + c_3 \lambda^2) \sin \frac{H}{f} + \frac{\lambda^4}{16\pi^2} \]

**Generically:**

\[ V(H) = -\alpha f^2 \sin^2 \frac{H}{f} + \beta f^4 \sin^4 \frac{H}{f} \]

**Where**

\[ -\alpha = g \frac{N_c}{16\pi^2} \frac{\lambda^2}{2} \]

\[ \beta = \left\{ \begin{array}{cc}
\frac{c_N}{16\pi^2} \frac{\lambda^2}{2} g^2 & \text{top} \\
\frac{c'}{16\pi^2} \frac{\lambda^2}{2} g^2 & \text{gauge boson}
\end{array} \right. \]  
*QCD: N + 3

Since \( \lambda_t > g \Rightarrow \) top quark contribution breaks EW symmetry!

**EWSB**

Recall \( u = \sin \frac{\theta}{2} \Rightarrow \) \( V(u) = -\frac{\alpha f^2}{2} u^2 + \beta u^4 \)

\[ \frac{dV(u)}{du} = 0 \Rightarrow \frac{u^2}{\beta} = \frac{\alpha f^2}{2\beta} \]

For \( \alpha \sim 8 \sim O(1) \) require \( f \sim u \)
But resonance masses \( m_0 \approx g_f f \) where \( 1 \leq g_f \leq 4\pi \)
⇒ contributions to EW precision observables

\[
S \sim \frac{s}{4\pi} \sin^2 \theta_W \sin^2 \theta_W \quad S = \frac{s}{4\pi}
\]

Vector resonances \( \phi \) contribute to \( S \) parameter

\[
S \sim \frac{m_0^2}{m_0^2} \Rightarrow \begin{cases} \left[ m_0 > 2.5 \text{ TeV} \right] \text{ or } \left[ f \geq 2.5 \text{ TeV} \right] \end{cases}
\]

\( T \)-parameter

\[
T = \frac{-t}{16\pi^2} \left( \left(D^4 H \right)^* H \right) \left( D^4 H \right) \quad T = \frac{t}{8\pi^2 e^2}
\]

Provided \( t \Rightarrow SU(2)_L \times SU(2)_R \) → custodial symmetry

⇒ \( T \) parameter \( \checkmark \) OK

Thus EWPT ⇒ \( f > v \)

⇒ tuning in Higgs potential (in \( \beta, \gamma \) coefficients)

\[
\begin{array}{c}
\text{Higgs mass} \\
\text{in } \beta, \gamma \text{ coefficients}
\end{array}
\]

\[
V(h) = -\alpha f^2 \sin^2 \left( \frac{<H> + h}{f} \right) + \beta \frac{f^4}{\sin^4 \left( \frac{<H> + h}{f} \right)}
\]

⇒ \( m_h^2 = 8\left(1 - \frac{\nu^2}{f^2}\right) \beta \nu^2 \)

Typically \( \beta \sim \frac{N_c}{16\pi^2} \gamma^2 \frac{g_0^2}{f^2} \)

⇒ \( m_h^2 \sim \frac{N_c}{16\pi^2} \frac{g_l^2}{f_l^2} \frac{g_R^2}{f_R^2} \)

\( \text{where } m_T = \text{ fermionic top partner mass} \)

Assuming \( g_T \sim g_r \) then LHC limit \( m_T > 2.5 \text{ TeV} \) or \( g_T > 3 \)

⇒ \( m_h > m_t \).
However no need to have $g_T \sim g_T$!

Choose $g_T < g_T$ to obtain Higgs mass $\sim 125$ GeV

$\Rightarrow m_T < m_T \Rightarrow$ light top-partners!

5+5 model $\quad \mathcal{O}_L = 5 \ ; \ \mathcal{O}_R = 5$

$5_{\frac{2}{3}} \rightarrow 2_{\frac{7}{6}} \oplus 2_{\frac{1}{2}} \oplus 1_{\frac{2}{3}} \quad $\text{so(6)$ \times$u(1)$_T$ reps.}$

possible top partner reps

$\frac{2}{3}$ charge!

Includes exotic fermions \[ \begin{cases} 2_{\frac{7}{6}} \Rightarrow q_{EM} = \frac{7}{6} + \frac{1}{2} = \frac{5}{3} \text{ or } \frac{2}{3}! \\ 2_{\frac{1}{2}} \Rightarrow q_{EM} = \frac{1}{2} + \frac{1}{2} = \frac{3}{3} \text{ or } -\frac{1}{3} \end{cases} \]

14+1 model $\quad \mathcal{O}_L = 14 \ ; \ \mathcal{O}_R = 1$

$14_{\frac{2}{3}} \rightarrow 3_{\frac{3}{2}} \oplus 3_{-\frac{1}{2}} \oplus 3_{-\frac{1}{2}} \oplus 2_{\frac{7}{6}} \oplus 2_{\frac{1}{2}} \oplus 1_{\frac{2}{3}}$

exotic fermions

Top partner limits:

\begin{itemize}
  \item e.g. $T_{\frac{2}{3}}$
  \item some sign dileptons
\end{itemize}

LHC (Run 1) : $m_T > 700 - 800$ GeV

Note $g_T$ constrained by Higgs mass

As lower limit on $m_T = g_T f$ increases $\Rightarrow f$ increases

$\Rightarrow$ tuning increases

(similar to stop mass limits in SUSY)
Higgs decays

Composite Higgs models can generate large $h \rightarrow Z\gamma$ rate

Dim 6 operators: [Giudice, Grooten, Pomarol, Rattazzi hep-ph/0703164]

\[ U_{h\bar{h}} = \frac{ig}{m_H} (D^\mu H)(D^\nu H) B_{\mu\nu} \]
\[ U_{h\bar{h}} = \frac{ig}{m_H} (D^\mu H) (D^\nu H) W_{\mu\nu} \]
not invariant under shift

\[ O_8 = \frac{g^2}{m^2} H^\dagger H G_{\mu\nu} G^{\mu\nu} \]
\[ O_7 = \frac{g^2}{m^2} H^\dagger H B_{\mu\nu} B^{\mu\nu} \]

\[ \begin{array}{c}
C_7 \Rightarrow h \rightarrow gg \\
C_8, C_9 \Rightarrow h \rightarrow \gamma\gamma \\
O_{h\bar{h}} - O_{h\bar{h}} \Rightarrow h \rightarrow Z\gamma \\
invariant under shift symmetry \Rightarrow \text{can be enhanced due to strong dynamics}
\end{array} \]

Ex. $SO(5)/SO(4)$: $U_{h\bar{h}} - O_{h\bar{h}}$ \{tree-level (spin-1 resonances) \{loop-level (composite fermions) \[Azezov et al 1308.2676 \]

Note: requires PRL breaking without contributing to $Z \rightarrow l\bar{l}\nu$
consistent with EWPT (since modified Higgs couplings contributes to $S$

Dark matter can be identified with a singlet scalar
requires nonminimal coset

e.g. $SO(6)/SO(5)$ has 5 NGBs = $4 + 1$

\[ \begin{array}{c}
singlet, \eta \\
Higgs doublet
\end{array} \]

Parity: $\eta \rightarrow -\eta$ from $O(6)/O(5)$

$\Rightarrow \eta$ stable (see Frigerio et al 1204.2809)
Gauge Coupling Unification [Ref: Agashev, Contino, Sundrum 2005]

Assume coset space $G/H$ and composite $t_R$.

Strong dynamics $\Rightarrow$ $(t_R, X^c) = \text{complete } H \text{ multiplet}$

extra fermion states in $H$ multiplet

To decouple $X^c$, introduce new top "companions" $X = \text{elementary fermions}$

Partial compositeness $\Rightarrow L = \lambda_X X^c \bar{\Phi}$

$\Rightarrow L = m_X X^c \bar{\Phi}$ where $m_X = \lambda_X f$ - Dirac mass term

Top companions split $H$ multiplet:

Elementary fermions $X$ provide new contribution to the running of SM gauge couplings:

$\alpha_i(\mu) - \alpha_j(\mu) = \text{SM} - \{H, t^c, \bar{t}^c\}$

$\text{composite Higgs, top}$

1-loop $\beta$ function coefficients:

$$b_1 - b_2 = \frac{94}{15} \quad \text{and} \quad b_3 = \frac{13}{3} \quad \Rightarrow \quad \frac{b_3}{b_1 - b_2} \approx 0.69 \quad (\text{cf. MSSM value } \approx 0.71)$$

Note 1: $H$ must contain $SU(5)$ $\Rightarrow$ composite sector contributes universally to running

2. Two-loop corrections can only be estimated

$$\frac{d\alpha_i}{d\ln \mu} = \frac{b_i}{2\pi} + \frac{B_{ij}}{2\pi} \alpha_j + \frac{C_{ijk} \lambda_k^2}{2\pi^3} \quad B \sim 9 \text{ top strong} \quad C \sim 3 \lambda_X \text{ top strong}.$$
**Flavor**

Partial compositeness: \[ \mathcal{L} = \lambda_i \overline{L}_i \Phi U_i^c + \lambda_i \overline{f}_i \Phi F_i \]

Assume \( \lambda_i \approx \lambda_j \approx O(1) \), i.e., no hierarchies in the couplings ("anarchic")

\( \Delta F = 1 \)

\[ b \rightarrow s \chi \quad \mathcal{L} \sim \frac{\lambda_3}{g^2} \frac{\lambda \chi}{m^2} \overline{u}_s \sigma^{\mu \nu} b \chi F^{\mu \nu} \quad \Rightarrow \quad f \gtrsim \frac{3-5 \text{ TeV}}{g_5} \quad \text{(depending on Re, Im parts)} \]

\[ \mathcal{L} \sim \frac{\lambda_3}{g^2} \frac{\lambda \chi}{m^2} \overline{s} \sigma^{\mu \nu} b \chi F^{\mu \nu} \quad \Rightarrow \quad f \gtrsim \frac{10 \text{ TeV}}{g_5} \]

[Note: Assumes free-level violation; Limits reduce by factor 10 for loop level]

\( \text{Re}(\epsilon'/\epsilon_k) \) \( \text{CP violation in } K^0 \rightarrow 2\pi \)

\[ \mathcal{L} \sim \frac{\lambda_3}{g^2} \frac{\lambda \chi}{m^2} \overline{s} \sigma^{\mu \nu} d_k G^{\mu \nu} \quad \Rightarrow \quad f \gtrsim \frac{13 \text{ TeV}}{g_5} \]

\( \Delta F = 2 \)

\[ \mathcal{L} \sim \frac{\lambda_3}{g^2} \frac{\lambda \chi}{m^2} \frac{1}{m^2} \left( \overline{d}_s \gamma^\mu f_i \right) \left( \overline{u}_k \gamma^\mu f_i \right) \]

**Neutron EDM**

\( \mathcal{O}_{\text{eff}} = C_{\text{eff}} \frac{m_e}{16 \pi^2} \overline{u}_s \gamma^\mu F_{\mu \nu} \gamma^5 d_k \quad \Rightarrow \quad f \gtrsim \frac{20-60 \text{ TeV}}{g_5} \quad (C_{\text{eff}} \neq 0) \)

\( \Rightarrow \quad f \gtrsim \frac{25-65 \text{ TeV}}{g_5} \quad (C_{\text{eff}} \neq 0) \)

**Lepton sector**

**Electron EDM**

\( \mathcal{O}_{\text{eff}} = C_{\text{eff}} \frac{m_e}{16 \pi^2} \overline{e}_s \gamma^\mu F_{\mu \nu} \gamma^5 e_k \quad \Rightarrow \quad f \gtrsim \frac{140 \text{ TeV}}{g_5} \)

**Muon EDM**

\[ \mathcal{L} = m_e \overline{f}_{\mu} \left( \frac{m_e \sigma^{\mu \nu} e_k}{\Lambda^2} + \frac{m_u \sigma^{\mu \nu} u_k}{\Lambda^2} \right) \quad \Rightarrow \quad f \gtrsim \frac{300 \text{ TeV}}{g_5} \]

To satisfy bounds clearly need additional structure

- e.g., flavor symmetry or heavy resonances (v.tuned)
## Comparison with SUSY models

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<td>(\lesssim 5%)</td>
<td>(\lesssim 0.5%) (MSSM) (3-5%) (low-scale SUSY breaking + NMSSM)</td>
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Summary

- Composite Higgs framework is an alternative solution to the hierarchy problem
  \[ \rightarrow \text{Higgs is a pseudo Nambu-Goldstone boson} \]

- Partial compositeness (via top quark) generates Higgs potential
  \[ \rightarrow 125 \text{ GeV Higgs} \Rightarrow \text{light (exotic) top partners } \gtrsim 700 \text{ GeV} \]
  \[ \rightarrow \text{Deviations in gauge boson/fermion couplings} \]
  \[ \rightarrow \text{Possibly large } h \rightarrow 2\gamma \]

- Incorporates dark matter & gauge coupling unification

- LHC Run 1 limits \( \Rightarrow \text{tuning} \leq 5\% \)

Future Directions/Questions

- Flavor bounds \( \Rightarrow f \gtrsim 10^{-100} \text{ TeV} \)
  \[ \rightarrow \text{partial compositeness only for 3rd generation?} \]
  \[ \text{or "split composite Higgs" (e.g. Bernard, TG, Santor, Gay, Spray: 1409.7391)} \]

- Composite twin Higgs \( \rightarrow \text{noncolored top partners} \)
  \[ \text{(ameliorates tuning)} \]
  \[ \text{(e.g. Barbieri et al 1501.07893 ; Lew, Tesi, Wang 1501.07890)} \]

- What is the underlying UV description?
  \[ \text{(for recent proposals see Bernard, TG, Santor, Gay 1311.6562}
  \text{Ferretti, Karateev 1312.5330)} \]