“Ugly Ducklings-Turned-Swans” in the Top Quark Sector

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1. Introduction

Ugly duckling = Swan

Before
1. Introduction

- Ugly duckling = Swan

Before

After
1. Introduction

- **LHC Run I**
  - Higgs discovery, SM precision measurement
  - SM having unexplained issues (e.g., neutrino mass, dark matter, gauge hierarchy puzzle etc)
  - Many new physics models: theory-motivated and phenomenology-motivated
  - No hint of new physics signature
1. Introduction

- **LHC Run II**

- LHC Run II will be marked by **precision studies** and searches for **small signals**

- Precision measurement by different approaches/methods

- New physics searches: searching for buried “Arkenstone” out of a tons of “treasure”

- New ideas/approaches
  - Providing different systematics
  - Facilitating discovery of new particles
1. Introduction

Ugly ducklings in physics

- “Ugly ducklings” in physics?
  - Some variables/approaches/techniques could be considered as problematic / not useful/not well under control (depending on the questions/channels at hand)
  - Likely to be avoided/disregarded/used with prescriptions to remove problematic issues in relevant analyses
1. Introduction

**Ugly ducklings in physics**

- “Ugly ducklings” in physics?
  - Some variables/approaches/techniques could be considered **as problematic /not useful/not well under control** (depending on the questions/channels at hand)
  - Likely to be avoided/disregarded/used with prescriptions to remove problematic issues in relevant analyses

With different points of view for them, such “Ugly Ducklings” in physics can be reborn as “**Beautiful Swans**” especially in the LHC era
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4. Extra Radiation
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2. “Ugly Ducklings” in Top Sector

- Why top sector?

- LHC as a top quark factory: top pairs: 950,000 fb
2. “Ugly Ducklings” in Top Sector

Why top sector?


- Top properties well measured/studied since its discovery at TeVatron, e.g., LHC combined \(e\mu\), 
  \[\sigma_{tt} = 240.6 \pm 1.4\text{(stat.)} \pm 5.7\text{(syst.)} \pm 6.2\text{(lumn.)}\]
  \[p\overline{b},\text{ CMS-PAS TOP-14-016, ATLAS-CONF-2014-054, } m_t = 173.34 \pm 1.4\text{(stat.)} \pm 5.7\text{(syst.)},\text{ World combination March 2014 (ATLAS, CDF, CMS, Do)}\]
2. “Ugly Ducklings” in Top Sector

Why top sector? (continued)

- Still rooms for NP to hide, e.g., $\Gamma_t = 2.0 \pm 0.5$ GeV, Particle Data Group (2012)
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- Largest quantum correction to higgs mass
  - Highly promising sector/window to new physics
2. “Ugly Ducklings” in Top Sector

Why top sector? (continued)

- Still rooms for NP to hide, e.g., $\Gamma_t = 2.0 \pm 0.5$ GeV, Particle Data Group (2012)
- Largest quantum correction to higgs mass
  - Highly promising sector/window to new physics
- New physics in the production level (faking $tt$ signature)
  - Better constrained/ruled out in the relevant parameter space of the new physics models (e.g. Czakon, Mitov, Papucci, Ruderman and Weiler ’14)
- New physics in the decay level (via rare decays of top)
  - Better chance to have new physics signals
2. “Ugly Ducklings” in Top Sector

- “Ugly ducklings” in the top sector
  - Three ugly duckling siblings
  - Energy distribution
    - NOT Lorentz invariant vs. longitudinal boost-invariant transverse quantities at hadron colliders
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- Extra radiation (mostly via QCD)
  - NOT fully understood vs. tree-level process (cf. DM search)
2. “Ugly Ducklings” in Top Sector

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- Three ugly duckling siblings
- Energy distribution
  - NOT Lorentz invariant vs. longitudinal boost-invariant transverse quantities at hadron colliders
- Extra radiation (mostly via QCD)
  - NOT fully understood vs. tree-level process (cf. DM search)
- Mis-bottom-tagging (mostly charm)
  - NOT easy to distinguish from bottom-initiated jets
Energy distribution
3. Energy Distribution

Motivation

- A simple 2-body decay of a heavy resonance $B$ into $A$ and massless visible $a$

- Energy of visible particle $a$ is **mono-chromatic** and **simple** function of masses in the rest frame of particle $B$

$$E^* = \frac{m_A^2 - m_B^2}{2m_A}$$

- $E^*$: energy of visible particle measured in the rest frame of particle $B$

- $E^*$ is measured, mass of $A$ is known $\rightarrow$ **mass of $B$ can be measured!** and vice versa

- Great to be on this special frame!
3. Energy Distribution

- 2-body decay kinematics in the lab frame

- Energy (not a Lorentz-invariant) of particle $a$ should be Lorentz-transformed

- Depending on $m_A$ and $m_B$ plus unknown boost factor $\gamma = 1/\sqrt{1 - \beta^2}$ and emission angle of particle $a$ from the axis of $\vec{\beta}$

$$E = E^* \gamma (1 + \beta \cos \theta^*)$$

- No longer fixed energy of particle $a$ in the lab frame, but a function of $\gamma, \theta^* \rightarrow$ becoming a distribution due to variation in them $\rightarrow$ information loss?!

Peak of such an energy distribution $=$ rest-frame energy
3. Energy Distribution

**Existence of energy peak – primer**

- Lorentz transformation: \( E = E^* \gamma (1 + \beta \cos \theta^*) \)
- Unpolarized/scalar mother particles
  - \( \cos \theta^* \) becomes flat \( \rightarrow E \) is also flat (simple chain rule)

\[
E^* \frac{\sqrt{1 - \beta}}{\sqrt{1 + \beta}} \quad \text{Number of events}
\]

\[
\frac{1}{2E^* \beta \gamma} \quad E
\]
3. Energy Distribution

- Existence of energy peak – primer (continued)

- Lower bound (upper bound) smaller (bigger) than $E^*$ (for any boost)
  - No other $E$ gets larger contribution from a given boost than does $E = E^*$
  - No other $E$ is contained in every rectangle

- Asymmetric on linear $E$ (symmetric on logarithmic $E$)
3. Energy Distribution

Existence of energy peak – primer (continued)

- Distribution in E: summing up the contributions from all relevant boost factors
  - “Stacking up” rectangles weighted by boost distribution (Lebesque-type integral)
  - Energy distribution has a unique peak at $E = E^*$ (Agashe, Franceschini, and DK ’12, also Stecker ‘71)
- Details of the boost distribution (depending on production mechanism, PDFs, mother masses...) NOT matters
  1) rise-and-fall
  2) rise-and-fall with non-zero at $\gamma = 1$
  3) more crazy (?)
3. Energy Distribution

- “Stacking up” rectangles

- $E^*$ must be the location of the peak!
3. Energy Distribution

- Bottom energy from top decay

- Bottom mass negligible: peak is expected **not** to shift

\[ E_b^* = \frac{m_t^2 - m_W^2 - m_b^2}{2m_t} = 68 \text{ GeV} \]

- ... maybe an “accident”?!
3. Energy Distribution

Different CM energy colliders

- “Invariant” (under boost distributions) feature in non-invariant energy distribution holds even with colliders of different center-of-mass energies and types

- Shape can change, while peak does NOT change
3. Energy Distribution

- Example pseudo-experiment at detector level

- Proof of the concept using 100 pseudo experiments from MadGraph5+Pythia+Delphes (ATLAS-2012-097)
- Fit with blue dots
- consistent with the input value
- Fit **NOT** spoiled by cuts or detector effects!!

\[ m_{\text{top}} = 173.1 \pm 2.5 \text{ GeV (stat.) with 5/fb LHC7} \]

- LO effects are well under control → **CMS at work!!!**
3. Energy Distribution

- Energy peak at production NLO
  - Recoil of the $t\bar{t}$ system by ISR
  - Top quark getting more boost → different boost distribution → change in width of distribution, but NO shift in the energy-peak!!

![Graph showing energy distribution](image-url)
3. Energy Distribution

- Energy peak at decay NLO
  - 3-body decay of top quark at decay NLO
  - Fraction of bottom energy carried away by final state radiation jet
    - Peak **shifts** to the lower energy regime!
3. Energy Distribution

- Energy peak in three-body decay
  - Energy of visible particle given by a distribution, **NOT** fixed unlike 2-body decay!
  - Each value of the rest-frame energy goes through similar argument in 2-body kinematics.

```
\[ E^*_1 \quad E^*_2 \quad \ldots \quad E^*_n \]
```
3. Energy Distribution

2 body vs. 3 body

- Peak position smaller than the maximum rest-frame energy (Agashe, Franceschini, DK, and Wardlow '12)
  - $E_{\text{peak}}$: model-dependent
  - Neglecting hard emission from a bottom quark, i.e., jet-veto
  - Safe from soft radiation off bottom (according to the detector-level simulation study)
  - (Typically) suppressed by $\frac{\alpha_s}{\pi}$, i.e., small perturbation in the LO phenomenon
3. Energy Distribution

**Motivation to top mass measurement**

- Beginning with an “example/test” to prove the principle
- Evolved into a “serious” measurement of top mass
- Why another method?!
  - (Theoretically) physical mass (like top mass measurement using the endpoints of kinematic variables) vs. Monte Carlo mass (like template method) etc.
  - (Experimentally) different systematics, independent measurement etc.

- Some doable challenges (Agashe, Franceschini, DK and Schulze, in progress)
  - 3-body decay due to FSR
  - Renormalization/factorization scale choice
  - Jet energy resolution/jet formation
3. Energy Distribution

- Production & decay NLO

- In real situation, jet definition is important; even ISR could affect the energy

- Large jet radius capturing more FSR jets, but more contaminated by ISR jets

- Small jet radius losing more FSR jets, but less contaminated by ISR jets
3. Energy Distribution

- Production & decay NLO: small R

  - Small R: \textit{R}=0.5 (anti-kt, MCFM)

  - Less contamination by ISR jets, but losing more FSR jets
    ✓ Energy peak shifts to the lower energy regime

  - Decay NLO sensitivity to the scale choice: ±1 GeV on the top mass
3. Energy Distribution

- Production & decay NLO: large R

- Large R: $R=1.0$ (anti-kt, MCFM)

- More contamination by ISR jets, but capturing more FSR jets
  - Energy peak shifts to the higher energy regime

- Decay NLO sensitivity to the scale choice: $\pm 1$ GeV on the top mass
3. Energy Distribution

- Production & decay NLO: “decent” R

  - “Decent” R: R=0.7 (anti-kt, MCFM)

  - Decent contamination by ISR jets, and
capturing decent number of FSR jets
    ✓ Cancellation between the two effects?
    ✓ NO shift in the energy peak

  - Decay NLO sensitivity to the scale choice:
    ± 0.5 GeV on the top mass
3. Energy Distribution

- Mild corrections from NLO

- Top decay still as in SM
- Theoretical systematics based on \((\text{small})\) parameters
  - \(\delta_{\text{prod}}\): PDF uncertainty, new physics in the production level (still unpolarized top)
  - \(f_{\text{pol}}\): new physics contribution with polarized top
  - \(\epsilon_{FSR}\): NLO effect, jet-veto
- Bottom jet energy peak correction
  \[ \frac{\delta E_b^*}{E_b^*} = f_{\text{pol}} + \epsilon_{FSR} \times \delta_{\text{prod}} \]
- \(\epsilon_{FSR}\) for QCD/SM production is \textbf{calculable} and has been being studied (Agashe, Franceschini, DK and Schulze, in progress)
  - Conventional methods \(\sim \delta_{\text{prod}}\) vs. energy-peak \(\sim \epsilon_{FSR} \times \delta_{\text{prod}}\)
Extra Radiation
4. Extra Radiation

Motivation

(For example,) separating $tW$ from dominant $t\bar{t}$ background
4. Extra Radiation

**Motivation**

- (For example,) separating $tW$ from dominant $t\bar{t}$ background

- Once a bottom is missed, they are
  - the same in the final state
  - kinematically very similar to each other

- Any “killer” kinematic variables to distinguish them from each other?
4. Extra Radiation

Quick review on $M_T^2$

- Stransverse mass ($M_{T2}$): a generalization of $M_T$  
  (Lester and Summers '99; Barr, Lester, and Stephens '03; Cho, Choi, Kim and Park '07)
  - full usage of both decay sides
  - MET relating both decay sides
  - bounded above
  - $M_{T2}^{max}$ as a simple function of mass parameters

$$M_{T2}^{max} = \frac{m_B^2 - m_A^2}{m_B}$$
4. Extra Radiation

- Subsystem $M_{T2}$

- More than one visible particle per decay side $\rightarrow$ richer structure/more $M_{T2}$ variables

- Various subsystems depending on particles whose mass is minimized and particles which are considered invisible (Burns, Kong, Matchev and Park '08)

- 3 symmetric subsystems and 3 asymmetric subsystems for $t\bar{t}$
4. Extra Radiation

- $tW$ vs. $t\bar{t}$ at LO

- Requiring 1 bottom-tagged jet + 2 opposite signed leptons
- Observed bottoms and leptons coming from the decay of the same particles in both $tW$ and $t\bar{t}$
  - ✓ Their typical momentum scale is similar
  - ✓ Any kinematic variables processed with visible 4 momenta are likely to develop similar distributions
- Impact of missing bottom jet
  - ✓ reshuffling of the distribution of overall initial state radiation
- No “killer” kinematic variables motivating machine learning procedure such as MVA or BDT
4. Extra Radiation

Results at LO

- Detector level simulation together with event selections of CMS collaboration
- Any kinematic variables seeming hopeless as expected
4. Extra Radiation

$tW$ vs. $t\bar{t}$ at “NLO”

- Attaching an extra jet into the leading order diagram of $tW$ process
  - Requiring 2 bottom-tagged jets + 2 opposite signed leptons or 1 bottom-tagged jet + 1 regular jet + 2 opposite signed leptons
- Additional jet requirement retrieving the leading order diagram of $t\bar{t}$
  - Relevant event topology well-defined $\rightarrow$ distributions in kinematic variables upper-bounded
- Event topology for $tW$ process ill-defined $\rightarrow$ distributions can be stretched far beyond the $t\bar{t}$ endpoints; $tW$ endpoints dictated by hardness of ISR jets (DK and Kong ’15)
4. Extra Radiation

- Results at “NLO”
  - Detector level simulation together with event selections of CMS collaboration
  - 2 bottom-tagged jets + 2 opposite signed leptons

- Large fraction of $tW$ events found beyond the kinematic endpoints for $t\bar{t}$ (dashed lines)
4. Extra Radiation

- Results at “NLO”
  - ROC curves: background rejection vs. signal acceptance
    - First four variables show good performance, while the other two show decent performance
    - 99.5% (99%) background rejection vs. 5% (20%) signal acceptance
4. Extra Radiation

- Results at “NLO”

- Detector level simulation together with event selections of CMS collaboration

- 1 bottom-tagged jet + 1 regular jet + 2 opposite signed leptons

- Again large fraction of $tW$ events found beyond the kinematic endpoints for $t\bar{t}$
4. Extra Radiation

- Results at “NLO”

- ROC curves: background rejection vs. signal acceptance

- First four variables show good performance, while the other two show decent performance
- More chance of accepting ISR jets in $t\bar{t}$: slightly worse performance than previous channel
4. Extra Radiation

Application to new physics

- Generically applicable to topology distinction between

\[ A\bar{A} \rightarrow (Bb)(\bar{B}\bar{b}) \rightarrow (Ccb)(\bar{C}\bar{c}b) \] vs. \[ A\bar{B} \rightarrow (Bb)(\bar{B}) \rightarrow (Ccb)(\bar{C}\bar{c}) \]

- New physics examples

1) \( \bar{\ell}\bar{\ell} \) vs. \( \bar{\ell}\tilde{\chi}_1^- \) (or \( \bar{\ell}\tilde{\chi}_1^+ \)) where \( \bar{\ell} \rightarrow \tilde{\chi}_1^+ b \rightarrow b\ell^+\bar{\nu} \) and similarly, \( \bar{\ell} \rightarrow \tilde{\chi}_1^- \bar{b} \rightarrow \bar{b}\ell^-\bar{\nu} \)

2) \( \bar{g}\bar{g} \) vs. \( \bar{g}\tilde{q} \) (or \( \bar{g}\tilde{q} \)) where \( \bar{g} \rightarrow q\bar{q} \rightarrow q\bar{q}\tilde{\chi}_1^0 \)
Mis-Bottom-Tagging
5. Mis-Bottom-Tagging

**Motivation**

- Given a signal process involving charm quark-induced jets in the final state, e.g., a rare decay process of top quark,
  
  $$pp \rightarrow t\bar{t} \rightarrow (bH^+)(bW^-) \rightarrow (bbc)(\bar{b}\ell^-\bar{\nu})$$

- its dominant background, i.e., semi-leptonic $t\bar{t}$
  
  $$pp \rightarrow t\bar{t} \rightarrow (bW^+)(bW^-) \rightarrow (bsc)(\bar{b}\ell^-\bar{\nu})$$

- typical event selection would be 3 bottom-tagged jets + 1 regular jet + 1 lepton + MET

- To increase the signal-over-background, 1 more bottom-tagged jet could be required based on the observation that (DK and Park, in progress)

  **mis-tagging rate for charm quark > mis-tagging rate for light quarks**
5. Mis-Bottom-Tagging

- Bottom-tagging efficiency

- Bottom-tagging efficiency ($\epsilon_b$): ~70%, mis-tagging efficiency for charm quark ($\epsilon_c$): ~20%, mis-tagging efficiency for light quarks ($\epsilon_s$): ~1%

- (Very rough and optimistic) estimation: $\bar{b}c$ (signal) vs. $\bar{s}c$ (background)

\[
\frac{S}{B} \sim \frac{\epsilon_b (1-\epsilon_c)}{(1-\epsilon_s)\epsilon_c} \quad (3 \text{ b-jets}) \rightarrow \frac{S}{B} \sim \frac{\epsilon_b \epsilon_c}{\epsilon_s \epsilon_c} \quad (4 \text{ b-jets}): \text{increased by a factor of ~25}
\]
5. Mis-Bottom-Tagging

Issues

- Combinatorics, $p_T$ and $\eta$ dependence of tagging efficiency

- Charm-quark tagging? ... mis-c-tagging rate for bottom quark, simultaneous requirement of bottom and charm quark taggings, etc... In general, analysis would be quite involved

- Small mass gap between top and charged Higgs: too soft a bottom jet to be detected
6. Summary

Conclusions

- Seemingly NOT useful variables/techniques/approaches can be reinterpreted as useful (with careful study) in the context of precision study and discovery potential
- 3 topics were discussed with physics examples in the top sector
  - Energy distribution
  - Extra radition
  - Mis-bottom tagging
- More “ugly ducklings” can be reborn as “beautiful swans”
Thank you!
Plateau in energy distribution

If the distribution starts from $\gamma \neq 1$, then the relevant energy distribution will develop a plateau in the middle of it.
No “accident” in $p_T$

- Peak and shape change
Formal proof

- First derivative
\[ f(E) = \int_{\frac{1}{2}(\frac{E}{E^*} + \frac{E^*}{E})}^{\infty} \frac{d\gamma}{2E^*\sqrt{\gamma^2 - 1}} g(\gamma) \]

\[ f'(E) = \frac{\text{sgn} \left( \frac{E^* - E}{2EE^*} \right)}{2E^*} g \left( \frac{1}{2} \left( \frac{E}{E^*} + \frac{E^*}{E} \right) \right) \]

- Vanishing derivative gives the extrema → Here this is the same as solving \( g = 0 \).

- Remember last assumption: \( g(\gamma) \neq 0 \) near \( \gamma = 1 \)
  
  → This is typical for particles produced at colliders.

- Two possibilities: \( g(1) = 0 \) or \( g(1) \neq 0 \)

- \( g(1) = 0 : f'(E=E^*) \propto g(1) = 0 \rightarrow f \) has a unique extremum at \( E = E^* \).

- \( g(1) \neq 0 : f'(E) \) flips its sign at \( E = E^* \) due to the sign function (from + to -). → the distribution has a **cusp** at \( E = E^* \) which appears as a peak.
Fitting function: functional properties of generic $f(E)$

- $f$ is a function with an argument of $\frac{1}{2} \left( \frac{E}{E^*} + \frac{E^*}{E} \right)$, i.e., even under $\frac{E}{E^*} \leftrightarrow \frac{E^*}{E}$ or $E \to \frac{E^*}{E}$.
  
  - clear from the expression of $f(E)$

\[
f(E) = \int_{\frac{1}{2} \left( \frac{E}{E^*} + \frac{E^*}{E} \right)}^{\infty} d\gamma \frac{g(\gamma)}{2E^* \sqrt{\gamma^2 - 1}}
\]

- $f$ is maximized at $E=E^*$.
  
  - proven heuristically and formally

- $f$ vanishes as $E$ approaches 0 or $\infty$.
  
  - the integral expression of $f(E)$ becomes trivial in those limits.

- $f$ becomes a $\delta$-function in some limiting case.
  
  - if any of mother particles are NOT boosted, i.e., the rest frame, then $f$ should return a $\delta$-function-like distribution.
Fitting function: proposal of a “simple” ansatz

\[ f(E) = \frac{1}{K_1(p)} \exp \left[ \frac{p}{2} \left( \frac{E}{E^*} + \frac{E^*}{E} \right) \right] \]

- \( K_1(p) \) : modified Bessel function of the second kind of order 1
- \( p \) : fitting parameter which encodes the width of the peak
- \( E^* \) as a fitting parameter can be extracted by fitting!
- All four properties are satisfied. → for the last property, use the asymptotic behavior of \( K_1(p) \)

\[ K_1(p) \xrightarrow{p \to \infty} \frac{e^{-p}}{\sqrt{p}} \left( 1 + O \left( \frac{1}{p} \right) \right) \]

- Proposed ansatz does not develop a cusp so that it is more suitable for the case of \( g(1)=0 \), e.g., pair-production of mothers (cf. the case of \( g(1)\neq0 \), single production of mothers).
A brief look into the massive case

- Energy of the visible particle should be Lorentz-transformed in a modified way.
  \[ E = E^*\gamma + p^*\sqrt{\gamma^2 - 1}\cos\theta^* \]

- Each rectangle’s coverage becomes shrunken.
  \[ E \in [E^*\gamma - p^*\sqrt{\gamma^2 - 1}, E^*\gamma + p^*\sqrt{\gamma^2 - 1}] \]
  \[ 2E^*\sqrt{\gamma^2 - 1} \rightarrow 2p^*\sqrt{\gamma^2 - 1} \]

- One modification: the lower bound is NOT smaller than \( E^* \) for some boost factors! (while the upper bound is still greater than \( E^* \) for any boost factor)
  → Our argument is not applicable for such boost factors, and \( E^* \) cannot be the location of the peak. \( : E_{\text{peak}} \geq E^* \rightarrow \) The critical boost factor can be calculable.
  \[ \gamma_{\text{cr}} = 2\gamma^* - 1 \text{ with } \gamma^* \text{ being the boost factor of the visible particle in the rest frame} \]
A brief look into the massive case

- For the top decay,
  \[ \gamma^* \approx 15 \quad \rightarrow \quad \gamma_{\text{cr}}^{\text{top}} \approx 450 \]
  
  \[ \rightarrow \] This value is not accessible given the current LHC-14TeV.

  \[ \rightarrow \] The peak still stays at \( E=E^* \).

- Another modification : symmetry property w.r.t \( E=E^* \) does NOT hold!

- Such a symmetry property implies \( \frac{E_{l_b}E_{u_b}}{E^*^2} = 1 \) for any \( \gamma \).

  \[ \rightarrow \] One possible estimator for deviation :
  \[ \delta_m = \frac{E_{l_b}E_{u_b}}{E^*^2} - 1 = \frac{m_b^2}{E^*^2} (\gamma^2 - 1) = \frac{\gamma^2 - 1}{\gamma^*^2} \]

  \[ \rightarrow \] \( \delta_m \) can be large for large \( m_b \) and \( \gamma \).

- For the top decay,
  \[ \gamma^* \approx 15 \quad \text{and typical} \quad \gamma \quad \text{of top quarks is roughly 1.2-1.4} \]

  \[ \rightarrow \] Violation of the symmetry property is negligible.
○ Production NLO: scale choice (preliminary)

- Very little sensitivity to the scale choice (less than 0.4 GeV on the top mass)
Mass measurement: general strategy

- Three unknowns: $m_A$, $m_B$, and $m_C \rightarrow$ three equations are needed.

$$E_{b \text{peak}} = \frac{m_C^2 - m_B^2}{2m_C}$$
$$E_{a \text{peak}} = \frac{m_B^2 - m_A^2}{2m_B}$$

$$m_{ab}^{\text{max}} = \sqrt{(m_C^2 - m_B^2)(m_B^2 - m_A^2)}$$

$$m_C = \frac{2 m_{ab}^{\text{max}}}{m_{ab}^{\text{max}} - 2 m_{ab}^{\text{max}} 4 E_{b \text{peak}}^2 - 16 E_{b \text{peak}}^2 E_{a \text{peak}} + 4 m_{ab}^{\text{max}} 4 E_{b \text{peak}}^2 E_{a \text{peak}}^2}$$

$$m_B = \frac{8 m_{ab}^{\text{max}}}{m_{ab}^{\text{max}} - 16 E_{b \text{peak}}^2 E_{a \text{peak}} + 4 m_{ab}^{\text{max}} 4 E_{b \text{peak}}^2 E_{a \text{peak}}^2}$$

$$m_A = \frac{4 m_{ab}^{\text{max}} E_{b \text{peak}} E_{a \text{peak}}}{m_{ab}^{\text{max}} - 16 E_{b \text{peak}}^2 E_{a \text{peak}} + 4 m_{ab}^{\text{max}} 4 E_{b \text{peak}}^2 E_{a \text{peak}}^2} \sqrt{4 m_{ab}^{\text{max}} 4 E_{b \text{peak}}^2 E_{a \text{peak}}^2 + 16 E_{b \text{peak}}^2 E_{a \text{peak}}^2 - m_{ab}^{\text{max}} 4}$$
New physics application: gluino decay

- Supersymmetry model
- Neutralino is our lightest stable particle (LSP) → R-parity conserving model.
- For the channel with the 1\textsuperscript{st} and 2\textsuperscript{nd} generation, the current bound on the gluino mass is too restricted (~1.5 TeV).
  → Production cross section of gluino might not be enough for mass measurement.
- 3\textsuperscript{rd} generation is more motivated by naturalness.
  → The current bound is ~1 TeV (assuming 1\textsuperscript{st} and 2\textsuperscript{nd} generations are heavier than gluino).
Energy distribution in the lab frame

- Each value of the rest-frame energy goes through similar argument in 2-body kinematics.
- The peak value is lower than the maximum rest-frame energy. (cf. the peak value is the same as the fixed rest-frame energy in 2-body kinematics)
- Reference values ($E^\text{max}_\text{peak}$ for 3-body vs. $E^\text{fixed}_\text{max}$ for 2-body) can be measured by other observables such as $M_{T2}$.
- **Peak = Reference value → 2-body decay → $Z_2$**
- **Peak < Reference value → 3-body decay → $Z_3$**
**Difference in $Z_2$ and $Z_3$**

- Two $Z_2$ charges: 0 or +1 ($\equiv -1$) vs. Three $Z_3$ charges: 0, +1 or +2 ($\equiv -1$)
- Under $Z_3$, a DM partner/mother can decay into 1DM or 2DM by $Z_3$ charge conservation.

\[
+1 = +1 + 0 + 0 + 0 + \ldots
\]

Present in **both** $Z_2$ and $Z_3$

\[
+1 = (-1) + (-1) + 0 + 0 + \ldots = -2 \equiv +1
\]

Present **only** in $Z_3$
Model consideration

- Decay of bottom partner into 1DM/2DM + bottom
- Major background: $Z(\rightarrow 2\nu) + 2b$
- Realistic cuts imposed
Sample result