Black hole entropy in loop quantum gravity

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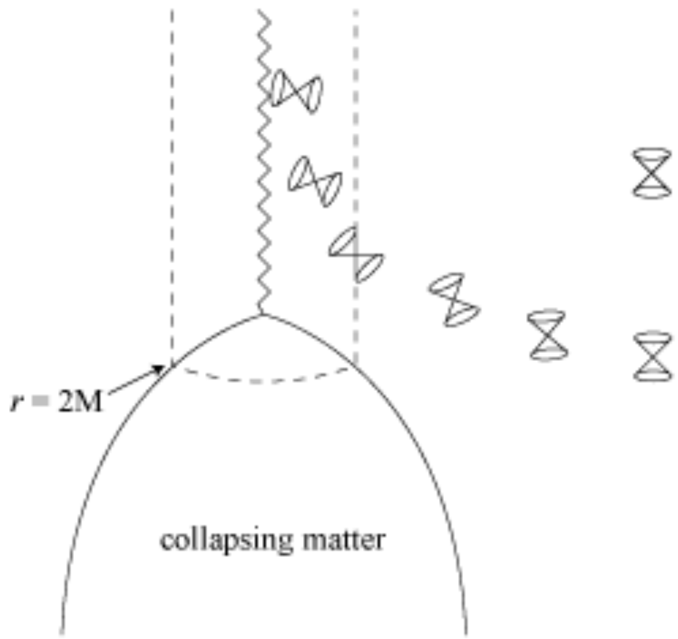
Black hole thermodynamics



[Bekenstein 72; Bardeen, Carter, Hawking 73; Hawking 74]

Black holes in their stationary phase behaves as thermodynamical systems:

$$S \longleftrightarrow A/(8\pi\alpha) \qquad T \longleftrightarrow \alpha\varkappa$$

But, in classical GR: $T=0$

Hawking radiation:
thermal emission of particles from a BH at

$$T = \frac{\kappa\hbar}{2\pi} \longrightarrow S = \frac{A}{4\ell_p^2}$$

Semiclassical result

☞ **Questions:**

Statistical physics: entropy of any system is given by $S = \ln N$

N = number of states of the system for the given macroscopic parameters

for a solar mass black hole
$N = e^S \sim 10^{10^{77}}$

1) Microscopic origin of the entropy?
2) Where do all these d.o.f. live?

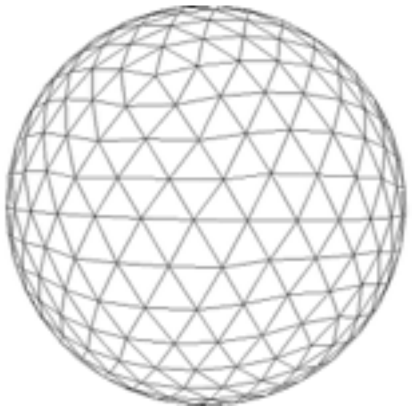
☞ Call for a quantum treatment of the gravitational dof

Weak holographic principle:

The entropy in the 1st law is the log of the number of states of the black hole that can affect the *exterior* [Bekenstein; Sorkin; Smolin; Jacobson…]

➥ The horizon carries some kind of information with a density approximately 1 bit per unit area

"It from Bit" [Wheeler]



What these bits of information represent depends on the deep structure of space-time

✧ The finiteness of the BH entropy hints at discreteness of space-time at the Planck scale

Outline

- Basic ingredients of LQG
- Quantization of an Isolated Horizon
- Entropy counting: results and open issues
- CFT/gravity correspondence

The LQG approach

Metric variables

Einstein-Hilbert action

$$I[g_{ab}] = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R$$

$$\kappa = 8\pi G$$

upon foliation of spacetime in terms of space-like three dimensional surfaces Σ

$$q_{ab},\ \pi^{ab} = \frac{1}{\sqrt{q}}(K^{ab} - Kq^{ab})$$

extrinsic curvature of Σ

symplectic structure

$$\{\pi^{ab}(x), q_{cd}(y)\} = 2\kappa\delta^a_{(c}\delta^b_{d)}\delta(x,y)$$

Hamiltonian

$$H(q_{ab},\pi^{ab},N_a,N) = N_aV^a(q_{ab},\pi^{ab}) + NS(q_{ab},\pi^{ab})$$

vanishes identically on solutions of the e.o.m.

$\Rightarrow$

Connection variables

Triad $e^i_a,\ i = 1,2,3\ \ su(2)$ indices

set of three 1-forms defining a frame at each point in Σ $\quad q_{ab} = e^i_a e^j_b \delta_{ij}$

densitized triad

$$E^a_i \equiv \frac{1}{2}\epsilon^{abc}\epsilon_{ijk}e^j_b e^k_c \qquad K^i_a \equiv \frac{1}{\sqrt{det(E)}}K_{ab}E^b_j\delta^{ij}$$

symplectic structure

$$\{E^a_j(x), K^i_b(y)\} = \kappa\delta^a_b\delta^i_j\delta(x,y)$$

spin connection

$$\partial_{[a}e^i_{b]} + \epsilon^i{}_{jk}\Gamma^j_{[a}e^k_{b]} = 0$$

Ashtekar-Barbero connection

$$A^i_a = \Gamma^i_a + \gamma K^i_a \qquad \{E^a_j(x), A^i_b(y)\} = \kappa\gamma\delta^a_b\delta^i_j\delta(x,y)$$

Hamiltonian

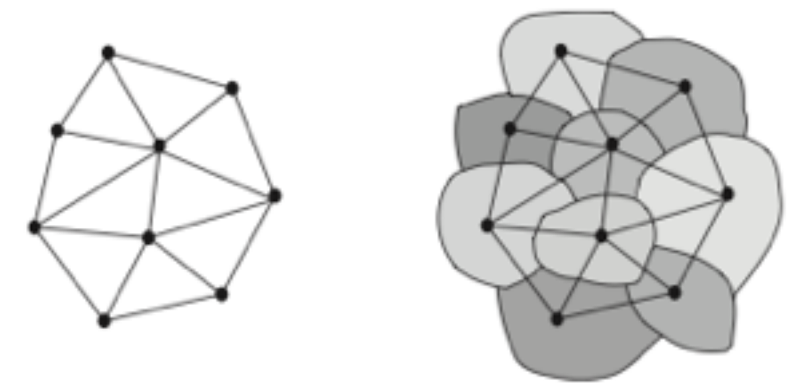
$$H = N_aV^a(E^a_j, A^j_a) + NS(E^a_j, A^j_a) + N^iG_i(E^a_j, A^j_a)$$

GR = background independent *SU(2)* gauge theory
(partly analogous to *SU(2)* Yang-Mills theory)

➢ Kinematical structure: holonomy along a path γ $\quad h_\gamma[A] = P\exp - \int_\gamma A$

Cylindrical functionals $\quad \Psi_{\Gamma,f}[A] = f(h_{\gamma_1}[A], \dots, h_{\gamma_{N^\Gamma_\ell}}[A])$

$$\begin{aligned}\langle \Psi_{\Gamma_1,f}, \Psi_{\Gamma_2,g}\rangle &\equiv \mu_{AL}(\overline{\Psi_{\Gamma_1,f}[A]}\Psi_{\Gamma_2,g}[A]) \\ &= \int \prod_{i=1}^{N^{\tilde\Gamma}_\ell} dh_i \overline{\tilde f(h_{\gamma_1}, \cdots, h_{\gamma_{N^{\tilde\Gamma}_\ell}})}\tilde g(h_{\gamma_1}, \cdots, h_{\gamma_{N^{\tilde\Gamma}_\ell}})\end{aligned}$$



$|\Gamma, j_l, v_n\rangle$

Spin network states basis: graphs colored with *SU(2)* spins

description of quantized geometries

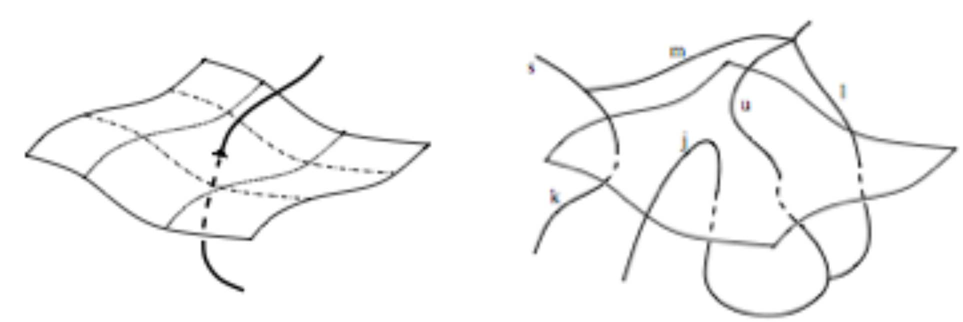
Peter-Weyl th. $\quad f(g) = \sum_j f_j^{mm'} \Pi^j_{mm'}(g)$

Fluxes $\quad \hat\Sigma^i_S(x) = \epsilon^i{}_{jk} \int_S \hat e^j(x) \wedge \hat e^k(x) = \int_S n_a \hat E^{ia}(x) = 8\pi\gamma\ell_P^2 \sum_{p\in\gamma\cap S} \delta(x, x_p)\hat J^i(p)$

with $\quad [\hat J^i(p), \hat J^j(p)] = \epsilon^{ij}{}_k \hat J^k(p)$

✶ Area operator:

$$\hat A_S|\Psi\rangle = \sqrt{\hat E^a_i n_a \hat E^b_j n_b \delta^{ij}}|\Psi\rangle = 8\pi\gamma\ell_P^2 \sum_{p\in\gamma\cap S} \sqrt{j_p(j_p+1)}\,|\Psi\rangle$$



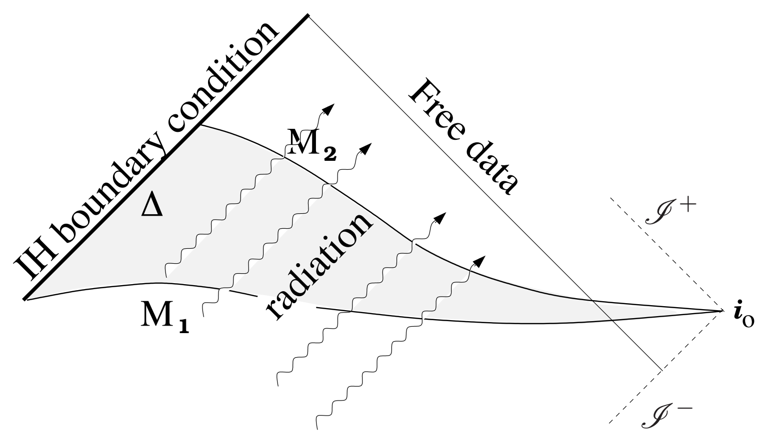
Spectral analysis
of geometrical operators ➥ Planck scale
discreteness

“Atoms” of quantum space = polymer-like excitations
of the gravitational field

Quasi local definition of BH

Isolated Horizons

IH boundary conditions



- $\Delta = S^2 \times \mathbb{R}$ null hyper-surface with vanishing expansion
- $\ell^a =$ normal future pointing null vector field with vanishing expansion within Δ
- Einstein's field equations hold at Δ

➥ $$\underleftarrow{F^i_{ab}}(A) = -\frac{\pi(1-\gamma^2)}{a_H}\underleftarrow{\Sigma^i_{ab}}$$

$$p = (\Sigma, A) \in \Gamma \quad \delta = (\delta\Sigma, \delta A) \in \mathrm{T_p}(\Gamma)$$

for the pull back of fields on the horizon δ = linear combinations of *SU(2)* gauge transformations and diffeomorphisms preserving the preferred foliation of Δ

The presymplectic structure

$$\kappa\,\Omega_M(\delta_1, \delta_2) = \int_M 2\delta_{[1}\Sigma_i \wedge \delta_{2]}K^i$$

is preserved in the presence of an IH
(no boundary term needed)

$$= \frac{1}{\gamma}\int_M 2\delta_{[1}\Sigma^i \wedge \delta_{2]}A_i \underbrace{- \frac{a_H}{\pi\gamma(1-\gamma^2)}\int_H \delta_1 A_i \wedge \delta_2 A^i}$$

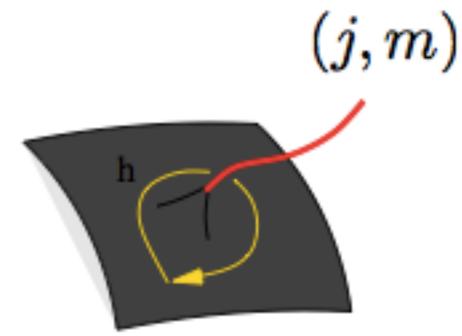
boundary term given by an *SU(2)* Chern-Simons presymplectic structure

The single intertwiner BH model

✧ Bulk theory: LQG Hilbert space associated to a fixed graph $\gamma \subset M$ with end points ps on H

$$\hat{a}_H |\{j_p, m_p\}_1^n; \cdots\rangle = 8\pi\gamma\ell_p^2 \sum_{p=1}^{n} \sqrt{j_p(j_p+1)} |\{j_p, m_p\}_1^n; \cdots\rangle$$

spin network states



boundary condition

$$-\frac{a_H}{\pi(1-\gamma^2)} \epsilon^{ab} \hat{F}^i_{ab} = 16\pi G\gamma \sum_{p \in \gamma \cap H} \delta(x, x_p) \hat{J}^i(p)$$

✧ Boundary theory: *SU(2)* Chern-Simons with punctures

$$S_{CS} + S_{int} = \frac{k}{4\pi} \int_{D\times\mathbb{R}} \mathrm{tr}[A \wedge dA + \frac{2}{3} A \wedge A \wedge A] + \lambda_j \int_c \mathrm{tr}[\tau_3(\Lambda^{-1} d\Lambda + \Lambda^{-1} A \Lambda)]$$

$\Lambda \in SU(2)$ particle d.o.f.

$S^i \in \mathfrak{su}(2)$ momentum conjugate to Λ

Poisson brackets:

$$\{A^i_a(x), A^j_b(y)\} = \delta_{ij}\epsilon_{ab} \frac{2\pi}{k} \delta^2(x-y), \quad a, b = 1, 2; \; x^0 = y^0$$

$$\{S^i, \Lambda\} = -\tau^i \Lambda\,, \quad \{S^i, S^j\} = i\epsilon^{ij}{}_k S^k$$

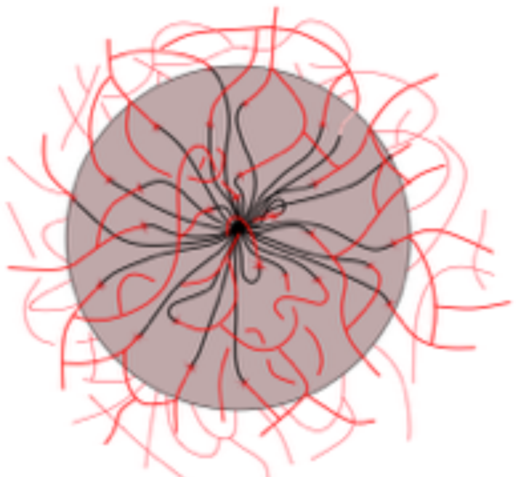
E.O.M. $\epsilon^{ab} F^i_{ab}(A(x)) = -\frac{2\pi}{k} S^i \delta^2(x-p)$

➢ Combinatorial quantization:

➥ $k \leftrightarrow a_H/(4\pi\ell_P^2\gamma(1-\gamma^2)), \quad S^i \leftrightarrow J^i\,, \quad \mathscr{H}^{CS}_{kin}(j_1 \cdots j_n) \leftrightarrow \mathrm{Inv}(\otimes_p j_p)$

Quantum BH dof described by a Chern-Simons theory on a punctured 2-sphere H

[Ashtekar, Baez, Corichi, Krasnov 99]
[Engle, Noui, Perez, DP 11]

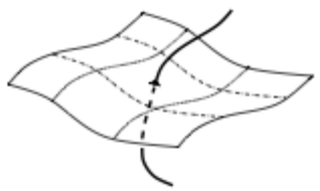


$\Rightarrow$

$$\dim[\mathscr{H}^{\mathrm{CS}}(\mathrm{j}_1 \dots \mathrm{j}_\mathrm{n})] = \dim[\mathrm{Inv}(\mathrm{j}_1 \otimes \cdots \otimes \mathrm{j}_\mathrm{n})]$$

we can model the IH by a single *SU(2)* intertwiner

BH entropy d.o.f. = polymer-like excitations of the gravitational field

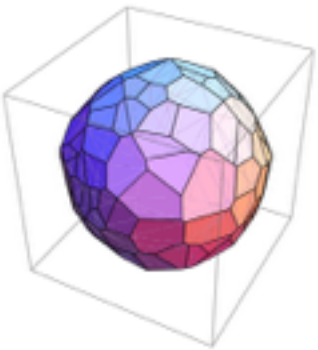


$$S = \ln \sum_{j_1,\dots,j_n} \dim[\mathscr{H}^{CS}(j_1 \cdots j_n)] = \frac{a_H}{4\ell_P^2}\frac{\gamma_0}{\gamma} - \frac{3}{2}\log a_H$$

Bekenstein-Hawking formula for $\gamma = \gamma_0\,, \quad \text{with } \gamma_0 = 0.274067\dots$

[Kaul, Majumdar 98]
[Agullo, Barbero, Diaz-Polo, Fernandez-Borja, Villasenor 08]
[Ghosh, Mitra 05]
[Livine, Terno 05]
[Engle, Noui, Perez, DP 11]

Semiclassical limit of the *SU(2)* intertwiner quantum geometry: tesselated surfaces
[Livine, Terno 05; Bianchi 10]



BH microstates $\Longleftrightarrow$ horizon *quantum shapes*

Quantum IH Temperature [DP 13]

[Frodden, Ghosh, Perez 11]: important role of the Unruh temp for a preferred family of stationary local observers at a proper fixed distance from the IH

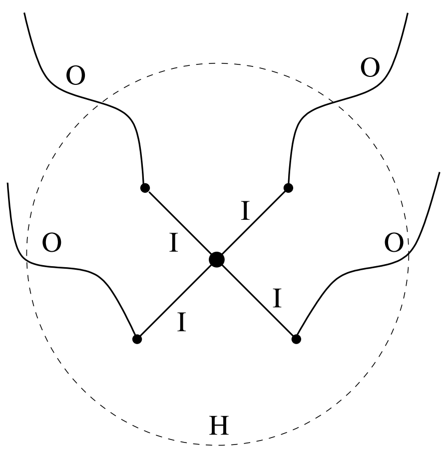
KMS-states = physical extension of Gibbs equilibrium thermal states to infinite dimensional quantum systems

Kubo-Martin-Schwinger

[Haag, Winnink, Hugenholtz]

$\mathscr{H}_{IH} \subset \bigotimes_p |j_p\rangle$: the horizon state $\Omega = \bigotimes_p |j_p\rangle\langle j_p|$ on which to impose $F^i(A) = -\frac{2\pi}{k}\Sigma^i$

$$|j_p\rangle = \sum_{m_p=-j_p}^{+j_p} |j_p, m_p\rangle_I \otimes |j_p, m_p\rangle_O$$ tracing over the internal dof $|j,m\rangle_I$



$$\hat{\rho} = \frac{1}{Z}\bigotimes_{p=1}^{N} P_O^{j_p} \underbrace{e^{i(\frac{2\pi}{k}-2\pi)\hat{\Sigma}_p}}_{\text{Boltzmann-like factor on each puncture}} P_O^{j_p}$$

where

projector $P_O^{j_p} = \sum_{m_p=-j_p}^{+j_p} |j_p, m_p\rangle_{O\,O}\langle j_p, m_p|$

$Z = \mathrm{tr}(\bigotimes_{p=1}^{N} P_O^{j_p}\, e^{i(\frac{2\pi}{k}-2\pi)\hat{L}_p}\, P_O^{j_p})$

partition function

✪ KMS-condition: given the complex correlation function $f_{AB}(z) = \hat{\rho}(\alpha_z(A)B)$ with $z \in \mathbb{C}$

$\alpha_z =$ one-parameter algebra automorphism generated by the boost operator (local horizon generator)

$$f_{AB}(-i\beta) = \hat{\rho}(\alpha_{-i\beta}(A)B) = \hat{\rho}(B\alpha_0(A)) = f_{BA}(0) \quad \Longrightarrow \quad \boxed{\text{geometrical notion of temperature}}$$

quantum correction

$\beta = 2\pi(1 - \frac{1}{k})$ and $\gamma = i$ [Frodden, Geiller, Noui, Perez 12]

☞ Thermality of the density matrix associated to the horizon quantum state originates from the entanglement between internal and external horizon dof

$$S_{Bol} = -\beta^2 \frac{\partial}{\partial \beta}\left(\frac{1}{\beta}\ln Z\right)$$

Boltzmann ent. = entanglement ent.

$$S_{ent} = -\mathrm{tr}(\hat\rho \ln \hat\rho)$$

[DP 13]

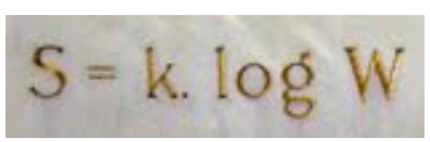
➥ $$S = \frac{a_H}{4\ell_P^2} + \mu N$$

quantum hair argued to be associated to a new horizon microscopic observable [Ghosh, Perez 11]

$$\mu \equiv \log\left[\sum_j (2j+1) e^{-2\pi i (1-\frac{1}{k})j}\right]$$

chemical potential

(call for a GFT description)



W = number of horizon \`quantum shapes'

⇦ **Intertwiner structure** encoding ⇨

Correlations of quantum geometry dof across the horizon

Carlip's proposal

- ➢ 2+1 gravity acquires new degrees of freedom in presence of a boundary (broken gauge invariance)
- ➢ In the Chern-Simons formulation, these are described by WZW theory
- ➢ new, dynamical "would-be gauge" d.o.f. can account for the BH entropy

attempt to describe the microphysics of BH in terms of a "dual" 2-dim Conformal Field Theory

Powerful method

Cardy formula:

$$S = 2\pi\sqrt{\frac{cL_0}{6}}$$

However, several open questions:

✻ what is the microscopic nature of the d.o.f.?

✻ where do the d.o.f. live?

✻ extension to higher dimensions?

☞ **Universality problem:**

(hidden) CFT symmetry underlying different microscopic approaches to BH entropy?

BH Entropy in LQG

$$S_{LQG} = \frac{A}{4\ell_p^2} + \mu N$$

Main open questions:

- Can inclusion of matter d.o.f. on the IH give the Bekenstein-Hawking formula?
  (see e.g. proposal of [Ghosh, Noui, Perez 13]: extra degeneracy due to entanglement entropy of matter)

- Is there a CFT lurking somewhere?
  (does LQG belongs to Carlip's `universality class'?)

- Are the previous two questions related??

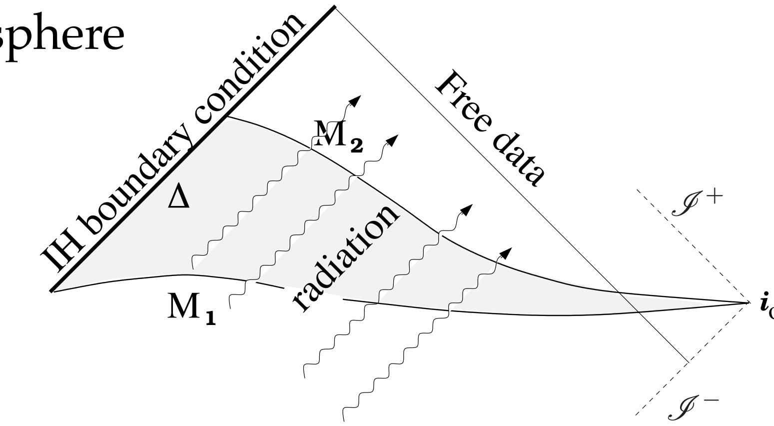
CFT/Gravity correspondence on the isolated horizon

in collaboration with Amit Ghosh

Nucl. Phys. B (in press), e–print: gr-qc/1405.7056

Kac-Moody Algebra

IH boundary conditions ⇨ SU(2) CS theory with punctures on the horizon 2-sphere



$$S_{CS} + S_{int} = \frac{k}{4\pi}\int_{D\times\mathbb{R}} \mathrm{tr}[A\wedge dA + \frac{2}{3}A\wedge A\wedge A] + \lambda_j \int_c \mathrm{tr}[\tau_3(\Lambda^{-1}d\Lambda + \Lambda^{-1}A\Lambda)]$$

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$$\{S^i,\Lambda\} = -\tau^i\Lambda\,, \quad \{S^i,S^j\} = i\epsilon^{ij}{}_k S^k$$

E.O.M. $$F^i_{12}(A(x)) = -\frac{2\pi}{k}S^i\delta^2(x-p)$$

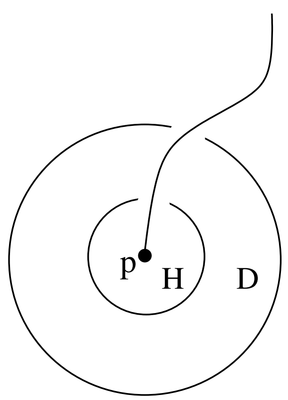
need of regularization

[Witten 89]

[Guadagnini, Martellini, Mintchev 89]

[Ashtekar, Baez, Krasnov 00]

[Noui, Perez 04]

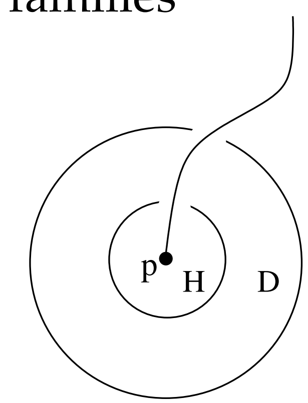


[Balachandran, Bimonte, Gupta, Stern 92] [Banados 96]

The algebra of gauge constraints leads to a set of charges at the boundaries whose Poisson bracket algebra is a classical Kac-Moody algebra.

(equivalent to the charges obtained by a reduction of the Chern-Simons theory to a boundary WZW theory)

The presence of the two boundaries ∂D and ∂H induces the presence of two families of observables, each localized on one boundary



✦ 2 sets of test functions:

$$\xi_N^{(D)i}(\theta)|_{\partial D} = e^{-iN\theta}\tau^i\,, \quad \xi_N^{(D)i}(\theta)|_{\partial H} = 0$$
$$\xi_N^{(H)i}(\theta)|_{\partial H} = e^{iN\theta}\tau^i\,, \quad \xi_N^{(H)i}(\theta)|_{\partial D} = 0$$

θ (mod 2π) is an angular coordinate on the two boundaries

✦ Kac-Moody generators: $q(\xi^{(B)}) = \dfrac{k}{\pi}\displaystyle\int_{D/H} \mathrm{tr}[d\xi^{(B)}A - \xi^{(B)}A\wedge A]\,, \quad B = D, H$

commutation relations of the quantum operators associated with these observables:

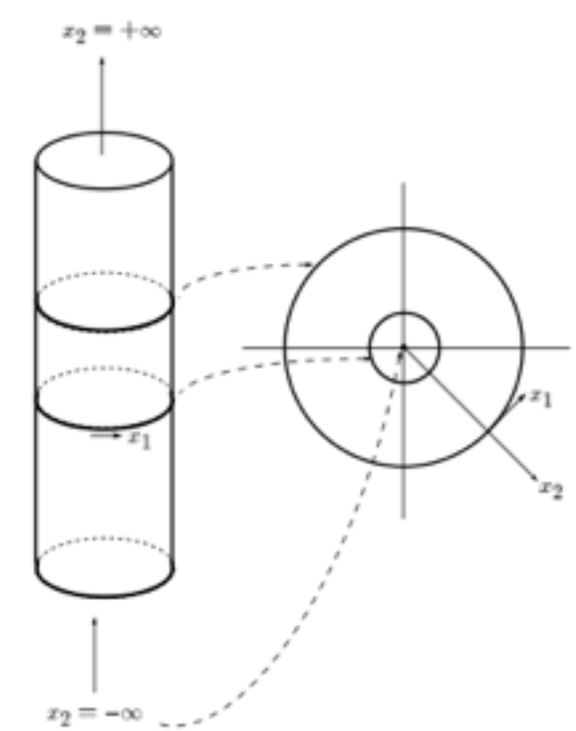
$$[\hat{q}_N^i, \hat{q}_M^j] = i\epsilon_{ijk}\hat{q}^k_{N+M} + N\frac{k}{2}\delta_{N+M,0}\delta_{ij}$$

Kac-Moody algebra

The currents $q_N^{(B)i}$ correspond to the modes of the holomorphic field $A^i(z)$

conformal map: $z = e^w, \;\; w = t_E + i\theta$

light-cone coordinate in Euclidean space



conformal primary field of weight 1

$$A^i(z) = \frac{1}{k}\sum_{N\in\mathbb{Z}} z^{-N-1} q_N^{(H)i}$$

the holomorphic Chern-Simons gauge connection can be identified with an affine current satisfying the Kac-Moody algebra

Virasoro Algebra

Kac-Moody algebra $\Rightarrow$ Virasoro algebra

Sugawara construction

Holomorphic stress-energy tensor (SET): $$\hat{T}(z) = \frac{1}{(k+2)} \sum_i (\hat{q}^i \hat{q}^i)(z)$$

SET Laurent expansion: $$\hat{T}(z) = \sum_{N \in \mathbb{Z}} \hat{L}_N z^{-N-2}$$

SET conformal dimension $h = 2$

Virasoro generators: $$\hat{L}_N = \frac{1}{(k+2)} \sum_i \sum_{M \in \mathbb{Z}} : \hat{q}^i_M \hat{q}^i_{N-M} :$$

$: \cdots :$ normal ordering

$\downarrow$

finite energy values in a highest weight representation

the $\hat{L}_N$'s perform diffeos of the boundaries ∂D, ∂H and they fulfill the Virasoro algebra

$$[\hat{L}_N, \hat{L}_M] = (N-M)\hat{L}_{N+M} + \frac{c}{12} N(N^2-1)\delta_{N+M,0}\,, \qquad N, M \in \mathbb{Z}$$

c = central charge: $[c, \hat{L}_N] = 0 \ \forall N \in \mathbb{Z}\,,$ for su(2) $c = \dfrac{3k}{k+2}$

✦ Energy operator: $$\hat{L}_0 = \frac{1}{(k+2)}\left(\hat{q}^i_0\hat{q}^i_0 + 2\sum_{M>0}\hat{q}^i_{-M}\hat{q}^i_M\right)$$

$$H \propto \hat{L}_0 + \hat{\bar{L}}_0$$ generator of dilations in the z-plane → time translation in the cylinder

➢ Fields in a CFT can be grouped into families $[\phi_n]$

- a single primary field ϕ_n
- an infinite set of secondary fields (descendants)

$\Big\}$ Irreps of the conformal group (primary field = highest weight)

In any given **highest weight representation** the spectrum of $\hat{L}_0$ is bounded from below and there is only one highest weight state $|v_j\rangle$ s.t.

$$\hat{L}_0|v_j\rangle = \underset{\text{conformal dimension}}{\Delta_j}|v_j\rangle\,, \quad \hat{L}_N|v_j\rangle = 0 \quad N > 0$$

All the other states in the given highest weight representation (c, Δ_j) can be constructed by repeated application of $\hat{L}_{-N},\ N > 0$ on $|v_j\rangle$

unitary representations: $c \geq 1,\ \Delta_j \geq 0$

$$c = \frac{3k}{k+2} \xrightarrow[\text{large } k]{} 3 \ ✓$$

Vertex Operators

The highest weight state $|v_j\rangle$ can be obtained from an 'absolute' vacuum $|0\rangle$ by application of a vertex operator

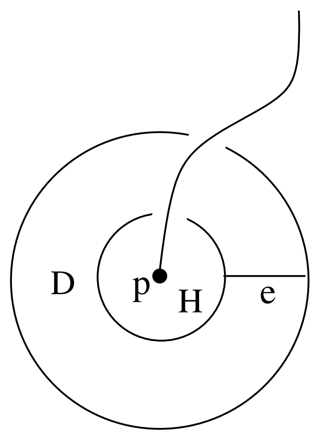
$$|v_j\rangle = \hat{V}_j|0\rangle$$

regularity of $\hat{T}(z)|0\rangle$ at $z=0$ implies $\hat{L}_N|0\rangle = 0, \quad N \geqslant -1$

$$\Rightarrow \quad \underbrace{\hat{L}_{-1}, \hat{L}_0, \hat{L}_1}_{\text{global conformal group}} |0\rangle = 0 \quad \text{the vacuum state is } SL(2,\mathbb{C}) \text{ invariant}$$

➢ The vertex operator $\hat{V}_j$ can be interpreted as a Wilson line

[Balachandran, Bimonte, Gupta, Stern 92]



The Wilson line along *e* creates an highest weight state of τ_3 charge j_3 $\Rightarrow$ holonomy = primary field

natural environment for spin network states

via the Sugarawa construction, the action of primary fields $\hat{V}_j$ on the vacuum $|0\rangle$ generates highest weight states for the representation of both Kac-Moody and Virasoro algebras

✦ In the case of the affine algebra:

$$F^i_{12}(A(x)) = -\frac{2\pi}{k} S^i \delta^2(x-p) \quad \rightarrow \quad \oint_{\partial H} A^i = -\frac{2\pi}{k} S^i \quad \Rightarrow \quad q_0^{(B)i} = -\frac{k}{2\pi}\oint_{\partial B} A^i = S^i$$

CS boundary condition Stokes th.

The zero modes $q_0^{(B)i}$ constitutes an *SU(2)* Lie algebra

The full infinite set of $q_N^{(B)i}$'s provides a so-called 'affinization' of this finite dim subalgebra

$$q_0^{(B)i}|v_j\rangle = \tau^i_{(j)}|v_j\rangle, \quad \text{with} \quad q_N^{(B)i}|v_j\rangle = 0 \ (N>0)$$

su(2) generators in the spin-*j* representation

➢ Identifying the operator $\hat{S}^i$ at the source *p* with the LQG flux operator $\hat{J}^i(p)$

Highest weight states ⇔ Spin network states

zero modes = gravitational d.o.f.
higher modes = new d.o.f. (matter)

Energy operator spectrum:

$$\hat{L}_0|v_j\rangle = \frac{1}{k+2}\tau^i_{(j)}\tau^i_{(j)}|v_j\rangle = \frac{1}{k+2} j(j+1)|v_j\rangle$$

Free Field Representation

The Wakimoto free field representation = affine extension of the monomial representation of the su(2) finite Lie algebra

su(2) generators in the Chevalley basis $\{h_0, e_0, f_0\}$ $\rightarrow$

Affine extension:

$\{h_0, e_0, f_0\}$ = zero modes of appropriate free bosonic fields (affine generators)

correct *SU(2)* Kac-Moody OPE at level *k*

Sugawara SET in terms of these currents = SET of a free-bosonic field with a non-zero background charge $-1/2\sqrt{k+2}$ (plus the ghost fields term)

Liouville theory??

also the central charge $c = \dfrac{3k}{k+2}$ can be recovered by summing up all the contributions

$q_0^{(B)i} = -\dfrac{k}{2\pi}\oint_{\partial B} A^i = J^i$ $\rightarrow$ gravitational d.o.f.

$q_N^{(B)i}\,,\quad N > 0$ $\rightarrow$ new matter d.o.f. (associated to the bosonic modes)

\`Affinization' of the gravitational SU(2) finite Lie algebra ➥ Infinite tower of new d.o.f.

CFT Partition Function

✦ Back to the cylinder, on to the torus: $z \;\rightarrow\; w = it_E + x \;\rightarrow$ identify 2 periods

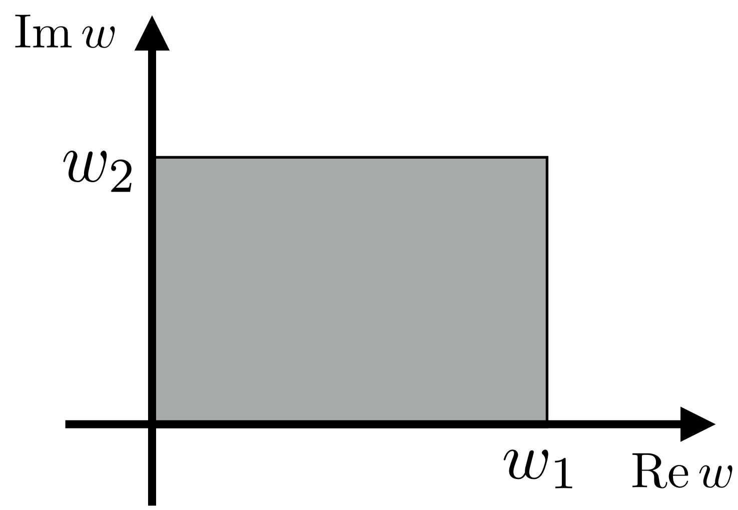
a torus on the complex w-plane

Hamiltonian (time translation)

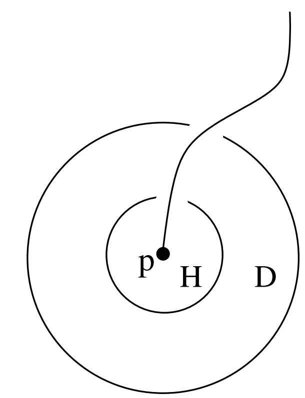
$$\hat{H} = \hat{L}_0 + \hat{\bar{L}}_0 - \frac{c}{12}$$

Momentum (space translation)

$$\hat{P} = i(\hat{L}_0 - \hat{\bar{L}}_0)$$



CFT properties depend only on the *modular parameter*: $\tau = \dfrac{w_2}{w_1}$



$$\tau = i/\epsilon_p$$

$$Z_p(\tau) = \mathrm{tr}\, e^{2\pi i \tau (\hat{L}_0 - \frac{c}{24})} e^{-2\pi i \bar{\tau} (\hat{\bar{L}}_0 - \frac{c}{24})}$$

via appropriate boundary conditions, $q_N^{(D)i} \;\rightarrow\; 0$ keep only holomorphic part to avoid over counting

➢ due to modular invariance: $\tau \;\rightarrow\; -1/\tau$ same torus ➡ notion of inverse temperature β associated to the periodicity of the rotational symmetry [DP 13]

system on a circle of circumference L with inverse temperature β $\Leftrightarrow$ system on a circle of circumference β with inverse temperature L

modular invariance $\Rightarrow$ $Z = \prod_{p=1}^{N} \sum_{j_p=0}^{k/2} \chi^k_{j_p}(\tau)$

characters of the Kac-Moody representations j's

$q \equiv e^{2\pi i \tau}$

$\tau = i\beta$

$\Delta_j = \frac{j(j+1)}{k+2}$

$$\chi^k_j(\tau) = \mathrm{tr}_{j,k}[q^{\hat{L}_0 - \frac{c}{24}} \overbrace{e^{2\pi i \hat{H}^3_0}}^{\text{account for extra Lie algebra symmetry}}]$$

$$= \frac{q^{\frac{j(j+1)}{2(k+2)}} \sum_{m\in\mathbb{Z}} (-)^{2j+m(k+2)} (2j+1+(k+2)m) q^{(k+2)m^2+(2j+1)m}}{\prod_{m=1}^{\infty}(1-q^m)^3}$$

[Goddard, Kent, Olive 86]

$\Downarrow$ semi-classical limit

$\epsilon_p \to 0$

$k \to \infty$

$$Z = \prod_{p=1}^{N} \sum_{j_p=0}^{k/2} (2j_p+1) e^{2\pi i \tau \Delta_{j_p}} e^{2\pi i j_p}$$

in general, $\mathrm{tr}[q^{L_0}] = \sum_j \rho(j) q^{\Delta_j}$ characterize the number of states $\rho(j)$ that occur at a given level Δ_j

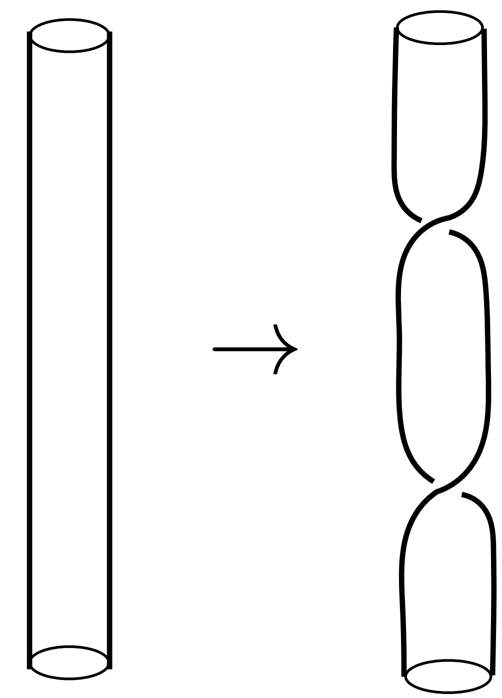
Holographic bound

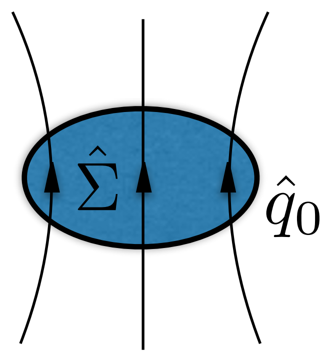
$\Rightarrow$ $\rho(j_p) = \exp\left(a_p/4\ell_P^2\right)$ with $\gamma = i$

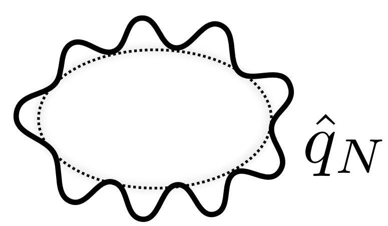
$(a_p = 8\pi \ell_P^2 \gamma j_p)$

- Regularization procedure introduces a new boundary at each puncture
- Infinite set of charges satisfying a Kac-Moody algebra (diffeos on the circle)
- Due to central extension would-be-gauge d.o.f. become physical
- IH boundary conditions → CFT/gravity correspondence

dynamics induced by L_0
=
particles self-interactions







Local conformal symmetry
at each puncture on the horizon

$\gamma = i$

ultimately related to
the horizon thermality

[Frodden, Geiller, Noui, Perez 12]
[DP 13]

Extra (matter) d.o.f.

↓

holographic degeneracy factor in Z in agreement
with Bekenstein-Hawking formula

Speculation:

If we see each spin network intertwiner as a micro-BH (see e.g. [Krasnov, Rovelli 09]), then this new regularization can provide an alternative way to couple matter dof in LQG

⇨ Unified CFT description of gravity and matter at the Planck scale

✧ Fundamental conformal invariance (as an alternative to lack of new physics at LHC)??

SM valid up to the Planck scale [Froggatt, Nielsen 95]:
top quark and Higgs masses predicted from the "Multiple Point Principle" assumption,
i.e. the Standard Model effective Higgs potential should have two degenerate minima (vacua),
one of which should be at the Planck scale, where it vanishes!

Scenario supported by the recent NNLO calculation of
[Degrassi, Di Vita, Elias-Miró, Espinosa, Giudice, Isidori, Strumia 13].

Nature started at the Planck level at a very distinguished point: free scalar theory in a new vacuum
(Higgs scalars are actually Goldstones of spontaneously broken conformal symmetry)
[Gorsky, Mironov, Morozov, Tomaras 14]

Implications for inflation??