Effective field theory, fluid dynamics, and the spontaneous breaking of space-time symmetries

w/ A. Nicolis, R. Rattazzi, J. Wang (hep-th/1011.6396)
w/ A. Nicolis, R. Porto, and J. Wang (hep-th/1211.6461)
w/ A. Nicolis (hep-th/1303.3289)
w/ A. Nicolis, R. Penco (hep-th/1311.6491)

+ L. Hui, S. Dubovsky, D. Son, R. Rosen, and others
Outline

1) Fluid dynamics from EFT perspective

2) Spontaneously broken space-time symmetries
Inspiration for fluids

Fluids are everywhere

- Nuclear scales (Quark Gluon Plasma)
- Human scales (glass of water, superfluid He)
- Terrestrial scales (geophysics, atmospheric dynamics)
- Cosmological scales (density perturbations)
Historically described by EOM→
Lagrangian description/EFT

- symmetries manifest
- QM: direct road to quantization
- ask model independent questions (vs Kinetic Theory)

Outstanding problems
- viscosity/entropy bound
  \[ \frac{\eta}{s} \geq \frac{1}{4\pi} \]
- turbulence
Punchline

**Perfect fluid**

- Lagrangian description exists* which you can take seriously as an EFT (symmetries, s.s.b. pattern, etc.)—not necessarily news

- Well defined quantum theory @ T=0?**

- Systematic classical perturbation theory: vortex–sound coupling

**Dissipation**

- Some foundational steps made

---

*S. Dubovsky, T. Gregoire, A. Nicolis & R. Rattazzi (hep-th/0512260)

**S. E., A. Nicolis, R. Rattazzi, J. Wang (hep-th/1011.6396)
Lagrangian for fluid dynamics

1) Degrees of freedom?
2) Symmetries
3) Construct the most general possible Lagrangian w/ 1) & 2)→ derivative expansion
Qualifications

- **Perfect** (dissipative effects higher order in derivatives) -- work in the far IR
- **Fully Relativistic**
- **Vortices in low energy theory** (not a super fluid)
Compressional (sound) modes

Transverse (vortex) modes are heavy*

*Rotons are tricky

Compressional (sound) modes

Transverse (vortex) modes are light

Super fluid

Perfect fluid
Degrees of Freedom

Long wavelength/low energy limit $\rightarrow$ fluid elements

$$\phi^I = \phi^I(\vec{x}, t), \quad I = 1, 2, 3$$
Degrees of Freedom

Long wavelength/low energy limit $\rightarrow$ fluid elements

\[ \phi^I = \phi^I(\vec{x}, t), \quad I = 1, 2, 3 \]

choose coordinate system

\[ \phi^I = x^I \]
Symmetries

- **Space-time:** Poincaré (scalars)

- **Internal symmetries:**
  - **Shift** \( \phi^I \rightarrow \phi^I + a^I \)
  - **Rotation** \( \phi^I \rightarrow O^I_J \phi^I \)

  Crystal or “jelly”

- **Volume-preserving diff**

  \( \phi^I \rightarrow \xi^I(\phi^J) \) \hspace{1cm} \text{with} \hspace{1cm} \det \left( \frac{\partial \xi^I}{\partial \phi^J} \right)
Lagrangian

- Shift $\rightarrow$ derivative

- Poincaré $\rightarrow$ \[ B^{IJ} = \partial_\mu \phi^I \partial^\mu \phi^J \]

- Rotation $\rightarrow$ SO(3) invar functions of $B^{IJ}$

- Vol-pres diffs $\rightarrow$ \[ B = \det(B^{IJ}) \]

\[ S = \int d^4x \ F(B) \]
Is this fluid dynamics?

Relativistic perfect fluids:

- **Stress tensor:**
  \[ T_{\mu\nu} = (\rho + P)u_\mu u_\nu + p\eta_{\mu\nu} \]

- **EOM:**
  \[ \partial^\mu T_{\mu\nu} = 0 \]

- **EOS:**
  \[ \rho(p) \]

Our action: Correct classical dynamics provided

\[
\begin{align*}
  u^\mu &= \frac{1}{6\sqrt{B}} \epsilon^{\mu\alpha\beta\gamma} \epsilon_{IJK} \partial_\alpha \phi^I \partial_\beta \phi^J \partial_\gamma \phi^K \\
  \rho &= -F(B) \\
  p &= F(B) - 2F'(B)B
\end{align*}
\]
Meet the Goldstones!

At a given $p$: \[ \phi^I = x^I + \pi^I \]

\[ \mathcal{L} \rightarrow \dot{\vec{\pi}}^2 - c_s^2 (\partial_I \pi^I)^2 + \text{interactions} \]

\[ c_s^2 = \left. \frac{2F''(B)B + F'(B)}{F'(B)} \right|_{B=1} = \left. \frac{dp}{d\rho} \right|_{B=1} \]

Longitudinal = sound \quad \omega = c_s k

Transverse = vortices \quad \omega = 0

Very Funny
Incompressibility:

measure of the pressure gradient needed to sustain a given density gradient, i.e.

\[
\frac{dp}{d\rho} = c_s^2
\]

incompressibility is a dynamical regime

\[
v_{\text{flow}} \ll c_s
\]

Only vortex dofs

incompressible limit \iff \lim c_s \to \infty
Incompressible limit:

continuity equation: \[ \nabla \cdot \vec{v} = 0 \]
\[ \vec{\omega} = \nabla \times \vec{v} \]

Euler equation: \[ \frac{\partial \vec{\omega}}{\partial t} = \nabla \times (\vec{v} \times \vec{\omega}) \]

Linear regime? \textbf{NO DYNAMICS}!

i.e. the dynamics are completely NL
Now that we have the Lagrangian we can do 3 things

- Use it
- Improve it
- Extend it
Vortex-Sound Interactions

What we are after: A systematic expansion that incorporates compression

Incompressible given vortex flow → Not-so incompressible vortex + sound

An expansion around: \( \frac{u_{\text{flow}}}{C_s} \ll 1 \)
Expand around 

\[ \vec{x}(\phi, t) = \vec{x}_0(\phi, t) + \vec{\psi}(\phi, t) \]

Longitudinal: \[ \vec{n}_0 \times \vec{\psi} = 0 \]

Inserting into S:

\[ S = S_{vn} + S_{(\partial \psi)m} + S_{vn,(\partial \phi)m} \]

For instance:

\[ S_{vn} = w_0 \int d^3x dt \left( -c^2 + \frac{v^2}{2} + \frac{(c^2 - c_s^2)v^4}{8c^4} + O(v^6) \right) \]

\[ S_{\psi vn} = w_0 \int d^3x dt \left( v_i \partial_t \psi^i + v_i (v \cdot \nabla) \psi^i - \frac{c_s^2 v^2 [\nabla \psi]}{2c^2} + \ldots \right) \]

Relativistic corrections! \[ \frac{c_s^2}{c^2} \sim 1 \]
\[ S_{\text{int}} = S(\partial \psi)^2 + S(\partial \psi)^3 + \ldots + S_{\partial \psi, v^n} + S(\partial \psi)^2, v^n + S(\partial \psi)^3, v^n + \ldots \]
Powerful formalism, what can we do with it?

Sound emitted by a turbulent source:

Lighthill ('52) w/ rel. corr.
power given by quadrupole like formula

Sound scattered off a turbulent source:

Lund & Rojas ('89) w/ rel. corr.
cross section given in terms of correlation functions of vorticity

Potential between vortices:

Known?

\[ V \sim \frac{l^3}{r^3} \frac{v^2}{c_s^2} E_{kin} \]

(w/ William Irvine to detect effect in vortex rings)
Dissipation: the general idea*

Local action, non-dissipative by construction:

\[ S[\phi, \chi] = S_0[\phi] + S_\chi[\chi] + S_{int}[\phi, \chi] \]

DOF we are keeping track of

DOF we will not keep track of

Observables of \( \phi \) only will detect `dissipative' effects corresponding to exciting the \( \chi \) modes

*S. E, A. Nicolis, R. Porto and J. Wang (hep-th/1211.6461)
\[ S[\phi, \chi] = S_0[\phi] + S_\chi[\chi] + S_{int}[\phi, \chi] \]

can be strong weakly coupled

\[ S_{int} = \int d^4x \sum_{n,m} \partial^n \phi^m(x) O_{n,m}(x) \]

as dictated by symmetry

Observables of \( \phi \) are mediated by correlation functions of the \( O \) 's
Simple Example: \[ \mathcal{L}_{\text{int}} = \lambda \phi \mathcal{O} \]

T-order (vac to vac) 2-pt function:

\[
\langle \phi(p)\phi(-p) \rangle = \langle \phi(p)\phi(-p) \rangle_0 + \lambda^2 \langle \phi(p)\phi(-p) \rangle^2_0 \langle \mathcal{O}(p)\mathcal{O}(-p) \rangle + \ldots
\]

or diagrammatically

\[ + \to \to \phi(p) \to \to \phi(-p) \to \to + \to \to \ldots \]
More general correlation functions (thermalized $\chi$ state) we need work with the In-In formalism
double the path (+,-) with (thermal) density matrix $\rho(\chi_0^+, \chi_0^-)$ for initial conditions

$$\Gamma[\phi^+, \phi^-]$$

$$\frac{\delta \Gamma[\phi^+, \phi^-]}{\delta \phi^+(x)} \bigg|_{\phi^+ = \phi^- = \langle \phi \rangle} = 0$$

$$\frac{\delta S_2}{\delta \phi} + i\lambda^2 \langle OO \rangle_{R} \ast \phi = 0$$

matches the standard LR result
Properties of the hydrodynamic $\chi$ sector

$\phi$ all the d.o.f. that propagate over long distances and times (hydrodynamic modes)

in the absence of external perturbations, thermalized, and have no long distance or late time correlations (fall off faster than any power) but gapless

$\chi$

$\langle \mathcal{O}(x)\mathcal{O}(x') \rangle \quad G(\omega, \vec{k}) = \text{FT} \langle \mathcal{O}(x)\mathcal{O}(x') \rangle$

admits a Taylor Series exp. about origin
Focus on retarded 2-pt function

\[ G_R(\vec{x}, t) \equiv \theta(t) \langle [\mathcal{O}(\vec{x}, t), \mathcal{O}(0)] \rangle \]

standard spectral representation arguments ⇒

\[ \text{Im}(iG_R) \quad \text{is odd,} \quad \text{Re}(iG_R) \quad \text{is even} \]

\[ \text{Im}(iG_R(\omega, \vec{k})) = \rho(\omega, \vec{k}) \sim A \omega \times \delta \cdots \delta , \quad \omega, k \to 0 \]

Extra time derivative!
How do our hydrodynamic modes couple to this sector?

turn on small perturbations about the background:

$$\phi^I(x) = x^I + \pi^I(x)$$

equivalent to performing a small, modulated, spatial translation

$$\phi^I_0(\vec{x}) \rightarrow \phi^I(\vec{x}, t) = \phi^I_0(\vec{x} + \vec{\pi}(\vec{x}, t))$$

\(\chi\) Live in the fluid, i.e. they undergo the same spatial translation

$$S_\chi[\chi] \rightarrow S_\chi[\chi] - \int d^4x \, \partial_\mu \pi^i T^\mu_i$$
Rediscovering Kubo’s relations:

EOM:
\[
\omega_0 \left( \omega^2 \pi^i - c_s^2 k^i k^j \pi^j \right) + i G_{ij}^R (\omega, \vec{k}) \pi^j = 0
\]

\[
G_{ij}^R (\omega, \vec{k}) = k_\mu k_\nu \langle T_{\chi i}^\mu T_{\chi j}^\nu \rangle
\]

\[
\text{Im}(i G_{ij}^R) \simeq \omega k^2 \left[ (A_0 + \frac{4}{3} A_2) P_{ij}^L + A_2 P_{ij}^T \right]
\]

putting this all back in the equations of motion

\[
\Delta \omega_L \simeq -i \frac{(A_0 + \frac{4}{3} A_2)}{2w_0} k^2
\]

\[
\Delta \omega_T \simeq -i \frac{A_2}{w_0} k^2
\]
which precisely matches the standard results provided

$$\zeta = A_0, \quad \eta = A_2$$

or, expressed using a funny limit (and similarly for $\eta$)

$$\zeta = \frac{1}{9} \delta_{ij} \delta_{kl} \lim_{\omega \to 0} \left[ \frac{1}{\omega} \lim_{k \to 0} (i \cdot \langle T^{ij} T^{kl} \rangle) \right]$$

Kubo's formula

But...... when we generalize there are problems...
Spontaneous breaking of space-time symmetries

Broken internal symmetry (PI)

Goldstone's Theorem: massless mode, stable

Mechanism independent

Broken s-t (and internal) symmetry

Less constrained: gap?, redundant dof?

Mechanism dependent (sort of)
Step back: fluids as a test case

Fluids spontaneously break space-time symmetries:

At a given $p$: \( \phi^I = x^I + \pi^I \)  

Is there a way to construct the theory for the Goldstones based on the symmetry breaking pattern alone?

Almost!*

*V. I. Ogievetsky 1970’s
1) Identify symmetry breaking pattern: $G \rightarrow H$

unbroken $= \begin{cases} \bar{P}_t \equiv P_t \\ \bar{P}_i \equiv P_i + Q_i \\ \bar{J}_{ij} \equiv J_{ij} + L_{ij} \end{cases}$

time translations
spatial translations
spatial rotations

broken $= \begin{cases} K_i \\ Q_i \\ M_{ij} \end{cases}$
boosts
internal shifts
internal $SL(3)$

Goldstones associated with each broken sym... TOO MANY!

*Nicolis, Penco, Rosen hep-th/1307.0517
2) Construct objects that transform covariantly (this is the coset construction):

\[ \Omega(x) = e^{ix^\mu \bar{P}_\mu e^i[\eta^i(x)K_i e^{i\pi^i(x)}Q_i e^{i\alpha^{ij}(x)}M_{ij}}} \]

used to couple to "matter" fields

\[ \Omega(x)^{-1} d\Omega(x) = ie^\mu_\alpha (\bar{P}_\mu + \omega^{ij}_\mu \bar{J}_{ij} + D_\mu \eta^i K_i + D_\mu \pi^i Q_i + D_\mu \alpha^{ij} M_{ij}) dx^\alpha \]

const. inv metric  \hspace{2cm} cov derivatives of Goldstones

*Nicolis, Penco, Rosen hep-th/1307.0517
3) Eliminate some Goldstones (inverse Higgs):

rule of thumb: \[ [\bar{P}, T_i] = T_j + \ldots \]

US:
\[
\begin{align*}
[\bar{P}_0, K_i] &= -i(\bar{P}_i - Q_i) \\
[\bar{P}_k, M_{ij}] &= -i\left(\delta_{ik}Q_j - \frac{1}{d}\delta_{ij}Q_k\right)
\end{align*}
\]

\[ D_0 \pi^i = 0 \quad \Rightarrow \quad \eta^i(\pi^i) \]
\[ D_i \pi^j - \frac{1}{d}\delta^j_i D_k \pi^k = 0 \quad \Rightarrow \quad \alpha^{ij}(\pi^i) \]

Left with only \[ D_1 \pi^1 \]

+ higher derivative terms

\[ \int d^d x G(D_1 \pi^1) \equiv \int d^d x F(B) \quad \text{w/} \quad \phi^I = x^I + \pi^I \]

*Nicolis, Penco, Rosen hep-th/1307.0517
What about other space-time breaking theories?

Many symmetry breaking patterns $\Rightarrow$ many different physical systems

Some known……. some (seemingly) not

What is this “inverse-Higgs constraint”?  

- Do we have to impose it? (NO)
- Are there systems where it is natural to NOT impose it? (YES)
- Over-counting interpretation? (NOT ALWAYS)
- New technology $\Rightarrow$ new (strongly coupled) systems

Nicolis, Penco, Piazza, and Rosen (1306.1240)
S. E., Nicolis, and Penco (1311.6491)
Conclusions

- New language $\iff$ New questions $\iff$ New effects $\iff$ New measurements!

- Still much work to be done (NL coupling to dissipative sector)

- A great deal of possible applications: cosmology, plasma physics, exotic condensed matter states, shocks?, all in a model independent fashion

- Interesting from a purely field-theoretic point of view: what can fluids teach us about QFT?

- Clarifying how to deal with SB s-t symmetries $\implies$ tools to deal with rich systems beyond fluids
What I didn’t mention:

- Other, fluid like, systems: superfluids, both 3 and 4
  
  Nicolis (1108.2513) + w/ Nicolis, and Penco (pending)

- Solid Inflation (cosmology application of this formalism)
  
  w/ A. Nicolis, and J. Wang (hep-th/1210.0569)

- Vortex lines (in superfluids and fluids) + rotons

- SUSY
  
  C. Hoyos, B. Keren-Zur, Y. Oz (1206.2958)

- How our understanding of dissipation has (slightly) improved...
QM of perfect fluids

Why are there no perfect fluids at $T=0$ in nature?

Fluid $\rightarrow T \rightarrow 0$ $\rightarrow$ Superfluid, Solid, Supersolid, Etc.

Theory inconsistent?
How can we show that the theory is inconsistent? Investigate strong coupling scale

Trick:

\[ c_T \neq 0 \]

consider transverse 2→2 scattering

\[ \sigma_{TT \rightarrow TT} \sim \frac{1}{k^2} \left( \frac{k^4}{w_0 c_T} \right) \quad c_T \rightarrow 0 \]

Also prove “Coleman-like” theorem: quantum fluctuations disrupt the semi-classical vacuum

\[ \phi^I = x^I \quad \text{no good} \]
Expanding around $\frac{v_{\text{flow}}}{c_s} \ll 1$

Expansion is most clear in the comoving coordinates $\vec{x}(\vec{\phi}, t)$

We can write:

$$S = -w_0 c^2 \int d^3 \phi dt \det J f((\det J^{-1}) \sqrt{1 - v^2/c^2})$$

where $J^i_j = \frac{\partial x^i}{\partial \phi^j}$ and $\vec{v} = \partial_t \vec{x}(\vec{\phi}, t)$

$$\mathcal{L} = -w_0 f(\sqrt{B}) \quad f'(1) = 1 \quad f''(1) = \frac{c_s^2}{c^2}$$
Why is this a useful starting point for our expansion?

Expand around

\[ \mathbf{x}(\phi, t) = \mathbf{x}_0(\phi, t) + \mathbf{\psi}(\phi, t) \]

at fixed time

is a v.p.d., i.e.

\[ \det J_0 = 1 \]

longitudinal as a function of \( \mathbf{x}_0 \), i.e.

\[ \nabla_0 \times \mathbf{\psi} = 0 \]

\[ \implies \det J = 1 + \nabla_0 \cdot \mathbf{\psi} + \frac{1}{2} \left[ (\nabla_0 \cdot \mathbf{\psi})^2 - (\nabla_0^i \psi^j)^2 \right] + \ldots \]

\[ \implies \mathbf{v} = \mathbf{v}_0 + \frac{D}{Dt} \mathbf{\psi} = \mathbf{v}_0 + (\partial_t + (\mathbf{v}_0 \cdot \nabla_0)) \mathbf{\psi}(\mathbf{x}_0, t) \]

Expanding:

\[ S = S_v + S_\psi + S_{vn} + (\partial_\phi)^m \]
Problems with dissipation:

When we include conserved charge (need additional scalar), it is no longer clear that we know what we are doing:

\[ S_{\text{int}} \overset{?}{=} - \int d^4x \left[ \partial_\mu \pi^i T^{\mu i}_\chi + y_0 \partial_\mu \pi^0 j^\mu_\chi \right] \]

same arguments

Kubo formula for heat conductivity

FAILS
However...

If we consider instead the “symmetry inspired” coupling:

\[ S_{\text{int}} \simeq -\int d^4x \left[ \partial_j \pi^i T_{\chi}^{ji} + B \partial_i \pi^0 j_{\chi}^i + C \partial_0 \pi^i j_{\chi}^i \right] \]

calculate using the Kubo formula for heat conductivity

\[ \text{Im} i \cdot \langle j^i j^j \rangle_K = \chi T \left( \frac{n}{\rho + p} \right)^2 \omega \cdot \delta^{ij} \]

match

\[ B = -C = -\frac{y_0 w_0}{b_0 F_b} = \frac{\mu (\rho + p)}{sT} \]

simple answer, can we understand better?