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Effective field theory, fluid dynamics, and the spontaneous breaking of space-time symmetries

w/ A. Nicolis, R. Rattazzi, J. Wang (hep-th/1011.6396)
w/ A. Nicolis, R. Porto, and J. Wang (hep-th/1211.6461)
w/ A. Nicolis (hep-th/1303.3289)
w/ A. Nicolis, R. Penco (hep-th/1311.6491)

+ L. Hui, S. Dubovsky, D. Son, R. Rosen, and others

Outline

1) Fluid dynamics from EFT perspective

2) Spontaneously broken space-time symmetries

Inspiration for fluids

Fluids are everywhere

- Nuclear scales (Quark Gluon Plasma)
- Human scales (glass of water, superfluid He)
- Terrestrial scales (geophysics, atmospheric dynamics)
- Cosmological scales (density perturbations)

Historically described by EOM--> Lagrangian description/EFT

- symmetries manifest
- QM: direct road to quantization
- CLASSICAL: Well adapted for perturbation theory (Ex: W. Goldberger, I. Rothstein arXiv:hep-th/0409156)
- ask model independent questions (vs Kinetic Theory)

Outstanding problems

- viscosity/entropy bound

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

- turbulence

Punchline

Perfect fluid

- Lagrangian description exists* which you can take seriously as an EFT (symmetries, s.s.b. pattern, etc.)—not necessarily news
- Well defined quantum theory @ T=0?**
- Systematic classical perturbation theory: vortex-sound coupling

Dissipation

- Some foundational steps made

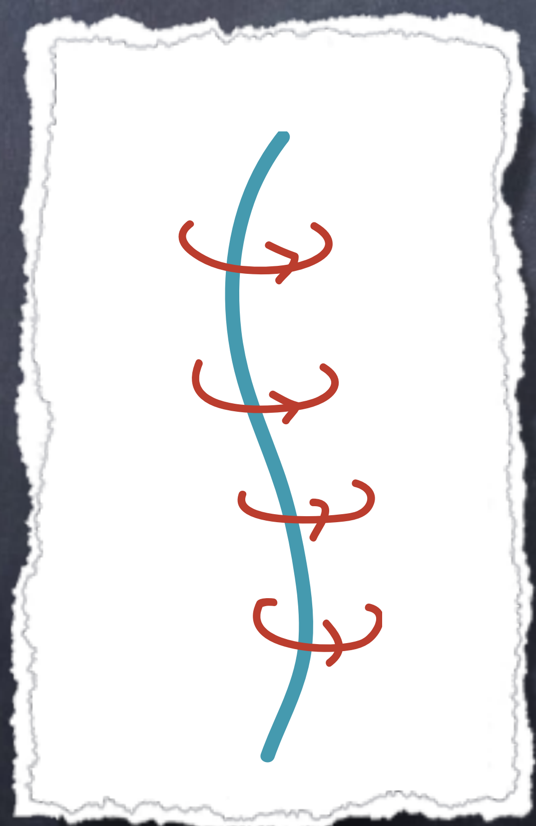
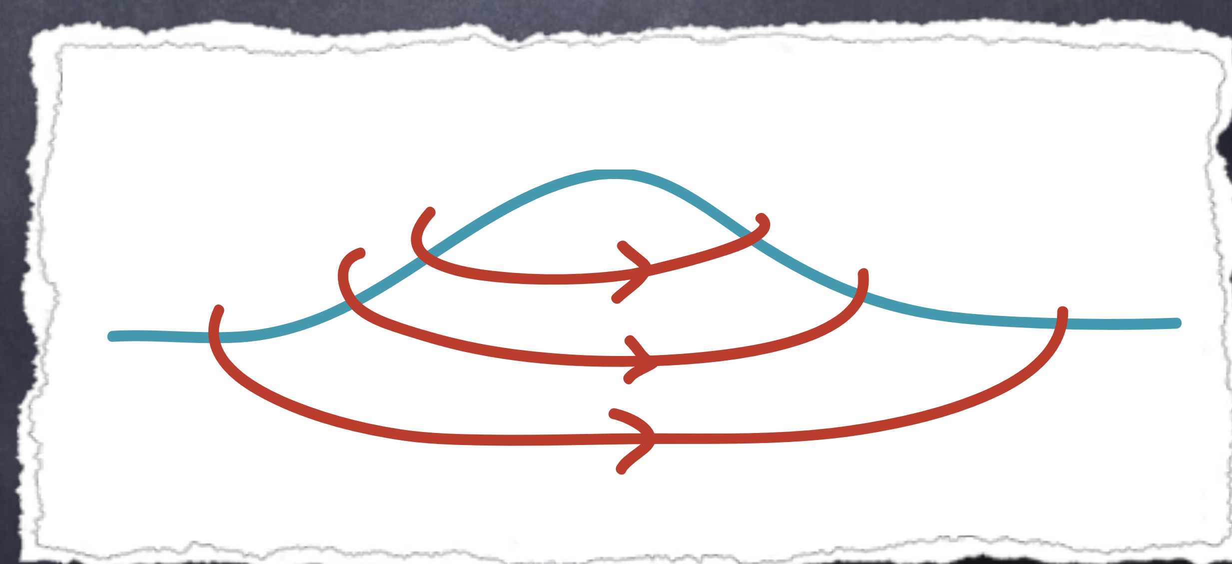
*S. Dubovsky, T. Gregoire, A. Nicolis & R. Rattazzi (hep-th/0512260)
**S. E., A. Nicolis, R. Rattazzi, J. Wang (hep-th/1011.6396)

Lagrangian for fluid dynamics

- ~~Navier-Stokes~~ EFT
    - 1) Degrees of freedom?
    - 2) Symmetries
    - 3) Construct the most general possible Lagrangian w/ 1) & 2)-> derivative expansion

Qualifications

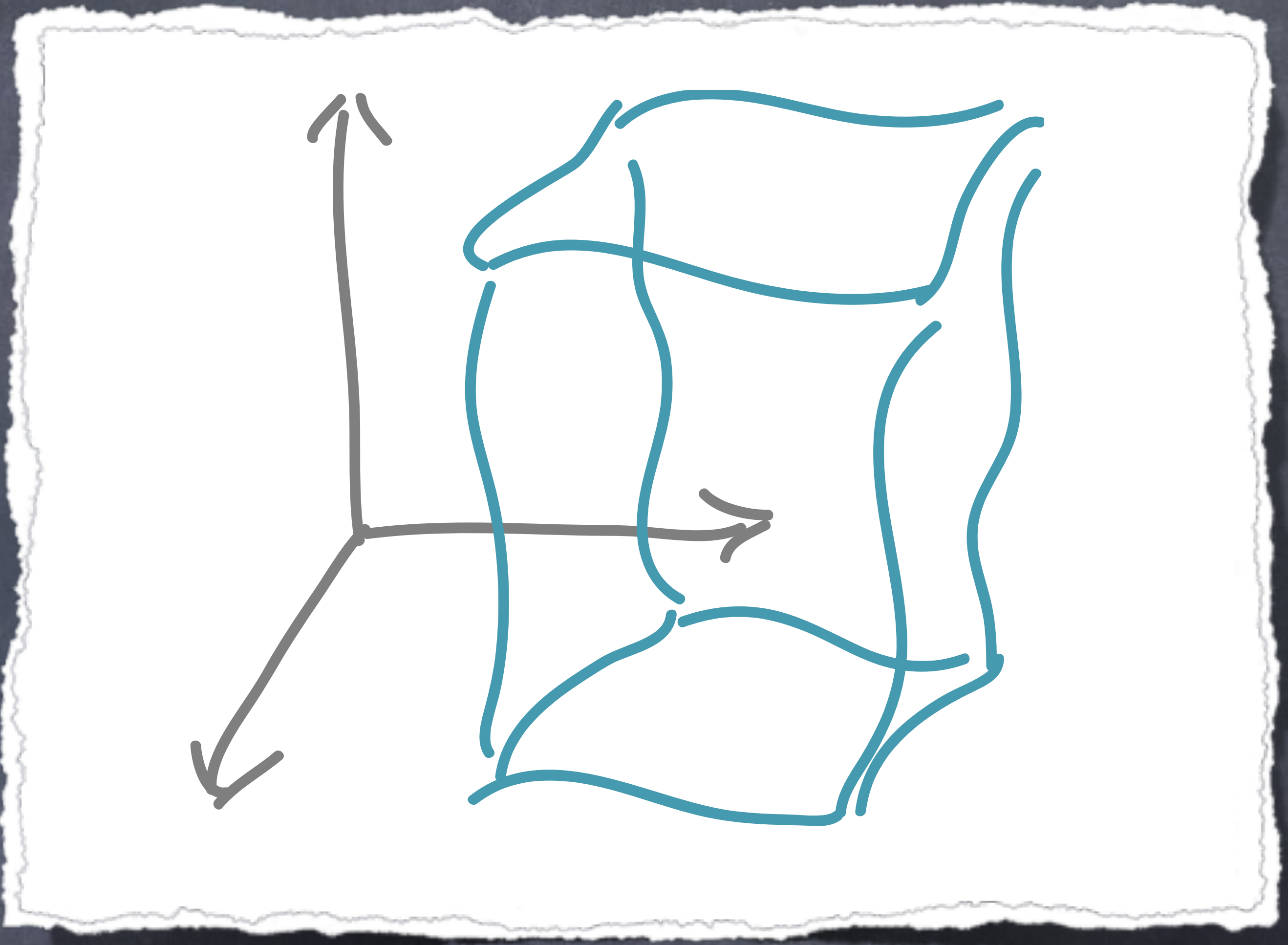
- **Perfect** (dissipative effects higher order in derivatives)--work in the far IR
- Fully **Relativistic**
- **Vortices in low energy theory** (not a super fluid)

Super fluid	Perfect fluid
• compressional (sound) modes	• compressional (sound) modes
• transverse (vortex) modes are **heavy***	• transverse (vortex) modes are **light**
	

*Rotons are tricky

Degrees of Freedom

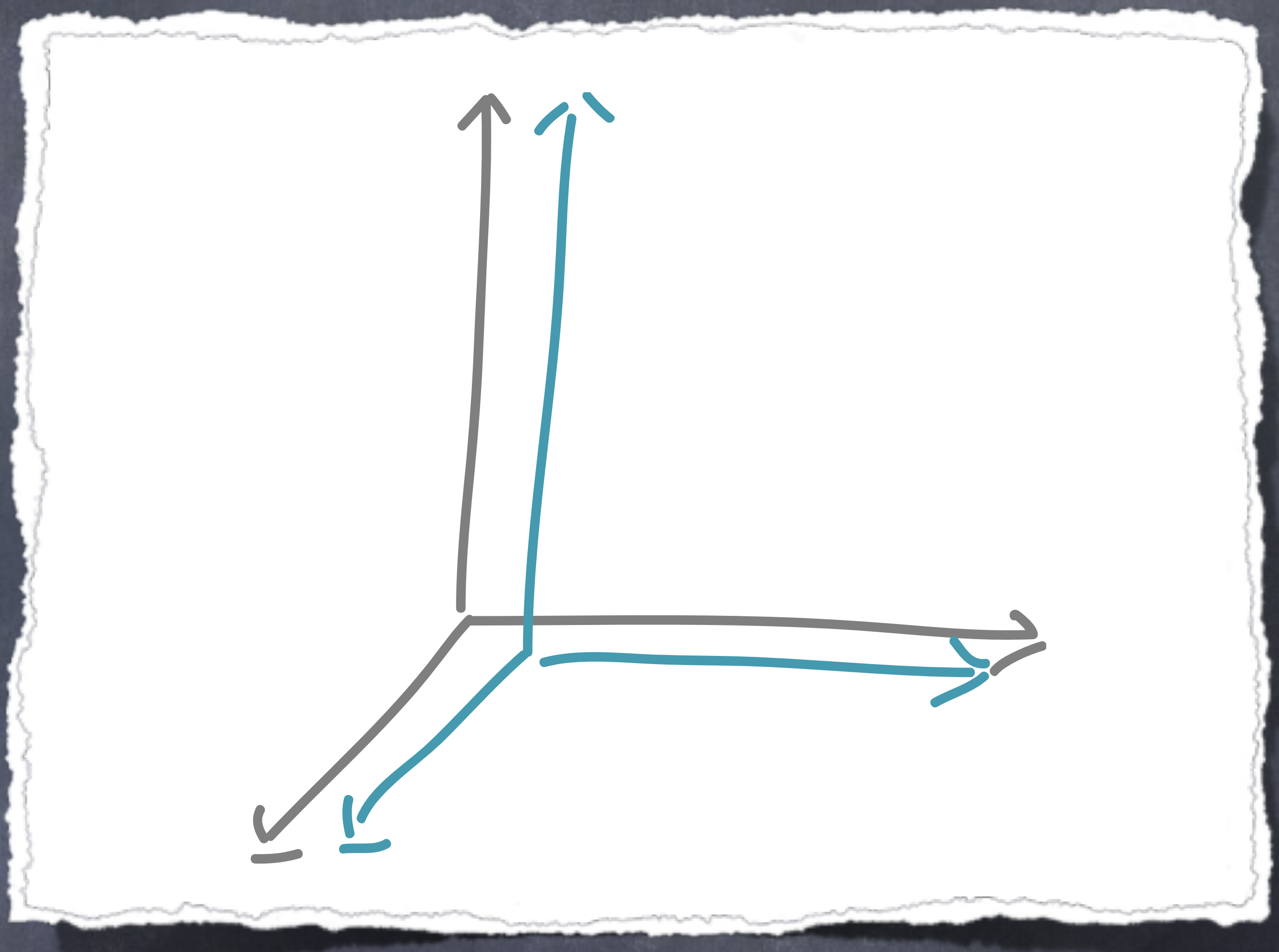
- Long wavelength/low energy limit-> fluid elements



$$\phi^I = \phi^I(\vec{x}, t), \quad I = 1, 2, 3$$

Degrees of Freedom

- Long wavelength/low energy limit-> fluid elements



$$\phi^I = \phi^I(\vec{x}, t), \quad I = 1, 2, 3$$

choose coordinate system

$$\phi^I = x^I$$

Symmetries

- Space-time: Poincaré (scalars)
- Internal symmetries:
  - Shift $\qquad \phi^I \to \phi^I + a^I$
  - Rotation $\qquad \phi^I \to O^I_J \phi^I$

  $\Longrightarrow$ Crystal or "jelly"

  - Volume-preserving diff

$$\phi^I \to \xi^I(\phi^J) \qquad \text{with} \qquad \det\left(\frac{\partial \xi^I}{\partial \phi^J}\right)$$

Lagrangian

- Shift -> derivative
- Poincaré -> $B^{IJ} = \partial_\mu \phi^I \partial^\mu \phi^J$
- Rotation -> SO(3) invar functions of B^{IJ}
- Vol-pres diffs -> $B = \det(B^{IJ})$

$$\Rightarrow \qquad S = \int d^4x \, F(B)$$

Is this fluid dynamics?

Relativistic perfect fluids:

- Stress tensor: $T_{\mu\nu} = (\rho + P) u_\mu u_\nu + p\eta_{\mu\nu}$
- EOM: $\partial^\mu T_{\mu\nu} = 0$
- EOS: $\rho(p)$

Our action: Correct classical dynamics provided

$$u^\mu = \frac{1}{6\sqrt{B}} \epsilon^{\mu\alpha\beta\gamma} \epsilon_{IJK} \partial_\alpha \phi^I \partial^J_\beta \phi \partial_\gamma \phi^K$$

$$\rho = -F(B)$$

$$p = F(B) - 2F'(B)B$$

Meet the Goldstones!

At a given p: $\phi^I = x^I + \underline{\pi^I}$ ← Goldstones

$$\mathcal{L} \to \dot{\vec{\pi}}^2 - c_s^2(\partial_I \pi^I)^2 + \text{interactions}$$

$$c_s^2 = \left.\frac{2F''(B)B + F'(B)}{F'(B)}\right|_{B=1} = \left.\frac{dp}{d\rho}\right|_{B=1}$$

Longitudinal = sound $\omega = c_s k$

Transverse = vortices $\omega = 0$ Very Funny

Why is fluid dynamics different/ vortices tricky?

Incompressibility:

measure of the pressure gradient needed to sustain a given density gradient, i.e.

$$\frac{dp}{d\rho} = c_s^2$$

⇒ incompressibility is a dynamical regime

$$v_{\text{flow}} \ll c_s$$

incompressible limit $\Longleftrightarrow \lim c_s \to \infty$

Only vortex dofs

Incompressible limit:

continuity equation: $\vec{\nabla} \cdot \vec{v} = 0$ $\qquad$ $\vec{\omega} = \vec{\nabla} \times \vec{v}$

Euler equation: $\frac{\partial}{\partial t}\vec{\omega} = \vec{\nabla} \times (\vec{v} \times \vec{\omega})$

Linear regime? NO DYNAMICS!

i.e. the dynamics are completely NL

Now that we have the Lagrangian we can do 3 things

- Use it
- Improve it
- Extend it

Vortex-Sound Interactions

What we are after: A systematic expansion that incorporates compression

Incompressible $\Rightarrow$ Not-so incompressible

given vortex flow $\Rightarrow$ vortex + sound

An expansion around: $$\frac{v_{\rm flow}}{c_s} \ll 1$$

Expand around $$\vec{x}(\vec{\phi},t)=\vec{x}_0(\vec{\phi},t)+\vec{\psi}(\vec{\phi},t)$$

Longitudinal: $\longrightarrow$ $\vec{\nabla}_0\times\vec{\psi}=0$

Inserting into S: $$S=S_{v^n}+S_{(\partial\psi)^m}+S_{v^n,(\partial\phi)^m}$$

For instance:

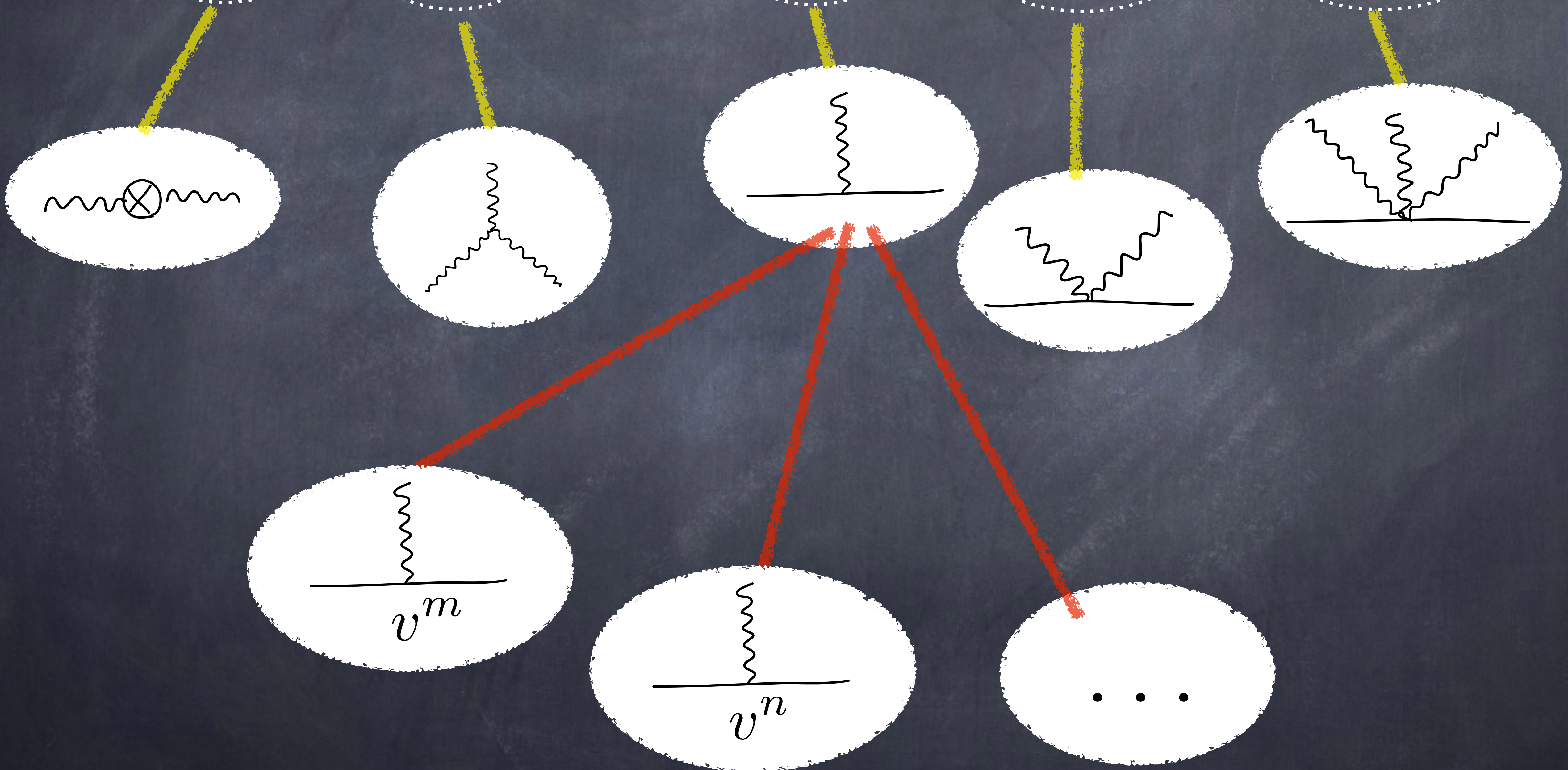
$$S_{v^n}=w_0\int d^3x dt\left(-c^2+\frac{v^2}{2}+\frac{(c^2-c_s^2)v^4}{8c^4}+\mathcal{O}(v^6)\right)$$

$$S_{\psi\ v^n}=w_0\int d^3x dt\ (v_i\partial_t\psi^i+v_i(v\cdot\nabla)\psi^i-\frac{c_s^2v^2[\nabla\psi]}{2c^2}+\ldots)$$

Relativistic corrections! $\frac{c_s^2}{c^2}\sim 1$

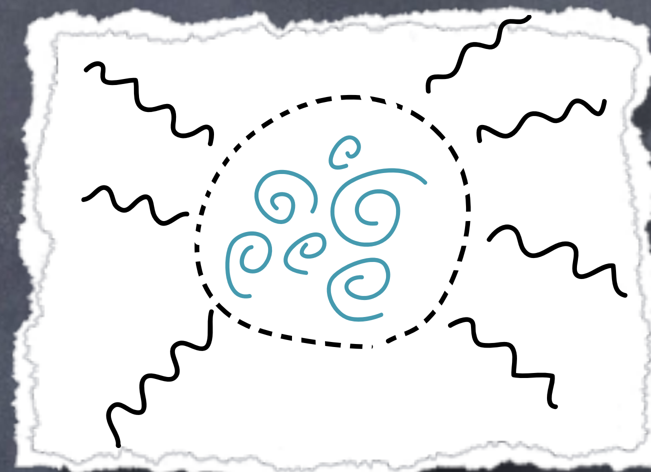
Pictorially

$$S_{int} = S_{(\partial\psi)^2} + S_{(\partial\psi)^3} + \ldots + S_{\partial\psi, v^n} + S_{(\partial\psi)^2, v^n} + S_{(\partial\psi)^3, v^n} + \ldots$$



Powerful formalism, what can we do with it?

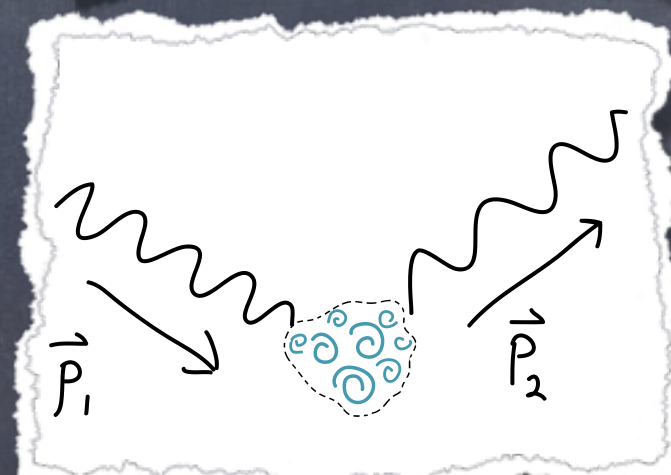
Sound emitted by a turbulent source:



Lighthill ('52) w/ rel. corr.

power given by quadrupole like formula

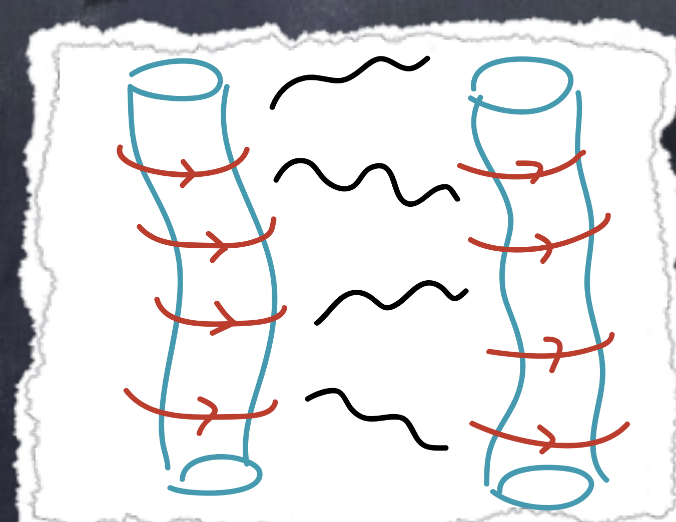
Sound scattered off a turbulent source:



Lund & Rojas ('89) w/ rel. corr.

cross section given in terms of correlation functions of vorticity

Potential between vortices:



Known?

$$V \sim \frac{l^3}{r^3}\frac{v^2}{c_s^2}E_{kin}$$

(w/ William Irvine to detect effect in vortex rings)

Dissipation: the general idea*

Local action, non-dissipative by construction:

$$S[\phi, \chi] = S_0[\phi] + S_\chi[\chi] + S_{int}[\phi, \chi]$$

DOF we will not keep track of

DOF we are keeping track of

Observables of ϕ only will detect `dissipative' effects corresponding to exciting the χ modes

*S. E, A. Nicolis, R. Porto and J. Wang (hep-th/1211.6461)

$$S[\phi,\chi] = S_0[\phi] + S_\chi[\chi] + S_{int}[\phi,\chi]$$

can be **strong** **weakly** coupled

$$S_{int} = \int d^4x \sum_{n,m} \partial^n \phi^m(x) \mathcal{O}_{n,m}(x)$$

as dictated by symmetry

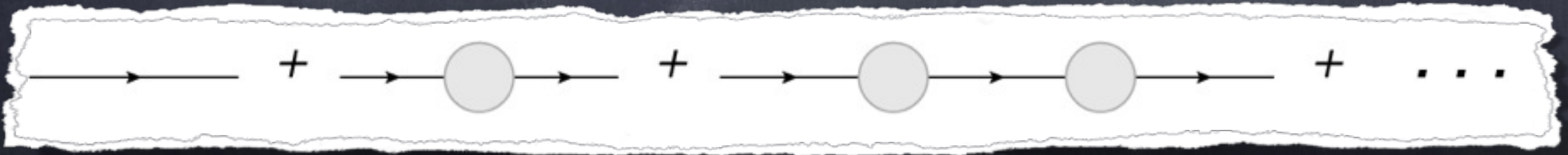
Observables of ϕ are mediated by correlation functions of the $\mathcal{O}$'s

Simple Example: $\mathcal{L}_{int} = \lambda \phi \mathcal{O}$

T-order (vac to vac) 2-pt function:

$$\langle \phi(p)\phi(-p) \rangle = \langle \phi(p)\phi(-p) \rangle_0 + \lambda^2 \langle \phi(p)\phi(-p) \rangle_0^2 \langle \mathcal{O}(p)\mathcal{O}(-p) \rangle + ...$$

or diagrammatically



More general correlation functions (thermalized χ state) we need work with the In-In formalism

double the path (+,-) with (thermal) density matrix $\rho(\chi_0^+, \chi_0^-)$ for initial conditions

$$\Rightarrow \quad \int \mathcal{D}\chi^+ \mathcal{D}\chi^-$$

$$\Rightarrow \quad \Gamma[\phi^+, \phi^-] \quad \Rightarrow \quad \left.\frac{\delta\Gamma[\phi^+, \phi^-]}{\delta\phi^+(x)}\right|_{\phi^+=\phi^-=\langle\phi\rangle} = 0$$

$$\Rightarrow \quad \frac{\delta S_2}{\delta\phi} + i\lambda^2 \langle\mathcal{O}\mathcal{O}\rangle_R * \phi = 0$$

matches the standard LR result

Properties of the hydrodynamic χ sector

ϕ all the d.o.f. that propagate over long distances and times (hydrodynamic modes)

χ in the absence of external perturbations, thermalized, and have no long distance or late time correlations (fall off faster than any power) but gapless

$$\langle \mathcal{O}(x)\mathcal{O}(x')\rangle \Rightarrow G(\omega,\vec{k}) = \mathrm{FT}\,\langle \mathcal{O}(x)\mathcal{O}(x')\rangle$$

admits a Taylor Series exp. about origin

Focus on retarded 2-pt function

$$G_R(\vec{x},t) \equiv \theta(t)\,\langle[\mathcal{O}(\vec{x},t),\mathcal{O}(0)]\rangle$$

standard spectral representation arguments =>

$\mathrm{Im}(iG_R)$ is odd, $\mathrm{Re}(iG_R)$ is even

$$\Rightarrow \quad \mathrm{Im}(iG_R(\omega,\vec{k}\,)) = \rho(\omega,\vec{k}\,) \simeq A\,\omega \times \delta\cdots\delta\;, \quad \omega,\; k \to 0$$

Extra time derivative!

How do our hydrodynamic modes couple to this sector?

turn on small perturbations about the background:

$$\phi^I(x) = x^I + \pi^I(x)$$

equivalent to performing a small, modulated, spacial translation

$$\phi_0^I(\vec{x}) \to \phi^I(\vec{x}, t) = \phi_0^I(\vec{x} + \vec{\pi}(\vec{x}, t))$$

χ Live in the fluid, i.e. they undergo the same spacial translation

S_{int}

$$\Rightarrow \quad S_\chi[\chi] \to S_\chi[\chi] - \int d^4x \, \partial_\mu \pi^i \, T_\chi^{\mu i}$$

Rediscovering Kubo's relations:

EOM: $$w_0\big(\omega^2\,\pi^i - c_s^2 k^i k^j\,\pi^j\big) + iG_R^{ij}(\omega,\vec{k})\,\pi^j = 0$$

$$G_R^{ij}(\omega,\vec{k}) = k_\mu k_\nu \langle T_\chi^{\mu i} T_\chi^{\nu j}\rangle$$

$$\Rightarrow \quad \mathrm{Im}\big(iG_R^{ij}\big) \simeq \omega\, k^2\big[(A_0 + \frac{4}{3}A_2)P_L^{ij} + A_2\,P_T^{ij}\big]$$

putting this all back in the equations of motion

$$\Rightarrow \quad \Delta\omega_L \simeq -i\,\frac{(A_0+\frac{4}{3}A_2)}{2w_0}k^2 \qquad \Delta\omega_T \simeq -i\,\frac{A_2}{w_0}\,k^2$$

which precisely matches the standard results provided

$$\zeta = A_0 \,, \qquad \eta = A_2$$

or, expressed using a funny limit (and similarly for η)

$$\zeta = \frac{1}{9}\delta_{ij}\delta_{kl} \lim_{\omega\to 0}\left[\frac{1}{\omega}\lim_{\vec{k}\to 0}\left(i\cdot\langle T^{ij}T^{kl}\rangle\right)\right]$$

Kubo's formula

But...... when we generalize there are problems...

Spontaneous breaking of space-time symmetries

Broken internal symmetry (PI) ⇨ Goldstone's Theorem: massless mode, **stable**

Mechanism **independent**

Broken s-t (and internal) symmetry ⇨ Less constrained: gap?, redundant dof?

Mechanism **dependent** (sort of)

Step back: fluids as a test case

Fluids spontaneously break space-time symmetries:

At a given p: $\phi^I = x^I + \underline{\pi^I}$ ← Goldstones

Is there a way to construct the theory for the Goldstones based on the symmetry breaking pattern alone?

Almost!*

*V. I. Ogievetsky 1970's

Fluids from broken symmetries alone: Coset construction*

1) Identify symmetry breaking pattern: $G \to H$

$$\text{unbroken} = \left\{ \begin{array}{lll} \bar{P}_t & \equiv P_t & \text{time translations} \\ \bar{P}_i & \equiv P_i + Q_i & \text{spatial translations} \\ \bar{J}_{ij} & \equiv J_{ij} + L_{ij} & \text{spatial rotations} \end{array} \right\}$$

$$\text{broken} = \left\{ \begin{array}{ll} K_i & \text{boosts} \\ Q_i & \text{internal shifts} \\ M_{ij} & \text{internal } SL(3) \end{array} \right\}$$

Goldstones associated with each broken sym... TOO MANY!

*Nicolis, Penco, Rosen hep-th/1307.0517

2) Construct objects that transform covariantly

(this is the coset construction):

$$\Omega(x) = e^{ix^\mu \bar{P}_\mu} e^{i\eta^i(x) K_i} e^{i\pi^i(x) Q_i} e^{i\alpha^{ij}(x) M_{ij}}$$

used to couple to "matter" fields

$$\Omega(x)^{-1} d\Omega(x) = i e^\mu_\alpha (\bar{P}_\mu + \omega^{ij}_\mu \bar{J}_{ij} + \mathcal{D}_\mu \eta^i K_i + \mathcal{D}_\mu \pi^i Q_i + \mathcal{D}_\mu \alpha^{ij} M_{ij}) dx^\alpha$$

const. inv metric

cov derivatives of Goldstones

*Nicolis, Penco, Rosen hep-th/1307.0517

3) Eliminate some Goldstones (inverse Higgs):

rule of thumb: $[\bar{P}, T_i] = T_j + \ldots$

US:
$$\begin{aligned} [\bar{P}_0, K_i] &= -i(\bar{P}_i - Q_i) \\ [\bar{P}_k, M_{ij}] &= -i\left(\delta_{ik} Q_j - \tfrac{1}{d}\delta_{ij} Q_k\right) \end{aligned}$$

$$\Rightarrow \quad \begin{aligned} \mathcal{D}_0 \pi^i &= 0 \longrightarrow \eta^i(\pi^i) \\ \mathcal{D}_i \pi^j - \tfrac{1}{d}\delta_i^j \mathcal{D}_k \pi^k &= 0 \longrightarrow \alpha^{ij}(\pi^i) \end{aligned}$$

Left with only $\mathcal{D}_1 \pi^1$ + higher derivative terms

$$\Rightarrow \quad \int d^d x G(\mathcal{D}_1 \pi^1) \equiv \int d^d x F(B) \quad \text{w/} \quad \phi^I = x^I + \pi^I$$

*Nicolis, Penco, Rosen hep-th/1307.0517

What about other space-time breaking theories?

Many symmetry breaking patterns==> many different physical systems

Some known…… some (seemingly) not

What is this "inverse-Higgs constraint"?

- Do we have to impose it? (NO)
- Are there systems where it is natural to NOT impose it? (YES)
- Over-counting interpretation? (NOT ALWAYS)
- New technology ==> new (strongly coupled) systems

Nicolis, Penco, Piazza, and Rosen (1306.1240)

S. E., Nicolis, and Penco (1311.6491)

Conclusions

- New language <==> New questions<==> New effects <==> New measurements!
- Still much work to be done (NL coupling to dissipative sector)
- A great deal of possible applications: cosmology, plasma physics, exotic condensed matter states, shocks?, all in a model independent fashion
- Interesting from a purely field-theoretic point of view: what can fluids teach us about QFT?
- Clarifying how to deal with SB s-t symmetries ==> tools to deal with rich systems beyond fluids

What I didn't mention:

- Other, fluid like, systems: superfluids, both 3 and 4 Nicolis (1108.2513) + w/ Nicolis, and Penco (pending)
- Solid Inflation (cosmology application of this formalism) w/ A. Nicolis, and J. Wang (hep-th/1210.0569)
- Vortex lines (in superfluids and fluids) + rotons
- SUSY C. Hoyos, B. Keren-Zur, Y. Oz (1206.2958)
- How our understanding of dissipation has (slightly) improved...

QM of perfect fluids

Why are there no perfect fluids at T=0 in nature?

Fluid

Superfluid

Solid

Supersolid

Etc.

$$T \longrightarrow 0$$

Theory inconsistent?

How can we show that the theory is inconsistent? Investigate strong coupling scale

Trick:

$$c_T \neq 0$$

consider transverse 2->2 scattering

$$\sigma_{TT\to TT} \sim \frac{1}{k^2}\left(\frac{k^4}{w_0 c_T}\right) \qquad c_T \to 0$$

strong coupling scale -> 0

Also prove "Coleman-like" theorem: quantum fluctuations disrupt the semi-classical vacuum

$$\phi^I = x^I \quad \text{no good}$$

Expanding around $\frac{v_{\text{flow}}}{c_s} \ll 1$

Expansion is most clear in the comoving coordinates

$$\vec{x}(\vec{\phi}, t)$$

We can write:

$$S = -w_0 c^2 \int d^3\phi dt \, \det J \, f((\det J^{-1})\sqrt{1 - v^2/c^2})$$

where $J^i_j = \frac{\partial x^i}{\partial \phi^j}$ and $\vec{v} = \partial_t \vec{x}(\vec{\phi}, t)$

$$\mathcal{L} = -w_0 f(\sqrt{B}) \qquad f'(1) = 1 \qquad f''(1) = \frac{c_s^2}{c^2}$$

Why is this a useful starting point for our expansion?

Expand around $$\vec{x}(\vec{\phi},t) = \vec{x}_0(\vec{\phi},t) + \vec{\psi}(\vec{\phi},t)$$

at fixed time is a v.p.d., i.e. $\det J_0 = 1$

longitudinal as a function of $\vec{x}_0$, i.e.

$$\nabla_0 \times \vec{\psi} = 0$$

$$\implies \det J = 1 + \nabla_0 \cdot \vec{\psi} + \frac{1}{2}\left[(\nabla_0 \cdot \vec{\psi})^2 - (\nabla_0^i \psi^j)^2\right] + \ldots$$

$$\implies \vec{v} = \vec{v}_0 + \frac{D}{Dt}\vec{\psi} = \vec{v}_0 + \left(\partial_t + (\vec{v}_0 \cdot \nabla_0)\right)\vec{\psi}(\vec{x}_0,t)$$

Expanding: $$S = S_v + S_\psi + S_{v^n,(\partial\phi)^m}$$

Problems with dissipation:

When we include conserved charge (need additional scalar), it is no longer clear that we know what we are doing:

same arguments ⇒ $$S_{\rm int} \stackrel{?}{\simeq} - \int d^4x \, [\partial_\mu \pi^i \, T^{\mu i}_\chi + y_0 \, \partial_\mu \pi^0 \, j^\mu_\chi]$$

⇒ Kubo formula for heat conductivity

FAILS

However...

If we consider instead the ``symmetry inspired" coupling:

$$S_{\rm int} \simeq -\int d^4x \left[\partial_j \pi^i \, T_\chi^{ji} + B\, \partial_i \pi^0 \, j_\chi^i + C\, \partial_0 \pi^i \, j_\chi^i\right]$$

calculate using the Kubo formula for heat conductivity

$$\mathrm{Im}\, i \cdot \langle j^i j^j \rangle_{\rm K} = \chi T \left(\frac{n}{\rho + p}\right)^2 \omega \cdot \delta^{ij}$$

match

$$\Rightarrow \quad B = -C = -\frac{y_0 w_0}{b_0 F_b} = \frac{\mu(\rho + p)}{sT}$$

simple answer, can we understand better?