Evidence for a New Particle on the Worldsheet of the QCD Flux Tube

Sergei Dubovsky
CCPP, NYU & ICTP, Trieste

today

Three parts to the story:

- ✴Dynamics of QCD flux tubes
- ✴Integrable quantum gravity
- ✴Crazy thoughts about EW hierarchy problem

SD, Raphael Flauger, Victor Gorbenko,
1203.1054, 1205.6805, 1301.2325, 1404.0037
Patrick Cooper, SD, Victor Gorbenko, Ali Mohsen, 1411.0703
+more to appear

SD, Victor Gorbenko, Mehrdad Mirbabayi
1305.6939

Why would one care about QCD ?

*Reasons **not** to care:*

- ✓We completely know the theory.
- ✓No room for surprises.
- ✓All "easy" results are already known.
  Need to work hard, and the progress will be only incremental.

Why would one care about QCD ?

Reasons to care:

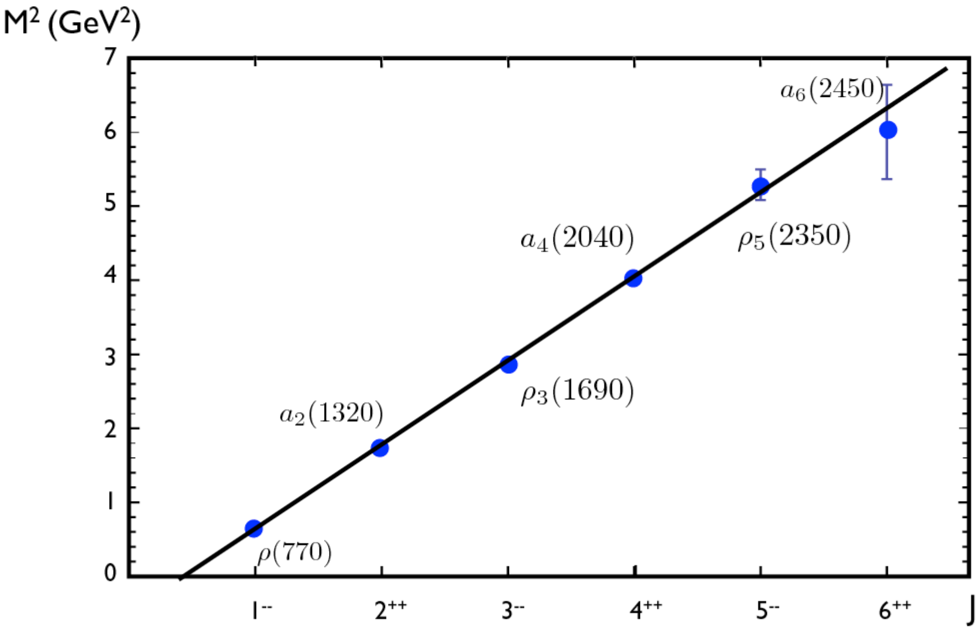
✓We completely know the theory !

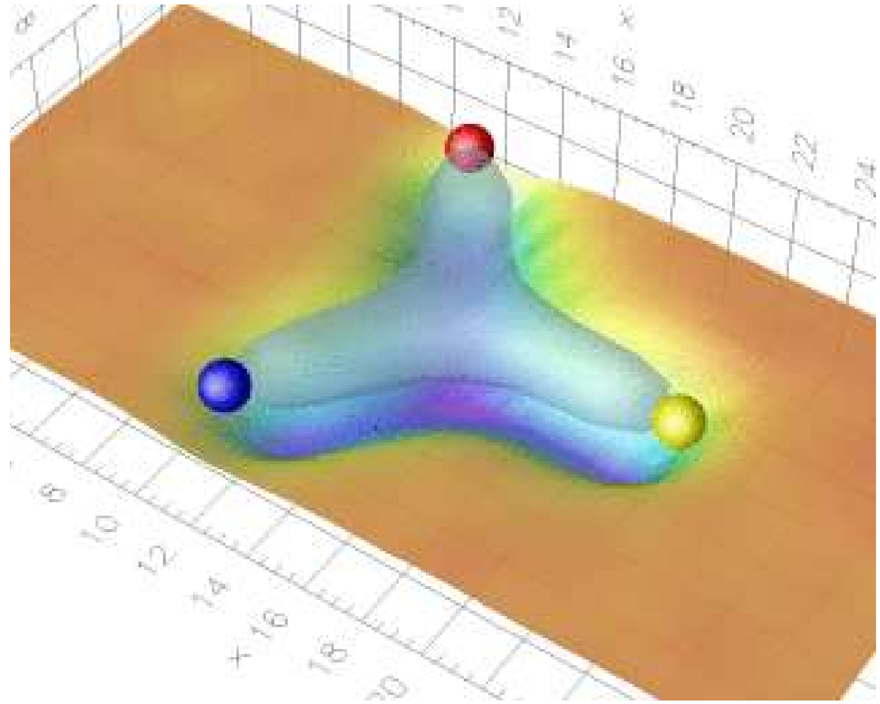
✓There is a 50 years old surprise, which is not quite understood yet.

✓There are "easy" qualitative results, still waiting to be discovered.

✓As an extra benefit we may learn something about gravity.

QCD is a theory of strings





Bissey et al, hep-lat/0606016

What can we say about this string theory?

Remarkable recent progress from top-down

✓Planar N=4 SYM string is integrable
✓Exact solution for the spectrum

Next Steps:

✓OPE coefficients
✓Is there a confining theory with an integrable string?

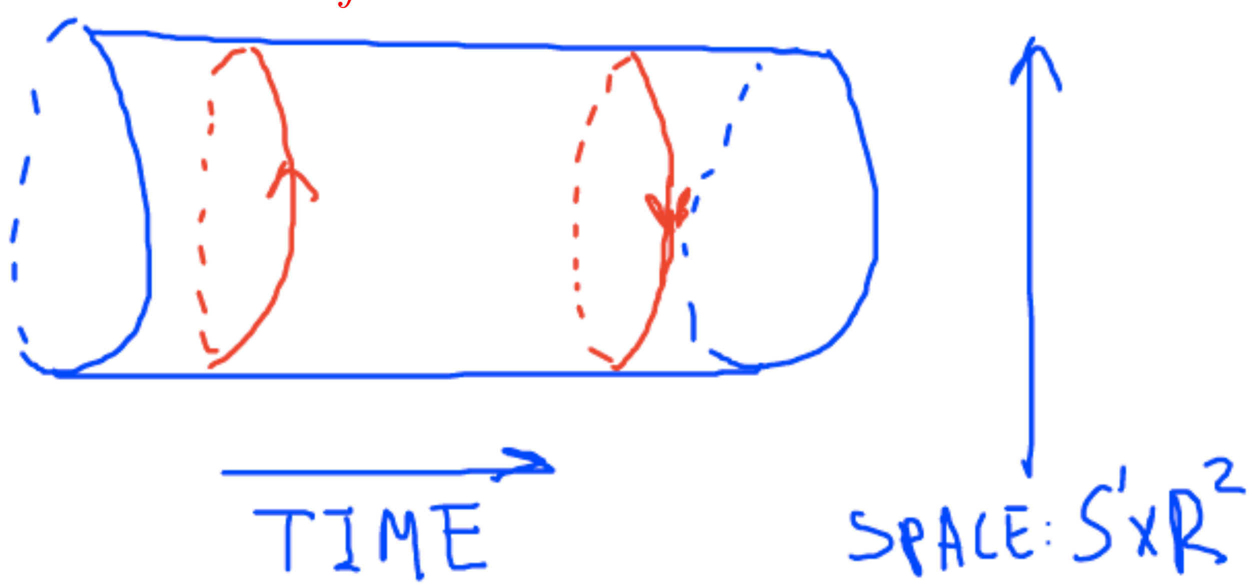
This talk: bottom up (EFT) approach:

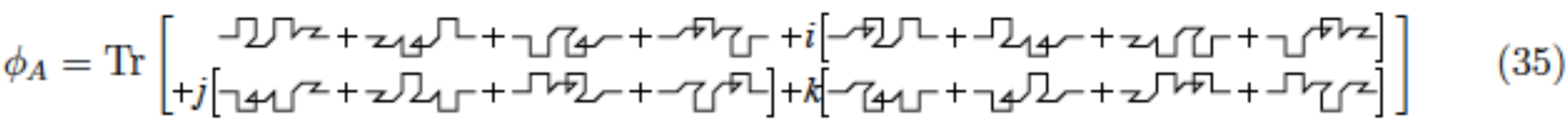
If you quack like a duck, you should be a perturbed duck

What is being measured?

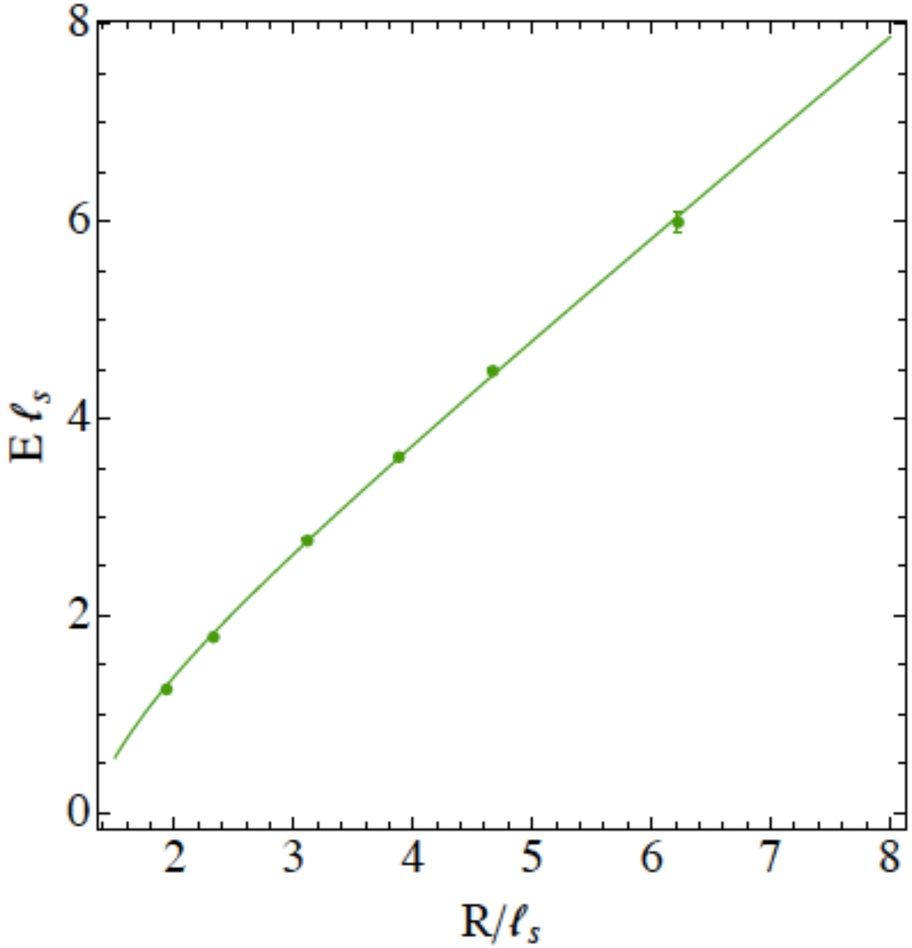
all the data from the papers by Athenodorou, Bringoltz and Teper

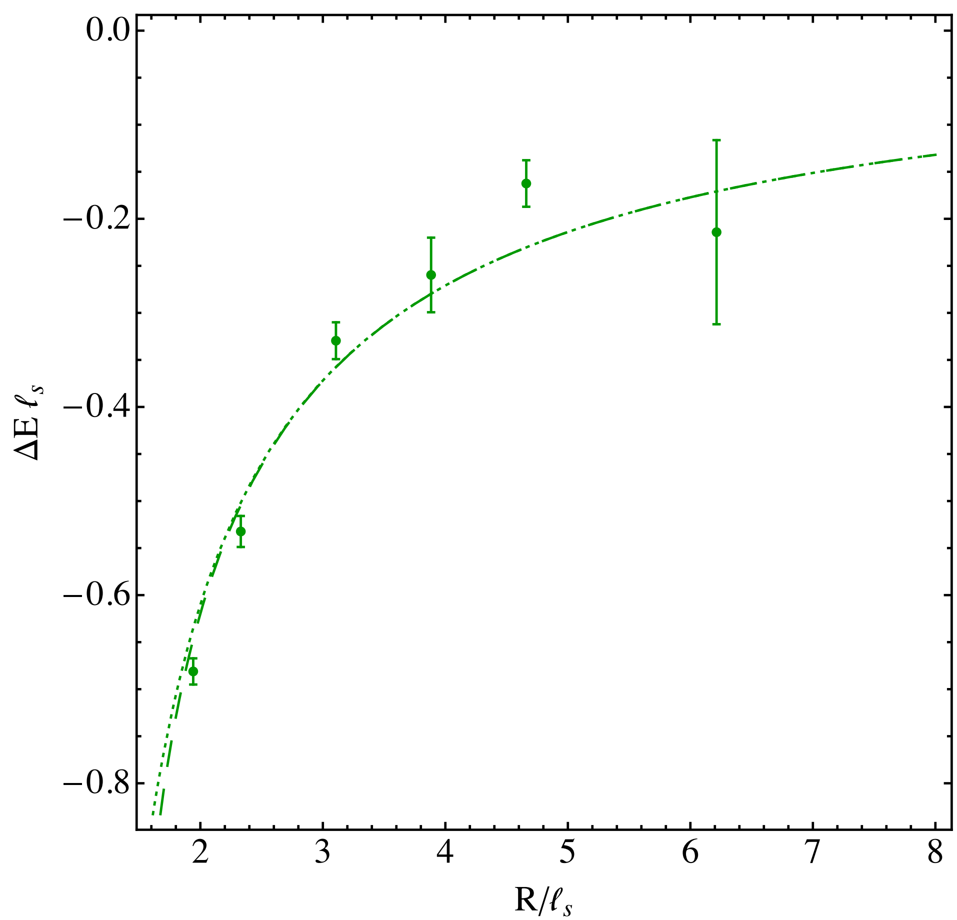
$\mathcal{O} = P\exp\{i \oint A\}$



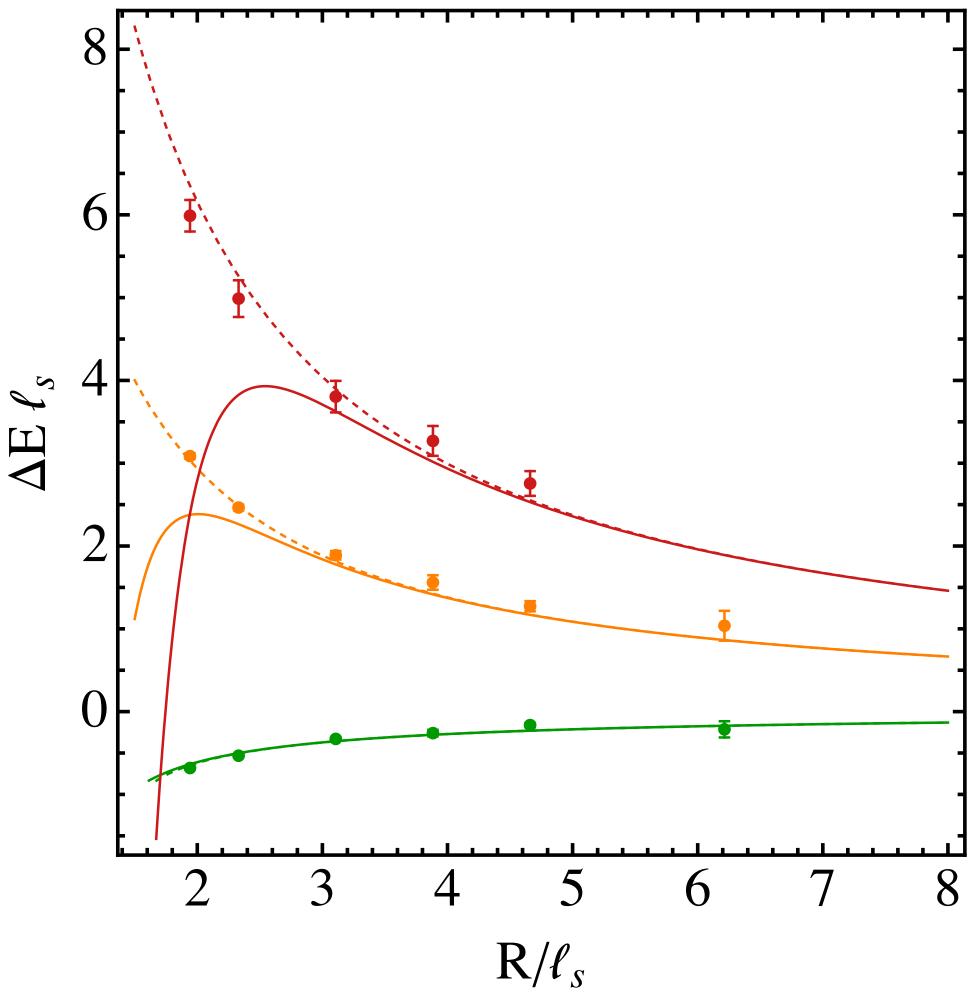


Puzzle #1: Remarkable agreement with a theory





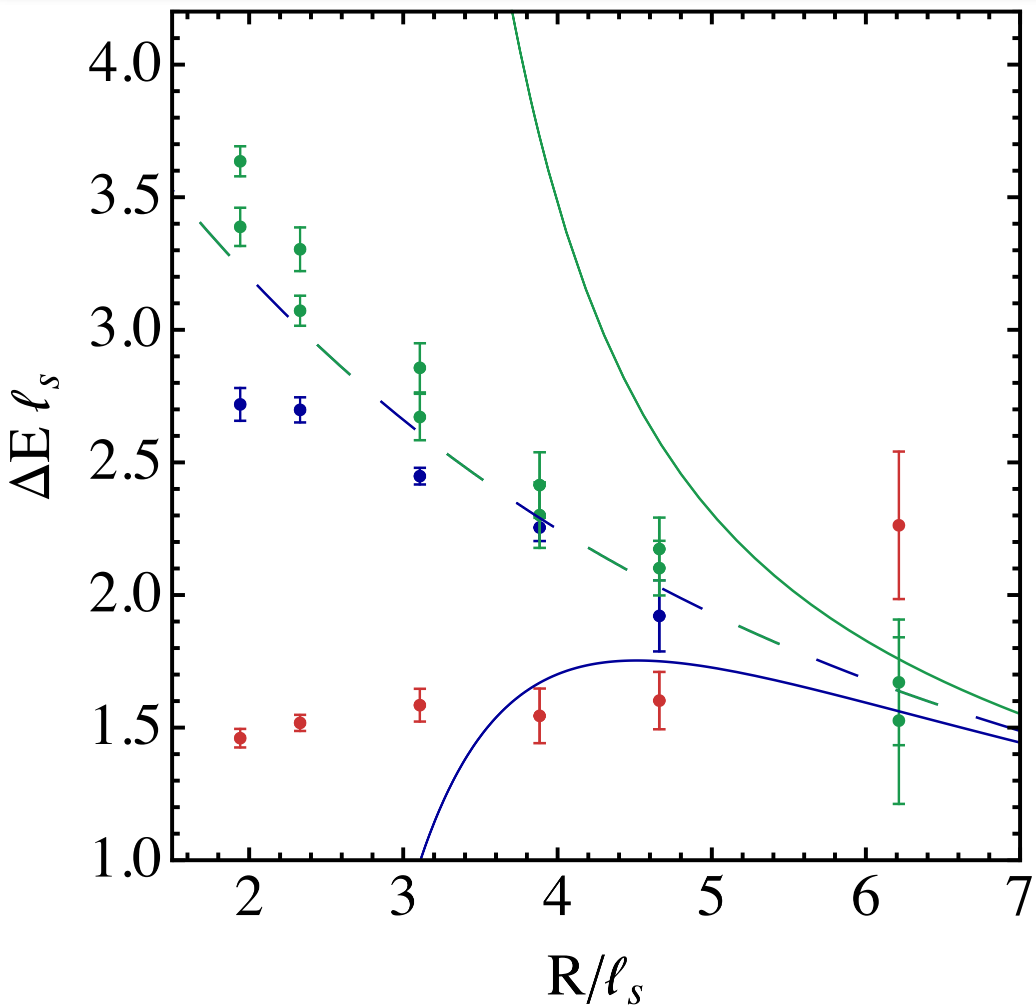
Puzzle #2: The theory is known to be wrong



Dashed --- light cone quantized bosonic string

Solid --- standard ℓ_s/R effective field theory expansion

Puzzle #3: More is going on



Dashed --- light cone quantized bosonic string

Solid --- standard ℓ_s/R effective field theory expansion

~~Nambu-Goto Spectrum~~

"Light Cone" or GGRT

$$E_{LC}(N,\tilde{N}) = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell_s^4} + \frac{4\pi}{\ell_s^2}\left(N+\tilde{N}-\frac{D-2}{12}\right)}$$

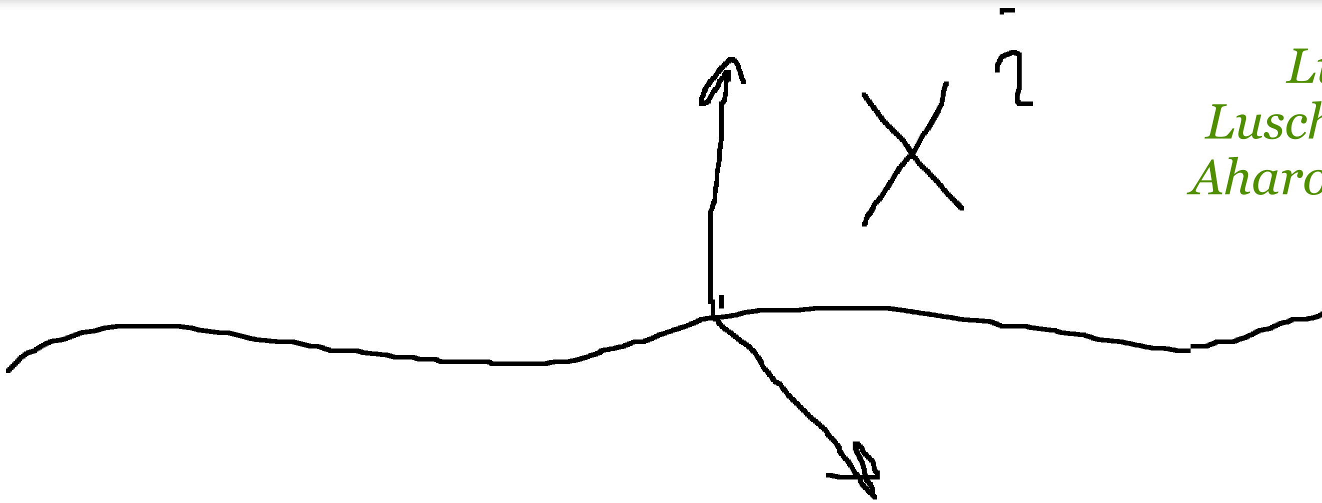
Comes from quantization in the light cone gauge

Goddard, Goldstone, Rebbi, Thorn'73 +winding

Crucial property: no splittings between different SO(D-2) multiplets

Consistent with target space Lorentz symmetry only at D=26. What it has to do with D=4 spectrum?

(Long) String as seen by an Effective Field Theorist



Luscher '81
Luscher, Weisz '04
Aharony et al '07-11
...

Theory of Goldstone Bosons

$$ISO(1, D-1) \to ISO(1,1) \times SO(D-2)$$

$$\delta_{\epsilon}^{\alpha i} X^j = -\epsilon(\delta^{ij}\sigma^{\alpha} + X^i \partial^{\alpha} X^j)$$

CCWZ construction

$$X^\mu = (\sigma^\alpha, X^i(\sigma)) \qquad h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$$

$$S_{string} = -\int d^2\sigma \sqrt{-\det h_{\alpha\beta}} \left(\ell_s^{-2} + \frac{1}{\alpha_0} \left(K^i_{\alpha\beta}\right)^2 + \ldots \right)$$

Nambu-Goto rigidity

Perturbatively:

$$S_{string} = -\ell_s^{-2} \int d^2\sigma \frac{1}{2}(\partial_\alpha X^i)^2 + c_2(\partial_\alpha X^i)^4 + c_3(\partial_\alpha X^i \partial_\beta X^j)^2 + \ldots$$

$$c_2 = -\frac{1}{8} \qquad c_3 = \frac{1}{4}$$

Interacting, in fact non-renormalizable, healthy effective field theory with cutoff ℓ_s

Why D=26 is special?

Theory is renormalizable (in some sense)

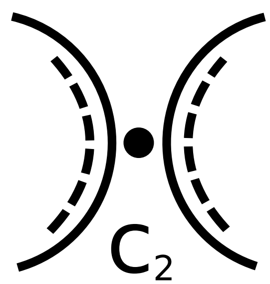
General SO(D-2) invariant amplitude:

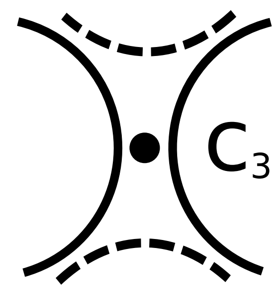
$$\mathcal{M}_{ij,kl} = A\delta_{ij}\delta_{kl} + B\delta_{ik}\delta_{jl} + C\delta_{il}\delta_{jk}$$

annihilation

$$A(s,t,u) = A(s,u,t) = B(t,s,u) = C(u,t,s)$$

Tree level:

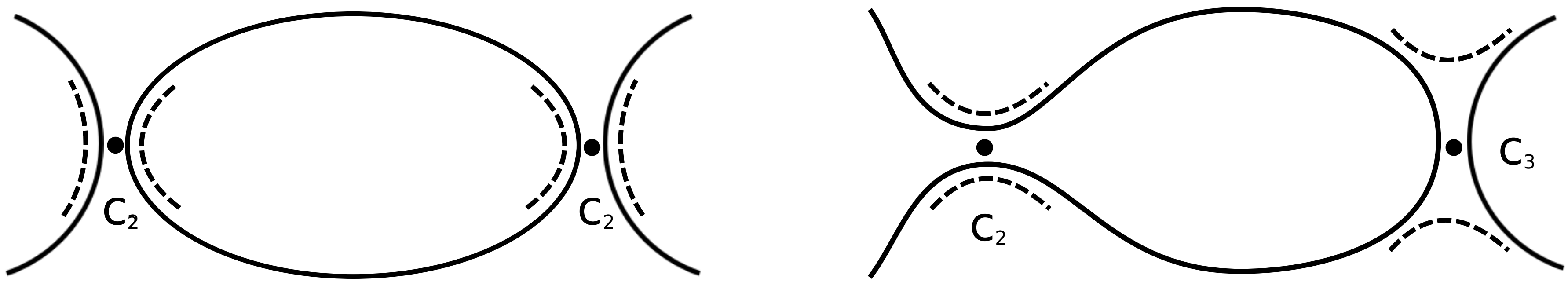




$$\mathcal{M}_{ij,kl} = -\frac{\ell_s^2}{2}(\delta^{ik}\delta^{jl}su + \delta^{il}\delta^{jk}st)$$

No annihilations for Nambu-Goto!

One-loop:



~18 diagrams

Finite part:

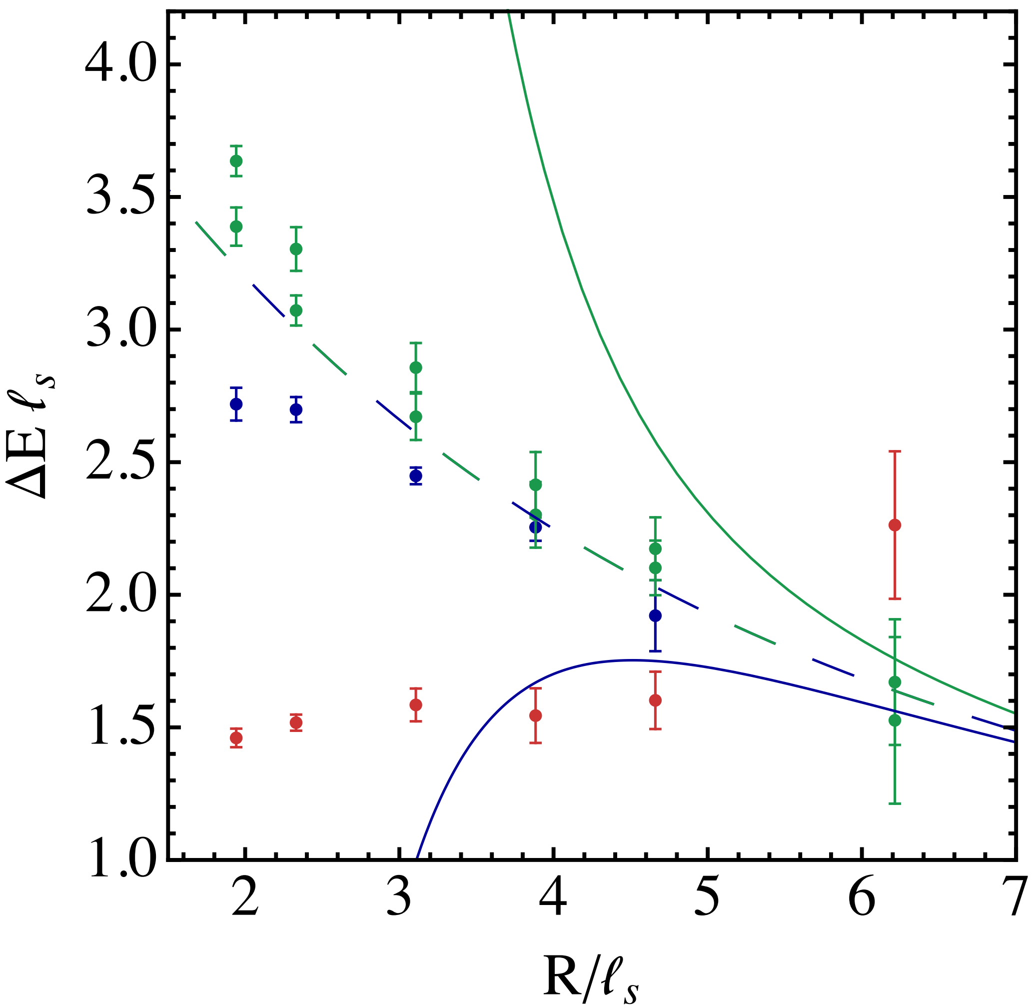
$$\mathcal{M}_{ij,kl} = -\ell_s^4 \frac{D-26}{192\pi}\left(s^3\delta_{ij}\delta_{kl} + t^3\delta_{ik}\delta_{jl} + u^3\delta_{il}\delta_{jk}\right)$$

$$+$$

$$-\frac{\ell_s^4}{16\pi}\left(\left(s^2 u \log\frac{t}{s} + s u^2 \log\frac{t}{u}\right)\delta_{ik}\delta_{jl} + \left(s^2 t \log\frac{u}{s} + s t^2 \log\frac{u}{t}\right)\delta_{il}\delta_{jk}\right)$$

Polchinski-Strominger interaction

gives rise to annihilations!



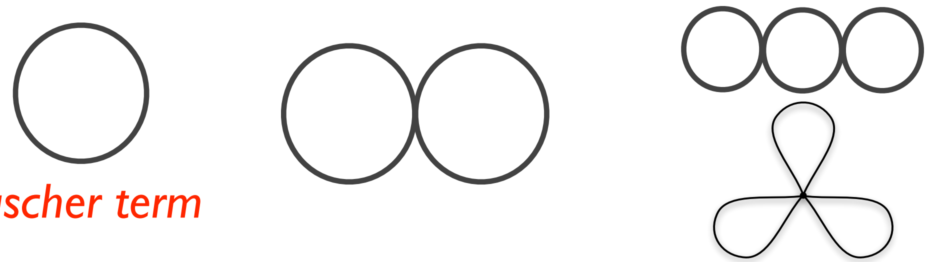
R^{-5} splittings in SO(D-2) multiplets

$$\mathcal{L}_{QCD\ string} = \mathcal{L}_{light\ cone} - \frac{D-26}{192\pi}\partial_\alpha\partial_\beta X^i \partial^\alpha\partial^\beta X^i \partial_\gamma X^j \partial^\gamma X^j + \ldots$$

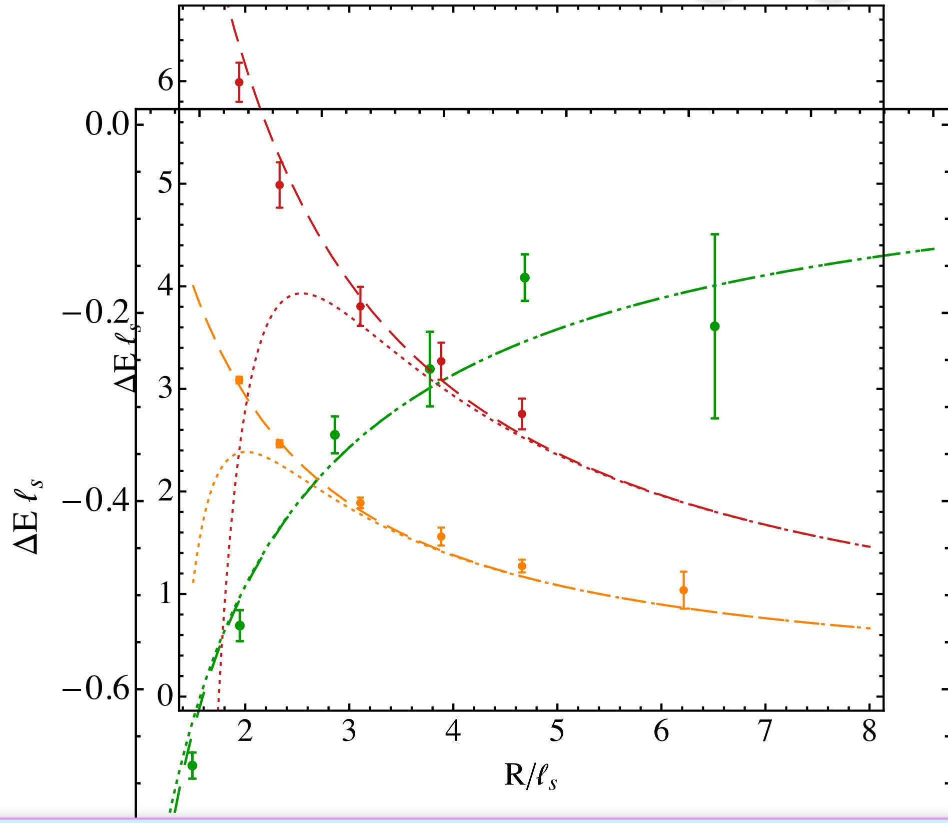
Explains the ground state data

$$E_0(R) = \frac{R}{\ell_s^2} - \frac{(D-2)\pi}{6R} - \frac{(D-2)^2\pi^2\ell_s^2}{72R^3} - \frac{(D-2)^3\pi^3\ell_s^4}{432R^5} + \text{non-universal terms}$$

classical



Luscher term



Need to work harder for excited states!

GGRT spectrum:

$$E_{LC}(N,\tilde{N}) = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell_s^4} + \frac{4\pi}{\ell_s^2}\left(N+\tilde{N} - \frac{D-2}{12}\right)}$$

ℓ_s/R expansion breaks down for excited states because 2π is a large number!

for excited states:

$$E = \ell_s^{-1}\mathcal{E}(p_i\ell_s, \ell_s/R)$$

Let's try to disentagle these two expansions

Finite volume spectrum in two steps:

1) Find infinite volume S-matrix

2) Extract finite volume spectrum from the S-matrix

1) is a standard perturbative expansion in $p\ell_s$

2) perturbatively in massive theories (Luscher)

exactly in integrable 2d theories through TBA

Relativistic string is neither massive nor integrable...

But approaches integrable GGRT theory at low energies!

GGRT S-matrix:

$$e^{2i\delta_{GGRT}(s)} = e^{is\ell_s^2/4}$$

- ✶Polynomially bounded on the physical sheet
- ✶No poles anywhere. A cut all the way to infinity with an infinite number of broad resonances
- ✶One can reconstruct the entire finite volume spectrum using Thermodynamic Bethe Ansatz

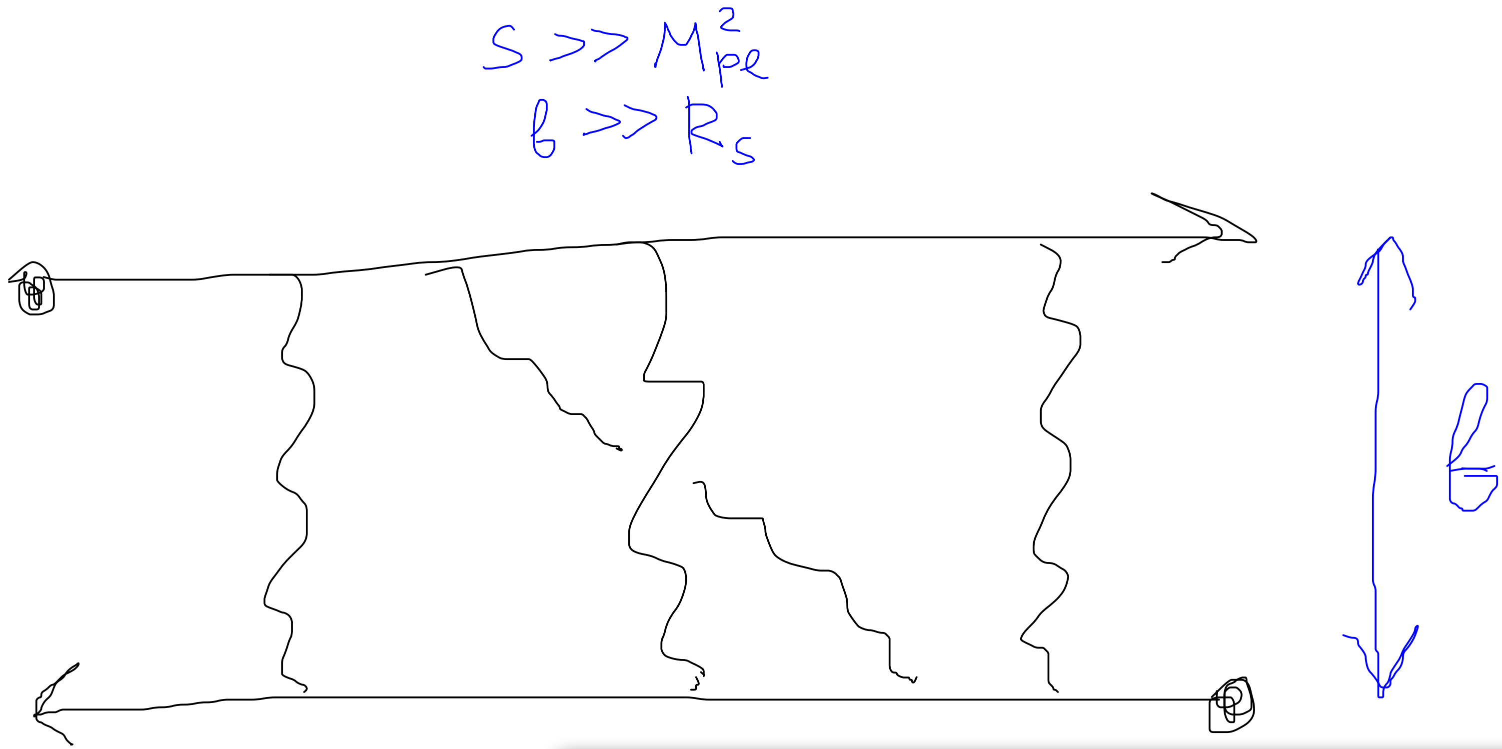
$$E(N,\tilde{N}) = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell^4} + \frac{4\pi}{\ell^2}\left(N+\tilde{N}-\frac{D-2}{12}\right)}$$

- ✶Does not go to a constant at infinity!

Integrable QG rather than QFT

Gravitational shock waves:

Dray, 't Hooft '85
Amati, Ciafaloni, Veneziano '88



Eikonal phase shift:

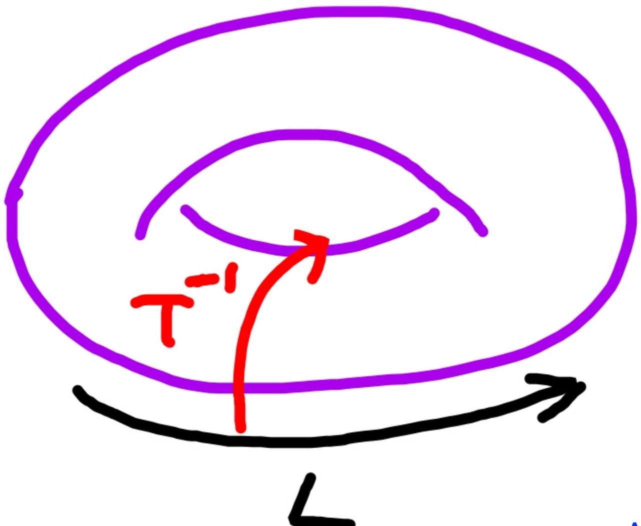
$$e^{i2\delta_{eik}(s)} = e^{i\ell^2 s/4}$$

$$\ell^2 \propto G_N b^{4-d}$$

Free string spectrum circa 2012

Thermodynamic Bethe Ansatz

Zamolodchikov '91



in thermodynamic (large L) limit

$$Z(T,L) = e^{-LE_0(1/T)} = e^{-Lf(T)/T}$$

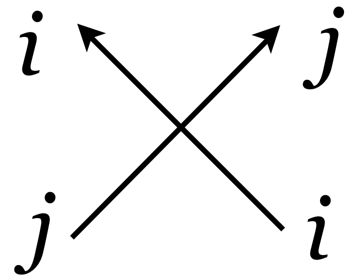
$$+$$

Asymptotic Bethe Ansatz

$$p_{k\,R}^{(i)} L + \sum_{i=1}^{D-2} \int_0^\infty 2\delta(p_{k\,R}^{(i)}, p)\rho_{1L}^{i}(p)dp = 2\pi n_{k\,R}^{(i)}$$

Asymptotic Bethe Ansatz

$$\Psi(x_1, x_2) = \langle 0 | X^i(x_1) X^j(x_2) | p_L^{(i)}, p_R^{(j)} | \rangle$$



$x_1 > x_2$ $\quad \Psi(x_1, x_2) = e^{-ip_L x_1} e^{ip_R x_2}$

$x_1 > x_2$ $\quad \Psi(x_1, x_2) = e^{-ip_L x_1} e^{ip_R x_2} e^{2i\delta(p_L, p_R)}$

periodicity: $\quad e^{-ip_{L,R}} = e^{2i\delta(p_L, p_R)}$

$$p_L + 2\delta(p_L, p_R) = 2\pi n_R$$

NB: particles are getting softer!

after taking the continuum limit
minimization of the free energy results in

$$\epsilon^i_L(p) = p\left[1 + \frac{\ell_s^2 T}{2\pi}\sum_{j=1}^{D-2}\int_0^\infty dp' \ln\left(1 - e^{-\epsilon^j_R(p')/T}\right)\right]$$

where

$$f = \frac{T}{2\pi}\sum_{j=1}^{D-2}\int_0^\infty dp' \ln\left(1 - e^{-\epsilon^j_L(p')/T}\right) + (L \to R)$$

reproduces the correct ground state energy

$$f(T) = \frac{1}{\ell_s^2}(\sqrt{1 - T^2/T_H^2} - 1)$$

$$T_H = \frac{1}{\ell_s}\sqrt{\frac{3}{\pi(D-2)}}$$

Excited States TBA

Dorey, Tateo '96

general idea: excited states can be obtained by analytic continuation of the ground state

finite size corrections

Asymptotic Bethe Ansatz

$$\hat{p}^{(i)}_{k\,L} R + \sum_{j,m} 2\delta(\hat{p}^{(i)}_{k\,L}, \hat{p}^{(j)}_{m\,R}) N^{(j)}_{m\,R} - i \sum_{j=1}^{D-2} \int_0^\infty \frac{dp'}{2\pi} \frac{d\,2\delta(i\hat{p}^{(i)}_{k\,L}, p')}{dp'} \ln\left(1 - e^{-R\epsilon^j_R(p')}\right) = 2\pi n^{(i)}_{k\,L}$$

$$\epsilon^i_L(p) = p + \frac{i}{R} \sum_{j,k} 2\delta(p, -i\hat{p}^{(j)}_{k\,R}) N^{(j)}_{k\,R} + \frac{1}{2\pi R} \sum_{j=1}^{D-2} \int_0^\infty dp' \frac{d\,2\delta(p,p')}{dp'} \ln\left(1 - e^{-R\epsilon^j_R(p')}\right)$$

$$E(R) = R + \sum_{j,k} p^{(j)}_{k\,L} + \sum_{j=1}^{D-2} \int_0^\infty \frac{dp'}{2\pi} \ln\left(1 - e^{-R\epsilon^j_L(p')}\right)$$

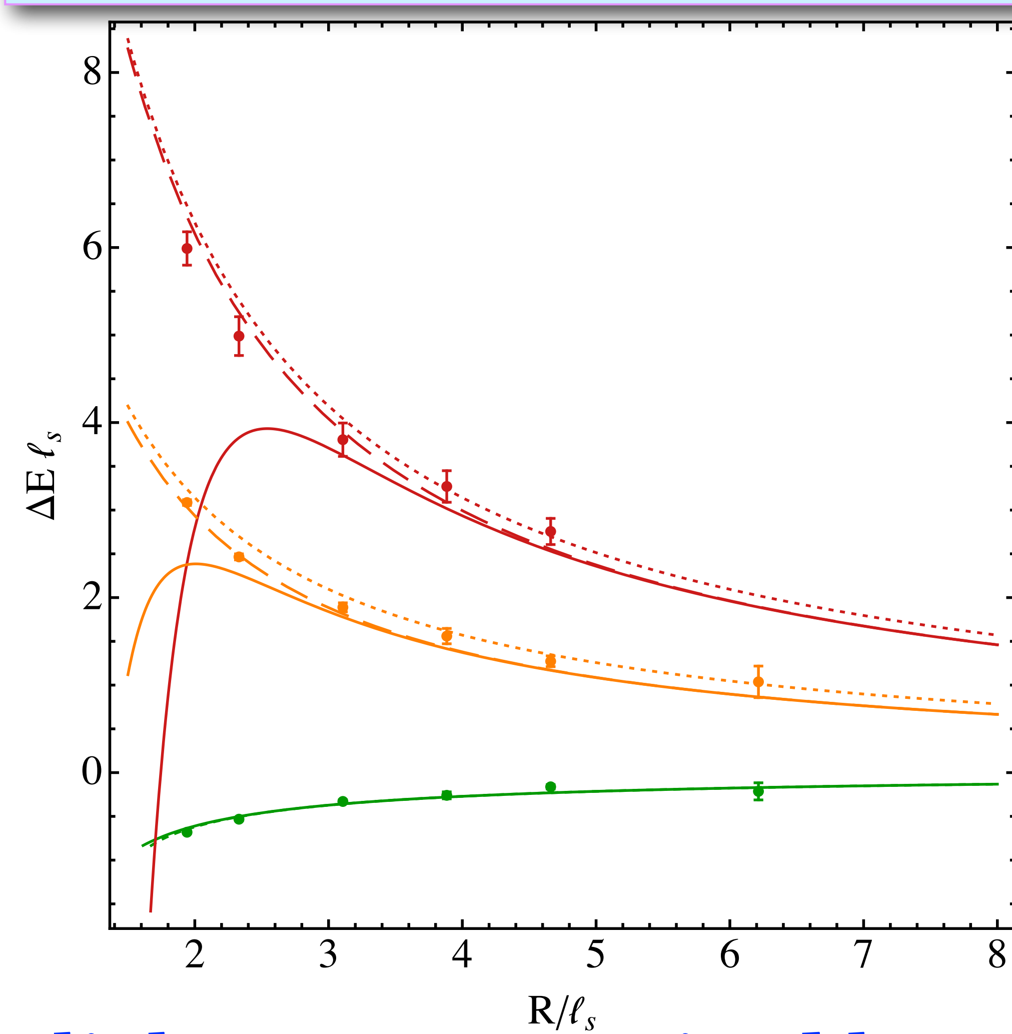
+right-movers

Exactly reproduces all of the light cone spectrum

The strategy is to incorporate corrections to the S-matrix into TBA equations.

Hard to do in full generality, but turns out possible at one-loop level with Polchinski-Strominger phase shift taken into account

Pure left-moving states

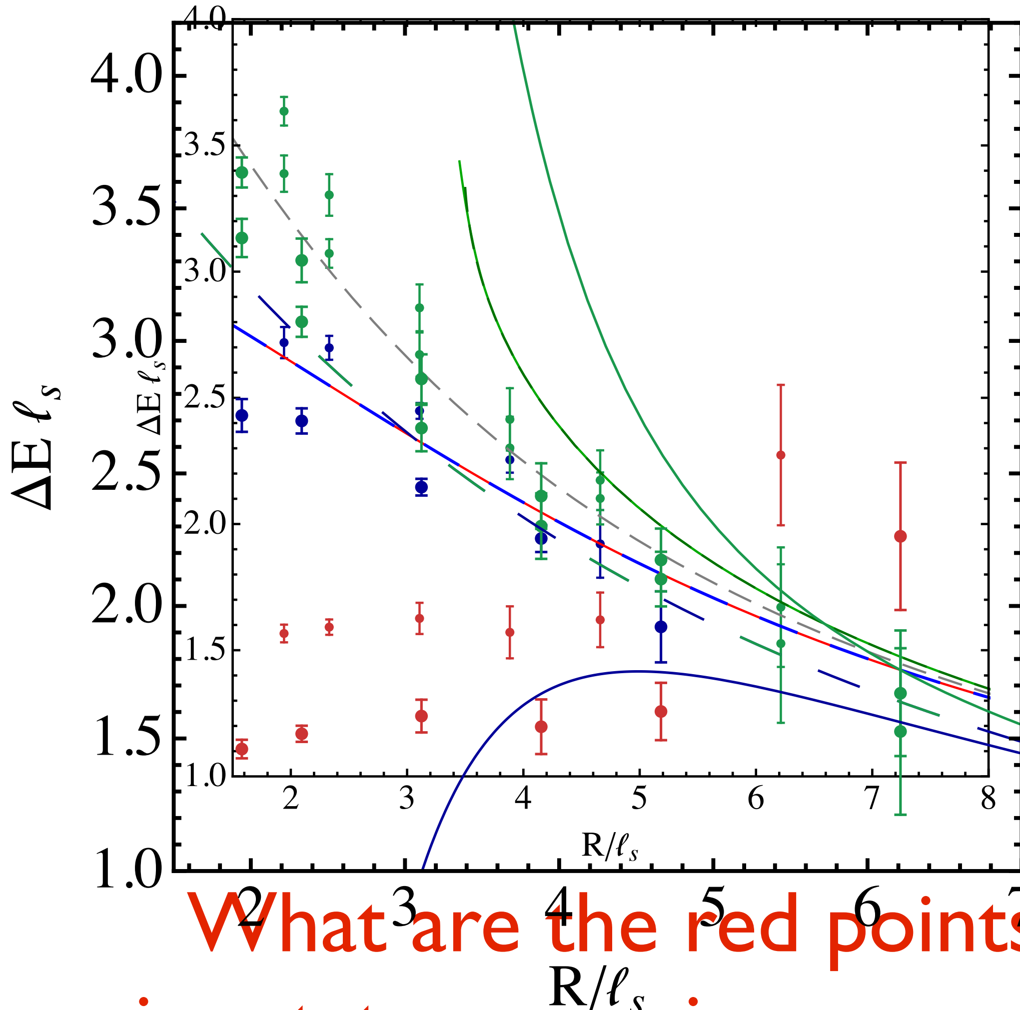


Dashed --- light cone quantized bosonic string

Solid --- standard ℓ_s/R effective field theory expansion

Dotted --- free theory (=ABA in this case)

Colliding left- and right-movers



What are the red points?

A new massive state appearing as a resonance in the antisymmetric channel!

see also arXiv:1007.4720
Athenodorou, Bringoltz, Teper

How do we include this massive state?

Contributes to scattering of Goldstones and changes the phase shifts. In particular, it appears as a resonance in the antisymmetric channel. We can calculate contributions from

$$S = \int d^2\sigma \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{2}m^2\phi^2 + \frac{\alpha}{8\pi}\phi\epsilon^{\alpha\beta}\epsilon_{ij}K^i_{\alpha\gamma}K^{j\ \gamma}_{\beta}$$

Full Calculation:

$$c\hat{p}R + 2\delta_{PS} + 2\delta_{res} = 2\pi$$

$$c = 1 + \ell_s^2 \frac{\hat{p}}{R} - \frac{\pi \ell_s^2}{6R^2 c}$$

$$2\delta_{res} = \sigma_1 \frac{\alpha^2 \ell_s^4 \hat{p}^6}{8\pi^2(4\hat{p}^2 + m^2)} + 2\sigma_2 \tan^{-1}\left(\frac{\alpha^2 \ell_s^4 \hat{p}^6}{8\pi^2(m^2 - 4\hat{p}^2)}\right)$$

$$2\delta_{PS} = \pm \frac{11\ell_s^4}{12\pi}\hat{p}^4$$

$$E = 2\hat{p} - \frac{\pi}{3Rc}$$

- **Equation (7) and unnumbered equation after equation (9)**

```
In[1]:= δPS[pl_, pr_, s_] = s 11 / 12 / Pi (pl pr) ^2;
δres[pl_, pr_, s1_, s2_, a_, m_] =
  s1 a^2 pl^3 pr^3 / (8 Pi^2) / (4 pl pr + m^2) + s2 If[pl pr < m^2 / 4, 2 ArcTan[a^2 (pr pl)^3 / (8 Pi^2 (m^2 - 4 pr pl))], 2 ArcTan[a^2 (pr pl)^3 / (8 Pi^2 (m^2 - 4 pr pl))] + 2 Pi];
```

- **Solution of quadratic equation (5)**

```
In[3]:= c[p_, R_] = (3 R (p + R) + Sqrt[3] Sqrt[R^2 (-2 π + 3 (p + R)^2)]) / (6 R^2);
```

- **Solution of equation (9) for 0--, 0++, and 2++ channels**

```
In[4]:= psol0mm[a_?NumberQ, m_?NumberQ, R_?NumberQ] := p /. FindRoot[c[p, R] p R + δPS[p, p, 1] + δres[p, p, 1, 1, a, m] == 2 Pi, {p, m / 2 - .1}];
psol0pp[a_?NumberQ, m_?NumberQ, R_?NumberQ] := p /. FindRoot[c[p, R] p R + δPS[p, p, 1] + δres[p, p, -1, 0, a, m] == 2 Pi, {p, m / 2 - .1}];
psol2pp[a_?NumberQ, m_?NumberQ, R_?NumberQ] := p /. FindRoot[c[p, R] p R + δPS[p, p, -1] + δres[p, p, 1, 0, a, m] == 2 Pi, {p, m / 2 - .1}];
```

- **Equation (6)**

```
In[7]:= WE[p_, R_] := -Pi / 3 / R / c[p, R]
```

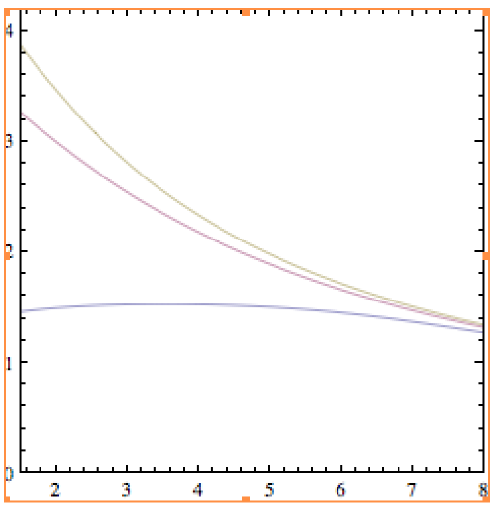
- **Equation (1)**

```
In[8]:= E0mm[a_?NumberQ, m_?NumberQ, R_?NumberQ] := 2 psol0mm[a, m, R] + WE[psol0mm[a, m, R], R]
E0pp[a_?NumberQ, m_?NumberQ, R_?NumberQ] := 2 psol0pp[a, m, R] + WE[psol0pp[a, m, R], R]
E2pp[a_?NumberQ, m_?NumberQ, R_?NumberQ] := 2 psol2pp[a, m, R] + WE[psol2pp[a, m, R], R]
```

- **Solid lines shown in Figure 2**

```
In[11]:= Plot[{E0mm[9.6, 1.85, r], E0pp[9.6, 1.85, r], E2pp[9.6, 1.85, r]}, {r, 1.5, 8}, PlotRange → {{1.5, 8}, {0, 4.2}}, Frame → True, AspectRatio → 1]
```

Out[11]=



How do we include this massive state?

Contributes to scattering of Goldstone's and changes the phase shifts. In particular, it appears as a resonance in the antisymmetric channel. We can calculate contributions from

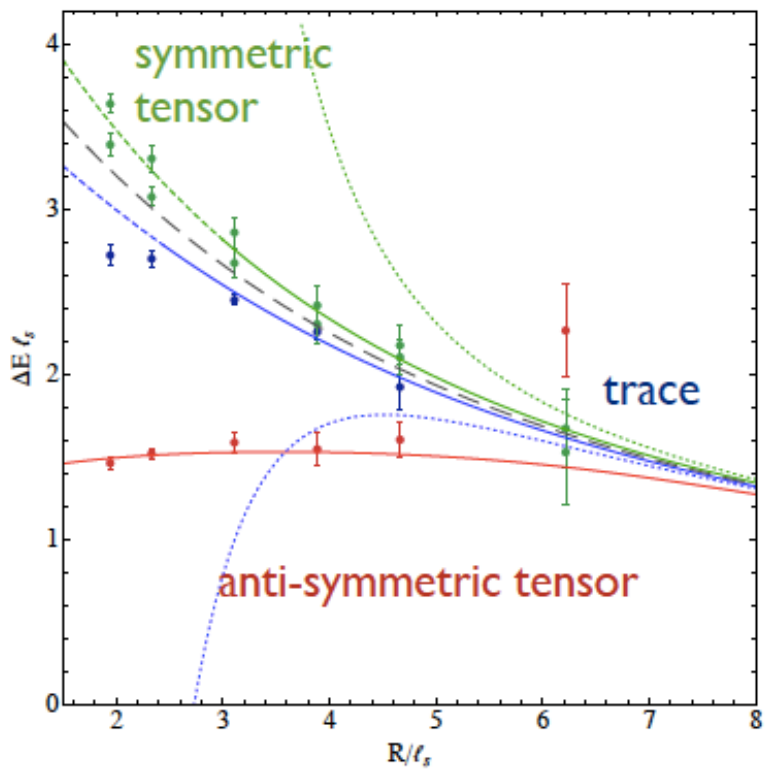
$$S = \int d^2\sigma \frac{1}{2}\partial_\alpha \phi \partial^\alpha \phi - \frac{1}{2} m^2 \phi^2 + \frac{\alpha}{8\pi} \phi \epsilon^{\alpha\beta} \epsilon_{ij} K^i_{\alpha\gamma} K^{j\ \gamma}_{\beta}$$

Including the resonant s-channel contribution

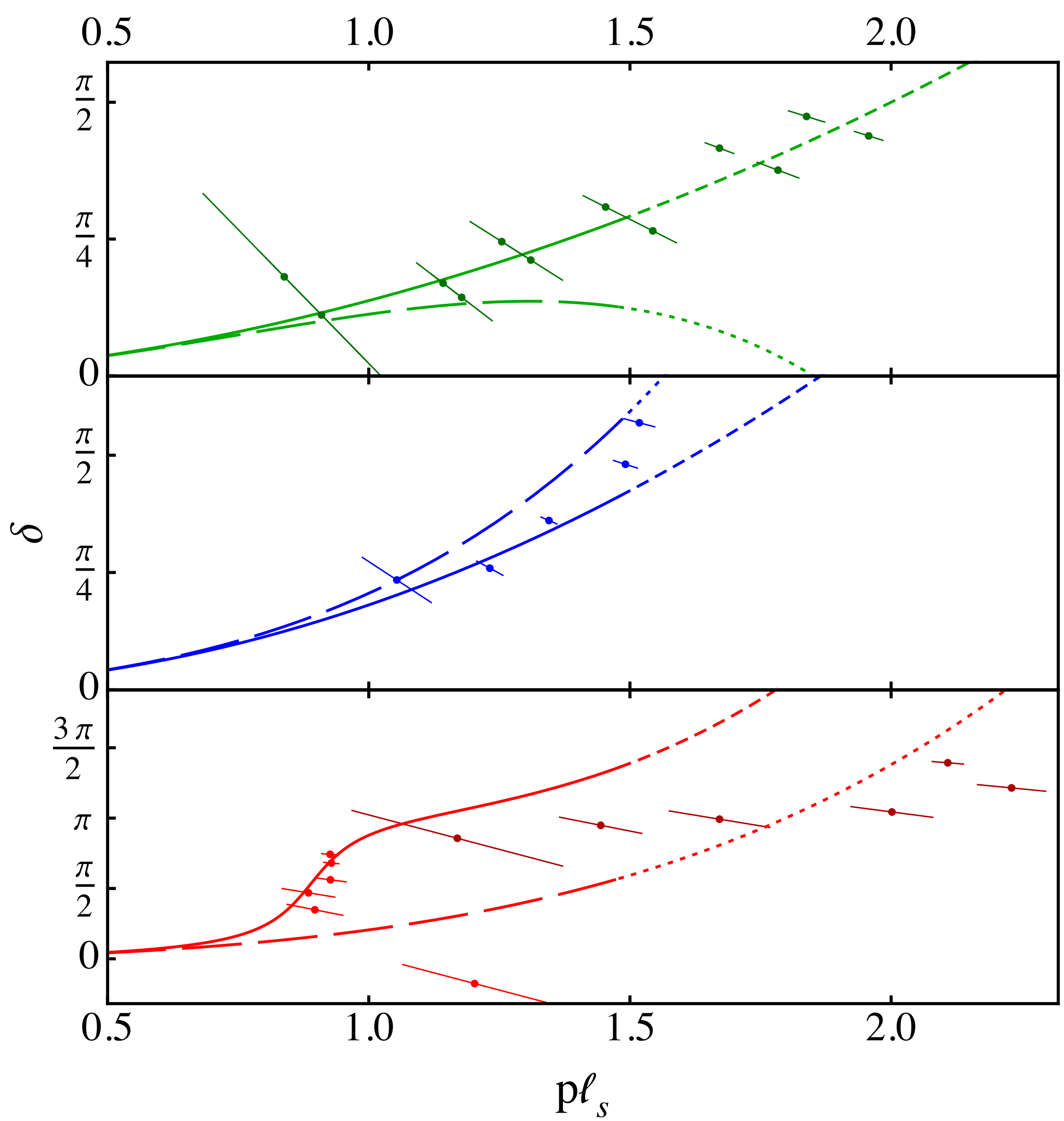
$$\delta(s) = \arctan\left(\frac{m\Gamma(s/m)^3}{m^2 - s}\right)$$

$$m \sim 1.85 \ell_s^{-1} \qquad \Gamma \sim 0.4 \ell_s^{-1}$$

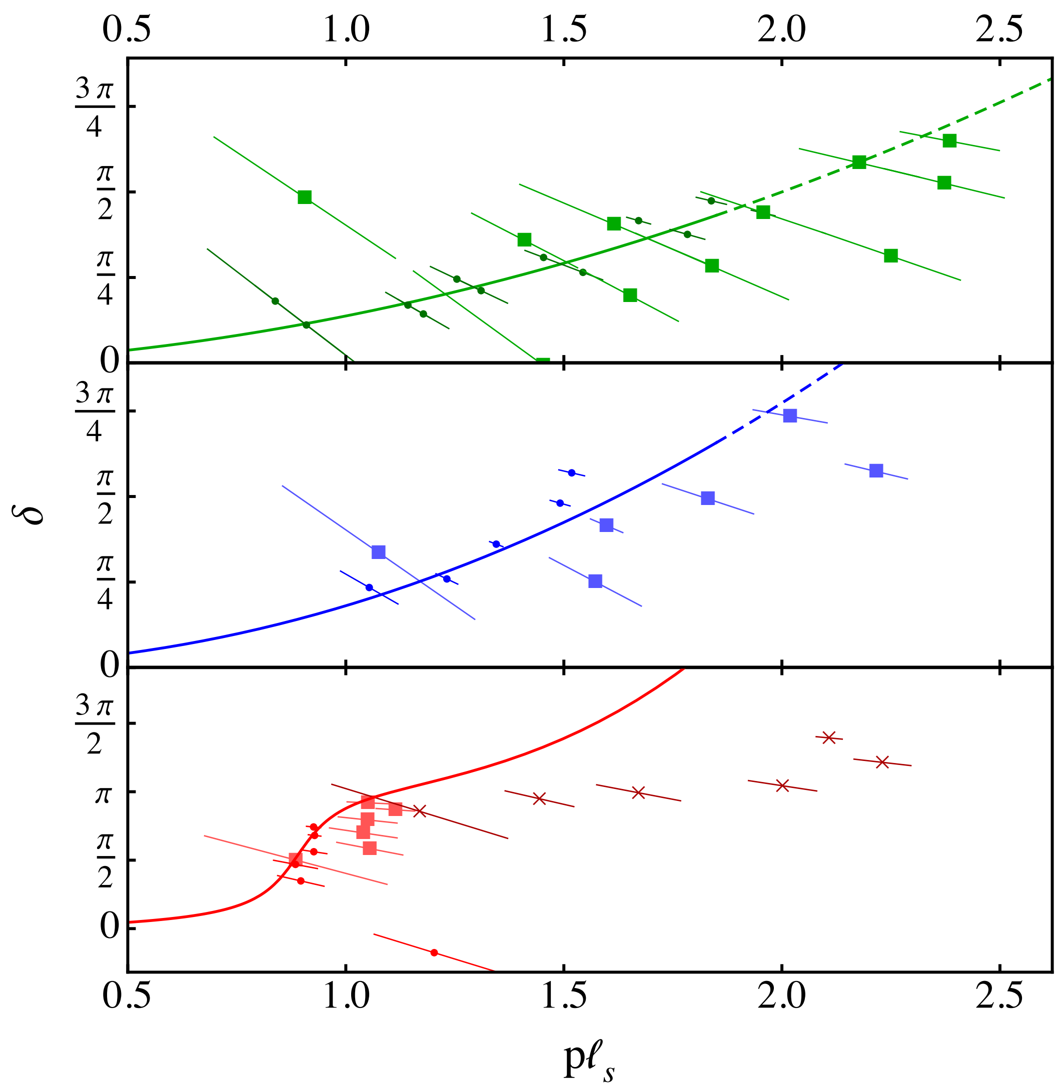
as well as perturbative non-resonant contributions in crossed channels



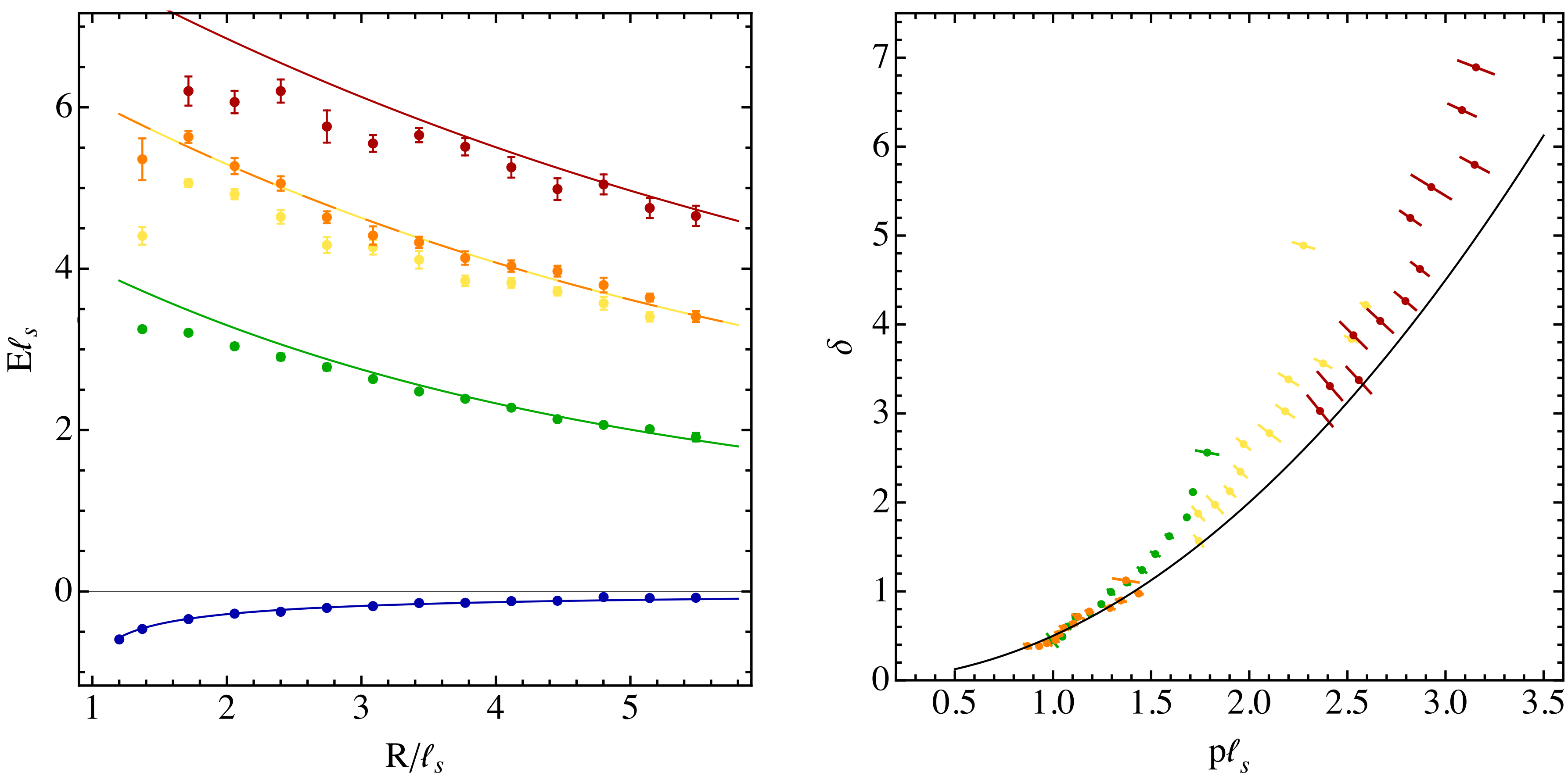
Reverting the logic: S-matrix from finite volume spectrum



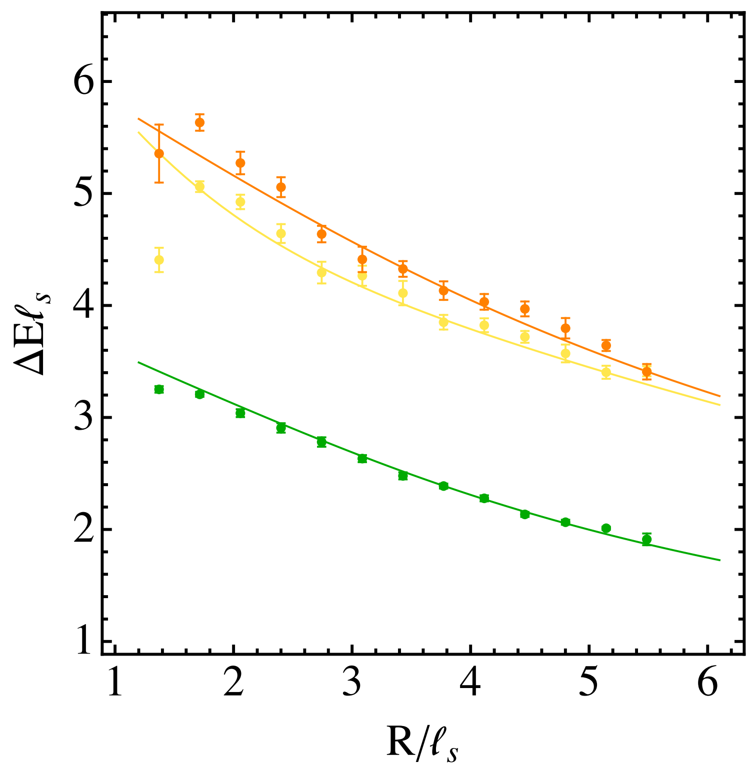
More states:



3D Yang-Mills

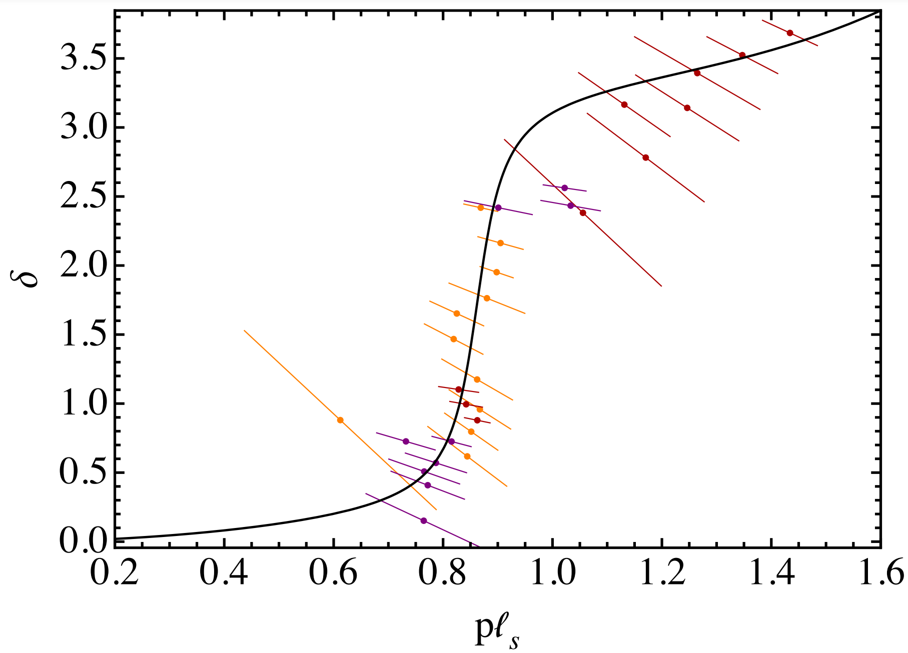


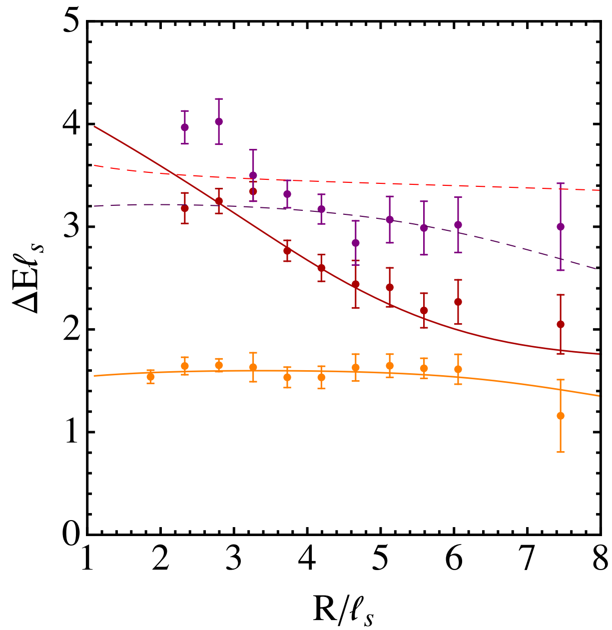
3D Yang-Mills



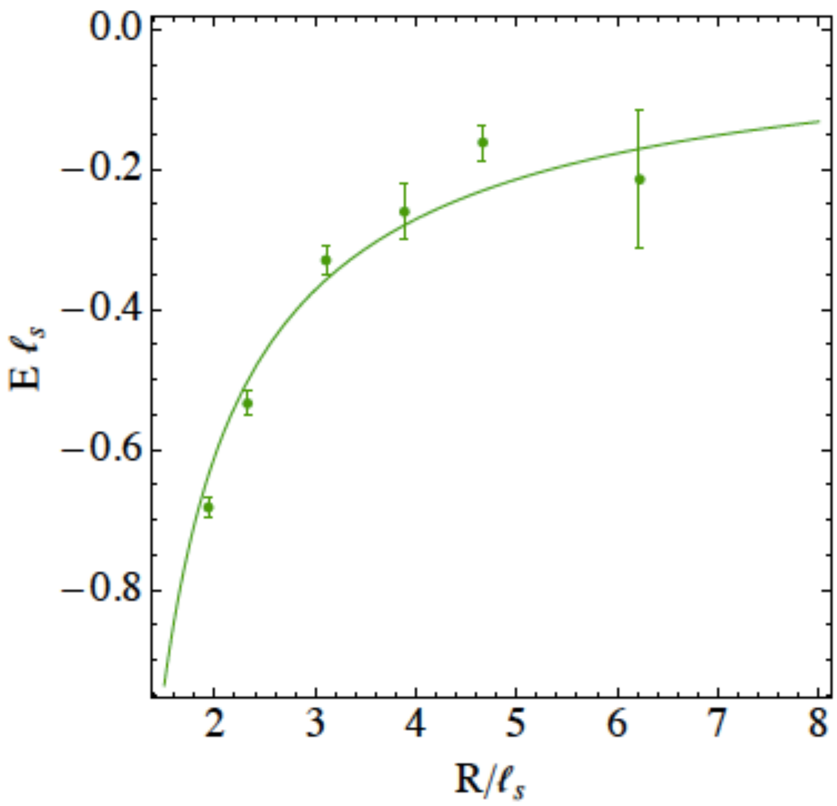
$$2\delta = 2\delta_{GGRT} + \frac{0.7 l_s^6}{(2\pi)^2} s^3$$

3A string in 3D SU(6) Yang-Mills



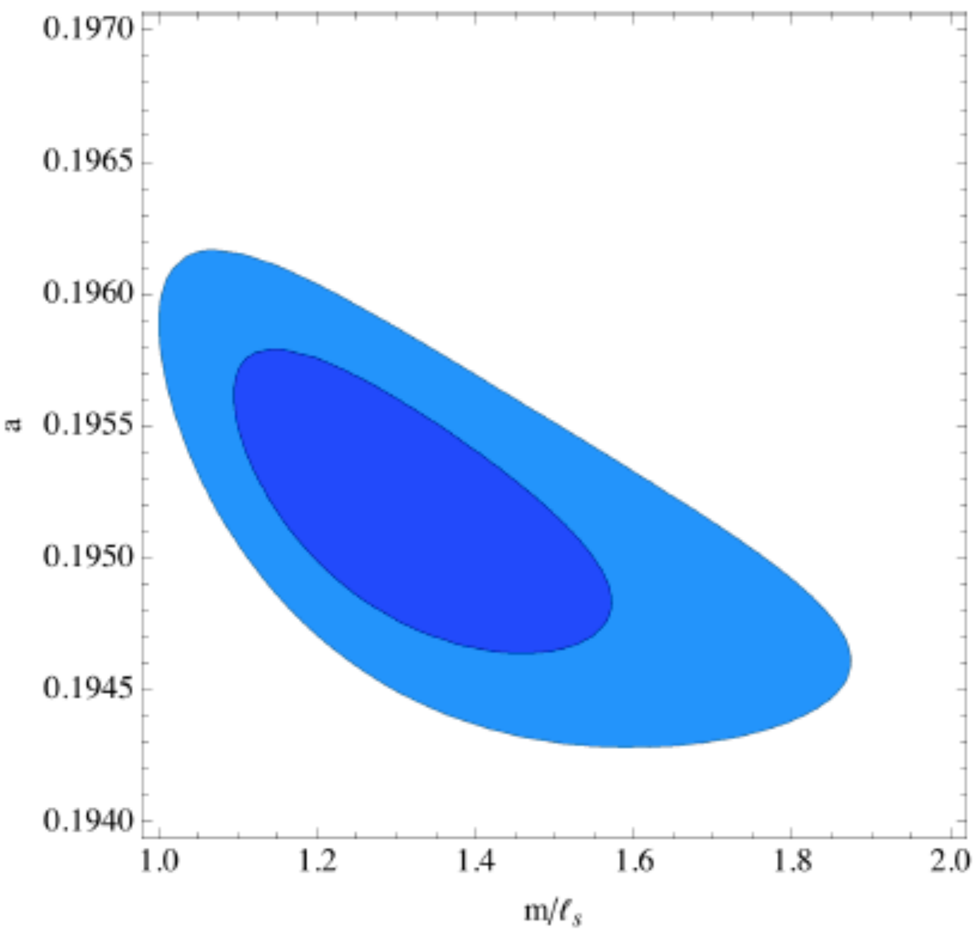


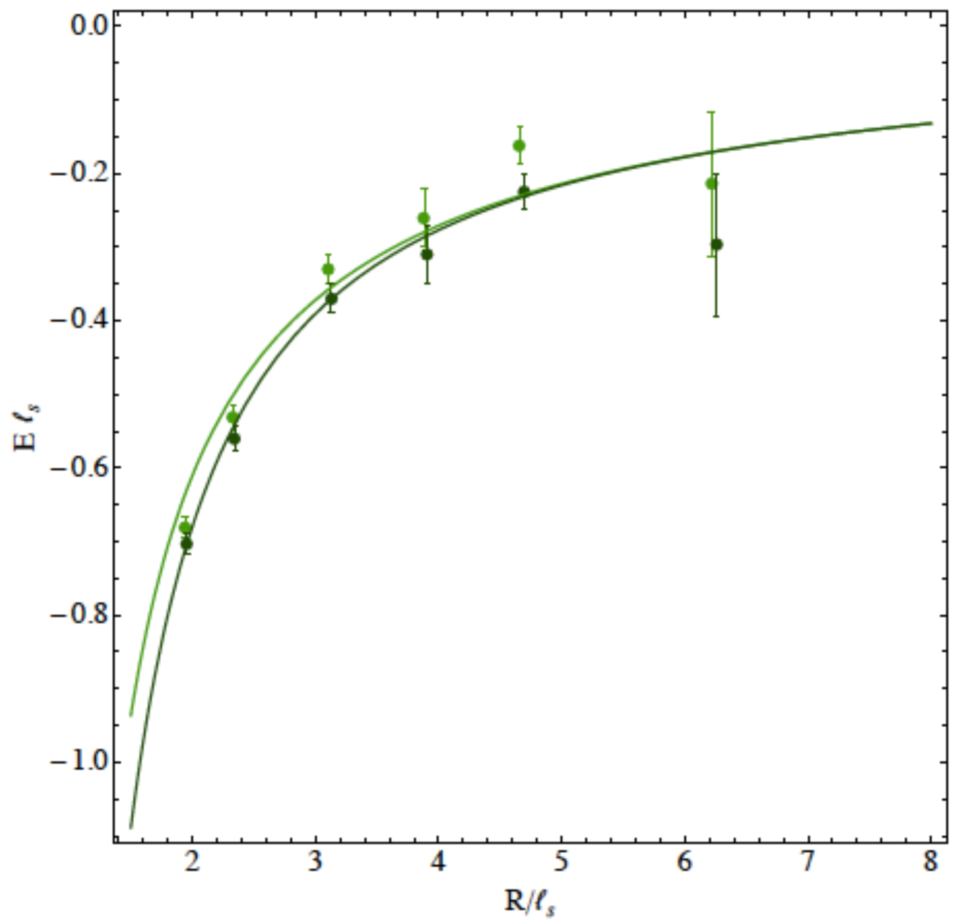
In 4D is this the lightest massive state, or there is a hidden valley?



A massive particle contributes into the Casimir energy

$$\Delta E(R) = -\frac{m}{\pi}\sum_n K_1(mnR)$$





$\Delta\chi^2 \approx 21$ for one new parameter. Remains to be seen whether this is due to "new physics" or systematics

Conclusions

- ✴ Even though the flux tubes studied on the lattice are not very long, at least some of their energy levels are under theoretical control.
- ✴ More to be understood about pseudoscalar state.
- ✴ Good chances to learn more about the worldsheet theory of the QCD string very soon.
- ✴ This is not unique to closed strings. One can extend this to open strings and make predictions for hybrid meson spectra (work in progress).