What Does Data Tell Us About the Theory of Inflation?

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HST: A Fully Quantum Model of Cosmology

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- Zero vorticity fluid so can be brought to co-moving gauge.

General Properties of Slow Roll and HST

- In co-moving gauge, gauge invariant fluctuations around approximately dS flat FRW are encoded in the metric.

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- Fluctuations of δH and γ, can be computed as expectation values

$$\langle \zeta_1 \ldots \zeta_n \gamma_1 \ldots \gamma_k \rangle = \mathrm{Tr}\ [\rho S_1 \ldots S_n T_1 \ldots T_k].$$

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- True in slow roll (Maldacena) as well as HST.

General Properties

- Maldacena's squeezed limit theorem says 3-point functions involving zero momentum scalar are smaller by a slow roll factor. $SO(1,4)$ invariance determines general form of all 3 point functions (Maldacena, Pimentel, McFadden, Skenderis, Trivedi, Mata, Raju, Shiraishi,Nitta,Yokoyama, Garriga, Vilenkin, Soda, Kodama, Nozawa,) so any 3 pt function involving scalars too small to be measured.

Contrasts Between Slow Roll and HST

- In slow roll, S and T are modeled as free fields in adiabatic Bunch Davies vacuum of FRW space-time, $H(t)$. Normalization of two point functions fixed $\propto (\frac{H(t)}{m_P})^2$. $SO(1,4)$ reps. picked at extreme limits of rep. theory. Normalization of $k-$point functions $\sim (\frac{H(t)}{m_P})^k$, but coefficients depend on terms in QUEFT. Choice of Bunch-Davies is massive fine tuning of initial conditions.

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- Representation theory of T the same in HST as slow roll, but S can be anything in scalar complementary series of unitary irreps, $\Delta_S \in [0, 3/2]$. Density matrix plausibly invariant for generic initial conditions, but not Bunch Davies. Order of magnitude normalizations $\sim (\frac{H}{m_P})^k$ where $\pi \frac{M_P^4}{H^2}$ is the entropy of the individual microsystems that are combined to make the $SO(1,4)$ invariant theories. Notice that, unlike the slow roll case, this factor is time independent and won't contribute to the tilt. *In particular, the tensor tilt is predicted to be zero in HST models.*

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- Unfortunately, we can choose different geometries $H_{slow\ roll}(t)$, $H_{HST}(t)$ which can partially mask these differences. Only the absence of tensor tilt differentiates between the two models decisively. But this is, as yet, unmeasured.

The Tensor Three Point Function

- Three functional forms allowed by $SO(1,4)$, one violates parity. Lowest order derivative expansion gives only one of these. Nominally, the second should be down by (H/m_P) because it comes from higher order terms in the derivative expansion of the bulk QUEFT.

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- Unfortunately, the prospect of measuring the tensor 3-point function seems remote.

Conclusions

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- Differences between Slow Roll and HST models, which are of a deep conceptual nature, can be masked by the flexibility of choosing the background FRW in both frameworks.
- Absence of tensor tilt, the only clear signal for HST in two point functions. Tensor 3 pt. function has crucial information, but good luck in measuring it this century :-).

Dynamics of HST Cosmology

- An HST cosmology is an infinite collection of quantum systems, labeled by a lattice on the initial value surface of a Big Bang universe. Nearest neighbors represent time-like trajectories, which maintain a space-like separation of one Planck unit throughout history. The rest of the geometry is determined by dynamical overlap conditions: at any time the intersection of the causal diamonds on two trajectories, will contain a maximal size causal diamond, corresponding to a Hilbert space of dimension $e^{\frac{A}{4L_P^2}}$. Dynamics in proper time of each trajectory must give a compatible density matrix in the overlap.

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- There exists a complete model obeying these rules, which has a coarse grained space-time interpretation as a flat FRW model with an equation of state that is the sum of a $p = \rho$ component and a cosmological constant. At late times, the local model has a time independent Hamiltonian, on a Hilbert space of finite dimension, $e^{\pi(\frac{M_P}{H})^2}$.

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- Model of inflation takes e^{3N_e} of these uncorrelated systems and couples them together gradually, starting with a "central observer" and moving out. The Hamiltonian is built to become the generator of $SO(1,4)$ in the limit N_e goes to infinity. Density matrix of the whole system plausibly becomes invariant. Corrections e^{-N_e} for correlation functions of a small number of observables.

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- Variables localized on a "fuzzy 3 sphere". Fluctuations come from local fluctuations of individual subsystems. For small k, $k-$point functions scale as $(n)^{-k}$, $(n = R_I L_P)$ by the usual rules of statistical mechanics.

Dynamics of HST Cosmology

- By the rules of HST, horizon radius at the end of inflation is $e^{\frac{3N_e}{2}} H^{-1}$. This must have enough entropy to account for entropy in fluctuations of CMB $\sim \delta T T^2 R^3$. This gives about 56 e-folds for unification scale inflation, which is what is indicated by the size of fluctuations. On the other hand $N_e \leq \frac{2}{3}\ln{(RH)}$ where R is our cosmological horizon. So $56 \leq N_e \leq 88$ in HST models. Probably on low end to account for post-inflationary expansion of the horizon.

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- We don't understand particle physics in HST language well enough to build an HST model of reheating, radiation and matter dominated eras.

Meta-Cosmological Problems

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- There is a fully consistent HST model in which a ball of radius R, evolves from $p = \rho$ to dS with radius $R = NL_P$, while the region outside it remains in the $p = \rho$ model. Jacobsonian geometry is black hole with dS interior embedded in $p = \rho$ FRW.

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- Einstein's equations (the hydrodynamics of space-time) have solutions with many widely separated black holes, with various radii, N_i, and relative initial velocities. Black holes may or may not collide.
- IF our inflationary model exists, we can supply each of these black holes with its own inflationary $1 \ll n_i \ll N_i$. Thus, we can have environmental selection of "good" values of (n, N) for our universe.

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- Given this, Weinberg's bound becomes relatively strong constraint on n: $n < C(\frac{\rho_0}{\Lambda})^{1/3}$, where ρ_0 is the dark matter density at the beginning of matter domination, and is plausibly fixed once N is fixed. Given the observed values, and the *a priori* constraint $n \gg 1$, this is pretty close to the observed value.

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- By a not particularly tuned choice of initial distribution of positions and velocities of black holes in the meta-model, we can arrange that our own little universe lives as long as we observe it to but a much shorter time than a Boltzmann brain recurrence time, before colliding with another island universe. The collision leads to drastic thermalization, followed by a much lower value of the c.c., for which no brains can survive.