Evidence for a New Particle on the Worldsheet of the QCD Flux Tube

Sergei Dubovsky
CCPP, NYU & ICTP, Trieste
Three parts to the story:

- Dynamics of QCD flux tubes
- Integrable quantum gravity
- Crazy thoughts about EW hierarchy problem

SD, Raphael Flauger, Victor Gorbenko, 1203.1054, 1205.6805, 1301.2325, 1404.0037
Patrick Cooper, SD, Victor Gorbenko, Ali Mohsen, 1411.0703
+more to appear

SD, Victor Gorbenko, Mehrdad Mirbabayi
1305.6939
Why would one care about QCD?

Reasons *not* to care:

- ✓ We completely know the theory.
- ✓ No room for surprises.
- ✓ All “easy” results are already known.
- ✓ Need to work hard, and the progress will be only incremental.
Why would one care about QCD?

Reasons to care:

✓ We completely know the theory!
✓ There is a 50 years old surprise, which is not quite understood yet.
✓ There are “easy” qualitative results, still waiting to be discovered.
✓ As an extra benefit we may learn something about gravity.
QCD is a theory of strings

What can we say about this string theory?

Bissey et al, hep-lat/0606016
Remarkable recent progress from top-down

✓ Planar N=4 SYM string is integrable
✓ Exact solution for the spectrum

Next Steps:
✓ OPE coefficients
✓ Is there a confining theory with an integrable string?

This talk: bottom up (EFT) approach:

If you quack like a duck, you should be a perturbed duck
What is being measured?

\[ \mathcal{O} = P \exp \{i \oint A\} \]

all the data from the papers by Athenodorou, Bringoltz and Teper

\[ \phi_A = \text{Tr} \left[ \frac{-iz_4 z + z_4^*}{+j[z_4 z + z_4^*]} \right] + i \left[ \frac{-iz_2 z + z_2^*}{+j[z_2 z + z_2^*]} \right] \]

(35)
Puzzle #1: Remarkable agreement with a theory
Puzzle #2: The theory is known to be wrong

Dashed --- light cone quantized bosonic string
Solid --- standard $\ell_s/R$ effective field theory expansion
Puzzle #3: More is going on

Dashed --- light cone quantized bosonic string
Solid --- standard \( \ell_s/R \) effective field theory expansion
Nambu-Goto Spectrum

“Light Cone” or GGRT

$E_{LC}(N, \tilde{N}) = \sqrt{\frac{4\pi^2 (N - \tilde{N})^2}{R^2}} + \frac{R^2}{\ell_s^4} + \frac{4\pi}{\ell_s^2} \left( N + \tilde{N} - \frac{D - 2}{12} \right)$

Comes from quantization in the light cone gauge

Goddard, Goldstone, Rebbi, Thorn’73 +winding

Crucial property: no splittings between different SO(D-2) multiplets

Consistent with target space Lorentz symmetry only at D=26. What it has to do with D=4 spectrum?
ISO(1, D \cdot 1) \rightarrow ISO(1, 1) \times SO(D - 2)

\delta_{\epsilon}^{\alpha i} X^j = -\epsilon (\delta^{ij} \sigma^\alpha + X^i \partial^\alpha X^j)
CCWZ construction

\[ X^\mu = (\sigma^\alpha, X^i(\sigma)) \quad h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu \]

\[ S_{\text{string}} = -\int d^2\sigma \sqrt{-\det h_{\alpha\beta}} \left( \ell_s^{-2} + \frac{1}{\alpha_0} (K^i_{\alpha\beta})^2 + \ldots \right) \]

Perturbatively:

\[ S_{\text{string}} = -\ell_s^{-2} \int d^2\sigma \frac{1}{2} (\partial_\alpha X^i)^2 + c_2 (\partial_\alpha X^i)^4 + c_3 (\partial_\alpha X^i \partial_\beta X^j)^2 + \ldots \]

\[ c_2 = -\frac{1}{8} \quad c_3 = \frac{1}{4} \]

Interacting, in fact non-renormalizable, healthy effective field theory with cutoff \( \ell_s \)
Why $D=26$ is special?

Theory is renormalizable (in some sense)
General SO(D-2) invariant amplitude:

\[ M_{ij,kl} = A \delta_{ij} \delta_{kl} + B \delta_{ik} \delta_{jl} + C \delta_{il} \delta_{jk} \]

**Annihilation**

\[ A(s, t, u) = A(s, u, t) = B(t, s, u) = C(u, t, s) \]

Tree level:

\[ M_{ij,kl} = -\frac{\ell^2_s}{2} (\delta_{ik} \delta_{jl} su + \delta_{il} \delta_{jk} st) \]

No annihilations for Nambu-Goto!
One-loop:

Finite part: \[ \mathcal{M}_{ij,kl} = -\ell_s^4 \frac{D - 26}{192\pi} \left( s^3 \delta_{ij} \delta_{kl} + t^3 \delta_{ik} \delta_{jl} + u^3 \delta_{il} \delta_{jk} \right) \]

Polchinski-Strominger interaction gives rise to annihilations!
$R^{-5}$ splittings in SO(D-2) multiplets

\[ \mathcal{L}_{\text{QCD string}} = \mathcal{L}_{\text{light cone}} - \frac{D - 26}{192\pi} \partial_\alpha \partial_\beta X^i \partial^\alpha \partial^\beta X^i \partial_\gamma X^j \partial^\gamma X^j + \ldots \]
Explains the ground state data

\[ E_0(R) = \frac{R}{\ell_s^2} - \frac{(D - 2)\pi}{6R} - \frac{(D - 2)^2\pi^2\ell_s^2}{72R^3} - \frac{(D - 2)^3\pi^3\ell_s^4}{432R^5} + \text{non-universal terms} \]

classical

Luscher term

Need to work harder for excited states!
GGRT spectrum:

\[
E_{LC}(N, \tilde{N}) = \sqrt{\frac{4\pi^2 (N - \tilde{N})^2}{R^2}} + \frac{R^2}{\ell_s^4} + \frac{4\pi}{\ell_s^2} \left( N + \tilde{N} - \frac{D - 2}{12} \right)
\]

\[\ell_s/R\] expansion breaks down for excited states because \(2\pi\) is a large number!

for excited states:

\[E = \ell_s^{-1} \mathcal{E}(p_i \ell_s, \ell_s/R)\]

Let’s try to disentangle these two expansions
Finite volume spectrum in two steps:

1) Find infinite volume S-matrix
2) Extract finite volume spectrum from the S-matrix

1) is a standard perturbative expansion in $p\ell_s$
2) perturbatively in massive theories (Luscher)
   exactly in integrable 2d theories through TBA

Relativistic string is neither massive nor integrable...
But approaches integrable GGRT theory at low energies!
GGRT S-matrix:

\[ e^{2i\delta_{GGRT}(s)} = e^{is\ell_s^2/4} \]

- Polynomials bounded on the physical sheet
- No poles anywhere. A cut all the way to infinity with an infinite number of broad resonances
- One can reconstruct the entire finite volume spectrum using Thermodynamic Bethe Ansatz
- Does not go to a constant at infinity!
Integrable QG rather than QFT

Gravitational shock waves:

\[ S \gg M_{Pl}^2 \]
\[ \ell \gg R_s \]

Eikonal phase shift:

\[ e^{i2\delta_{eik}(s)} = e^{i\ell^2 s/4} \]

\[ \ell^2 \propto G_N b^{4-d} \]

Dray, 't Hooft '85
Amati, Ciafaloni, Veneziano '88
In the saddle point approximation the integral is dominated by the densities we are aware of where they are fermions. Appearing in the thermodynamic Bethe Ansatz are bosons unlike any other physical examples. In thermodynamic (large L) limit, it is interesting to note that for the long string the particles notice that the expression for the energy includes the bulk cosmological constant. Regarding entropy, and finite equation with left- and right-movers interchanged. These equations receive corrections at where momentum implies that the level densities for the long string are in fact independent of flavor and where we have substituted $2kR_i$. The partition function can then be written as a functional integral over the particle, as follows from (1). Notice that this equation and $\exp L_{ij}$, $H_{ij}$ that minimize the free that become exact in the thermodynamic limit. Introducing the level densities $p_i$, this becomes the TBA constraint

$$Z(T, L) = e^{-LE_0(1/T)} = e^{-Lf(T)/T}$$

Asymptotic Bethe Ansatz

$$\sum_{i=1}^{D-2} \int_0^\infty 2\delta(p_{kR}^{(i)}, p)\rho_{1L}^i(p)dp = 2\pi n_{kR}^{(i)}$$
Asymptotic Bethe Ansatz

\[ \Psi(x_1, x_2) = \langle 0 | X^i(x_1)X^j(x_2) | p^{(i)}_L, p^{(j)}_R \rangle \]

\[ x_1 > x_2 \quad \Psi(x_1, x_2) = e^{-ip_L x_1} e^{ip_R x_2} \]

\[ x_1 > x_2 \quad \Psi(x_1, x_2) = e^{-ip_L x_1} e^{ip_R x_2} e^{2i\delta(p_L, p_R)} \]

periodicity:

\[ e^{-ip_{L,R}} = e^{2i\delta(p_L, p_R)} \]

\[ p_L + 2\delta(p_L, p_R) = 2\pi n_R \]

NB: particles are getting softer!
after taking the continuum limit
minimization of the free energy results in

\[
\epsilon_i^j(p) = p \left[ 1 + \frac{\ell_s^2 T}{2\pi} \sum_{j=1}^{D-2} \int_0^\infty dp' \ln \left( 1 - e^{-\epsilon_R(p')/T} \right) \right]
\]

where

\[
f = \frac{T}{2\pi} \sum_{j=1}^{D-2} \int_0^\infty dp' \ln \left( 1 - e^{-\epsilon_R(p')/T} \right) + (L \to R)
\]

reproduces the correct ground state energy

\[
f(T) = \frac{1}{\ell_s^2} \left( \sqrt{1 - T^2 / T_H^2} - 1 \right) \quad T_H = \frac{1}{\ell_s} \sqrt{\frac{3}{\pi(D-2)}}
\]
Excited States TBA

Dorey, Tateo ’96

**general idea:** excited states can be obtained by analytic continuation of the ground state

\[ \hat{p}^{(i)}_{kL} R + \sum_{j,m} 2\delta(\hat{p}^{(i)}_{kL}, \hat{p}^{(j)}_{mR}) N^{(j)}_{mR} - i \sum_{j=1}^{D-2} \int_{0}^{\infty} \frac{dp'}{2\pi} \frac{d2\delta(i\hat{p}^{(i)}_{kL}, p')}{dp'} \ln \left( 1 - e^{-R\epsilon^{j}_{R}(p')} \right) = 2\pi n^{(i)}_{kL} \]

Asymptotic Bethe Ansatz

\[ \epsilon^{i}_{L}(p) = p + \frac{i}{R} \sum_{j,k} 2\delta(p, -i\hat{p}^{(j)}_{kR}) N^{(j)}_{kR} + \frac{1}{2\pi R} \sum_{j=1}^{D-2} \int_{0}^{\infty} dp' \frac{d2\delta(p, p')}{dp'} \ln \left( 1 - e^{-R\epsilon^{j}_{R}(p')} \right) \]

\[ E(R) = R + \sum_{j,k} p^{(j)}_{kL} + \sum_{j=1}^{D-2} \int_{0}^{\infty} \frac{dp'}{2\pi} \ln \left( 1 - e^{-R\epsilon^{j}_{L}(p')} \right) \]

+right-movers

Exactly reproduces all of the light cone spectrum
The strategy is to incorporate corrections to the S-matrix into TBA equations.

Hard to do in full generality, but turns out possible at one-loop level with Polchinski-Strominger phase shift taken into account.
Pure left-moving states

Dashed --- light cone quantized bosonic string
Solid --- standard $\ell_s/R$ effective field theory expansion
Dotted --- free theory (=ABA in this case)
Colliding left- and right-movers

What are the red points?
A new massive state appearing as a resonance in the antisymmetric channel!

see also arXiv:1007.4720
Athenodorou, Bringoltz, Teper
How do we include this massive state?

Contributes to scattering of Goldstones and changes the phase shifts. In particular, it appears as a resonance in the antisymmetric channel. We can calculate contributions from

\[
S = \int d^2 \sigma \frac{1}{2} \partial_\alpha \phi \partial^\alpha \phi - \frac{1}{2} m^2 \phi^2 + \frac{\alpha}{8\pi} \phi \epsilon^{\alpha \beta} \epsilon_{ij} K^i_{\alpha \gamma} K^j_{\beta \gamma}
\]
\[ c \hat{p} R + 2 \delta_{PS} + 2 \delta_{res} = 2\pi \]

\[
c = 1 + \ell_s^2 \frac{\hat{p}}{R} - \frac{\pi \ell_s^2}{6R^2c} \]

\[
2 \delta_{res} = \sigma_1 \frac{\alpha^2 \ell_s^4 \hat{p}^6}{8\pi^2(4\hat{p}^2 + m^2)} + 2\sigma_2 \tan^{-1} \left( \frac{\alpha^2 \ell_s^4 \hat{p}^6}{8\pi^2(m^2 - 4\hat{p}^2)} \right) \]

\[
2 \delta_{PS} = \pm \frac{11\ell_s^4}{12\pi} \hat{p}^4 \]

\[
E = 2\hat{p} - \frac{\pi}{3Rc} \]
Dear Editor,

we resubmit the revised version of our Letter. We thank the referee for his comments and criticism. We have made significant changes to our manuscript to address them and feel that these changes have made it much easier for the interested reader to follow and to reproduce our results.

In my previous report I have asked the authors to rewrite their manuscript, improve their presentation, give a more quantitative discussion of their results and to clearly display the formulas used for the plots. The revised version still displays all the weaknesses characterizing the original version. In particular, this work is very qualitative, leaving the reader with the impossibility of judging the results presented.

We have made several significant changes to address this. We have included five additional equations, the dispersion relation of the pseudo-particles (in the text before (5)), (5), (6), (9) and the unnumbered equation following (9). These equations can immediately be used to reproduce our results as can be seen from the Mathematica notebook shown below.

\[ \text{Equation (7) and unnumbered equation after equation (9)} \]

\[
\delta \rho_s \left[ \rho_l, \rho_r, a \right] = s_{11} / 12 / \pi (p_l p_r)^2;
\]
\[
\delta \rho_s \left[ \rho_l, \rho_r, s_{1l}, s_{2l}, a, m \right] =
\]
\[
s_1 a^2 p_l^3 p_r^3 / (8 \pi^2) / (4 p_l p_r + m^2) + s_2 \text{If} \left[ p_l p_r < a^2 / 4, 2 \text{ArcTan} \left[ \frac{a^2}{8 \pi^2 \left( m^2 - 4 p_l p_r \right)} \right], 2 \text{ArcTan} \left[ \frac{a^2}{8 \pi^2 \left( m^2 - 4 p_l p_r \right)} \right] + 2 \pi \right];
\]

\[ \text{Solution of quadratic equation (5)} \]

\[
\frac{R (p + R)}{6 R^2} = \frac{3 R (p + R) + \sqrt{3} \sqrt{R^2 - 2 \pi + 3 (p + R)^2}}{6 R^2};
\]

\[ \text{Solution of equation (9) for } 0-, 0++, \text{ and } 2++ \text{ channels} \]

\[
\text{psol0mm} \left[ a ? \text{NumberQ}, m ? \text{NumberQ}, R ? \text{NumberQ} \right] := p \cdot \text{FindRoot} \left[ \text{c} \left[ p, R \right] p R + \delta \rho_s \left[ p, p, 1 \right] + \delta \rho_s \left[ p, p, 1, a, m \right] = 2 \pi, \left\{ p, m / 2 - 1 \right\} \right];
\]
\[
\text{psol0pp} \left[ a ? \text{NumberQ}, m ? \text{NumberQ}, R ? \text{NumberQ} \right] := p \cdot \text{FindRoot} \left[ \text{c} \left[ p, R \right] p R + \delta \rho_s \left[ p, p, 1 \right] + \delta \rho_s \left[ p, p, -1, 0, a, m \right] = 2 \pi, \left\{ p, m / 2 - 1 \right\} \right];
\]
\[
\text{psol2pp} \left[ a ? \text{NumberQ}, m ? \text{NumberQ}, R ? \text{NumberQ} \right] := p \cdot \text{FindRoot} \left[ \text{c} \left[ p, R \right] p R + \delta \rho_s \left[ p, p, -1 \right] + \delta \rho_s \left[ p, p, 1, 0, a, m \right] = 2 \pi, \left\{ p, m / 2 - 1 \right\} \right];
\]

\[ \text{Equation (6)} \]

\[
\text{WE} \left[ p, R \right] := -\pi / 3 / R / \text{c} \left[ p, R \right]
\]

\[ \text{Equation (1)} \]

\[
\text{Eo0mm} \left[ a ? \text{NumberQ}, m ? \text{NumberQ}, R ? \text{NumberQ} \right] := 2 \text{psol0mm} \left[ a, m, R \right] + \text{WE} \left[ \text{psol0mm} \left[ a, m, R \right], R \right]
\]
\[
\text{Eoopp} \left[ a ? \text{NumberQ}, m ? \text{NumberQ}, R ? \text{NumberQ} \right] := 2 \text{psol0pp} \left[ a, m, R \right] + \text{WE} \left[ \text{psol0pp} \left[ a, m, R \right], R \right]
\]
\[
\text{Ezpp} \left[ a ? \text{NumberQ}, m ? \text{NumberQ}, R ? \text{NumberQ} \right] := 2 \text{psol2pp} \left[ a, m, R \right] + \text{WE} \left[ \text{psol2pp} \left[ a, m, R \right], R \right]
\]

\[ \text{Solid lines shown in Figure 2} \]

\[
\text{Plot} \left[ \left\{ \text{Eo0mm} \left[ 9.6, 1.85, x \right], \text{Eoopp} \left[ 9.6, 1.85, x \right], \text{Ezpp} \left[ 9.6, 1.85, x \right] \right\}, \left\{ x, 1.5, 8 \right\}, \text{PlotRange} \rightarrow \left\{ \left\{ 1.5, 8 \right\}, \left\{ 0, 4.2 \right\} \right\}, \text{Frame} \rightarrow \text{True}, \text{AspectRatio} \rightarrow 1 \right]
\]
How do we include this massive state?

Contributes to scattering of Goldstone’s and changes the phase shifts. In particular, it appears as a resonance in the antisymmetric channel. We can calculate contributions from

\[ S = \int d^2 \sigma \frac{1}{2} \partial_\alpha \phi \partial^\alpha \phi - \frac{1}{2} m^2 \phi^2 + \frac{\alpha}{8\pi} \phi \epsilon^{\alpha\beta} \epsilon_{ij} K^i_{\alpha\gamma} K^j_{\beta\gamma} \]

Including the resonant s-channel contribution

\[ \delta(s) = \arctan \left( \frac{m \Gamma(s/m)^3}{m^2 - s} \right) \]

\[ m \sim 1.85 \ell_s^{-1} \quad \Gamma \sim 0.4 \ell_s^{-1} \]

as well as perturbative non-resonant contributions in crossed channels
Reverting the logic: S-matrix from finite volume spectrum
More states:
3D Yang-Mills
Figure 14: This plot shows the energies of the states included in the fit for the correction to the scattering phase shift at order \( \ell_s^6 \). The lines show the theoretical prediction. For the second excited two-particle state, only data for six longest strings is included in the fit because the phonon momenta become too large.

Including all data points with \( \ell_s \approx 2 \), and taking the error bars at face value, we find

\[
2\delta = 2\delta_{GGRT} + \frac{0.7\ell_s^6}{(2\pi)^2}s^3
\]
There is strong motivation for further high precision lattice studies of the properties of flux tubes. We feel that the most important conclusion to be drawn from the current paper is that the physical size of the compact dimension becomes comparable to the size of the massive phase shift plot. The natural explanation for the origin of this break is that it occurs when the winding corrections due to resonance become large. This interpretation is supported by observing that a very similar break at the same values of $R/\ell_s$ appears also in the lightest glueball energy plot \cite{7}, suggesting that the size of the resonance is roughly equal to the size of the massive glueball.

The corresponding points also show up very far from the theory curve on the corresponding phase shift plot. The left panel shows the energy as a function of string length for the lowest (orange) level at $k_s=2$ strings. This motivates further high precision lattice measurements of these states. The right panel shows the phase shift extracted from the data. Notice an interesting feature exhibited by the predictions for the 2-particle states, dashed lines represent 4-particle states. The solid lines are the theory prediction for even number of phonons and zero total momentum. The fundamental flux tube tension is equal to $f_d = \frac{s_0}{2\pi}$ and the $3$A string in 3D SU(6) Yang-Mills is given by $f_3 = \frac{s_1}{2\pi}$. The $6$A string is given by $f_6 = \frac{s_2}{2\pi}$, while the $8$A string data does not exhibit such a break. Perhaps only the shortest point in Fig. 17 may be considered as an indication for the beginning of the break. This is in agreement with the $3\ell_s$ data — average broken.
In 4D is this the lightest massive state, or there is a hidden valley?

A massive particle contributes into the Casimir energy

\[ \Delta E(R) = -\frac{m}{\pi} \sum_n K_1(mnR) \]
\[ \Delta \chi^2 \approx 21 \] for one new parameter. Remains to be seen whether this is due to “new physics” or systematics.
Conclusions

✴ Even though the flux tubes studied on the lattice are not very long, at least some of their energy levels are under theoretical control.

✴ More to be understood about pseudoscalar state.

✴ Good chances to learn more about the worldsheet theory of the QCD string very soon.

✴ This is not unique to closed strings. One can extend this to open strings and make predictions for hybrid meson spectra (work in progress).