Exclusive Processes and New Perspectives for QCD

Valparaiso, Chile  May 19-20, 2011

Stan Brodsky

Jack Gunion Fest
March 28-29, 2014
University of California, Davis

31 Joint Papers!!
Composite Theory of Large Angle Scattering and New Tests of Parton Concepts
Published in Phys.Lett. B39 (1972) 649
SLAC-PUB-1037

Composite Theory of Inclusive Scattering at Large Transverse Momenta
Published in Phys.Rev. D6 (1972) 2652
SLAC-PUB-1053

Large Angle Scattering and the Interchange Force
Published in Phys.Rev. D8 (1973) 287
SLAC-PUB-1183
DOI: 10.1103/PhysRevD.8.287

The Connection Between Regge Behavior And Fixed Angle Scattering
Richard Blankenbecler, Stanley J. Brodsky (SLAC), J.F. Gunion (MIT, LNS), R. Savit (SLAC).
Published in Phys.Rev. D8 (1973) 4117
SLAC-PUB-1294

- Pioneering papers on Hard Exclusive Processes
- Fixed-Angle Scaling, Angular Dependence
- Dominance of Quark Interchange
- Reggeons recede to negative integers!

\[ \frac{d\sigma}{dt}(AB \rightarrow CD) \]

at high transverse momentum

\[ \pi p \rightarrow \pi p \]

\[ pp \rightarrow pp \]

\[ \gamma p \rightarrow K \Lambda \]

\[ \alpha_R(t) \rightarrow -1 \text{ at large negative } t \]
Constituent Interchange Model (CIM)

*Blankenbecler, Gunion, sjb* (1972)

Analog of (electron) spin exchange in atom atom scattering
Analog of (electron) spin exchange in atom atom scattering

Constituent Interchange Model (CIM)
Blankenbecler, Gunion, sjb (1972)
\[
\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}
\]

\[
M(t, u)_{\text{interchange}} \propto \frac{1}{ut^2}
\]

\[
M(s, t)_{A+B\rightarrow C+D}
\]

\[
= \frac{1}{2(2\pi)^3} \int d^2k \int_0^1 \frac{dx}{x^2(1-x)^2} \Delta \psi_C(\vec{k}_\perp - x\vec{t}_\perp, x) \psi_D(\vec{k}_\perp + (1-x)\vec{t}_\perp, x) \psi_A(\vec{k}_\perp - x\vec{t}_\perp + (1-x)\vec{d}_\perp, x) \psi_B(\vec{k}_\perp, x)
\]

\[
\Delta = s - \sum_i \frac{k_{\perp i}^2 + m_i^2}{x_i}
\]

\textit{Agrees with electron exchange in atom-atom scattering in nonrelativistic limit}
AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions.

\[ M(t, u)_{\text{interchange}} \propto \frac{1}{ut^2} \]

\[ \frac{d\sigma}{dt}(K^+ p \rightarrow K^+ p) \propto \frac{1}{s^2 u^2 t^4} \]

**Non-linear Regge behavior:**

\[ \alpha_R(t) \rightarrow -1 \]

\[ \frac{d\sigma}{dt}(MB \rightarrow MB) = \frac{F(\theta_{cm})}{s^8} \text{ at fixed } \theta_{cm} \]

**Test of BBG Quark Interchange Mechanism**
Test of BBG Quark Interchange Mechanism in $pp \rightarrow pp$

\[ \frac{d\sigma}{dt} (pp \rightarrow pp) \propto \frac{1}{s^2 u^4 t^4} \]

\[ \frac{d\sigma}{dt} (pp \rightarrow pp) = \frac{F(\theta_{cm})}{s^{10}} \quad \text{at fixed } \theta_{cm} \]

\[ \alpha_R(t) \rightarrow -2 \]
Quark Interchange
(Spin exchange in atom-atom scattering)

\[ \frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2} \]

\[ M(t, u)_{\text{interchange}} \propto \frac{1}{ut^2} \]

\[ M(s, t)_{\text{gluon exchange}} \propto sF(t) \]
Comparison of Exclusive Reactions at Large $t$

B. R. Baller, (a) G. C. Blazey, (b) H. Courant, K. J. Heller, S. Heppelmann, (c) M. L. Marshak, E. A. Peterson, M. A. Shupe, and D. S. Wahl (d)

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D. S. Barton, G. Bunce, A. S. Carroll, and Y. I. Makdisi
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and

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(Received 28 October 1987; revised manuscript received 3 February 1988)

Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.: $\pi^\pm p \rightarrow p\pi^\pm, rp^\pm, \pi^+\Delta^\pm, K^+\Sigma^\pm, (\Lambda^0/\Sigma^0)K^0, K^\pm p \rightarrow pK^\pm; p^\pm p \rightarrow pp^\pm$. By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.
Convolution of Quark-Gluon Scattering Amplitude with LF Wavefunctions

\[ s + t + u = 2M_p^2 + 2M_K^2 \]

\[ \vec{r}_\perp \cdot \vec{q}_\perp = 0 \]

\[ t = -\vec{q}_\perp^2 \]

\[ \psi_p(x_i, \vec{k}_\perp i, \lambda_i) \]

BBG: Light-Front Wavefunctions (frame-independent)
Light-Front Wavefunctions: **rigorous representation of composite systems in quantum field theory**

\[ x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} \]

**Dirac: Front Form**

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

**Invariant under boosts! Independent of** \( P^\mu \)

Causal, Frame-independent, Simple Vacuum, Current Matrix Elements are overlap of LFWFS

Fixed \( \tau = t + z/c \)

\[ x_i P^+, x_i \vec{P}_{\perp} + \vec{k}_{\perp i} \]

\[ \sum_i^n x_i = 1 \]

\[ \sum_i^n \vec{k}_{\perp i} = \vec{0}_{\perp} \]
\[ K^\mu = (P^+, \frac{M^2_K + r^2 + q^2}{P^+}, \vec{q}_\perp + \vec{r}_\perp) \]
\[ K'^\mu = (P^+, \frac{M^2_K + r^2}{P^+}, \vec{r}_\perp) \]

\[ P^\pm = P^0 \pm P^3 \]

\[ P^\mu = (P^+, P^-, \vec{P}_\perp) = (P^+, \frac{M^2_p}{P^+}, \vec{0}_\perp) \]
\[ P'^\mu = (P^+, \frac{M^2_p + q^2}{P^+}, \vec{q}_\perp) \]

\[ s + t + u = 2M^2_p + 2M^2_K \]

\[ u = -\vec{r}^2 \]

\[ t = -\vec{q}^2 \]

\[ \psi_p(x, \vec{k}_\perp, \lambda) \]

**BBG: Remarkable LF Frame**

**Bj: “Fool’s ISR Frame”**

Ideal for QCD factorization proofs
Single A+=0 Gauge
**Light-Front Wavefunctions**

Dirac’s Front Form: Fixed $\tau = t + z/c$

$$\psi(x_i, \vec{k}_\perp i, \lambda_i)$$

Invariant under boosts. Independent of $P^\mu$

$$H_{LF}^{QCD} |\psi >= M^2 |\psi >$$

Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space
**Exact frame-independent formulation of nonperturbative QCD!**

\[
L_{QCD} \rightarrow H_{LF}^{QCD}
\]

\[
H_{LF}^{QCD} = \sum_i \left[ \frac{m^2 + k^2}{x} \right]_i + H_{LF}^{int}
\]

\[H_{LF}^{int}: \text{Matrix in Fock Space}\]

\[
H_{LF}^{QCD} \left| \Psi_h \right> = \mathcal{M}_h^2 \left| \Psi_h \right>
\]

\[
| p, J_z > = \sum_{n=3} \psi_n(x_i, k_{i\perp}, \lambda_i) | n; x_i, k_{i\perp}, \lambda_i >
\]

**Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions**

**LFWFs: Off-shell in P- and invariant mass**

**DLCQ, BLFQ**
\[ \langle p + q | j^+(0) | p \rangle \geq 2p^+ F(q^2) \]

\[ q_{\perp}^2 = Q^2 = -q^2 \]

\[ q^+ = 0 \quad \vec{q}_{\perp} \]

Form Factors are Overlaps of LFWFs

Fixed \( \tau = t + z/c \)

Drell & Yan, West
Drell, sjb
Exact LF formula!
Calculation of proton form factor in Instant Form

\[ \langle p + q | J^\mu(0) | p \rangle \]

- Need to boost proton wavefunction from \( p \) to \( p + q \): Extremely complicated dynamical problem; particle number changes
- Need to couple to all currents arising from vacuum!! Remains even after normal-ordering
- Each time-ordered contribution is frame-dependent
\[ \psi_n(x_i, k_{\perp i}, \lambda_i) \]

- **Light Front Wavefunctions:**

- Transverse density in momentum space
- Transverse density in position space

**Light Front Wavefunctions:**

- **GTMDs**
- **TMFFs**
- **GPDs**
- **TMDs**
- **TMSDs**
- **PDFs**
- **FFs**

- **Momentum space**
  \[ \vec{k}_{\perp} \leftrightarrow \vec{z}_{\perp} \]
- **Position space**
  \[ \vec{b}_{\perp} \leftrightarrow \vec{\Delta}_{\perp} \]

**Transverse density in position space**

**Longitudinal**

**Transverse**

**Sivers, T-odd from lensing**

**Lorce, Pasquini**
• LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics

• LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian

• Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors

• Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, .... modulo `lensing’ from ISIs, FSIs

• Cannot compute current matrix elements using instant form from eigensolutions alone -- need to include vacuum currents!

• Hadron Physics without LFWFs is like Biology without DNA!
Counting Rules:
Inspired by BBG

\[
\frac{d\sigma}{dt}(s, t) = \frac{F(\theta_{cm})}{s^{[n_{tot}-2]}} \quad s = E_{cm}^2
\]

\[
F_H(Q^2) \sim \left[ \frac{1}{Q^2} \right]^{n_H-1}
\]

\[
n_{tot} = n_A + n_B + n_C + n_D
\]

Fixed \( t/s \) or \( \cos \theta_{cm} \)

pQCD predicts the leading-twist scaling behavior of fixed-CM angle exclusive amplitudes

\[
s, -t >> m_\ell^2
\]

Non-Perturbative Proof from AdS/CFT: Polchinski and Strassler
Distribution Amplitudes
(gauge and frame-independent)

\[ K^+ \to K^+ \]

\[ \phi_K(x, \tilde{Q}^2) \]

\[ \phi_K(x, \tilde{Q}^2) \]

\[ \phi_P(x_1, x_2; \tilde{Q}^2) \]

\[ \phi_P(x_1, x_2; \tilde{Q}^2) \]

\[ p \to p' \]

\[ M = \int \prod d x_i d y_i \phi_F(x, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \phi_I(y_i, Q) \]

Distribution Amplitudes
(gauge and frame-independent)
PQCD and Exclusive Processes

\[ M = \int \prod dx_i dy_i \phi_F(x, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \phi_I(y_i, Q) \]

- Iterate kernel of LFWFs when at high virtuality; distribution amplitude contains all physics below factorization scale
- Rigorous Factorization Formulae: Leading twist
- Underly Exclusive B-decay analyses
- Distribution amplitude: gauge invariant, OPE, evolution equations, conformal expansions
- BLM/PMC scale setting: sum nonconformal contributions in scale of running coupling
- Derive Dimensional Counting Rules/ Conformal Scaling

Inspired by BBG Factorization
Quark-Counting: \[ \frac{d\sigma}{dt}(pp \rightarrow pp) = \frac{F(\theta_{CM})}{s^{10}} \]

Best Fit
\[ n = 9.7 \pm 0.5 \]

Reflects underlying conformal scale-free interactions
Counting Rules: $n = 9 - 2 = 7$

$$\frac{d\sigma}{dt} (\gamma p \rightarrow MB) = \frac{F(\theta_{cm})}{s^7}$$

**Conformal interactions; AdS/QCD**
Scaling behavior in exclusive meson photoproduction from Jefferson Lab at large momentum transfers

\[ -0.95 \leq \cos \theta_{\text{c.m.}} \leq 0.95. \]
Hard Exclusive Processes

- PQCD Factorization
- Convolution of Hadron Distribution Amplitudes with Hard QCD
- Leading Twist: Counting Rules
- Hadron Helicity Conservation
- Color Transparency
- BBG Quark Interchange
- Absence of Landshoff Amplitudes
- Puzzle: Huge Krisch $R_{NN}$
Hadron Distribution Amplitudes

$$\phi_M(x, Q) = \int^Q d^2 \vec{k} \, \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

\[\sum_{i} x_i = 1\]

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons
- ERBL Evolution Equations from PQCD, OPE,
- Conformal Invariance
- Compute from valence light-front wavefunction in light-cone gauge
- Anomalous Dimensions, OPE

$$k^2_\perp < Q^2$$

Fixed $\tau = t + z/c$

$Lepage, sjb$

Efremov, Radyushkin

Sachrajda, Frishman Lepage,
Braun, Gardi
ERBL Evolution of Meson Distribution Amplitude

\[
x_1 x_2 Q^2 \frac{\partial}{\partial Q^2} \tilde{\phi}(x_i, Q) = C_F \frac{\alpha_s(Q^2)}{4\pi} \left\{ \int_0^1 [dy] V(x_i, y_i) \tilde{\phi}(y_i, Q) - x_1 x_2 \tilde{\phi}(x_i, Q) \right\}
\]

where \( \tilde{\phi} = x_1 x_2 \phi \)

\[
V(x_i, y_i) = 2 \left[ x_1 y_2 \theta(y_1 - x_1) \left( \delta_{h_1 h_2} + \frac{\Delta}{y_1 - x_1} \right) + (1 \leftrightarrow 2) \right]
\]

\[
= V(y_i, x_i),
\]

and \( \Delta \tilde{\phi}(y_i, Q) \equiv \tilde{\phi}(y_i, Q) - \tilde{\phi}(x_i, Q) \).

\[
\phi(x_i, Q) = x_1 x_2 \sum_{n=0}^{\infty} a_n C_n^{3/2} (x_1 - x_2) \left( \ln \frac{Q^2}{\Lambda^2} \right)^{\gamma_n}
\]

where

\[
\gamma_n = \frac{C_F}{\beta} \left( 1 + 4 \sum_{k=2}^{n+1} \frac{1}{k} - \frac{2 \delta_{h_1 h_2}}{(n+1)(n+2)} \right) \geq 0.
\]

Fixed \( \tau = t + z/c \)

\[
\phi_M(x, Q) \quad x = x_1
\]

\[
b_\perp \sim 1/Q \quad x_2 = 1 - x
\]

AdS/QCD

Evolves from \( \sqrt{x(1-x)} \) to \( x(1-x) \)

\[
\phi_\pi(x) = \frac{4}{\sqrt{3\pi}} f_\pi \sqrt{x(1-x)}
\]
Timelike proton form factor in PQCD

\[
G_M(Q^2) \rightarrow \frac{\alpha_s^2(Q^2)}{Q^4} \sum_{n,m} b_{nm} \left( \log \frac{Q^2}{\Lambda^2} \right)^{\gamma_n^B + \gamma_n^B} \\
\times \left[ 1 + \mathcal{O} \left( \alpha_s(Q^2), \frac{m^2}{Q^2} \right) \right]
\]

Lepage and Sjb
Unexpected spin effects in pp elastic scattering

D.G. Crabb et al., PRL 41, 1257 (1978)
Spin Correlations in Elastic $p - p$ Scattering

$R_{NN}$

$p_{lab}$

$\mu_b$ near Charm Threshold

Ratio reaches 4:1!

Heppelmann et al.

Breakdown of Color Transparency


“The results challenge the prevailing theory that describes the proton’s structure and forces”

de Teramond & sjb: $B=2$ Resonance near Charm Threshold

$|uuduudc\bar{c}>$
Pioneering papers on DVCS

Interference with Bethe-Heitler

J=0 Fixed Pole in Real Part

Gauge Invariance, Leading Twist, Regge behavior
Leading-Twist Contribution to Real Part of DVCS

\[ T = -2 \sum_q \frac{e_q^2}{x_q} \gamma \cdot \bar{\gamma} \]

\[ T \propto s^0 F_{C=+}(t = 0) \]

Origin of ‘D-Term’ in QCD
s-independent ‘J=0 fixed pole’

Analytic continuation in \( \alpha_R \)

Close, Gunion, sjb
Szczepaniak, Llanes Estrada, sjb

LF Instantaneous interaction

Damashek, Gilman
Diffractive leptoproduction of vector mesons in QCD

Stanley J. Brodsky (SLAC), L. Frankfurt (Tel Aviv U.), J. F. Gunion (UC, Davis), Alfred H. Mueller (Columbia U.), M. Strikman (Penn State U.)

Jan 1994 - 34 pages

DOI: 10.1103/PhysRevD.50.3134
SLAC-PUB-6412, CU-TP-617, UCD-93-36
e-Print: hep-ph/9402283 | PDF

- Factorization Principle
- LF Wave Function, Distribution Amplitude
- $s, 1/Q^6$ dependence, $\sigma_L/\sigma_T$
AdS/QCD Holographic Wave Function for the $\rho$ Meson and Diffractive $\rho$ Meson Electroproduction

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We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive $\rho$-meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron–proton collider.

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

Fixed $\tau = t + z/c$

\[ \begin{array}{c}
\text{\(x, \vec{k}_\perp\)} \\
\text{\(1 - x, -\vec{k}_\perp\)}
\end{array} \]
AdS/QCD Holographic Wave Function for the $\rho$ Meson and Diffractive $\rho$ Meson Electroproduction

$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{\kappa^2 x(1-x)}{2}}$

Prediction from Light-Front Holography

$\gamma^* p \rightarrow \rho^0 p'$

J. R. Forshaw, R. Sandapen
**Atomic Physics from First Principles**

\[ \mathcal{L}_{QED} \rightarrow H_{QED} \]

\[ (H_0 + H_{int}) \psi = E \psi \]

\[ \left[ -\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r}) \right] \psi(\vec{r}) = E \psi(\vec{r}) \]

\[ \left[ -\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{l(l+1)}{r^2} + V_{\text{eff}}(r, S, \ell) \right] \psi(r) = E \psi(r) \]

\[ V_{\text{eff}} \rightarrow V_C(r) = -\frac{\alpha}{r} \]

**Coupled Fock states**

**Effective two-particle equation**

**Includes Lamb Shift, quantum corrections**

**Spherical Basis** \( r, \theta, \phi \)

**Coulomb potential**

**Semiclassical first approximation to QED \( \rightarrow \)** Bohr Spectrum
**Light-Front QCD**

\[ \mathcal{L}_{QCD} \xrightarrow{H_{QCD}^{LF}} H_{QCD}^{LF} \]

\[(H_{LF}^0 + H_{LF}^I)|\Psi| \geq M^2|\Psi| >\]

\[\left(\frac{\vec{k}^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF}\right) \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)\]

\[\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1 + 4L^2}{4\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)\]

**AdS/QCD:**

\[U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)\]

**Semiclassical first approximation to QCD**

**Confining AdS/QCD potential!**

**Sums an infinite # diagrams**
\[ \zeta^2 = x(1 - x)b_\perp^2. \]

**Light-Front Schrödinger Equation**

\[
\left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)
\]

**Unique Confinement Potential!**

**Conformal Symmetry of the action**

**Confinement scale:**

\[ \kappa \approx 0.6 \text{ GeV} \]

\[ 1/\kappa \approx 1/3 \text{ fm} \]

**de Alfaro, Fubini, Furlan:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!
Meson Spectrum in Soft Wall Model

- Effective potential: \[ U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2 (J - 1) \]

- LF WE

\[
\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (J - 1) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)
\]

- Normalized eigenfunctions

\[ \langle \phi | \phi \rangle = \int d\zeta \phi^2(z)^2 = 1 \]

\[ \phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2) \]

- Eigenvalues

\[ M_{n,J,L}^2 = 4\kappa^2 \left( n + \frac{J + L}{2} \right) \]
Massless pion in Chiral Limit!  

Same slope in n and L!

\[ M^2_{n,L,S} = 4\kappa^2 \left( n + L + S/2 \right) \]

Mass ratio of the \( \rho \) and the \( a_1 \) mesons: coincides with Weinberg sum rules

\[ m_q = 0 \]

G. de Teramond, H. G. Dosch, sjb
\( M^2 = 4\kappa^2 \left( n + \frac{J + L}{2} \right) \)

\( \kappa = 0.59 \text{ MeV} \)

\( \kappa = 0.54 \text{ MeV} \)
Orbital and Radial Excitations

\[ M^2 = 4\kappa^2 \left( n + \frac{J + L}{2} \right) \]

Weisberger

\[ \delta M^2 = \sum_i < \frac{m_i^2}{x_i} > \]

\[ m_q = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV} \]

Kaon Spectrum

de Tèramond, Dosch, sjb
Remarkable Features of Light-Front Schrödinger Equation

- Relativistic, frame-independent
- QCD scale appears - unique LF potential
- Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter
- Zero-mass pion for zero mass quarks!
- Regge slope same for n and L -- not usual HO
- Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry
- Phenomenology: LFWFs, Form factors, electroproduction
- Extension to heavy quarks

\[ U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \]
**Prediction from AdS/QCD: Meson LFWF**

\[ \psi_M(x, k_\perp) \]

**Note coupling**

\[ k_\perp^2, x \]

\[ \psi_M(x, k_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}} \]

\[ \phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)} \]

\[ f_\pi = \sqrt{P_{qq}} \sqrt{\frac{3}{8}} \kappa = 92.4 \text{ MeV} \]

**“Soft Wall” model**

massless quarks

**Fast Wall**

de Teramond, Cao, sjb

**Provides Connection of Confinement to Hadron Structure**
• LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = M^2 |\phi\rangle$ is a LF wave equation for $\phi$

\[
\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} \right) \phi(\zeta) = M^2 \phi(\zeta)
\]

- kinetic energy of partons
- confinement

• Critical value $L = 0$ corresponds to lowest possible stable solution, the ground state of the LF Hamiltonian

• Relativistic and frame-independent LF Schrödinger equation: $U$ is instantaneous in LF time

• A linear potential $V_{eff}$ in the instant form implies a quadratic potential $U_{eff}$ in the front form at large $q\bar{q}$ separation (thus linear Regge trajectories for small quark masses!)

\[
U_{eff} = V_{eff}^2 + 2\sqrt{p^2 + m_q^2} V_{eff} + 2 V_{eff} \sqrt{p^2 + m_q^2}
\]

• Result follows from comparison of invariant mass in the instant form in the CMS, $P = 0$, with invariant mass in front form in the constituent rest frame (CRF): $p_q + p_{\bar{q}} = 0$

Trawinski, de Teramond, Dosch, Glazek, sjb
• As Simple as Schrödinger Theory in Atomic Physics

• Relativistic, Frame-Independent, Color-Confining

• Confinement in QCD -- What sets the QCD mass scale?

• QCD Coupling at all scales

• Hadron Spectroscopy

• Light-Front Wavefunctions

• Form Factors, Structure Functions, Hadronic Observables

• Constituent Counting Rules

• Hadronization at the Amplitude Level

• Insights into QCD Condensates

• Chiral Symmetry
$\phi(z)$

**AdS5: Conformal Template for QCD**

- **Light-Front Holography**

  Fixed $\tau = t + z/c$

\[ \psi_n(x_i, k_{\perp i}, \lambda_i) \]

Duality of AdS$_5$ with LF Hamiltonian Theory

**Light Front Wavefunctions:**

- **Light-Front Schrödinger Equation**
- **Spectroscopy and Dynamics**

$k_{\perp}(\text{GeV})^{1.4}$
Exclusive Processes and New Perspectives for QCD

The Holographic Correspondence

• In the "semiclassical" approximation to QCD with massless quarks and no quantum loops the function is zero and the approximate theory is scale and confomal invariant.

• Isomorphism of $SO(4, 2)$ of confocal QCD with the group of isometries of AdS space $d^2 = R^2 z^2 (\mu dx \mu d z^2)$.

• Semiclassical correspondence as a first approximation to QCD strongly coupled at all scales.

• $x^\mu \rightarrow z$, maps scale transformations into the holographic coordinate $z$.

• Different values of $z$ correspond to different scales at which the hadron is examined: AdS boundary at $z = 0$ corresponds to the QCD UV scale separation limit.

• There is a maximum separation of quarks and a maximum value of $z$ at the IR boundary.

• Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

• Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

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\( \psi(x, \vec{b}_\perp) \quad \leftrightarrow \quad \phi(z) \)

\[
\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}
\]

\[
\psi(x, \zeta) = \sqrt{x(1-x)\zeta^{-1/2}} \phi(\zeta)
\]

\((\mu R)^2 = L^2 - (J - 2)^2\)

**Light-Front Holography**: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion.
Light-Front Holography

- **AdS\(_5\)/CFT\(_4\)**  Duality between AdS\(_5\) and Conformal Gauge Theory in 3+1 at fixed LF time  [G. de Téramond, H. G. Dosch, sjb](#)

  Valery E. Lyubovitskij, Tanja Branz, Thomas Gutsche, Ivan Schmidt, Alfredo Vega

- **AdS\(_4\)/CFT\(_3\)** Construction from Collective Fields”  [Robert de Mello Koch, Antal Jevicki, Kewang Jin, João P. Rodrigues](#)

- “Exact holographic mapping and emergent space-time geometry”  [Xiao-Liang Qi](#)

- Ehrenfest arguments:  [Glazek and Trawinski](#)
Dilaton-Modified AdS/QCD

\[ ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2) \]

- Soft-wall dilaton profile breaks conformal invariance \( e^{\varphi(z)} = e^{+\kappa^2 z^2} \)
- Color Confinement
- Introduces confinement scale \( \kappa \)
- Uses AdS\(_5\) as template for conformal theory
AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

\[
\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)
\]

\[
U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)
\]

Positive-sign dilaton

Dosch, de Teramond, sjb

Identical to Light-Front Bound State Equation!

\[
z \quad \leftrightarrow \quad \zeta = \sqrt{x(1-x)b_{\perp}^2}
\]
Introduce “Dilaton” to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

\[ ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right) \]

where \( \varphi(z) \to 0 \) at small \( z \) for geometries which are asymptotically AdS

- Gravitational potential energy for object of mass \( m \)

\[ V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z} \]

- Consider warp factor \( \exp(\pm \kappa^2 z^2) \)

- Plus solution: \( V(z) \) increases exponentially confining any object in modified AdS metrics to distances \( \langle z \rangle \sim 1/\kappa \)

\[ e^{\varphi(z)} = e^{+\kappa^2 z} \]

Klebanov and Maldacena

Positive-sign dilaton

- de Teramond, sjb

Stan Brodsky

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March 28-29, 2014

Exclusive Processes and New Perspectives for QCD
$Q^2 F_\pi(Q^2)$

$Q^2$ GeV$^2$
Photon-to-pion transition form factor

\[ Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2f_\pi \]

Lepage, sjb

F.-G. Cao, G. de Teramond, sjb

F. G. Cao, G. de Teramond, sjb

Belle

BaBar

CLEO

CELLO

Free current; Twist 2

Dressed current; Twist 2

Dressed current; Twist 2+4

Photon-to-pion transition form factor

Boundary propagator is

\[ \text{The new results again disagree with} \]

\[ \text{The simple valence} \]

\[ \text{components} \]

\[ \text{by introducing the dressed current} \]

\[ \text{In the case of soft wall potential} \]

\[ \text{the EM bulkitoi} \]

\[ \text{where} \]

\[ \text{The results calculated with} \]

\[ \text{Inserting the pion wave function} \]

\[ \text{The results computed with the free current and the experimental data at lowi} \]

\[ \text{and} \]

\[ \text{BrodskyiLepage's asymptotic prediction for the pion TFFw} \]

\[ \text{Noticing that the second term in Eqk d} \]

\[ \text{Thus the anomaly result} \]

\[ \text{In the amplitude d} \]

\[ \text{representation} \]

\[ \text{V} \]

\[ \text{v} \]

\[ \text{q} \]

\[ \text{f} \]

\[ \text{boundary propagator is} \]

\[ \text{0.00} \]

\[ \text{0.05} \]

\[ \text{0.10} \]

\[ \text{0.15} \]

\[ \text{0.20} \]

\[ \text{0.25} \]

\[ \text{0.30} \]

\[ \text{0.01} \]

\[ \text{0.02} \]

\[ \text{0.03} \]

\[ \text{0.04} \]

\[ \text{0.05} \]

\[ \text{0.06} \]

\[ \text{0.07} \]

\[ \text{0.08} \]

\[ \text{0.09} \]

\[ \text{0.10} \]

\[ \text{0.11} \]

\[ \text{0.12} \]

\[ \text{0.13} \]

\[ \text{0.14} \]

\[ \text{0.15} \]

\[ \text{0.16} \]

\[ \text{0.17} \]

\[ \text{0.18} \]

\[ \text{0.19} \]

\[ \text{0.20} \]

\[ \text{0.21} \]

\[ \text{0.22} \]

\[ \text{0.23} \]

\[ \text{0.24} \]

\[ \text{0.25} \]

\[ \text{0.26} \]

\[ \text{0.27} \]

\[ \text{0.28} \]

\[ \text{0.29} \]

\[ \text{0.30} \]
Current Matrix Elements in AdS Space (SW)

- Propagation of external current inside AdS space described by the AdS wave equation

\[
\left[ z^2 \partial_z^2 - z \left( 1 + 2 \kappa^2 z^2 \right) \partial_z - Q^2 z^2 \right] J_\kappa(Q, z) = 0.
\]

- Solution bulk-to-boundary propagator

\[
J_\kappa(Q, z) = \Gamma \left( 1 + \frac{Q^2}{4\kappa^2} \right) U \left( \frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2 \right),
\]

where \( U(a, b, c) \) is the confluent hypergeometric function

\[
\Gamma(a) U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1 + t)^{b-a-1} dt.
\]

- Form factor in presence of the dilaton background \( \varphi = \kappa^2 z^2 \)

\[
F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).
\]

- For large \( Q^2 \gg 4\kappa^2 \)

\[
J_\kappa(Q, z) \to z Q K_1(zQ) = J(Q, z),
\]

the external current decouples from the dilaton field.
Dressed soft-wall current brings in higher Fock states and more vector meson poles.
Timelike Pion Form Factor from AdS/QCD and Light-Front Holography

\[ F_\pi(s) = (1 - \gamma) \left(1 - \frac{s}{M_\rho^2}\right) + \gamma \left(1 - \frac{s}{M_\rho^2}\right)^2 \left(1 - \frac{s}{M_{\rho'}^2}\right) \left(1 - \frac{s}{M_{\rho''}^2}\right) \]

\[ M_{\rho_n}^2 = 4\kappa^2 \left(\frac{1}{2} + n\right) \]

\[ \gamma = 0.17 \]

Prescription for Timelike poles:

\[ \frac{1}{s - M^2 + i\sqrt{s}\Gamma} \]

14\% four-quark probability

Frascati data
\[ \log |F_\pi(s)| \]

**spacelike**

**timelike**

**JLab**

**Frascati**

**BaBar ISR**

\[ q^2 \text{ (GeV}^2) \]
AdS/QCD Soft-Wall Model

\[ \zeta^2 = x(1-x)b^2_\perp. \]

Light-Front Schrödinger Equation

\[ \left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta) \]

Confinement scale:

\[ \kappa \simeq 0.6 \, \text{GeV} \]

\[ \frac{1}{\kappa} \simeq \frac{1}{3} \, \text{fm} \]

• de Alfaro, Fubini, Furlan:

Unique Confinement Potential!

• de Tèramond, Dosch, sjb

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!
Uniqueness

\[ U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \quad e^{\varphi(z)} = e^{+\kappa^2 z^2} \]

- $\zeta^2$ confinement potential and dilaton profile unique!
- Linear Regge trajectories in $n$ and $L$: same slope!
- Massless pion in chiral limit! No vacuum condensate!
- Conformally invariant action for massless quarks retained despite mass scale
Uniqueness of Dilaton

\[ \varphi_p(z) = \kappa^p z^p \]

\[ m_\pi^2 / \kappa^2 \]

Pion is massless in chiral limit iff \( p=2! \)

\[ e^{\varphi(z)} = e^{+\kappa^2 z^2} \]

Dosch, de Teramond, sjb

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Exclusive Processes
and New Perspectives for QCD
**QCD Lagrangian**

Fundamental Theory of Hadron and Nuclear Physics

\[ \mathcal{L}_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f \]

\[ iD_\mu = i\partial_\mu - g A_\mu \quad G^{\mu\nu} = \partial^\mu A_\nu - \partial^\nu A_\mu - g [A^\mu, A^\nu] \]

*Classically Conformal if \( m_q = 0 \)*

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

Scale-Invariant Coupling Renormalizable
Asymptotic Freedom
Color Confinement

QCD Mass Scale from Confinement not Explicit

**Gunion Fest, UC Davis**
**March 28-29, 2014**

Exclusive Processes and New Perspectives for QCD
Conformal Invariance in Quantum Mechanics.

V. DE ALFARO

Istituto di Fisica Teorica dell'Università - Torino
Istituto Nazionale di Fisica Nucleare - Sezione di Torino

S. FUBINI and G. FURLAN (*)

CERN - Geneva

(ricevuto il 3 Maggio 1976)

Summary. — The properties of a field theory in one over-all time dimension, invariant under the full conformal group, are studied in detail. A compact operator, which is not the Hamiltonian, is diagonalized and used to solve the problem of motion, providing a discrete spectrum and normalizable eigenstates. The role of the physical parameters present in the model is discussed, mainly in connection with a semi-classical approximation.
\[ G|\psi(\tau)\rangle = i \frac{\partial}{\partial \tau} |\psi(\tau)\rangle \]

\[ G = uH + vD + wK \]

\[ G = H_\tau = \frac{1}{2} \left( - \frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4} x^2 \right) \]

Retains conformal invariance of action despite mass scale!

\[ 4uw - v^2 = \kappa^4 = [M]^4 \]

Identical to LF Hamiltonian with unique potential and dilaton!

\[ \left[ - \frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta) \]

\[ U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \]
What determines the QCD mass scale $\Lambda_{QCD}$?

- Mass scale does not appear in the QCD Lagrangian (massless quarks)
- Dimensional Transmutation? Requires external constraint such as $\alpha_s(M_Z)$
- dAFF: Confinement Scale $\kappa$ appears spontaneously via the Hamiltonian: $G = uH + vD + wK \quad 4uw - v^2 = \kappa^4 = [M]^4$
- The confinement scale regulates infrared divergences, connects $\Lambda_{QCD}$ to the confinement scale $\kappa$
- Only dimensionless mass ratios (and M times R ) predicted
- Mass and time units [GeV] and [sec] from physics external to QCD
- New feature: bounded frame-independent relative time between constituents
dAFF: New Time Variable

\[ \tau = \frac{2}{\sqrt{4uw - v^2}} \arctan \left( \frac{2tw + v}{\sqrt{4uw - v^2}} \right), \]

- Identify with difference of LF time \( \Delta x^+/P^+ \) between constituents
- Finite range
- Measure in Double Parton Processes

J.F. Gunion and Z. Kunszt
**Interpretation of Mass Scale $\kappa$**

- Does not affect conformal symmetry of QCD action
- Self-consistent regularization of IR divergences
- Determines all mass and length scales for zero quark mass
- Compute scheme-dependent $\Lambda_{\overline{MS}}$ determined in terms of $\kappa$
- Value of $\kappa$ itself not determined -- place holder
- Need external constraint such as $f_\pi$
Diffractive Excitation in QCD
G. Bertsch (Santa Barbara, KITP), Stanley J. Brodsky (SLAC & Santa Barbara, KITP),
A.S. Goldhaber, J.F. Gunion (Santa Barbara, KITP).
SLAC-PUB-2748, NSF-ITP-81-34
DOI: 10.1103/PhysRevLett.47.297

• Pioneering paper on Diffractive QCD
• Color Transparency and Opacity
• Diffractive DiJet Production
• Measure LFWF
Diffractive Dissociation of Pion into Quark Jets

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus

Nucleus left Intact!
Two-gluon exchange measures the second derivative of the pion light-front wavefunction.
**E791 Diffractive Di-Jet transverse momentum distribution**

Two Components:
confinement plus gluon exchange

**Gaussian behavior**
predicted by AdS/QCD

\[
\psi_M(x, k_T) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_T^2}{2\kappa^2 x(1-x)}}
\]

\[
\frac{d\sigma}{dk_T} \propto e^{-\frac{k_T^2}{\kappa^2}} \quad (x \sim \frac{1}{2})
\]

**High transverse power-law fall-off consistent with PQCD**

ERBL Evolution \( k_T^{-6.5} \)

relative jet transverse momentum \( k_T = 2k_\perp \)
Color Transparency

A. H. Mueller, sjb
Bertsch, Gunion, Goldhaber, sjb

• Fundamental test of gauge theory in hadron physics

• Small color dipole moments interact weakly in nuclei

• Complete coherence at high energies

• Clear Demonstration of CT from Diffractive Di-Jets
Key Ingredients in the E791 Experiment

Small color-dipole moment pion not absorbed; interacts with each nucleon coherently

QCD COLOR Transparency

\[ M_A = A \cdot M_N \]

\[ \frac{d\sigma}{dt}(\pi A \rightarrow q\bar{q}A') = A^2 \cdot \frac{d\sigma}{dt}(\pi N \rightarrow q\bar{q}N') \cdot F_A^2(t) \]

Target left intact

Diffraction, Rapidity gap

Gunion, Frankfurt, Mueller, Strikman, sjb
Frankfurt, Miller, Strikman
- Fully coherent interactions between pion and nucleons.

- Emerging Di-Jets do not interact with nucleus.

\[ M(A) = A \cdot M(N) \]

\[ \frac{d\sigma}{dq^2} \propto A^2 \quad q^2 \sim 0 \]

\[ \sigma \propto A^{4/3} \]

Nuclear coherence

\[ F_A^2(q^2) \sim e^{-\frac{1}{3}R^2_Aq^2} \]
Measure pion LFWF in diffractive dijet production
Confirmation of color transparency

A-Dependence results: \( \sigma \propto A^\alpha \)

<table>
<thead>
<tr>
<th>( k_t ) range (GeV/c)</th>
<th>( \alpha )</th>
<th>( \alpha ) (CT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25 &lt; ( k_t ) &lt; 1.5</td>
<td>1.64 ± 0.06 -0.12</td>
<td>1.25</td>
</tr>
<tr>
<td>1.5 &lt; ( k_t ) &lt; 2.0</td>
<td>1.52 ± 0.12</td>
<td>1.45</td>
</tr>
<tr>
<td>2.0 &lt; ( k_t ) &lt; 2.5</td>
<td>1.55 ± 0.16</td>
<td>1.60</td>
</tr>
</tbody>
</table>

\( \alpha \) (Incoh.) = 0.70 ± 0.1

*Conventional Glauber Theory Ruled Out!*

Factor of 7
Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

\[
S = -\frac{1}{4} \int d^4x \, dz \sqrt{g} \, e^{\varphi(z)} \frac{1}{g_5^2} G^2
\]

Flow equation

\[
\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)
\]

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

YM coupling $\alpha_s(\zeta) = \frac{g_{YM}^2(\zeta)}{4\pi}$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

Coupling measured at momentum scale $Q$

\[
\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta \, d\zeta \, J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)
\]

Solution

\[
\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) \, e^{-Q^2/4\kappa^2}.
\]

where the coupling $\alpha_s^{AdS}$ incorporates the non-conformal dynamics of confinement
Running Coupling from Light-Front Holography and AdS/QCD

Analytic QCD Coupling, defined at all scales, IR Fixed Point

\[ e^{\varphi(z)} = e^{+\kappa^2 z^2} \]

\[ \alpha_s^{AdS}(Q)/\pi = e^{-Q^2/4\kappa^2} \]

\[ \kappa = 0.54 \text{ GeV} \]

Two Components

Gaussian + \[ \frac{4\pi}{\beta_0 \log Q^2} \]

\[ \varphi(z) \]

Q (GeV)


Sublimated gluons below 1 GeV

Crossing point
Two-Components in QCD

• Scale-Invariant Contribution from Gluonic Interactions

• Non-Perturbative Color-Confining Interaction from AdS/QCD and dAFF
  \[ U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1) \]

• Crossover at \( \tilde{Q} \sim 2\kappa \sim 1.2 \text{ GeV} \)

• Phenomenology: Cross-over seen in Cornell potential, diffractive dijets, and running coupling

• Sets starting point for ERBL evolution of distribution amplitude and DGLAP evolution of structure functions
AdS/QCD and Light-Front Holography

\[ \mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left( n + \frac{J + L}{2} \right) \]

- Zero mass pion for \( m_q = 0 \) (\( n=J=L=0 \))
- Regge trajectories: equal slope in \( n \) and \( L \)
- Form Factors at high \( Q^2 \): Dimensional counting
  \[ [Q^2]^{n-1} F(Q^2) \rightarrow \text{const} \]
- Space-like and Time-like Meson and Baryon Form Factors
- Running Coupling for NPQCD
  \[ \alpha_s(Q^2) \propto e^{-\frac{Q^2}{4\kappa^2}} \]
- Meson Distribution Amplitude
  \[ \phi_\pi(x) \propto f_\pi \sqrt{x(1-x)} \]

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Exclusive Processes and New Perspectives for QCD

Stan Brodsky

SLAC National Accelerator Laboratory
Hadron Multiplicity in Color Gauge Theory Models
Published in Phys.Rev.Lett. 37 (1976) 402-405
SLAC-PUB-1749, UCD-76-5
DOI: 10.1103/PhysRevLett.37.402

● Pioneering paper on color effects in Jet Production

● Key Prediction verified at

\[ \frac{dN}{dy}\big|_g = 9 \frac{dN}{dy}\big|_q \]

On The multiplicity difference between quark and gluon jets
Published in Phys.Rev. D49 (1994) 4503-4509
• We write the Dirac equation

\[
(\alpha \Pi(\zeta) - M) \psi(\zeta) = 0,
\]

in terms of the matrix-valued operator \( \Pi \)

\[
\Pi_\nu(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + \frac{1}{2} \gamma_5}{\zeta} - \kappa^2 \zeta \gamma_5 \right),
\]

and its adjoint \( \Pi^\dagger \), with commutation relations

\[
\left[ \Pi_\nu(\zeta), \Pi^\dagger_{\nu'}(\zeta) \right] = \left( \frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma_5.
\]

• Solutions to the Dirac equation

\[
\psi_+(\zeta) \sim z^{\frac{1}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_n^\nu(\kappa^2 \zeta^2),
\quad \nu = L + 1
\]

\[
\psi_-(\zeta) \sim z^{\frac{3}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu+1}(\kappa^2 \zeta^2).
\]

• Eigenvalues

\[
M^2 = 4\kappa^2 (n + \nu + 1).
\]
Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL 94, 201601 (2005)]

• Nucleon LF modes

\[ \psi_+(\zeta)_{n,L} = \kappa^{2+L} \sqrt{\frac{2n!}{(n + L)!}} \zeta^{3/2+L} e^{-\kappa^2 \zeta^2/2} L_{n}^{L+1} (\kappa^2 \zeta^2) \]

\[ \psi_-(\zeta)_{n,L} = \kappa^{3+L} \frac{1}{\sqrt{n + L + 2}} \sqrt{\frac{2n!}{(n + L)!}} \zeta^{5/2+L} e^{-\kappa^2 \zeta^2/2} L_{n}^{L+2} (\kappa^2 \zeta^2) \]

• Normalization

\[ \int d\zeta \psi_+^2(\zeta) = \int d\zeta \psi_-^2(\zeta) = 1 \]

• Eigenvalues

\[ M_{n,L,S=1/2}^2 = 4\kappa^2 (n + L + 1) \]

• “Chiral partners”

\[ \frac{M_{N(1535)}}{M_{N(940)}} = \sqrt{2} \]
Figure 2:
Orbital and radial baryon excitation spectrum. Positive-parity spin-$\frac{1}{2}$ nucleons (a) and spectrum gap between the negative-parity spin-$\frac{3}{2}$ and the positive-parity spin-$\frac{1}{2}$ nucleons families (b). Minus parity $N$ (c) and plus and minus parity $\Delta$ families (d), for $\sqrt{\lambda} = 0.49$ GeV (nucleons) and 0.51 GeV (Deltas). The predictions for the daughter trajectories for $n = 1$, $n = 2$, ..., are also shown in this figure. Only confirmed PDG states are shown. The lowest state $N(1440)$ and the $N(1710)$ are well accounted for as the first and second radial excited states of the proton. The newly identified state, the $N(1900)$ is depicted here as the first radial excitation of the $N(1720)$. The model is successful in explaining the parity degeneracy observed in the light baryon spectrum, such as the $L = 2$, $N(1680) - N(1720)$ pair in Fig. 2(a). In Fig. 2(b) we compare the positive parity spin-$\frac{1}{2}$ parent nucleon trajectory with the negative parity $\Delta$ trajectories.
Chiral Features of Soft-Wall AdS/QCD Model

- Boost Invariant
- Trivial LF vacuum! No condensate, but consistent with GMOR
- Massless Pion
- Hadron Eigenstates (even the pion) have LF Fock components of different $L^z$
- Proton: equal probability $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$
  
  \[ J^z = +1/2 : < L^z > = 1/2, < S^z > = 0 \]
- Self-Dual Massive Eigenstates: Proton is its own chiral partner.
- Label State by minimum $L$ as in Atomic Physics
- Minimum $L$ dominates at short distances
- AdS/QCD Dictionary: Match to Interpolating Operator Twist at $z=0$. 

No mass-degenerate parity partners!
Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors
  
  \[ F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2, \]
  \[ F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2, \]
  
  where the effective charges \( g_+ \) and \( g_- \) are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have \( S^z = +1/2 \). The two AdS solutions \( \psi_+(\zeta) \) and \( \psi_- (\zeta) \) correspond to nucleons with \( J^z = +1/2 \) and \(-1/2\).

- For \( SU(6) \) spin-flavor symmetry
  
  \[ F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2, \]
  \[ F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) \left[ |\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right], \]
  
  where \( F_1^p(0) = 1, \ F_1^n(0) = 0 \).
• Compute Dirac proton form factor using SU(6) flavor symmetry

\[ F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q, z) \Psi_+(z) \]

• Nucleon AdS wave function

\[ \Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n + L)!}} z^{7/2+L} L_n^{L+1} (\kappa^2 z^2) e^{-\kappa^2 z^2 / 2} \]

• Normalization \( (F_1^p(0) = 1, \ V(Q = 0, z) = 1) \)

\[ R^4 \int \frac{dz}{z^4} \Psi_+^2(z) = 1 \]

• Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

\[ V(Q, z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1 - x)^2} x \frac{Q^2}{4\kappa^2} e^{-\kappa^2 z^2 x/(1-x)} \]

• Find

\[ F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_{\rho}^2}\right)\left(1 + \frac{Q^2}{M_{\rho'}^2}\right)} \]

with \( M_{\rho n}^2 \to 4\kappa^2(n + 1/2) \)
Predict hadron spectroscopy and dynamics

Excited Baryons in Holographic QCD

G. de Teramond & sjb

Gunion Fest, UC Davis
March 28-29, 2014

Exclusive Processes
and New Perspectives for QCD

Stan Brodsky
Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs

\[ F_2^p(Q^2) = 0.49 \text{ GeV} \]

$\kappa = 0.49 \text{ GeV}$

Harmonic Oscillator Confinement
Normalized to anomalous moment

G. de Teramond, sjb

Gunion Fest, UC Davis
March 28-29, 2014

Stan Brodsky
SLAC National Accelerator Laboratory
Using $SU(6)$ flavor symmetry and normalization to static quantities.
Nucleon and flavor form factors in a light front quark model in AdS/QCD

Dipankar Chakrabarti, Chandan Mondal

1Department of Physics, Indian Institute of Technology Kanpur, Kanpur-208016, India.
Nucleon Transition Form Factors

\[ F_{1N \rightarrow N^*}^P(Q^2) = \frac{\sqrt{2}}{3} \frac{Q^2}{M^2_\rho} \left( 1 + \frac{Q^2}{M^2_\rho} \right) \left( 1 + \frac{Q^2}{M^2_{\rho'}} \right) \left( 1 + \frac{Q^2}{M^2_{\rho''}} \right). \]

Proton transition form factor to the first radial excited state. Data from JLab.
\[ |p, S_z > = \sum_{n=3}^{\infty} \Psi_n(x_i, k_{\perp i}, \lambda_i) |n; k_{\perp i}, \lambda_i > \]

**sum over states with n=3, 4, ...constituents**

The Light Front Fock State Wavefunctions

\[ \Psi_n(x_i, k_{\perp i}, \lambda_i) \]

are boost invariant; they are independent of the hadron’s energy and momentum \( P^\mu \).

The light-cone momentum fraction

\[ x_i = \frac{k^+_i}{p^+} = \frac{k^0_i + k^z_i}{P^0 + P^z} \]

are boost invariant.

\[ \sum_{i}^{n} k^+_i = P^+, \sum_{i}^{n} x_i = 1, \sum_{i}^{n} k^z_i = 0^+ . \]

**Intrinsic heavy quarks**

\( s(x), c(x), b(x) \) at high \( x \) !

Mueller: gluon Fock states

BFKL Pomeron

Hidden Color

\( \bar{s}(x) \neq s(x) \)

\( \bar{u}(x) \neq \bar{d}(x) \)
Intrinsic Chevrolets!

Intrinsic Chevrolets At The SSC
DOI/ER/40048-21 P4, C84/06/23
C84-06-23 (Snowmass Summer Study 1984:0227)

Heavy Particle Production At The SSC
SLAC-PUB-3300, C84/02/13
Invited paper given at Conference: C84-02-13 (SSC/DPF Workshop 1984:100)

A Higher Twist Correction To Heavy Quark Production
OITS-359, C87/03/08
Invited talk given at Conference: C87-03-08 (Moriond 1987: Hadrons:85)

The Physics of Heavy Quark Production in Quantum Chromodynamics
Published in Phys.Rev. D36 (1987) 2710
SLAC-PUB-4193, UCD-87-7
DOI: 10.1103/PhysRevD.36.2710

Heavy Quark Production Processes In QCD
Published in eConf C840723 (1984) 025
SLAC-PUB-3527, C84-07-23, SSI-1984-025
Invited talk given at Conference: C84-07-23 (SLAC Summer Inst.1984:603) Proceedings

- Pioneering Papers on Intrinsic Heavy Quark Fock States of Hadrons

- Rigorous scaling law from OPE:

\[ \frac{1}{M^2 Q} \]

Novel SUSY and Higgs Production Mechanisms
**HERMES: Two components to \( s(x, Q^2) \)!**

Comparison of the HERMES \( x(s(x) + \bar{s}(x)) \) data with the calculations based on the BHPS model. The solid and dashed curves are results of the BHPS model. The values of \( \mu \) are 0.5 GeV and 0.3 GeV, respectively. The normalizations of the calculations are adjusted to fit the data at \( x > 0.1 \) with statistical errors only, denoted by solid circles.

\[
s(x, Q^2) = s(x, Q^2)_{\text{extrinsic}} + s(x, Q^2)_{\text{intrinsic}}
\]
Proton Self Energy
Intrinsic Heavy Quarks

\[ x_Q \propto \left( m_Q^2 + k_{\perp}^2 \right)^{1/2} \]

Probability (QED) \( \propto \frac{1}{M_4^4} \)

Probability (QCD) \( \propto \frac{1}{M_Q^2} \)

Collins, Ellis, Gunion, Mueller, sjb
Polyakov, et al.
Proton 5-quark Fock State: *Intrinsic Heavy Quarks*

**Diagram:**
- Proton (p) entering on the left.
- Intrinsic Heavy Quarks (Q) and (Q̅) at the center.
- Fixed LF time (dotted line).

*QCD predicts Intrinsic Heavy Quarks at high x!*

*Minimal off-shellness*

Mathematical Expression:

\[ x_Q \propto (m_Q^2 + k_{\perp}^2)^{1/2} \]

**Probability (QED):**

\[ \propto \frac{1}{M_\ell^4} \]

**Probability (QCD):**

\[ \propto \frac{1}{M_Q^2} \]

*References:*
- Collins, Ellis, Gunion, Mueller, sjb
- Polyakov, et al.
**HERMES:** Two components to $s(x, Q^2)$!

Comparison of the HERMES $x(s(x) + \bar{s}(x))$ data with the calculations based on the BHPS model. The solid and dashed curves are obtained by evolving the BHPS result to $Q^2 = 2.5$ GeV$^2$ using $\mu = 0.5$ GeV and $\mu = 0.3$ GeV, respectively. The normalizations of the calculations are adjusted to fit the data at $x > 0.1$ with statistical errors only, denoted by solid circles.

$$s(x, Q^2) = s(x, Q^2)_{\text{extrinsic}} + s(x, Q^2)_{\text{intrinsic}}$$
\textbf{QCD:} (1/m_{Q^2}) \textit{scaling:} predict IC !

\begin{figure}
\centering
\includegraphics[width=\textwidth]{plot.png}
\caption{Calculations of the $\bar{c}(x)$ distributions based on the BHPS model. The solid curve corresponds to the calculation using Eq. 1 and the dashed and dotted curves are obtained by evolving the BHPS result to $Q^2 = 75$ GeV$^2$ using $\mu = 3.0$ GeV, and $\mu = 0.5$ GeV, respectively. The normalization is set at $P_5^{c\bar{c}} = 0.01$.}
\end{figure}

\begin{verbatim}
\textbf{Intrinsic Charm}
\end{verbatim}

\textit{W. C. Chang and J.-C. Peng}

\textit{arXiv: 1105.2381}

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First Evidence for Intrinsic Charm

Measurement of Charm Structure Function!


**DGLAP / Photon-Gluon Fusion: factor of 30 too small**

Two Components (separate evolution):

\[ c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}} \]
Fluctuation in Proton QCD: Probability $\sim \frac{\Lambda_{QCD}^2}{M_Q^2}$

$|uudc\bar{c}\rangle$ vs. $|e^+e^-\ell^+\ell^-\rangle$ Fluctuation in Positronium QED: Probability $\sim \frac{(m_e\alpha)^4}{M_\ell^4}$

Collins, Ellis, Gunion, Mueller, sjb; Polyakov, et. al

$c\bar{c}$ in Color Octet

Distribution peaks at equal rapidity (velocity)
Therefore heavy particles carry the largest momentum fractions

$$\vec{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

**High x charm!** Charm at Threshold

**Action Principle:** Minimum KE, maximal potential
Properties of Non-Perturbative Five-Quark Fock-State

- Dominant configuration: same rapidity
- Heavy quarks have most momentum
- Correlated with proton quantum numbers
- Duality with meson-baryon channels
- Strangeness asymmetry at $x > 0.1$
- Maximally energy efficient
Leading Hadron Production from Intrinsic Charm

Coalescence of Comoving Charm and Valence Quarks Produce $J/\psi$, $\Lambda_c$ and other Charm Hadrons at High $x_F$

Spectator counting rules
Blankenbecler, sjb

$$\frac{dN}{dx_F} \propto (1 - x_F)^{2n_{spect} - 1}$$

Gunion Fest, UC Davis
March 28-29, 2014

Exclusive Processes and New Perspectives for QCD
Large $x_F$ production close to the maximum allowed by phase space!

Spectator counting rules

leaves 2 spectator quarks

$$\Lambda_c(cu\bar{d})$$

$$\frac{d\sigma}{dx_F}(pA \rightarrow \Lambda_c X) \sim (1 - x_F)^p$$
• EMC data: \( c(x, Q^2) > 30 \times \text{DGLAP} \)
  \( Q^2 = 75 \text{ GeV}^2, \ x = 0.42 \)

• High \( x_F \) \( pp \rightarrow J/\psi X \)

• High \( x_F \) \( pp \rightarrow J/\psi J/\psi X \)

• High \( x_F \) \( pp \rightarrow \Lambda_c X \)

• High \( x_F \) \( pp \rightarrow \Lambda_b X \)

• High \( x_F \) \( pp \rightarrow \Xi(ccd)X \) (SELEX)

Explain Tevatron anomalies: \( p\bar{p} \rightarrow \gamma cX, ZcX \)

Interesting spin, charge asymmetry, threshold, spectator effects

Important corrections to B decays; Quarkonium decays

Gardner, Karliner, sbj
Intrinsic Charm Mechanism for Inclusive High-\(X_F\) Higgs Production

Higgs can have > 80\% of Proton Momentum!

Also: intrinsic strangeness, bottom, top

New production mechanism for Higgs

AFTER: Higgs production at threshold!
Intrinsic Heavy Quark Contribution to Inclusive Higgs Production

\[
\frac{d\sigma}{dx_F}(pp \rightarrow HX) [fb]
\]

LHC: \(\sqrt{s} = 14\text{TeV}\)

Tevatron: \(\sqrt{s} = 2\text{TeV}\)

Requires Forward Acceptance at the LHC
Charm at Threshold

- Intrinsic charm Fock state puts 80% of the proton momentum into the electroproduction process
- 1/velocity enhancement from FSI
- CLEO data for quarkonium production at threshold
- Krisch effect shows B=2 resonance
- All particles produced at small relative rapidity -- resonance production
- Many exotic hidden and open charm resonances will be produced at JLab (12 GeV)
Do heavy quarks exist in the proton at high $x$?

**Conventional wisdom:**

Heavy quarks generated only at low $x$
via DGLAP evolution
from gluon splitting

$$s(x, \mu_F^2) = c(x, \mu_F^2) = b(x, \mu_F^2) \equiv 0$$

at starting scale $Q_0^2 = \mu_F^2$

**Conventional wisdom is wrong even in QED!**
Analysis of Particle Production at Large Transverse Momentum
Published in Phys.Rev. D12 (1975) 3469-3487
SLAC-PUB-1585
DOI: 10.1103/PhysRevD.12.3469

Physical Effects of Hadronic Bremsstrahlung. Reactions at Large and Small Momentum Transfers
Published in Phys.Rev. D10 (1974) 2153
SLAC-PUB-1378

- Theory of Direct Subprocesses
- Exclusive-Inclusive Connection with CIM
- Fixed-\(x_T\) Scaling, Spectator Counting Rules
- Regge Behavior at large \(t\)
**Crucial Test of Leading -Twist QCD:**  
**Scaling at fixed $x_T$**

\[ E \frac{d\sigma}{d^3p} (pp \rightarrow HX) = \frac{F(x_T, \theta_{cm})}{p_T^{n_{eff}}} \]

**Parton model:** \( n_{eff} = 4 \)

**As fundamental as Bjorken scaling in DIS**

**scaling law:** \( n_{eff} = 2 \, n_{active} - 4 \)
$pp \rightarrow \gamma X$

$$E \frac{d\sigma}{d^3p}(pp \rightarrow \gamma X) = \frac{F(\theta_{cm},x_T)}{p_T^4}$$

$gu \rightarrow \gamma u$

$n_{active} = 4$

$n_{eff} = 2n_{active} - 4$

$n_{eff} = 4$
The $p+p$ collisions at $\sqrt{s} = 20-1800$ GeV and $\sqrt{s} = 20-200$ GeV are shown in the graph. The data points from the collaborations D0, CDF, UA2, UA1, UA6, PHENIX, R806, R110, E706, NA24, and WA70 are plotted. The graph shows the $x_T$-scaling of direct photon production, consistent with PQCD.
\[
E \frac{d\sigma}{d^3 p}(pp \rightarrow HX) = \frac{F(x_T, \theta_{CM})}{p_T} \frac{n_{eff}}{n_{eff}}
\]

\[
E \frac{d\sigma}{d^3 p}(pp \rightarrow pX) = \frac{F(x_T, \theta_{CM})}{p_T^{12}}
\]

"Trend consistent with RHIC at small \(x_T\)"
Direct Higher-Twist Contribution to Hadron Production

\[ \frac{d\sigma}{d^3p/E} = \alpha_s^3 f_\pi^2 \frac{F(x_\perp, y)}{p_{\perp}^6} \]

No Fragmentation Function
\[ E \frac{d\sigma}{d^3p} (pp \rightarrow H \bar{X}) = \frac{F(x_T, \theta_{CM} = \pi/2)}{p_T^{n_{eff}}} \]

Leading-twist prediction fails at ISR, FNAL, RHIC, CDF!

\[ x_T = 2p_T/\sqrt{s} \]
Protons less absorbed in nuclear collisions than pions because of dominant color transparent higher twist process.
Scale dependence

Pion scaling exponent extracted vs. \( p_\perp \) at fixed \( x_\perp \)

2-component toy-model

\[
\sigma^{\text{model}}(pp \rightarrow \pi \ X) \propto \frac{A(x_\perp)}{p_\perp^4} + \frac{B(x_\perp)}{p_\perp^6}
\]

Define effective exponent

\[
n_{\text{eff}}(x_\perp, p_\perp, B/A) \equiv -\frac{\partial \ln \sigma^{\text{model}}}{\partial \ln p_\perp} + n^{\text{NLO}}(x_\perp, p_\perp) - 4
\]

\[
= \frac{2B/A}{p_\perp^2 + B/A} + n^{\text{NLO}}(x_\perp, p_\perp)
\]

Arleo, Hwang, Sickles, sjb
RHIC/LHC predictions

PHENIX results

Scaling exponents from $\sqrt{s} = 500$ GeV preliminary data

Magnitude of $\Delta$ and its $x_\perp$-dependence consistent with predictions

Arleo, Hwang, Sickles, sjb
Two-Dimensional Confinement

Interesting feature

\[ U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \]

\[ \vec{\zeta}_\perp = \vec{b}_\perp \sqrt{x(1-x)} \]

confinement in plane of pair
Ridge in high-multiplicity p p collisions

Two-particle correlations: CMS results

- Ridge: Distinct long range correlation in \( \eta \) collimated around \( \Delta \Phi \approx 0 \) for two hadrons in the intermediate \( 1 < p_T, q_T < 3 \text{ GeV} \)

Raju Venugopalan
Possible origin of same-side CMS ridge in p p Collisions

Bjorken, Goldhaber, sjb

\[ \vec{V} = \sum_{i=1}^{N} [\cos 2\phi_i \hat{x} + \sin 2\phi_i \hat{y}] \]
\[ O = C(\alpha_s(\mu_0^2)) + B(\beta \log \frac{Q^2}{\mu_0^2}) + D\left(\frac{m_q^2}{Q^2}\right) + E\left(\frac{\Lambda_{QCD}^2}{Q^2}\right) + F\left(\frac{\Lambda_{QCD}^2}{m_Q^2}\right) + G\left(\frac{m_q^2}{m_Q^2}\right) \]

**QCD Observables**

- **Scale-Free Conformal Series**
- **Running Coupling Effects**
- **Intrinsic Heavy Quarks**
- **Higher Twist from Hadron Dynamics**
- **Light by Light Loops**

**BLM/PMC: Absorb β-terms into running coupling**

\[ O = C(\alpha_s(Q^*^2)) + D\left(\frac{m_q^2}{Q^2}\right) + E\left(\frac{\Lambda_{QCD}^2}{Q^2}\right) + F\left(\frac{\Lambda_{QCD}^2}{m_Q^2}\right) + G\left(\frac{m_q^2}{m_Q^2}\right) \]
Principle of Maximum Conformality (PMC)

- Sets pQCD renormalization scale correctly at every finite order
- Predictions are scheme-independent
- Satisfies all principles of the renormalization group
- Agrees with Gell Mann-Low procedure for pQED in Abelian limit
- Shifts all $\beta$ terms into $\alpha_s$, leaving conformal series
- Automatic procedure: $R_\delta$ scheme
- Number of flavors $n_f$ set
- Eliminates $n!$ renormalon growth
- Choice of initial scale irrelevant
- Eliminates unnecessary systematic error -- conventional guess is scheme-dependent, disagrees with QED
- Reduces disagreement with pQCD for top/anti-top asymmetry at Tevatron from 3$\sigma$ to 1$\sigma$
Set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution...

**PMC/BLM**

No renormalization scale ambiguity!

Result is independent of Renormalization scheme and initial scale!

QED Scale Setting at $N_C=0$

Eliminates unnecessary systematic uncertainty

$\delta$-Scheme automatically identifies $\beta$-terms!

---

Choose renormalization scheme; e.g. $\alpha_s^R(\mu_R^{\text{init}})$

Choose $\mu_R^{\text{init}}$; arbitrary initial renormalization scale

Identify $\{\beta_i^R\} - \text{terms using } n_f - \text{terms}$ through the PMC -- BLM correspondence principle

Shift scale of $\alpha_s$ to $\mu_R^{PMC}$ to eliminate $\{\beta_i^R\} - \text{terms}$

Conformal Series

Result is independent of $\mu_R^{\text{init}}$ and scheme at fixed order

---

**Principle of Maximum Conformality**

Xing-Gang Wu, Matin Mojaza
Leonardo di Giustino, SJB
AdS/QCD Soft-Wall Model

\[ \zeta^2 = x(1 - x)b_\perp^2. \]

Light-Front Schrödinger Equation

\[ \left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta) \]

\text{Confinement scale:} \quad \kappa \simeq 0.6 \text{ GeV}

\[ 1/\kappa \simeq 1/3 \text{ fm} \]

Unique Confinement Potential!

Conformal Symmetry of the action

\text{Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!}

- de Alfaro, Fubini, Furlan:
An analytic first approximation to QCD

AdS/QCD + Light-Front Holography

• As Simple as Schrödinger Theory in Atomic Physics

• LF radial variable $\zeta$ conjugate to invariant mass squared

• Relativistic, Frame-Independent, Color-Confining

• Unique confining potential!

• QCD Coupling at all scales: Essential for Gauge Link phenomena

• Hadron Spectroscopy and Dynamics from one parameter

• Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules

• Insight into QCD Condensates: Zero cosmological constant!

• Systematically improvable with DLCQ-BLFQ Methods
Advantages of the Front Form

- Light-Front Time-Ordered Perturbation Theory: Elegant, Physical
- Frame-Independent
- Few LF Time-Ordered Diagrams (not n!) -- all $k^+$ must be positive
- $J^z$ conserved at each vertex
- Cluster Decomposition -- only proof for relativistic theory
- Automatically normal-ordered; LF Vacuum trivial up to zero modes
- Renormalization: Alternate Denominator Subtractions: Tested to three loops in QED
- Reproduces Parke-Taylor Rules and Amplitudes (Stasto-Cruz)
- Hadronization at the Amplitude Level with Confinement
Light-Front vacuum can simulate empty universe

- Independent of observer frame
- Causal
- Lowest invariant mass state $M=0$.
- Trivial up to $k^+=0$ zero modes-- already normal-ordering
- Higgs theory consistent with trivial LF vacuum (Srivastava, sjb)
- QCD and AdS/QCD: “In-hadron”condensates (Maris, Tandy Roberts) -- GMOR satisfied.
- QED vacuum; no loops
- Zero cosmological constant from QED, QCD
Predict Hadron Properties from First Principles!

QCD Lagrangian

Hadron Masses and Observables

Lattice Gauge Theory

Effective Field Theory Methods
SCET, ChPT, ...

Conformal Invariance

Bethe-Salpeter
Dyson Schwinger

Light-Front Hamiltonian

PQCD Evolution Equations
Counting Rules

DLCQ/BLFQ

AdS/QCD!

Light-Front Holography

Bound-State Dynamics!
Confinement!
Exclusive Processes and New Perspectives for QCD

March 28-29, 2014
University of California, Davis

Happy Birthday #70!!