Beyond the Standard Model in the LHC Era
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With the discovery of the Higgs boson the Standard Model may be complete. As you have heard from Carena’s lectures, the limited information we currently have about the Higgs is consistent with its interpretation as a single Higgs doublet. We will learn much more over the next few years.

It seems likely that we have a complete description of nature up to scales of several hundred GeV or even higher. Thinking of the Standard Model as an effective field theory, we know all of the degrees of freedom and the values of the parameters; the cutoff is close to a TeV or even higher.

To date, there are no deviations from the resulting picture. So why aren’t we satisfied? And where, if we expect new phenomena, might we look for them?
**ATLAS Preliminary not in combination**

**CMS Preliminary not in combination**

\[ W, Z, H \rightarrow bb \]
\[ H \rightarrow \tau\tau \]
\[ H \rightarrow WW^{(*)} \rightarrow \ell\nu\ell\nu \]
\[ H \rightarrow ZZ^{(*)} \rightarrow llll \]
\[ H \rightarrow \gamma\gamma \]

**Combined**

**ATLAS**

- \( \mu = 0.2 \pm 0.7 \) A3*
- \( \mu = 1.4^{+0.5}_{-0.4} \) A2*
- \( \mu = 0.99^{+0.31}_{-0.28} \) A1
- \( \mu = 1.43^{+0.40}_{-0.35} \) A1
- \( \mu = 1.55^{+0.33}_{-0.28} \) A1
- \( \mu = 1.33^{+0.21}_{-0.18} \) A1

**CMS Prel.**

- \( \mu = 1.15 \pm 0.62 \) C1
- \( \mu = 1.10 \pm 0.41 \) C1
- \( \mu = 0.87 \pm 0.29 \) C6**
- \( \mu = 0.68 \pm 0.20 \) C1
- \( \mu = 0.92 \pm 0.28 \) C1
- \( \mu = 0.77 \pm 0.27 \) C1

**Figure:** Early stage LHC tests of Higgs particle
Mysteries of the Standard Model

Despite its successes, the Standard Model is deeply unsatisfying at several levels.

- The basic structure is mysterious. Why repetitive generations? Why not a simple group? [Why the degrees of freedom?]
- What determines the many couplings of the theory (17?) [From where the parameters? Why hierarchies among dimensionless numbers?]
- Why is CP such a good symmetry of the strong interactions? [Strong CP Problem]
- What accounts for the large hierarchies which we know must exist? $M_p = 2 \times 10^{18} \text{GeV}; M_{\text{gut}} = 2 \times 10^{16} \text{GeV}; M_\nu = 10^{14} \text{GeV}; M_H = 125 \text{ GeV}$. [“Hierarchy problem”: usually reserved for the small value of $M_H/M_W$]
- What constitutes the dark matter (no candidate within the Standard Model)?
- Dark energy: what is it? Why is there so little of it? Why is $M_{\text{gut}}$ so much larger than $M_H$?
Accelerator Experiments: discovery (Higgs, neutrino masses and mixings), exclusions (susy, extra dimensions, technicolor...); limits on rare processes

Non-Accelerator experiments: searches for dark matter (WIMPs, axions), limits on proton decay, neutrino properties.

Astrophysics experiments and observations: CMB, cosmic ray studies, supernovae... Features of inflation, dark energy, dark matter...
Frameworks for Theoretical Speculations (and formulating the questions and constraining the speculations!)

1. Field theory and Effective Field Theory
   - dynamics
   - symmetries – gauge, continuous (approximate) global, discrete

2. String theory (in a generalized sense, to be explained)

In these lectures, we will touch on some of these issues.
Plan for the Lectures

We want to take stock of where we are now. We will focus mostly on speculative ideas for solutions of the problems and questions we have enumerated above. Our time is limited and some of the topics are covered by other speakers.

Field Theory Viewpoint and Tools:

1. Naturalness principle
2. Dark energy/cosmological constant
3. Dynamical Solutions of the hierarchy problem
4. Supersymmetry and the Hierarchy Problem; problems and prospects
5. Strong CP: the nature of the problem, models in field theory, their problems. Axion dark matter and axion cosmology
Many of these questions require an “ultraviolet complete" theory. So we briefly look at the “view from the mountaintop": string theory

1. Generalities about string theory: its successes and limitations.
2. Some general lessons for Beyond the Standard Model Physics
   (a.) Axions – problems
   (b.) Absence of global symmetries
   (c.) Natural inflation – problematic
3. More speculative: moduli, role in cosmology, inflatons?
4. Still more speculative: landscape. Implications for naturalness?
Our current theories of the laws of nature are best viewed as tentative, *effective* field theories, valid at energies below some scale at which new degrees of freedom or other phenomena might manifest themselves. Naturalness, from this perspective, is the assertion that features of this effective field theory should not be extremely sensitive to the structure of the underlying theory.
Familiar examples include the electron mass in QED. QED requires a cutoff for its definition, $\Lambda$. Dimensional analysis suggests $m_e \approx \Lambda$. But this is not the case. Perturbative corrections are small, as you heard in Shadmi’s lectures:

$$\delta m_e = \frac{3}{4\pi} m_e^{(0)} \log(\Lambda/m_e)$$  \hspace{1cm} (1)

This is because the theory, in the limit $m_e \to 0$, becomes more symmetric. In four component language:

$$\psi \to e^{i\omega \gamma^5} \psi$$  \hspace{1cm} (2)

is a symmetry. In two components:

$$e \to e^{i\omega} e \quad \bar{e} \to e^{i\omega} \bar{e}$$  \hspace{1cm} (3)
't Hooft gave a sharp statement of this *Naturalness Principle*: a constant of nature (a quantity appearing in an (effective) lagrangian should be small only if the theory becomes more symmetric as that quantity tends to zero.

Examples in the SM include: the quark and lepton Yukawa couplings. Theory has a big symmetry if these vanish ($SU(3)^5$).

Mass scales in QCD ($m_p, ..$) vanish as $\Lambda_{qcd} = e^{-\frac{8\pi^2}{b_0 g^2}}$ In limit $g \rightarrow 0$, theory has an additional symmetry, scale (conformal) invariance.
There are several quantities in the SM which fails ’t Hooft’s test. The first is the mass of the Higgs particle, which is tied to the scale at which the symmetry of the electroweak theory is broken. In the simplest version of the SM, the potential of the Higgs field is

\[ V(\phi) = -\mu^2 |\phi|^2 + \frac{\lambda}{4} |\phi|^4. \]  

(4)

Assuming that this potential describes the recently observed Higgs particle (and measurements to date are consistent with this picture), we know the values of \( \mu \) and \( \lambda \):

\( \mu \approx 89 \text{ GeV}; \ \lambda \approx 0.13. \)
Dimensional analysis would predict $\mu^2 \approx M_p^2$, and there is no enhancement of the symmetry of the theory if we take $\mu^2 \to 0$. The strongest coupling of the Higgs field in the SM is its Yukawa coupling to the top quark: $\mathcal{L}_{ttH} = y_t H Q_3 \bar{t}$
This is given by:

$$\delta \mu^2 = -6y_t^2 \int_0^\Lambda \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} = - \frac{6y_t^2}{16\pi^2} \Lambda^2. \tag{5}$$
Yukawa couplings and their hierarchies are consistent with ’t Hooft’s naturalness principle. There is one small dimensionless parameter in the SM which appears to violate ’t Hooft’s condition.

\[ \mathcal{L}_\theta = \frac{\theta}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}. \]  

\[ \tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}. \]
A total derivative; does not affect equations of motion (easy to check in QED – exercise; also true in non-abelian theories). However, it does have physical effects. Using current algebra one can compute the electric dipole moment of the neutron, $d_n$:

$$d_n = 5.2 \times 10^{-16} \theta \text{ cm.}$$

(7)

$d_n < 3 \times 10^{-26} \text{ e cm}$ so $\theta < 10^{-10}$. If nature respected CP in the absence of $\theta$, this small value of a dimensionless number would be natural in the sense of ’t Hooft. But nature violates CP; indeed, the phase appearing in the CKM matrix is of order one. So, like the Higgs mass, this number cries out for an explanation.
A cosmological constant is a dimension zero term in the effective action, even more problematic than the dimension two Higgs mass term:

\[ \mathcal{L}_\Lambda = \int d^4x \sqrt{g} \Lambda. \quad (8) \]

Assuming that the observed dark energy is a cosmological constant, we have \( \Lambda \approx 10^{-47} \text{ GeV}^4 \). Why not \( \Lambda \approx M_p^4 \), roughly 120 orders of magnitude larger?
This estimate is reinforced by a simple-minded calculation in field theory. Even if the vacuum energy vanishes classically, there is a quantum contribution to the energy, which is just a sum of the zero point energy for bosons and the energy of the filled Dirac sea for fermions,

\[ \Lambda = \sum_{\text{helicities}} (-1)^F \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m^2}. \] (9)

Here \((-1)^F\) is +1 for bosons and −1 for fermions. Each term in the sum is quartically divergent. Taking \(M_p\) as the cutoff yields the naive estimate.
In the case of supersymmetric theories, things are somewhat better. The number of bosonic and fermionic degrees of freedom is the same, and the leading divergence cancels. But one gets a result proportional to the fourth power of the supersymmetry-breaking scale. Even for the lowest conceivable SUSY breaking scale (TeV), this is many orders of magnitude larger than the observed dark energy.

Global SUSY; even with susy breaking, vacuum energy divergence is only logarithmic; \((-1)^F m^2 = 0\). (Locally: \(m_{3/2}^2\), so \(m_{3/2}^2 \Lambda^2 |F|^2\).
There is no proposal to understand the small value of the dark energy in 'thooftian terms; General Relativity does not become more symmetric as $\Lambda \to 0$. Calculations in string theory, the only framework we have where the dark energy may be calculable, are consistent with expectations based on dimensional analysis.

The value of the c.c. is remarkable in another way. While small in particle physics units, it is substantial in units relevant to the present cosmological epoch; indeed, the c.c. has just become important "recently" (the past few billion years), and it will dominate the energy density "forever".

These three naturalness problems motivate many speculations about physics Beyond the Standard Model. In the rest of these lectures, we will consider possible implications of each.
Technicolor was the first proposal to understand the hierarchy problem. Closely parallels the understanding of the hierarchy between the proton mass and the Planck scale. They proposed that electroweak symmetry breaking arises due to a condensate of fermions in some new strong interactions, similar to QCD but with a scale of order 1 TeV. Susskind dubbed this solution *technicolor*. 
Consider the SM without the Higgs particle, and with only a single generation of quarks and leptons

\[ Q = \begin{pmatrix} u \\ d \end{pmatrix}; \quad \bar{u} \quad \bar{d}; \quad L = \begin{pmatrix} \nu \\ e \end{pmatrix}; \quad \bar{e}. \] (10)

The theory possesses a global symmetry

\[ SU(2)_L \times SU(2)_R \times U(1) \times U(1). \] 

\( SU(2)_L \) is just the \( SU(2) \) of weak interactions; \( SU(2)_R \) is an approximate symmetry under which \( \bar{u} \) and \( \bar{d} \) transform as a doublet. The \( U(1) \) of the SM is a combination of the diagonal generator of the \( SU(2)_R \) as well as one of these \( U(1) \)'s. The strong interactions break the symmetry to the diagonal subgroup, (isospin), as well as a \( U(1) \).

\[ \langle \bar{q}_f q_g \rangle = \Lambda^3 \delta_{fg} \] (11)
Because the $SU(2)_L \times U(1)$ subgroup of this symmetry is gauged, the $W$ and $Z$ gain mass, and the photon remains massless. Using the non-linear lagrangian description of chiral symmetry breaking, where the pions are described by a matrix of fields with a simple transformation property under the $SU(2)_L \times SU(2)_R$:

$$\Sigma = e^{i \frac{\pi^a \sigma^a}{2f_\pi}} ; \quad \Sigma \rightarrow U_L \Sigma U_R$$  \hspace{1cm} (12)

The lagrangian for $\Sigma$ is:

$$\mathcal{L}_\Sigma = f_\pi^2 \text{Tr} \left( D_\mu \Sigma D_\mu \Sigma^\dagger \right).$$  \hspace{1cm} (13)

One finds (exercise!) that the gauge boson masses are just those of the SM, with the Higgs expectation value, $v$, replaced by $f_\pi$. 
The technicolor hypothesis just replaces the ordinary quarks by techniquarks, and color by a new interaction, \( f_\pi \rightarrow F_{TC} = \nu \). This theory solves the hierarchy problem both in the sense that there are no longer quadratic divergences (loosely the divergences are cut off at the technicolor scale), and also in that it provides an explanation of the weak scale, analogous to the QCD explanation of the proton mass: 

\[
F_{tc} = Me - \frac{8\pi^2}{b_{tc}g_{tc}(M)^2}.
\]
While a beautiful idea, this proposal runs into a number of difficulties. First, in this simple form, it has no mechanism to account for the masses of quarks and leptons. One can try to resolve this problem by introducing further gauge interactions, whose role is to break the chiral symmetries which protect fermion masses. The resulting models are quite baroque, requiring many gauge groups and intricate dynamics, but aesthetic objections aside, they run into serious issues with flavor changing neutral current processes. Put simply, the Standard Model possesses a variety of approximate symmetries due to small quark masses, and these account, for example, for the small rate for $K \leftrightarrow \bar{K}$ mixing; it is difficult to mimic this phenomenon in a strongly interacting theory.
Prior to the Higgs discovery, other serious problems have long been noted, especially difficulties with precision studies of the Standard Model. The existence of a Higgs much lighter than 1 TeV, and with width less than a few GeV, is particularly difficult to understand in a Technicolor framework. Most proposals to understand this assume that the technicolor theory is nearly conformal over a range of scales, with a light, SM-like Higgs a consequence.

This and other dynamical proposals will appear in Tony Ghergetta’s lectures.
MSSM: A supersymmetric generalization of the SM.

1. Gauge group $SU(3) \times SU(2) \times U(1)$; corresponding (12) vector multiplets.
2. Chiral field for each fermion of the SM: $Q_f, \bar{u}_f, \bar{d}_f, L_f, \bar{e}_f$.
3. Two Higgs doublets, $H_U, H_D$.
4. Superpotential contains a generalization of the Standard Model Yukawa couplings:

$$W_y = y_U H_U Q \bar{U} + y_D H_D Q \bar{D} + y_L H_D \bar{E}. \quad (14)$$

$y_U$ and $y_D$ are $3 \times 3$ matrices in the space of generations.
Soft Breaking Parameters

Need also breaking of supersymmetry, potential for quarks and leptons. Introduce *explicit soft breakings*:

1. **Soft mass terms for squarks, sleptons, and Higgs fields:**

   \[ \mathcal{L}_{\text{scalars}} = Q^* m_Q^2 Q + \bar{U}^* m_U^2 \bar{U} + \bar{D}^* m_D^2 \bar{D} \]
   \[ + L^* m_L^2 L + \bar{E}^* m_E \bar{E} \]
   \[ + m_{H_U}^2 |H_U|^2 + m_{H_D}^2 |H_D|^2 + B_\mu H_U H_D + \text{c.c.} \]

   \( m_Q^2, m_U^2, \) etc., are matrices in the space of flavors.

2. **Cubic couplings of the scalars:**

   \[ \mathcal{L}_A = H_U Q A_U \bar{U} + H_D Q A_D \bar{D} \]
   \[ + H_D L A_E \bar{E} + \text{c.c.} \]

   The matrices \( A_U, A_D, A_E \) are complex matrices.

3. **Mass terms for the U(1) \((b)\), SU(2) \((w)\), and SU(3) \((\lambda)\) gauginos**

   \[ m_1 bb + m_2 ww + m_3 \lambda \lambda \]
1. $\phi\phi^*$ mass matrices are $3 \times 3$ Hermitian (45 parameters)
2. Cubic terms are described by 3 complex matrices (54 parameters)
3. The soft Higgs mass terms add an additional 4 parameters.
4. The $\mu$ term adds two.
5. The gaugino masses add 6.

There appear to be 111 new parameters.
But Higgs sector of SM has two parameters. In addition, the supersymmetric part of the MSSM lagrangian has symmetries which are broken by the general soft breaking terms (including $\mu$ among the soft breakings):

1. Two of three separate lepton numbers
2. A “Peccei-Quinn” symmetry, under which $H_U$ and $H_D$ rotate by the same phase, and the quarks and leptons transform suitably.
3. A continuous "$R$" symmetry, which we will explain in more detail below.

Redefining fields using these four transformations reduces the number of parameters to 105.
Constraints

Direct searches (LEP, Fermilab, now LHC) severely constrain the spectrum. E.g. squark, gluino masses $> \mathrm{TeV}$, charginos of order 100's of GeV. Spectrum must have special features to explain

1. Absence of Flavor Changing Neutral Currents (suppression of $K \leftrightarrow \bar{K}$, $D \leftrightarrow \bar{D}$ mixing, $\mu \rightarrow e + \gamma, \ldots$)

2. Suppression of $CP$ violation ($d_n$; phases in $K\bar{K}$ mixing).

Might be accounted for if spectrum highly degenerate, $CP$ violation in soft breaking suppressed. Or if scale of soft masses is large. E.g. for susy scale 30 TeV, the constraints from $K$ meson physics are quite mild. But constraints from $\mu \rightarrow e + \gamma$ are still large, require suppression of off-diagonal terms in mass matrices.
Low energy supersymmetry as a solution to the hierarchy problem

Two aspects:

1. Cancellation of quadratic divergences (illustrated in Yael Shadmi’s lectures).
2. Supersymmetric theories susceptible to appearance of large ratios of scales.
We will begin by focussing on the first of these, and consider the Higgs mass in supersymmetric theories and questions of fine tuning or naturalness, especially in light of the measured Higgs mass. We will simply take the soft breaking masses as parameters, subject to experimental constraints.

Rather than write a general formula for the Higgs mass, which you can find in the reviews Yael cited, let’s work out the mass in a particular limit, which happens to give the largest possible value. In:

\[ V(H_U, H_D) = m_1^2 |H_U|^2 + m_2^2 |H_d|^2 + m_3^2 H_U H_D + \text{c.c.} \]

\[ + \frac{g^2}{8} (H_U^* \tau^a H_U - H_D^* \tau^a H_D)^2 + \frac{g'^2}{8} (H_U^* H_U - H_D^* H_D)^2 \]

take

\[ m_1^2 < 0, m_2^2 > 0, m_1^2, m_3^2 \ll m_2^2 \]

The \[ < H_U > \gg < H_D > \]. \[ \tan \beta \sim \frac{m_3^2}{m_2^2} \].
This is referred to as the *decoupling limit*. The quartic coupling of the Higgs \( H_U \) is determined entirely by the \( D^2 \) terms in the potential, which are completely fixed in terms of the gauge couplings. You can readily check that the physical Higgs mass, in this limit, is \( M_Z^2 \). This is, in fact, a rigorous upper bound on the Higgs mass at tree level.

Fortunately (for supersymmetry partisans) this is not the end of the story; quantum corrections can be large.
At one loop, there is a correction to the Higgs mass:

$$\delta \lambda = \frac{12y_t^4}{16\pi^2} \log\left(\frac{\tilde{m}_t}{m_t}\right)$$  \hspace{1cm} (18)

More detailed studies yield results of the sort shown in the figure.
\( m_H \approx 125 \text{ GeV}, \) requires that the stop be quite heavy, 8 TeV or more (alternatively one can tune “A-Parameter” and obtain a lower stop mass). This has troubling implications for naturalness.

In addition to the top quark loop, there is now a loop containing a stop which tames the quadratic divergence of the SM.

**Figure**: Additional correction to Higgs mass from stops.
For simplicity, calling the mass of each of these scalars $\tilde{m}_t^2$, gives

$$\delta m_H^2 = 3y_t^2 \int \frac{d^4 k}{(2\pi)^4} \left( -\frac{1}{k^2 + m_t^2} + \frac{1}{k^2 + \tilde{m}_t^2} \right). \quad (19)$$

The minus sign in the first term is the usual minus sign in field theory for fermion loops. The leading quadratic divergence cancels, leaving only a logarithmically divergent term:

$$\delta m_H^2 = -\frac{3y_t^2}{16\pi^2} \tilde{m}_t^2 \log(\Lambda^2 / \tilde{m}_t^2). \quad (20)$$

Here $\Lambda$ is an ultraviolet cutoff, and we have assumed $m_t^2 \ll \tilde{m}_t^2$, consistent with exclusions from the Large Hadron Collider (LHC), which we will discuss shortly. This is closely parallel to the situation for the electron mass in QED.
If we substitute 8 TeV on the right hand side for $\tilde{m}_t$, and take $\Lambda = 10^{16}$ GeV, then we have that the correction to the Higgs mass parameter is of order $10^4 M_Z^2$, a tuning of parameters of a part in $10^4$. 
Problem in MSSM: quartic coupling is small. Add a singlet, \( S \), with

\[
W = \lambda S H_U H_D + \ldots
\]

so have \( |\lambda H_U H_D|^2 \) in potential. But \( \lambda \) limited if require perturbative up to high scales (\( \lambda < 0.7 \) roughly), which helps somewhat, but don’t get large radiative corrections in regime of small \( \beta \), where this effect is largest. So some improvement in fine tuning, but still tuned.

Other ideas of this sort include “non-decoupling D terms.”
This leaves us with questions:

1. How much fine tuning is too much? Should we give up on supersymmetry? If we modify our theories in some way, can we reduce the tuning? Are things just somewhat tuned, with supersymmetry at 10 TeV?

2. What is the effect of this higher scale on some of the flavor issues and other constraints?
Under certain circumstances (so-called “anomaly mediation”) gaugino masses lighter by a loop factor than squarks, sleptons. Picture where all squarks and sleptons at 100 TeV, say, while gauginos at few TeV or less. Still tuned, but virtues that couplings still unify, dark matter candidate. Problems of flavor are significantly reduced.
Non-Renormalization Theorems and the Susceptibility of Supersymmetry to Dynamical Breaking

Quite generally, supersymmetric theories have the property that, if supersymmetry is not broken at tree level, then to all orders of perturbation theory, there are no corrections to the superpotential and to the gauge coupling functions. These theorems were originally proven by examining detailed properties of Feynman diagrams, but they can be understood far more simply. Consider

\[ W = \frac{m}{2} \phi^2 + \frac{\lambda}{3} \phi^3. \]  

(21)

Suppose first \( \lambda = 0 \). Then \( R \) symmetry, \( R_\phi = 1 \). Adding \( \lambda \), think of as a (vev of) superfield. \( R_\lambda = -1 \). \( W \) holomorphic. So only \( \lambda \phi^3 \) allowed, i.e. *no corrections in powers of \( \lambda \).* (Kinetic terms...
This has a generalization to gauge theories. Write

\[ \mathcal{L} = -\frac{1}{32\pi^2} \int d^3 \theta \tau W^2_{\alpha}. \]  

(22)

\[ \tau = \frac{8\pi^2}{g^2} + i\theta. \]

In perturbation theory, \( \tau \rightarrow \tau + i\alpha \) is a symmetry \( \Leftarrow W \) independent of \( \tau \) (but gauge coupling renormalization?).
Beyond perturbation theory:

\[ \langle \lambda \lambda \rangle = \Lambda^3 = e^{-\tau/N} \]  

Since

\[ \mathcal{L} = \int d^2 \theta W^2_\alpha \]  

this correspondence to the appearance of a (constant) superpotential. Violates the perturbative nr theorems.
Beyond perturbation theory, however, these theorems break down. So have the possibility that

\[ E = c e^{-\frac{8\pi^2}{g^2}} \]

and very large hierarchies as a result.

Many examples known, but seem somewhat special (require chiral fields with special properties). In last few years, it has become clear that \textit{metastable} supersymmetry breaking is generic.
Nelson-Seiberg theorem: generically supersymmetry breaking requires a continuous $R$ symmetry, as in O’Raifeartaigh Model. But don’t expect global symmetries in nature (more later).
Aproximate Continuous R Symmetries from Discrete Symmetries

OR model:

\[ W = X_2(A_0^2 - f) + mA_0 Y_2 \]  \hspace{1cm} (25)

(subscripts denote \( R \) charges). If, e.g., \( |m^2| > |f| \), \( F_X = f \).

Can arise as low energy limit of a model with a discrete \( R \) symmetry:

\[ X_2 \rightarrow e^{2\pi i N} X_2; \quad Y_2 \rightarrow e^{2\pi i N} Y_2; \quad A_0 \rightarrow A_0. \]  \hspace{1cm} (26)

Allows \( \delta W = \frac{X^{N-n} Y^{n+1}}{M_p^{N-2}} \). \( N \) susy vacua far away. Approximate, accidental \( R \) symmetry. SUSY breaking metastable.
This structure becomes dynamical if \( f, m \) expectation values of dynamical fields, e.g.

Gaugino condensation: \( \langle \lambda \lambda \rangle X_2 \) generates \( f \) when \( \langle \lambda \lambda \rangle \) condenses ("gaugino condensation"). "Retrofitting".
Usual approach to model building: some set of fields (O’Raifeartaigh Model, model of DSB) break susy. “Communicated” to fields of MSSM by:

1. (Super) gravity interactions: scales $F/M_p = m_{3/2}$ for effective size of supersymmetry breaking.

2. Gauge Mediation: Fields with non-vanishing $F$ components couple to ordinary gauge interactions, which in turn couple to the fields of the MSSM.

Both tuned at this point. Each has virtues, drawbacks.
Supergravity (brief)

In units with $M_p = 1$ (here $M_p$ is the reduced Planck mass, approximately $2 \times 10^{18}$ GeV):

$$V = e^K \left[ D_i W g^{i\bar{i}} D_{\bar{i}} W^* - 3 |W|^2 \right]. \quad (27)$$

$D_i W \equiv F_i$ is the order parameter for susy breaking:

$$D_i W = \frac{\partial W}{\partial \phi_i} + \frac{\partial K}{\partial \phi_i} W. \quad (28)$$

If supersymmetry is unbroken, space time is Minkowski (if $W = 0$), It is AdS if ($W \neq 0$). If supersymmetry is broken and space is approximately flat space ($\langle V \rangle = 0$), then

$$m_{3/2} = \approx \langle e^{K/2} W \rangle. \quad (29)$$
Simplest realizations of SUSY: tuned at $10^2$ level or worse

Large masses, however, ameliorate flavor problems.

Well understood mechanisms to dynamically break the symmetry

Large hierarchies contemplated
1. The Strong CP Problem
2. Possible Solutions
3. Axion Models
4. Astrophysical Constraints on Axions
5. Cosmology of Axions: conventional
6. Theory of axion detection
7. The Problem of Axion Quality
8. Axions in String Theory
9. Cosmology of Axions: non-conventional
10. Concluding thoughts on likelihood of axions, where to look
QCD a well-understood theory. One outstanding puzzle: can add to QCD lagrangian

$$\mathcal{L}_\theta = \frac{\theta}{16\pi^2} G\tilde{G}.$$  \hspace{1cm} (30)

Here

$$\tilde{G}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}; \quad G\tilde{G} = 2\vec{E} \cdot \vec{B}.$$  \hspace{1cm} (31)

This term is a total derivative (easy to see in QED), and it is tempting to ignore it. But it turns out to have physical effects, and it violates \textit{parity}, and thus CP.
The possibility of a non-zero $\theta$ is related to another possible source of $CP$ violation: $\gamma_5$ terms in the quark lagrangian (in the language of four component fermions; in terms of two component fermions, this corresponds to complex masses). For a single quark, the issue is a term of the form:

$$\mathcal{L}_m = m(\cos(\theta)\bar{q}q + i \sin \theta \bar{q}\gamma_5 q).$$  \hspace{1cm} (32)$$

The use of $\theta$ here is not an accident; a transformation $q \to e^{-i\frac{\theta}{2}\gamma_5}q$ gets rid of the would-be $CP$-violating mass term, but at the price – as a result of the anomaly – of inducing a $\theta G\tilde{G}$ term of the type described above.
Anomaly in the current $j^\mu = \bar{q} \gamma^\mu \gamma^5 q$

\[
j^\mu = \bar{q} \gamma^\mu \gamma^5 q
\]

\[
\partial_\mu j^\mu = m\bar{q}q + \frac{g^2}{16\pi^2} F \tilde{F}
\]
One can show, using the properties of the chiral lagrangian (current algebra) that $\theta$ leads to an electric dipole moment for the neutron:

$$d_n = g_{\pi NN} \frac{\theta m_u m_d}{f_\pi (m_u + m_d)} \langle N_f | \bar{q} \gamma^a q | N_f \rangle \ln(m_p/m_\pi) \frac{1}{4\pi^2 m_p}$$

$$= 5.2 \times 10^{-16} \theta \text{cm}$$

(this is calculated in an approximation which becomes more and more reliable as the masses of the light quarks become smaller).

From the experimental limit, $d_n < 3 \times 10^{-26} \text{ e cm}$, one has $\theta < 10^{-10}$. 
This is a puzzle. Why such a small dimensionless number?

$\theta \to 0$: strong interactions preserve CP. If not for the fact that the rest of the SM violates CP, would be \textit{natural}.
1. $m_u = 0$ If true, $u \rightarrow e^{-i \frac{\theta}{2} \gamma^5} u$ eliminates $\theta$ from the lagrangian. An effective $m_u$ might be generated from non-perturbative effects in the theory (Georgi, McArthur; Kaplan, Manohar) Could result as an accident of discrete flavor symmetries (Banks, Nir, Seiberg), or a result of “anomalous” discrete symmetries as in string theory (M.D.)

2. CP exact microscopically, $\theta = 0$; spontaneous breaking gives the CKM phase but leads, under suitable conditions, to small effective $\theta$ (Nelson, Barr). In critical string theories, CP is an exact (gauge) symmetry, spontaneously broken at generic points in typical moduli spaces. A plausible framework.
Problems with each of these solutions:

1. $m_u = 0$. Lattice computations seem to rule out (the required non-perturbative effects do not seem to be large enough).

   \[ m_u = 2.15(15) \text{ MeV}, \quad m_d = 4.7(20) \text{ MeV}, \quad m_s = 93.5(2.5) \text{ MeV}, \]
   \[ m_c \approx 1.15 - 1.35 \text{ GeV}, \quad m_b \approx 4.1 - 4.4 \text{ GeV}, \quad m_t \approx 174.3 \pm 5 \text{ GeV}. \]

2. Spontaneous CP: special properties required to avoid large $\theta$ once CP is spontaneously broken. What would single out such theories?

3. Axions: Our focus today. We will see promise and limitations.
In a somewhat streamlined language, the Peccei-Quinn proposal was to replace $\theta$ by a dynamical field: $\theta \rightarrow \frac{a(x)}{f_a}$

It is assumed that $a \rightarrow a + \omega f_a$ is a good symmetry of the theory, violated only by effects of QCD. Without QCD, $\theta$ can take any value.

In QCD by itself, the energy is necessarily stationary when

$$\theta_{\text{eff}} = \left\langle \frac{a}{f_a} \right\rangle = 0.$$  \hfill (34)

This is simply because $CP$ is a good symmetry of QCD if $\theta = 0$, so the vacuum energy (potential) must be an odd function of $\theta$. 
One can do better, calculating, again using what we know about chiral symmetry in QCD, the axion potential:

\[ V(a) = -m_\pi^2 f_\pi^2 \frac{\sqrt{m_u m_d}}{m_u + m_d} \cos\left(\frac{a}{f_a}\right) \]  

(35)

This gives, for the axion mass:

\[ m_a = 0.6 \text{ meV} \left(\frac{10^{10} \text{ GeV}}{f_a}\right) . \]  

(36)
Peccei and Quinn actually constructed a model for this phenomenon, which was a modest extension of the Standard Model with an extra Higgs doublet. They didn’t phrase the problem in quite the way I did above, and didn’t appreciate that their model had a light, pseudoscalar particle, $a$. This was quickly recognized by Weinberg and Wilczek, who calculated its mass and the properties of its interactions. It quickly become clear that the original axion idea was not experimentally viable.
The Invisible Axion

But in the more general picture described above, the problems with the axion are easily resolved. The strength of the axion’s interactions are proportional to $1/f_a$. This is because of the Peccei-Quinn symmetry. The symmetry requires that axion interactions appear only with derivatives of the axion field; on dimensional grounds, these come with powers of $\partial_\mu / f_a$ (momenta $q_\mu / f_a$). QCD terms which break the symmetry also come with powers of $1/f_a$. So if $f_a$ is large enough, the axion will be hard to detect (it becomes “harmless” or “invisible”).

The scale, $f_a$, might be associated with some high scale of physics ($M_{\text{gut}}$? $M_p$? – more later).
Sample couplings

1. Axion to two photons (notation of PDG):

$$\mathcal{L}_{\gamma\gamma} = \frac{1}{4} G_{a\gamma\gamma} \ a \ F \tilde{F}$$  \hspace{1cm} (37)

where now $F$ is the \textit{electromagnetic} field strength.

$$G_{a\gamma\gamma} = \frac{\alpha}{2\pi} \left( \frac{E}{N} - \frac{4}{3} \frac{4 + z}{1 + z} \right) \frac{1 + z}{\sqrt{z}} \frac{m_a}{m_\pi f_\pi} \ z = \frac{m_u}{m_d}$$  \hspace{1cm} (38)

$E, \ N$ are the electromagnetic and QCD anomalies.

2. Axion to quarks, leptons:

$$\mathcal{L}_{\text{aff}} = \frac{C_f}{2f_a} \bar{\psi}_f \gamma^\mu \gamma^5 \psi_f \ \partial_\mu a.$$ \hspace{1cm} (39)

The detailed coefficients depend on the model.
Two Benchmark models

**DFSZ**

Add to the Standard Model an additional Higgs doublet (e.g. as in supersymmetry), i.e. two doublets, $H_u, H_d$, plus a singlet, $\phi$. Impose the Peccei-Quinn symmetry:

$$\phi \rightarrow e^{i\alpha} \phi; \quad H_u \rightarrow e^{-i\frac{\alpha}{2}}H_u; \quad H_d \rightarrow e^{-i\frac{\alpha}{2}}H_d$$  \hspace{1cm} (40)

Require potential such that

$$\langle \phi \rangle = \frac{f_a}{\sqrt{2}}.$$  \hspace{1cm} (41)

This breaks the PQ symmetry spontaneously.

(Pseudo-)Goldstone boson:

$$a = \sqrt{2}\text{Im} \phi + \frac{v_u}{f_a} \text{Im}H_U^0 + \frac{v_d}{f_a} \text{Im}H_D^0.$$
a couples to $G\tilde{G}$, $F\tilde{F}$. Also couples to leptons, quarks.

\[
\frac{E}{N} = \frac{8}{3}; \quad C_e = \frac{\cos^2 \beta}{3} \quad \tan \beta = \frac{\langle H_u \rangle}{\langle H_d \rangle}.
\]  

(42)

As expected, as $f_a$ becomes large, the axion’s interactions with other particles become weaker. Once $f_a \gg \frac{\text{GeV}}{G_F}$, unobservable in accelerator experiments.
KSVZ Model

Here one has a field, $\phi$, and a new quark, $q$ and $\bar{q}$, which will be very heavy. $q$ and $\bar{q}$ are assumed to carry color but to be $SU(2) \times U(1)$ singlets. In two component language, the Peccei-Quinn symmetry is assumed to be

$$\phi \rightarrow e^{i\alpha} \phi \quad q \rightarrow e^{-i\frac{\alpha}{2}} q' \quad \bar{q} \rightarrow e^{-i\frac{\alpha}{2}} \bar{q}. \quad \mathcal{L}_{\phi\bar{q}q} = \lambda \phi \bar{q}q$$

(43)

$\phi$ is assumed to have an expectation value:

$$\langle \phi \rangle = \frac{f_a}{\sqrt{2}}. \quad (44)$$

The imaginary part of $\phi$ is the axion:

$$\phi = \frac{1}{\sqrt{2}} f_a + ia.$$ 

(45)
But these are just two of a wealth of possible modes, characterized by the coefficients above. These two, however, are often used as benchmarks to characterize the capabilities of different experimental detection schemes, as well as to illustrate the range of possible astrophysical phenomena.
Axion interactions are “semi weak”, in the sense that cross sections go as $1/f_a^2$, as opposed to weak interactions which behave as $1/v^4$. So even for large $f_a$, reaction rates can be comparable to those for neutrinos. This raises a worry about stars, where various processes can produce axions. If interaction rates are large compared to those for neutrinos, excessive amounts of energy will be carried off by axions. More detailed studies in particular astrophysical environments place lower limits on $f_a$. 
Sources of Astrophysical Constraints

Partial list:

1. The sun
2. Red Giants, Globular Clusters
3. SN 1987a
4. White dwarfs
Primakoff process, axion bremsstrahlung.
Axion Luminosity

In sun:

\[ L_a = G_{a\gamma\gamma}^2 \times 1.85 \times 10^{17} L_\odot \]  \hspace{1cm} (46)

so

\[ G_{a\gamma\gamma} < 7 \times 10^{-10}. \]  \hspace{1cm} (47)

Stronger constraint from globular clusters, 7 → 1.
Paradoxically, because the axion is so weakly interacting, it can play a significant role in the early universe.

In an FRW universe:

$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0 \quad (48)$$

$a$ is overdamped for $H > m_a$; oscillates for $H < m_a$.

Because there is nothing special about the point $a = 0$, initially the axion might be a homogeneous field, with non-zero $a = a_0$ ($\theta = \theta_0$) (more about this assumption later).
$m_a$ is small; e.g. for

$$f_a = 10^{16}\text{GeV}, \quad m_a = 10^{-9}\text{eV}; \quad f_a = 10^{11}\text{GeV} \quad m_a = 6 \times 10^{-4}\text{eV}. \quad (49)$$

$H = 10^{-9}, 10^{-4}\text{eV}$ when the temperature of the universe is about $1, 10^2\text{GeV}$. This is late compared to, e.g., the likely times of inflation.
Solving the axion equation of motion

\[ \ddot{\theta} + 3 \dot{H} \dot{\theta} + m_a^2 \theta = 0 \]  
(50)

Seek a solution of form, for large \( t \):

\[ \theta(t) = \theta_0 f(t) \cos(m_a t) \]  
(51)

with \( f(t) \) slowly varying. For radiation/matter dominated eras:

\[ \begin{align*}
H &= \frac{1}{2t}; f = \left( \frac{t_0}{t} \right)^{3/4} \\
H &= \frac{2}{3t}; f = \left( \frac{t_0}{t} \right)
\end{align*} \]
(52)

Each of these solutions falls off as \( 1/R(t)^3 \). In other words, the system behaves like pressure-less dust, a collection of zero momentum axions.
When the axion starts to oscillate, it constitutes a fraction of the energy density of order $\theta_0^2 \frac{f_a^2}{M_p^2}$; with $f_a = 10^{11}$ this is about $10^{-14}$.

During the radiation dominated era, the energy density in matter (axions) falls off as $T^3$, as opposed to the radiation, which falls off as $T^4$. The usual matter-radiation equality occurs for $T \approx \text{eV}$. At this time, the energy density in axions, indeed, is of order the energy density in radiation.

Larger $f_a$: matter domination too early. Smaller: axions only a fraction of the dark matter.
So indeed for $f_a \approx 10^{11}$ GeV and $\theta_0 \approx 1$, the axions come to dominate the energy density of the universe at the approximate time of matter-radiation equality. More careful calculation takes into account the temperature dependence of the axion mass, and yields:

$$\Omega_a h^2 = 0.11 \theta_0^2 \left( \frac{f_a}{5 \times 10^{11} \text{ GeV}} \right)^{1.184}.$$  \hspace{1cm} (53)

So from the combination of astrophysical and cosmological considerations, the axion decay constant/mass lies in a rather narrow range. At the high end of this range, the axion constitutes the dark matter.
So far, we have assumed that the axion, initially, takes the same value throughout the universe. Within the framework of inflationary big-bang cosmology, this only makes sense if the Peccei-Quinn transition (the “turning on” of the expectation value of $\phi$) occurred before inflation, so that the initial value of the field is the same everywhere in the observable universe.

If this is not the case, the production of axions leads to similar limits, but involves more complicated processes, including the production of axion strings.
For $f_a > 10^8$ GeV, the axion is extremely weakly interacting. In scattering experiments, it is produced rarely and detection is essentially impossible.

However, if we assume that the axion constitutes the dark matter, we are living in a sea of axions, and we might hope to detect them. The main interaction at our disposal is the interaction with the electromagnetic field characterized by $G_{a\gamma\gamma}$: In particular, in a strong magnetic field, an axion can convert into a photon. If the magnetic field is in a cavity, this means we can hope to produce a cavity excitation.
Axion Detection Process

\[ G_{a\gamma\gamma} \bar{F} \bar{F} = G_{a\gamma\gamma} \bar{E} \cdot \bar{B} \]
There are many challenges. The axion is quite narrow and we don’t know its mass with anything like precision. So one needs to be able to sweep through many small frequency steps. One needs a cavity of very high quality. The most impressive effort of this type is the ADMX experiment at the University of Washington.
Figure 2: Exclusion ranges as described in the text. The dark intervals are the approximate CAST and ADMX search ranges, with green regions indicating the planned reach of future upgrades. Limits on coupling strengths are translated into limits on $m_A$ and $f_A$ using $z = 0.56$ and the KSVZ values for the coupling strengths. The "Beam Dump" bar is a rough representation of the exclusion range for standard or variant axions. The "Globular Clusters" and "White Dwarfs" ranges uses the DFSZ model with an axion-electron coupling corresponding to $\cos^2\beta = 1/2$. The "Cold Dark Matter" range is particularly uncertain; ranges for pre-inflation and post-inflation Peccei-Quinn transitions are shown. Figure adapted from [50].

At the moment we prefer to interpret these results as an upper limit $\alpha_{Aee} < \sim 10^{-27}$ shown in Figure 2. Similar constraints derive from the measured duration of the neutrino signal of the supernova SN 1987A. Numerical simulations for a variety of cases, including axions and Kaluza-Klein...
For $y$, $\bar{q} > 0$ and $\bar{q} < 0$, we require $\delta q$ and $\delta q$, respectively. We can parametrize the QA (2) as 

$$
\frac{\delta \phi}{\lambda} \int \phi \left[ \begin{array}{c}
\frac{\delta \phi}{\lambda} \int \phi \\
\frac{\delta \phi}{\lambda} \int \phi
\end{array} \right] U \left( \begin{array}{c}
\frac{\delta \phi}{\lambda} \int \phi \\
\frac{\delta \phi}{\lambda} \int \phi
\end{array} \right) U

= \left( \begin{array}{c}
\frac{\delta \phi}{\lambda} \int \phi \\
\frac{\delta \phi}{\lambda} \int \phi
\end{array} \right)
$$

as given in Eq. (101), while $q$ is the momentum difference between the axion and the photon, defined as

$$
q = m_a \cos(2\pi f_\gamma) - m_a \cos(2\pi f_\phi)
$$

where $m_a$ is the axion mass, $f_\gamma$ is the photon frequency, and $f_\phi$ is the axion frequency.

Generically, one needs a Planckian scale $Q\text{CDM}$ axion decay constant can be in the intermediate scale. This possibility may be realizable in some compactification schemes of M theory.

Axions produced in the nuclear core of the sun will be a QA if the following coupling, which the following coupling scales they all rely on the Primako and the sun is expected to follow a thermal distribution with an average energy of $\langle E \rangle = 67\text{keV}$.

The probability of a solar axion converting into a photon as it passes through the sun is expected to be

$$
\Phi_{\gamma\gamma} = \frac{P_{\gamma\gamma}}{P_{\gamma\gamma}} = \frac{\gamma_{\gamma\gamma}}{\gamma_{\gamma\gamma}}
$$

where $\gamma_{\gamma\gamma} = T_\gamma$, the integrated flux at $\epsilon_{\gamma\gamma}$. The solar axion helioscope, first described in 1983, free-streamed out and can possibly be detected on Earth.

Recently, however, it was shown that the MI axion may be a QA if the quintessential axion decay constant $\lambda_{\text{QCD}}$ is extremely small.

FIG. 15: A cartoon for the $F_a$ bounds.
The axion cosmology we have described assumes that the universe was in thermal equilibrium at very early times, times much shorter than the axion mass.

There are reasons to question this assumption. For example, suppose that nature is approximately supersymmetric. Then the axion has a scalar superpartner, the saxion. This particle is long lived. If it decays through the two photon interaction (or its superpartners), its lifetime is of order

\[
\Gamma = \left( \frac{\alpha}{4\pi} \right)^2 \frac{m_{saxion}^3}{f_a^2} = \left( 10^5 \text{ s} \right)^{-1} \frac{m_{saxion}^3}{\text{TeV}^3} \left( \frac{10^{16}}{f_a} \right)^2
\]  

(54)

where we have taken the grand unified scale as our benchmark axion decay constant. Even at $10^{11}$ GeV, this is after the axions start to oscillate.
The saxion, when it decays, “reheats” the universe. This temperature should be higher than the temperature at which nucleosynthesis occurs (say 10 MeV).

When the axion starts to oscillate, the universe is dominated by the saxion. It’s energy density is of order $m_a^2 f_a^2$, while the total energy is of order $m_a^2 M_p^2$, so the axion energy fraction is approximately

$$f_a^2 / M_p^2$$  \hspace{1cm} (55)

After the saxion decays to radiation, the fractional energy density grows with $1/T$. So between 10 MeV and 1 eV, it grows by $10^7$. This gives

$$f_a < 10^{15}$$ \hspace{1cm} (56)

a much weaker limit than before. Could be weaker still.
Finally, we turn to a theoretical question: Why are there axions at all? More precisely, why should there be a Peccei-Quinn symmetry, and how good a symmetry does this have to be?

General belief (supported by studies of string theory): a theory of quantum gravity does not possess (exact) global symmetries.

Then hopeless? No: symmetry might be an accidental consequence of other symmetries. Example: discrete symmetries.

\[ \phi \rightarrow \phi e^{\frac{2\pi i}{N}}. \]  \hspace{1cm} (57)

So leading symmetry breaking terms in potential might take the form:

\[ \mathcal{L}_{\text{symm-breaking}} = \frac{\phi^N}{M_p^{N-4}} \]  \hspace{1cm} (58)
If \( N \) is large, these terms would seem very small. But they have to be extremely small to insure the smallness of \( \theta \). One needs, e.g., the linear term in the \( a \) potential

\[
V = \frac{1}{2} m_a^2 a^2 + \Gamma a + \ldots
\]  

(59)

to be such that

\[
\frac{\Gamma}{m_a^2} < 10^{-10} f_a
\]

(60)

This translates into a requirement that \( N > 12 \), if \( f_a = 10^{11} \); even larger for larger \( f_a \).

Why should this be? Within field theory, doesn’t seem particularly plausible. But in string theory the situation is different. Pause and consider whether string theory might have anything to teach us about this and similar issues.
Status of string theory: In ten, eleven dimensions: theories of quantum gravity in Minkowski space; with supersymmetry appear sensible perturbatively and non-perturbatively.

When compactify space dimensions, obtain "models" or theories". If preserve enough supersymmetry, again expect perturbatively and non-perturbatively sensible. Typically exhibit moduli.

With less supersymmetry (or none), models with chiral fermions, gauge groups similar to those of SM. If \( N = 1 \) SUSY and flat space in leading approximation, question of supersymmetry breaking not well understood. Moduli fixed? Cosmological constant small? Little is known. More discussion when we consider landscape.
1. String theory does not admit global continuous symmetries.
2. String models often exhibit discrete symmetries, sometimes quite intricate.
3. String Models typically possess axions, potentially with the quality required to solve the strong CP problem.
4. (Appropriate for a SUSY school:) String theories often exhibit low energy supersymmetry (arguably these are the only string theories for which we can claim any understanding).
5. String theories typically exhibit (pseudo) moduli (scalar fields with (almost) no potential). Might be both problem and opportunity for cosmology.
We have seen that axions, from the perspective of effective field theory, are surprising. It has long been known that axions are common in string theory, indeed axion-like objects seem ubiquitous. What insight do they give and what expectations do they lead to?
Examples:

1. Heterotic string contains an axion (always) which couples universally to all of the gauge groups.

2. In heterotic and other string theories, antisymmetric tensor fields in higher dimensions become pseudoscalars in four dimensions with axion type couplings.

3. All of these fields exhibit approximate, continuous shift symmetries, \( a \rightarrow a + \omega f_a \). They typically exhibit exact discrete shift symmetries, \( a \rightarrow a + 2\pi f_a \). The breaking of the continuous symmetries is suppressed, at weak coupling, by \( e^{-2\pi/\alpha} \).
So string theory has axions suitable for solving the strong CP problem. But what about their cosmology?

1. $f_a$ large; in what is imagined a typical string phenomenology, $f_a \geq 10^{15}$ GeV.

2. If approximate supersymmetry, axions accompanied by saxions, other “moduli”. Can dilute axions as above. May require surprisingly low $f_a$. 

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Michael Dine  
Beyond the Standard Model in the LHC Era
Detecting string scale axions

Clearly challenging. Recent proposals by P.Graham, S. Rajendran, and others.

Strategies involve noting that if axions are the dark matter, \( \theta \propto \cos(m_a t) \), and using (searching for) time varying dipole moments. Some prototype experiments under discussion. Workshop at ICTP last June. Prototype experiments proposed and funded.
While the value of the c.c. is extremely small in particle physics units, the c.c. is just the right size that it has only “recently" become important (the past few billion years), and it will dominate the energy density “forever". No persuasive dynamical explanation offered.

Weinberg imagined that the observed universe is part of a larger structure, subsequently dubbed a “multiverse", in which the c.c. can take a range of values, essentially randomly distributed. If one could take an inventory of this multiverse, one would find that only in some regions are there observers. This criterion, know as the *anthropic principle*, is much like arguing that observers (e.g. humans) are only found in a very tiny fraction of the volume of the universe, on the surfaces of planets with liquid water.
At a minimum, Weinberg argued, a universe supporting observers should contain galaxies. In our universe, galaxies formed about 1 billion years or so after the big bang; we understand this as the time required for small primordial density fluctuations (presumably formed during an epoch of inflation) to grow and become non-linear. If the c.c. were so large that it dominated the energy density 1 billion years after the big bang, structure would not form.

This argument predicted a c.c. somewhat larger than actually discovered. More refined forms of the argument come closer to the observed value.
Do there exist physical theories in which such a possibility is realized? The number of possible configurations which must be surveyed is enormous; given the small value of the c.c. in typical particle physics units, one might imagine that there should be at least $10^{120}$ such states.

Various scenarios. In string theory with some compactified dimensions, there are many types of quantized flux (analogous to magnetic flux in QED, sometimes hundreds or thousands) which can take many values, giving the potential for vast numbers of possible states. In each of the resulting states, the low energy degrees of freedom and the parameters of the lagrangian will take different values. If there are enough such states, the parameters will be densely distributed. The existence of such a landscape or discretuum of vacua remains conjectural. If this mechanism is operative, then at least for the c.c., our notions of naturalness are not correct.
Implications of a Landscape

Other quantities besides the c.c. might be anthropically determined, it is plausible that the TeV scale is anthropically selected. If the Higgs mass-squared were much larger than it is, one would either electroweak symmetry would be unbroken, or it would be broken and and the $W$’s and $Z$ extremely heavy. In either case, life would likely be impossible. If stars existed at all, their properties would be quite different than those in our universe, affecting important quantities like the abundance of heavy elements.

Other possibilities include light quark masses, gauge couplings and the dark matter density. It is hard to see how features of heavy quarks, $\theta$, might be anthropic.
The prospect of a landscape calls the naturalness principle into question. But a landscape *might* be a setting in which the concept is sharp. If we understood the *statistics* of theories (degrees of freedom, parameters) we might be able to make predictions based on correlations between phenomena.

Since this is a pre-SUSY school, illustrate with supersymmetry.
In flux landscape, different classes of states identified and something about their statistics known. I like to refer to these as "branches" of the landscape. We can ask on each whether there is a correlation of the supersymmetry breaking scale with the value of the weak scale

1. Non-supersymmetric states – obviously not.
2. Supersymmetric states with non-dynamical breaking of supersymmetry: statistics favor high scale supersymmetry breaking, even with low weak scale.
3. Supersymmetric states with dynamical supersymmetry breaking – uniform distribution of supersymmetry breaking scale (logarithmically).
4. Supersymmetric states with dynamical breaking of supersymmetry and dynamical breaking of $R$ symmetries – low scales favored.
But what about the principle itself? Are symmetries favored?
Seems (literally) unlikely. A symmetry only if all fluxes transforming under the symmetry vanishes – an exponentially small fraction of the exponentially large number of states.
This argument may not hold for supersymmetry, where *stability* might favor the symmetry.

1. With small c.c., supersymmetry favors *classical* stability.
2. With small c.c., small susy breaking favors *quantum* stability.
We need to be alert to LHC discoveries. Conceivably evidence for SUSY, composite Higgs, or some other phenomenon anticipated to resolve hierarchy problem.

Alert to LHC surprises: we are exploring a new energy regime, esp. for particles with electroweak interactions, possibly for dark matter candidates.

We need to be alert to other shortcomings of the SM: dark matter, strong CP about which we may discover clues and develop better theoretical ideas and settings.

We need to think through lessons from other theoretical structures – string theory, higher dimensional theories, others(?). We have seen that string theory provides insights and constraints. Perhaps better ideas about hierarchy, generations, other puzzles.