Testing effective conformal dominance in 2D-QCD with an adjoint fermion

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hep-th/1308.4980

Outline

- Motivation
  - decoupling of operators in a broken CFT
- Our laboratory
  - the 2d QCD model with an adjoint fermion
- The basis of conformal quasi-primary operators
- Results
  - The single particle

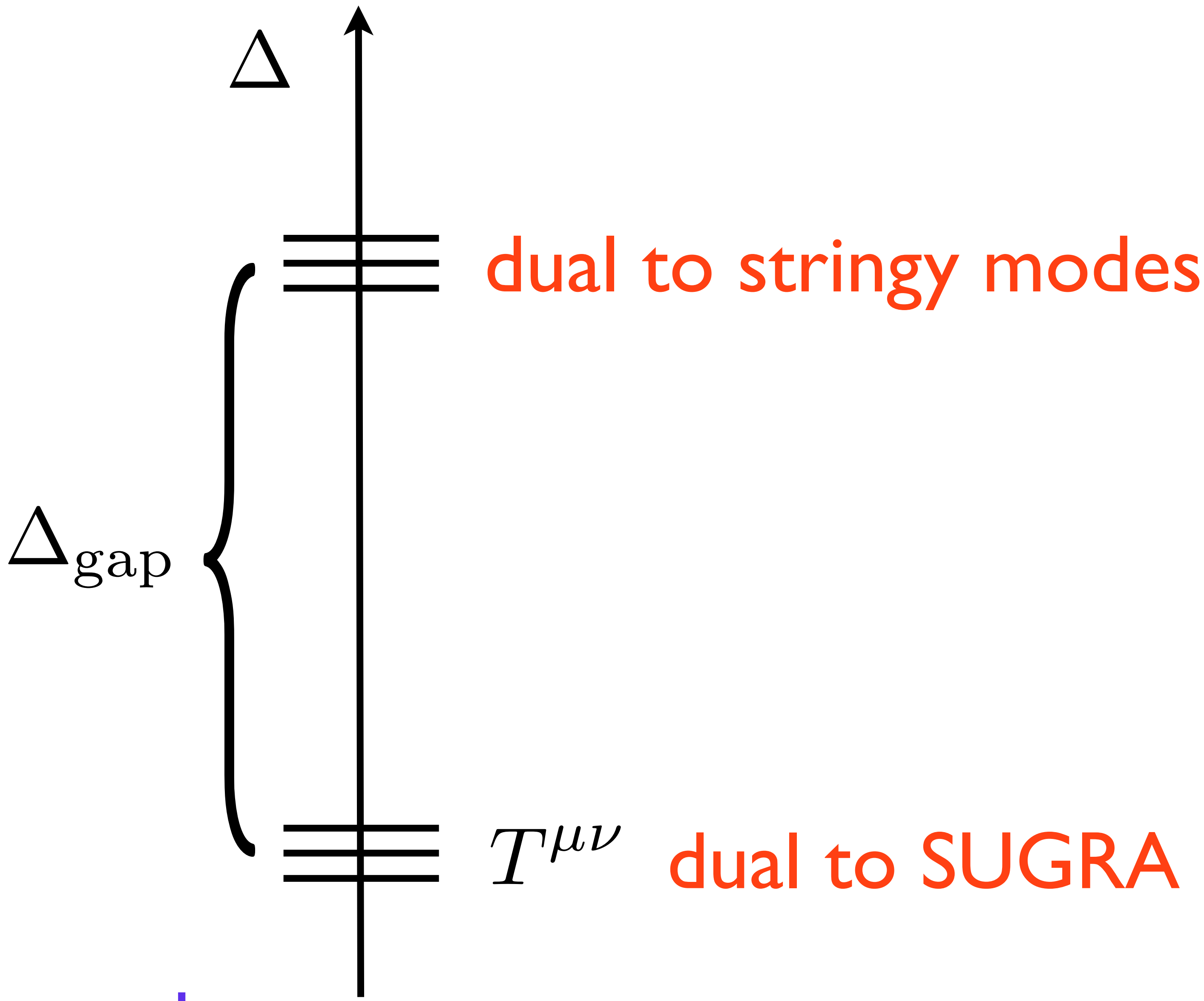
    Exponential convergence of the spectrum
  - The continuum

    An expansion parameter : $e^{-\Delta_{max}}$
- Conclusions

Motivation

Decoupling of higher-dim operators from the lightest states happens in systems with a large gap in op dim.

$$\mathcal{O}_\Delta(\lambda x) = \lambda^{-\Delta}\mathcal{O}(x)$$



in SUGRA background
dual to a confining gauge theory

e.g. hep-th/0007191, hep-th/0003136

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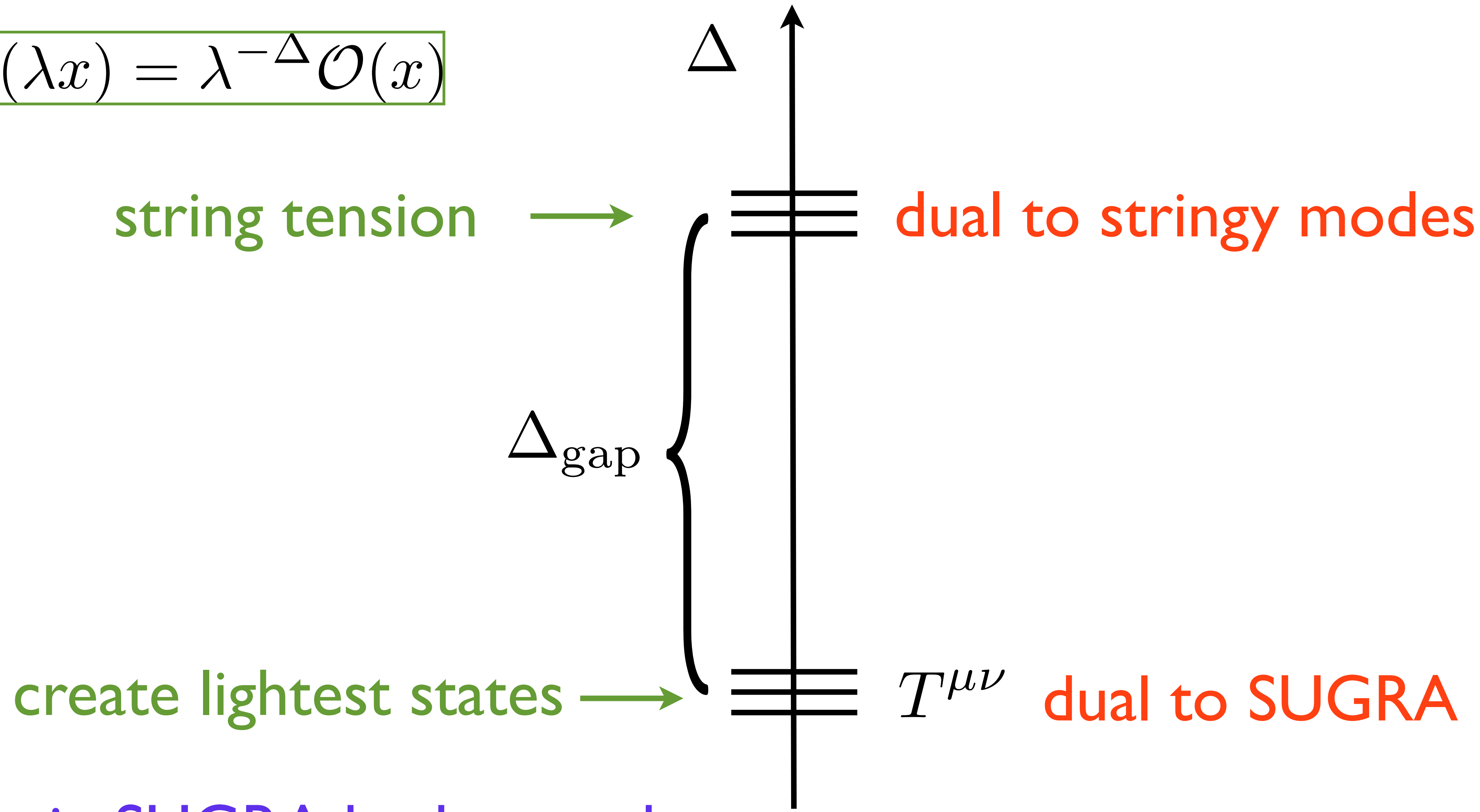

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dual to a confining gauge theory

e.g. hep-th/0007191, hep-th/0003136

Motivation

Can the decoupling be exponential?

Fitzpatrick, Kaplan, Katz, Randall, hep-th/1304.3448

In the presence of a mass gap

$$\langle \mathcal{O}_2(r)\mathcal{O}_1(0)\rangle \approx f(\Delta_1,\Delta_2)\frac{e^{-mr}}{r^{d-2}}$$

In many cases, the decoupling is exponential

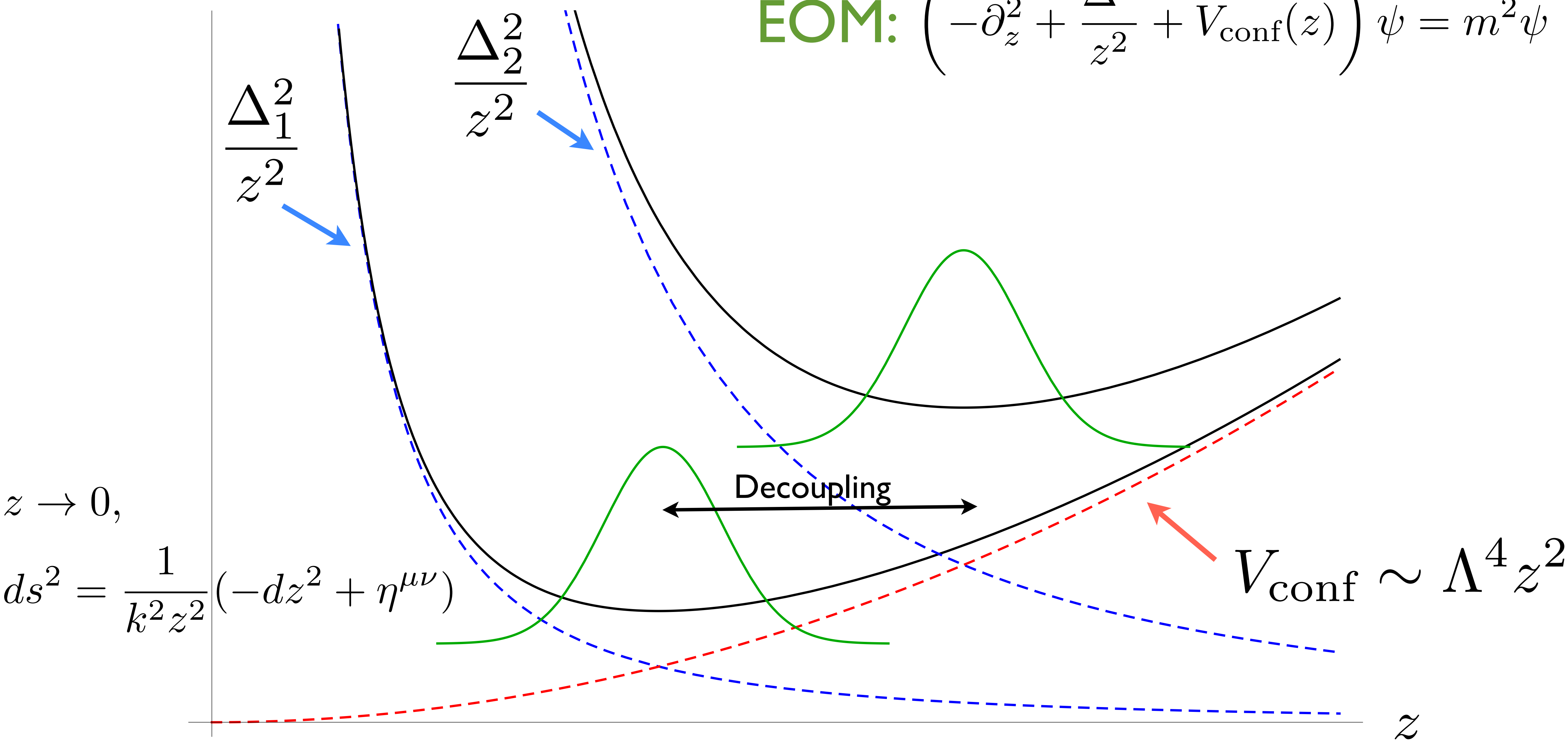
If $\Delta = \Delta_2 \gg \Delta_1$ $\qquad f_{\Delta_1}(\Delta) \sim \exp\left[-\lambda\Delta^p\right]$

Motivation

AdS/CFT: Fitzpatrick, Kaplan, Katz, Randall, hep-th/1304.3448

In a CFT broken by a single scale, high dimensional operators can decouple from the lightest states exponentially fast, $e^{-\Delta}$.

EOM: $\left(-\partial_z^2 + \frac{\Delta^2}{z^2} + V_{\rm conf}(z)\right)\psi = m^2\psi$



Motivation

Can there be an exponential decoupling if there is a mild / no gap?

$$|\psi_0\rangle = \mathcal{O}_{\Delta_1}|\Omega\rangle$$

$$\langle\Omega|\mathcal{O}_{\Delta}|\psi_0\rangle \sim \exp(-\lambda\Delta^p)$$

Motivation

Can there be an exponential decoupling if there is a mild / no gap?

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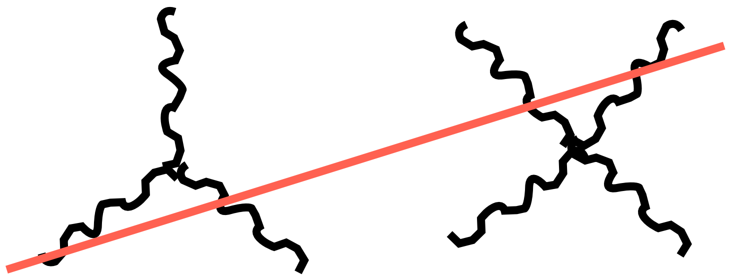
Yes, e.g. in 2d QCD at large N

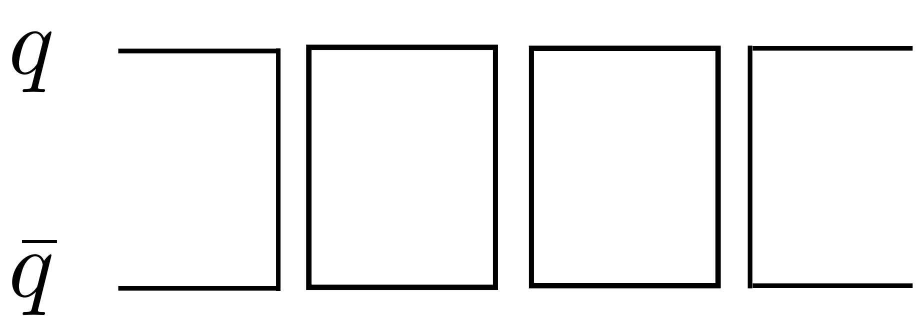
2d QCD models

't Hoooft Model Nucl. Phys. B75 (1974) 461

Simple : gluons have no dof, use light-cone gauge.



Large N : planar diagrams



2d QCD models

't Hooft Model

Decoupling of high-dim op from lightest states

Katz, Okui, hep-th/0710.3402

$$\langle \Omega | \bar{q} \partial^k q | \psi_0 \rangle \sim \exp(-k)$$

$$\langle \Omega | \mathcal{O}_\Delta | \psi_0 \rangle \sim \exp(-\lambda \Delta^p)$$

However,
at large N, reduces to QM, no particle # violation (fundamental fermions).

2d QCD models

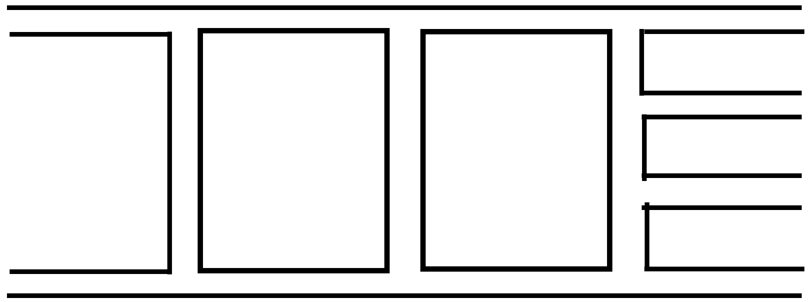
2d QCD with an adjoint fermion

e.g. Dalley, Klebanov, hep-th/9209049

$$S_{\mathrm{f}} = \int d^2x \, \mathrm{Tr} \left[i\Psi^T \gamma^0 \gamma^\alpha D_\alpha \Psi - m\Psi^T \gamma^0 \Psi - \frac{1}{4g^2} F_{\alpha\beta} F^{\alpha\beta} \right]$$

at large N - planar, has particle # violation.

more like real QCD.

$\mathrm{Tr}(qq)$  $\mathrm{Tr}(qqqq)$

No analytic control, numerical results obtained from Discrete Light-cone Quantization (DLCQ) method.

Effective conformal dominance

Suppose

$$\langle\Omega|\mathcal{O}_\Delta|\psi_0\rangle \sim \exp(-\lambda\Delta^p)$$

then taking a basis $|\psi_\Delta\rangle \equiv \mathcal{O}_\Delta|\Omega\rangle$

with

$$\Delta < \Delta_{max},$$

one expects accuracy

$$\delta M^2 \sim \exp(-\lambda^{'}\Delta_{max})$$

This suggests an expansion parameter: Δ_{max}

Effective conformal dominance

Although motivated by holography, this method is entirely field theoretic.

Construct the basis $\mathcal{O}_{\Delta_i} \to |\psi_{\Delta_i}\rangle \equiv \mathcal{O}_{\Delta_i}|\Omega\rangle$

to calculate the spectrum $\langle\psi_{\Delta_i}|M^2|\psi_{\Delta_j}\rangle \equiv M^2_{i,j}$.

$\langle p_1, p_2, ...|\psi_\Delta\rangle$ ~ poly. in p_i's

Δ ~ degree of poly.

Decoupling → Low-lying parton wavefunction dominated by low-degree poly.

2d QCD with an adjoint fermion

$$S = \int d^2x \mathrm{Tr}\left(i\Psi^T\gamma^0\gamma^\alpha D_\alpha\Psi - m\Psi^T\gamma^0\Psi - \frac{1}{4g^2}F_{\alpha\beta}F^{\alpha\beta}\right)$$

$$S = \int dx^+ dx^- \mathrm{Tr}\left(i\psi\partial_+\psi + i\chi\partial_-\chi + \frac{1}{2g^2}(\partial_- A_+)^2 + 2A_+\psi\psi\right)$$

light cone coord.

$x^\pm = (x^0 \pm x^1)/\sqrt{2}$

$$P^+ = \int dx^- \mathrm{Tr}\left(i\psi\partial_-\psi\right)$$

$$[P^+, P^-] = 0$$

$$P^- = \int dx^- \mathrm{Tr}\left(-2g^2\psi^2\frac{1}{\partial_-^2}\psi^2\right)$$

$$M^2 = 2P^+P^-$$

2d QCD with an adjoint fermion

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Non-dynamical

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left mover and right mover do not mix

(like left/right handed fermions in 4D)

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$$[P^+, P^-] = 0$$

↓

eigenfunctions of P^+ and P^-

2d QCD with an adjoint fermion

Eigenfunction $\psi_k(x_1, x_2, ..., x_k) = \langle p_1, p_2, ..., p_k | \psi \rangle$ $x_i = p_i / \sum p_j$

2d QCD with an adjoint fermion

Eigenfunction $\psi_k(x_1, x_2, ..., x_k) = \langle p_1, p_2, ..., p_k|\psi\rangle \qquad x_i = p_i / \sum p_j$

$$\langle p_1, p_2, ..., p_k|2P^+P^-|\psi\rangle = \frac{g^2N}{\pi(x_1+x_2)^2}\int_0^{x_1+x_2} dy\psi_k(y, x_1+x_2-y, x_3, ..., x_k)$$
$$+\frac{g^2N}{\pi}\int_0^{x_1+x_2}\frac{dy}{(x_1-y)^2}\left[\psi_k(x_1,x_2,x_3,...,x_k)-\psi_k(y,x_1+x_2-y,x_3,...,x_k)\right]$$
$$+\frac{g^2N}{\pi}\int_0^{x_1}dy\int_0^{x_1-y}dz\psi_{k+2}(y,z,x_1-y-z,x2,...,x_k)\left[\frac{1}{(y+z)^2}-\frac{1}{(x_1-y)^2}\right]$$
$$+\frac{g^2N}{\pi}\psi_{k-2}(x_1+x_2+x_3,x_4,...,x_k)\left[\frac{1}{(x_1+x_2)^2}-\frac{1}{(x_2+x_3)^2}\right]$$
$$\pm\ \text{cyclic permutations of } (x_1,x_2,...,x_k)$$

dim of g = 1

2d QCD with an adjoint fermion

Eigenfunction $\psi_k(x_1, x_2, ..., x_k) = \langle p_1, p_2, ..., p_k | \psi \rangle$ $\quad x_i = p_i / \sum p_j$

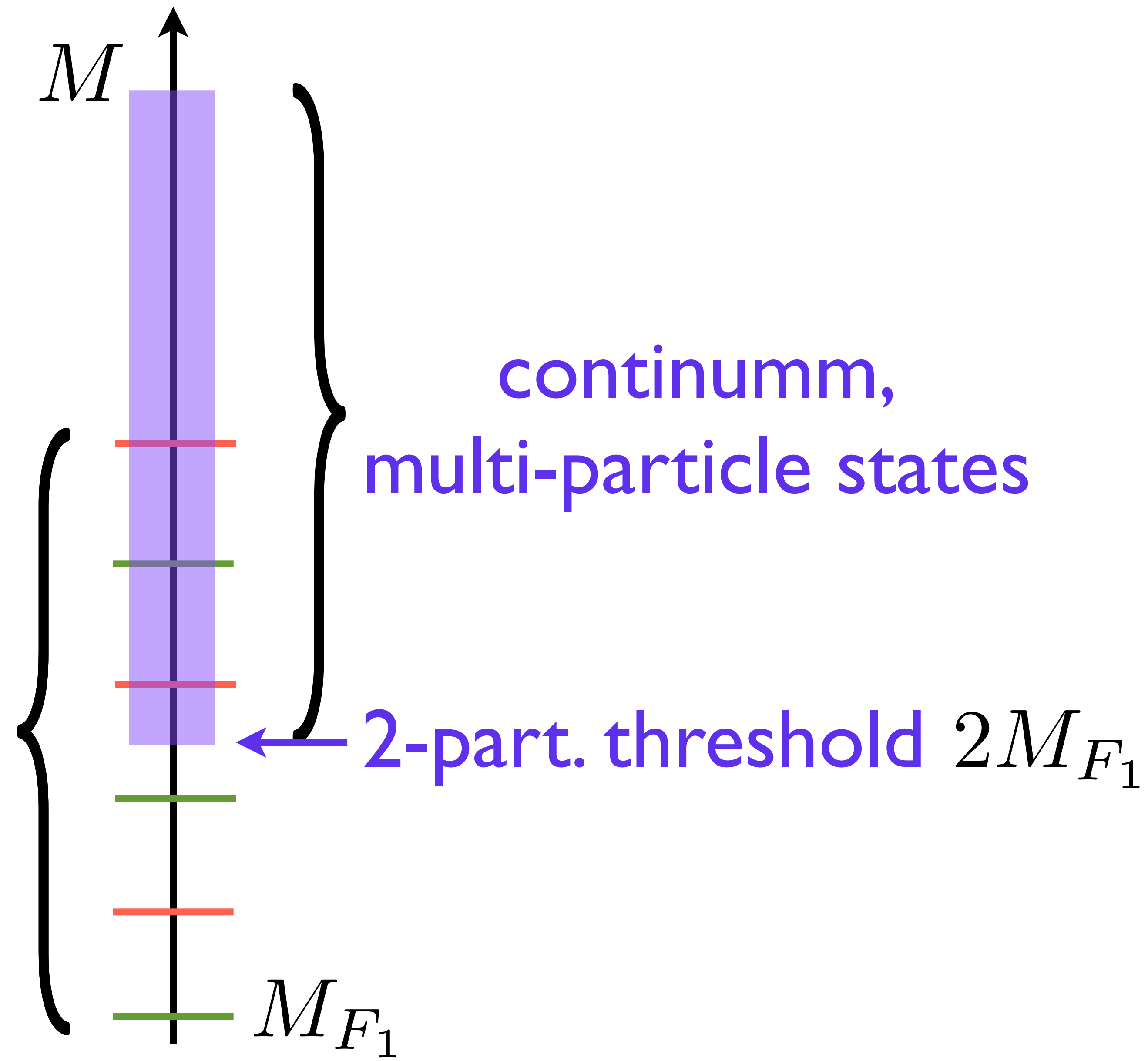
$$\begin{aligned}
\langle p_1, p_2, ..., p_k | 2P^+P^- | \psi \rangle =& \frac{g^2 N}{\pi (x_1 + x_2)^2} \int_0^{x_1+x_2} dy \psi_k(y, x_1 + x_2 - y, x_3, ..., x_k) \\
&+ \frac{g^2 N}{\pi} \int_0^{x_1+x_2} \frac{dy}{(x_1 - y)^2} \left[\psi_k(x_1, x_2, x_3, ..., x_k) - \psi_k(y, x_1 + x_2 - y, x_3, ..., x_k) \right] \\
&+ \frac{g^2 N}{\pi} \int_0^{x_1} dy \int_0^{x_1 - y} dz \psi_{k+2}(y, z, x_1 - y - z, x2, ..., x_k) \left[\frac{1}{(y+z)^2} - \frac{1}{(x_1 - y)^2} \right] \\
&+ \frac{g^2 N}{\pi} \psi_{k-2}(x_1 + x_2 + x_3, x_4, ..., x_k) \left[\frac{1}{(x_1 + x_2)^2} - \frac{1}{(x_2 + x_3)^2} \right] \\
&\pm \text{cyclic permutations of } (x_1, x_2, ..., x_k)
\end{aligned}$$

parton # changed by 2

dim of g = 1

2d QCD with an adjoint fermion

Spectrum



previously solved numerically by DLCQ

Bhanot, Demeterfi, Klebanov, hep-th/9307111,
Gross, Hashimoto, Klebanov, hep-th/9710240.

The basis - primary operators

Construct the basis:

$$|\psi_\Delta\rangle = \mathcal{O}_\Delta(x^-, x^+ = 0)|\Omega\rangle$$

Define primary operators in the UV:

$$\mathcal{O}_{n+k/2} \equiv \frac{1}{N^{k/2}} \sum_{\sum s_i = n} c_{s_1, s_2, \ldots, s_k} \mathrm{Tr}\left(\partial_-^{s_1}\psi_1 \partial_-^{s_2}\psi_2 \ldots \partial_-^{s_k}\psi_k\right)$$

Gauge singlet

satisfying the Killing equation:

$$[K^-, \mathcal{O}_{n+k/2}(x^-)] = i\left((x^-)^2\partial_- + x^-(2n+k)\right)\mathcal{O}_{n+k/2}(x^-)$$

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constraints on the coefficients:
orthonormal and cyclicity

The basis - technicality

$$\mathcal{O}_{n+k/2} \equiv \frac{1}{N^{k/2}} \sum_{\sum s_i = n} c_{s_1, s_2, \ldots, s_k} \mathrm{Tr}\left(\partial_-^{s_1}\psi_1 \partial_-^{s_2}\psi_2 \ldots \partial_-^{s_k}\psi_k\right)$$

Quantization of the fermion at constant "time" x^+ :

$$\psi_{ij} = \frac{1}{2\sqrt{\pi}} \int_0^\infty dp^+ \left(b_{ij}(p^+) e^{-ip^+x^-} + b^\dagger_{ji}(p^+) e^{ip^+x^-} \right)$$

$$f(p_1, p_2, \ldots, p_k) = \langle p_1, p_2, \ldots, p_k | \tilde{\mathcal{O}}_{n+k/2} | 0 \rangle$$

Killing eq. $\longrightarrow$ differential eq. of the poly. $f(p_1, p_2, \ldots, p_k)$

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$$\downarrow$$

$$\mathcal{O}_{n+k/2}$$

The basis - primary operators

T sym $\psi_{ij} \to \psi_{ji}$ ➡ Hilbert space (T - even / T - odd) $\otimes$ (Bosons / Fermions)

T - even sector, the lowest 5 operators:

$$\mathcal{O}_1 \sim \mathrm{Tr}\left((\partial\psi)\psi - \psi\partial\psi\right),$$
$$\mathcal{O}_2 \sim \mathrm{Tr}\left((\partial^3\psi)\psi - 9(\partial^2\psi)\partial\psi\right) \pm ...,$$
$$\mathcal{O}_3 \sim \mathrm{Tr}\left((\partial\psi)(\partial\psi)\psi\psi\right) \pm ...,$$
$$\mathcal{O}_4 \sim \mathrm{Tr}\left((\partial\psi)\psi\psi\psi\psi\psi\right) \pm ...,$$
$$\mathcal{O}_5 \sim \mathrm{Tr}\left((\partial^2\psi)\psi\psi\psi\psi\psi - 2(\partial\psi)\psi(\partial\psi)\psi\psi\psi\right) \pm ...$$

$\partial : \partial_-$

The mass matrix

$$\delta(P - P^{'})M^2_{i,j} = \int dxdy e^{iPx - iP^{'}y} \langle \mathcal{O}_i(x)|2P^+P^-|\mathcal{O}_j(y)\rangle$$

For previous example, the mass matrix (T-even) has a basis of the lowest 5 operators.

$$M^2_{i,j} = \begin{pmatrix} 12. & 3.05 & 4.83 & 0 & 0 \\ 3.05 & 51.3 & -7.38 & 0 & 0 \\ 4.83 & -7.38 & 44.3 & 0 & 0 \\ 0 & 0 & 0 & 56. & 0 \\ 0 & 0 & 0 & 0 & 72. \end{pmatrix} \Bigg\} \; \Delta_{max} = 5$$

$\downarrow$

Eigenstates w/ $M^2(\Delta_{max} = 5)$

The size of the basis

Bosonic sector

Δ_{max}	2	3	4	5	6	7	8	9
T-even	1	1	4	5	16	27	75	153
T-odd	0	1	2	6	12	31	66	165

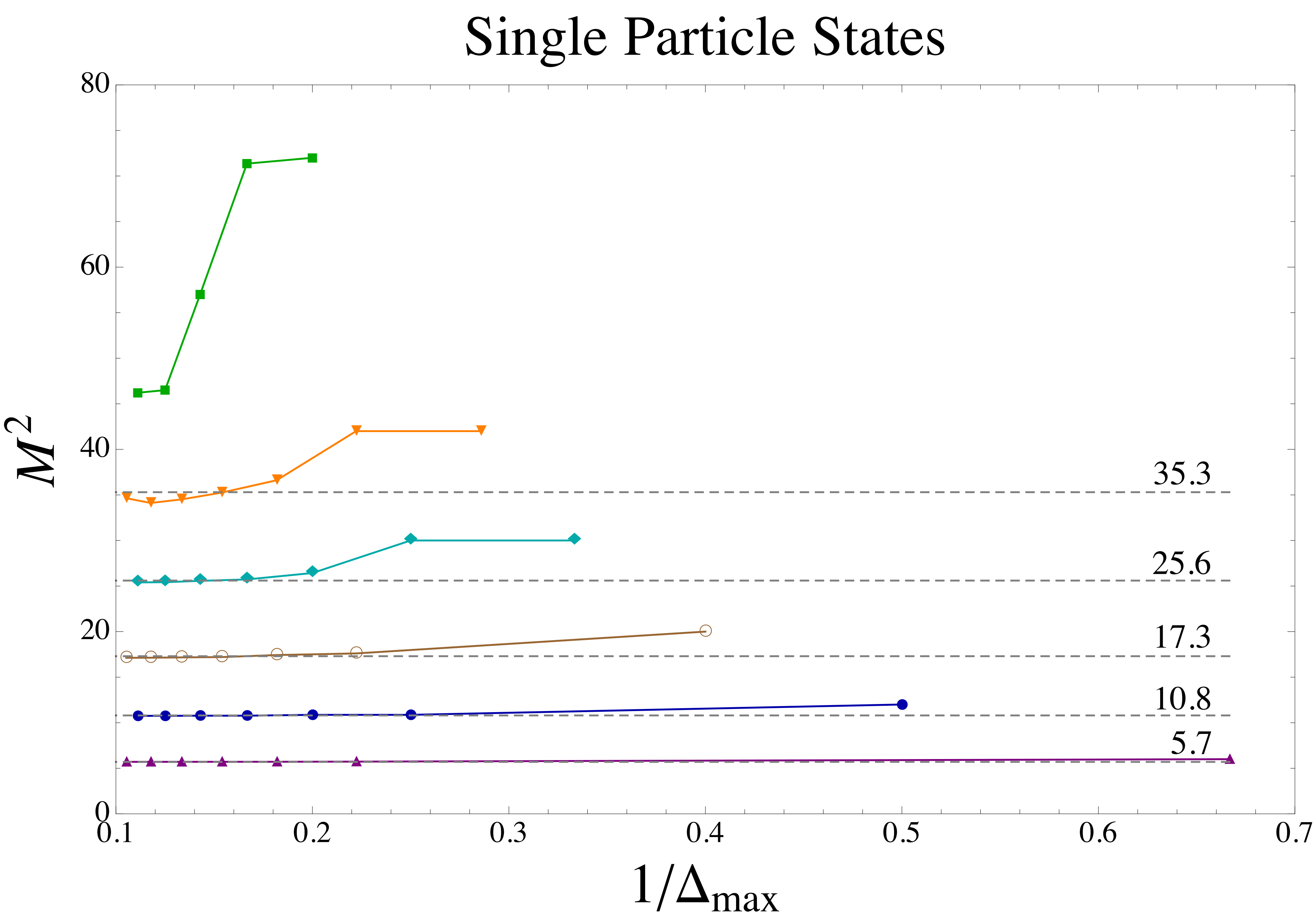
Fermionic sector

Δ_{max}	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5
T-even	0	1	1	5	7	22	42	111	235
T-odd	1	1	3	4	11	18	51	99	257

DLCQ: ~ 6700 states

(hep-th/9710240)

Results - single particle states



bosons: up to $\Delta_{max} = 9$

fermions: up to $\Delta_{max} = 9.5$

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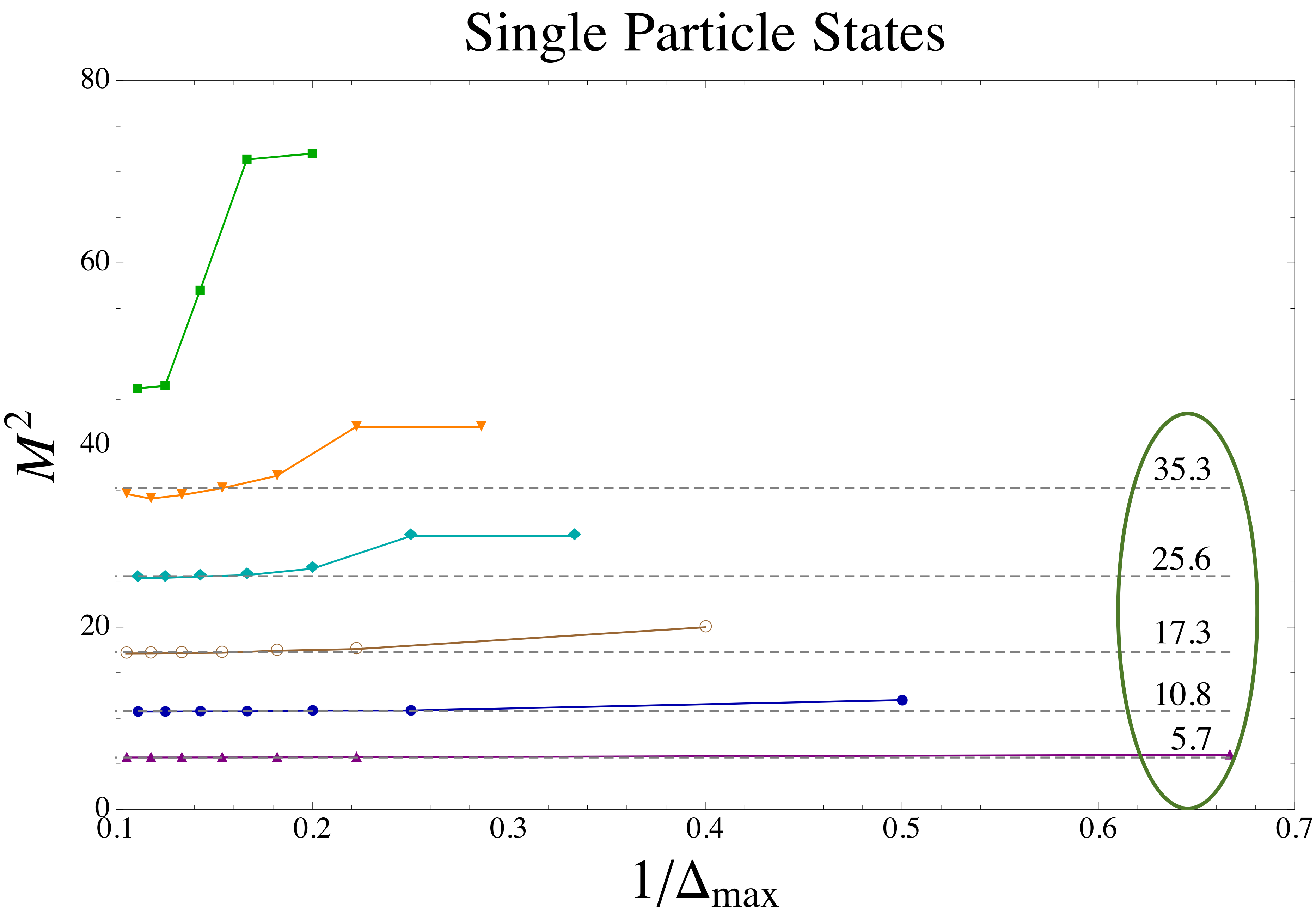

DLCQ results
hep-th/9710240

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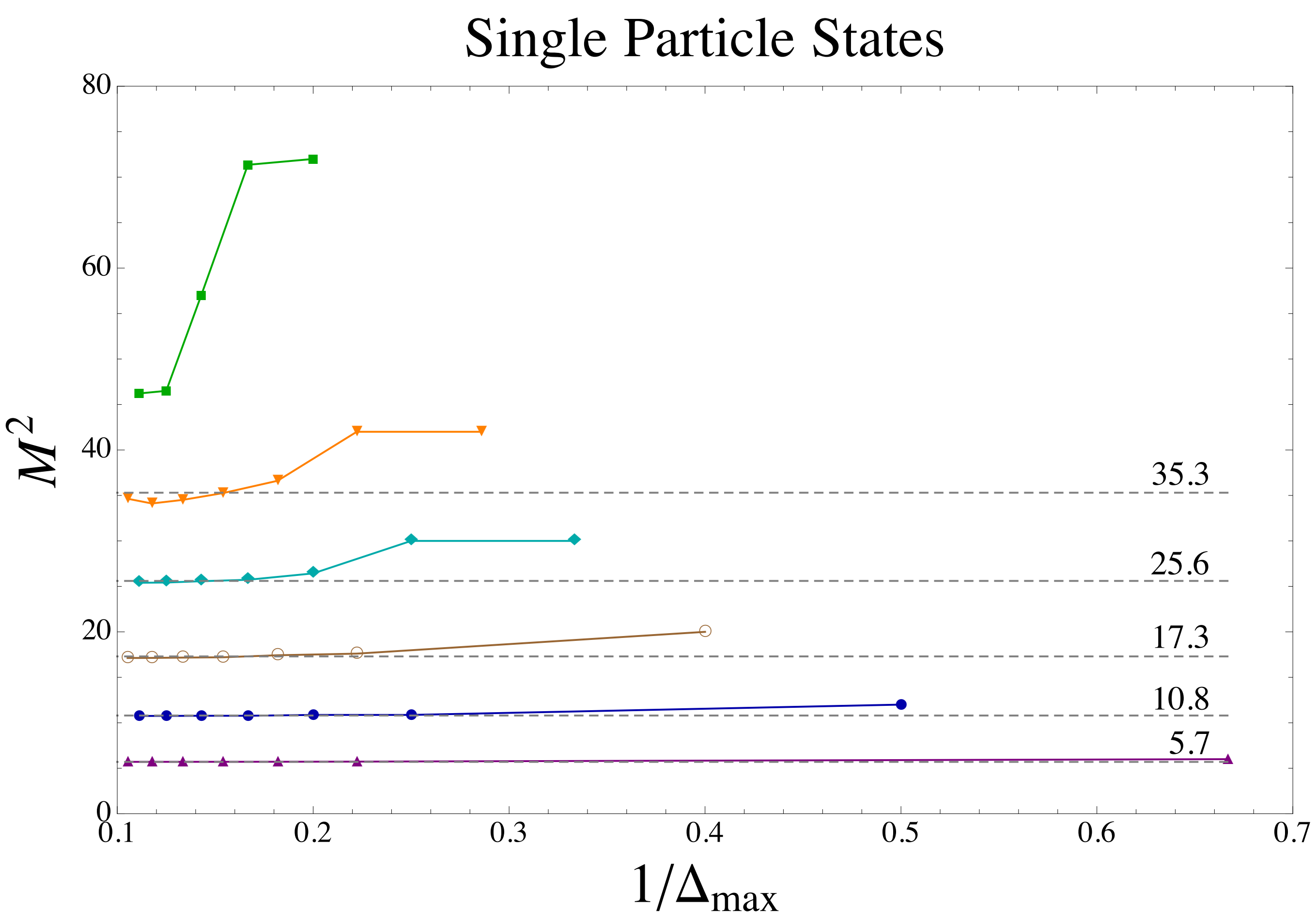
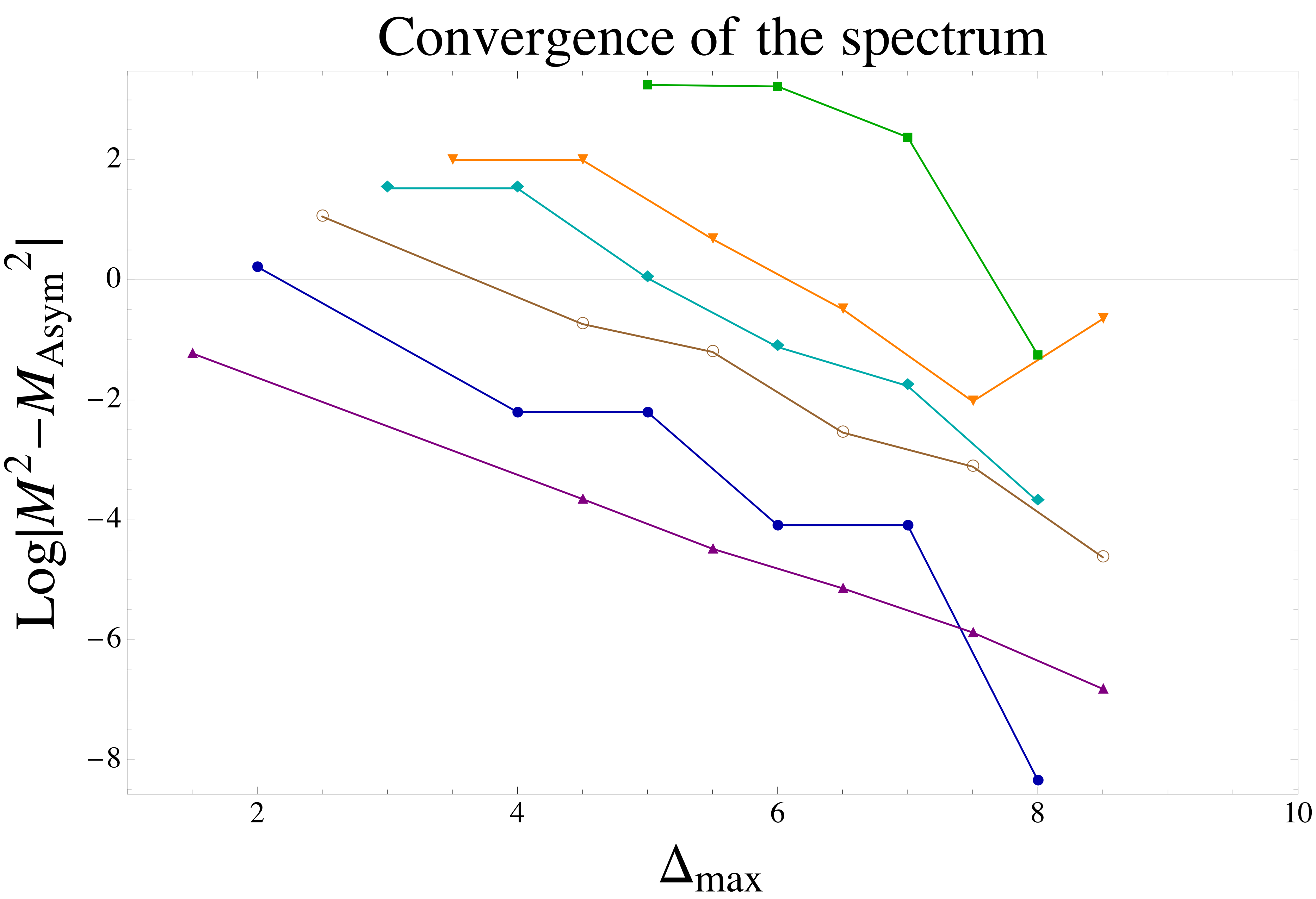


Exponential decoupling: $\delta M^2 \sim \exp(-\lambda' \Delta_{max})$?

Results - effective conformal dominance



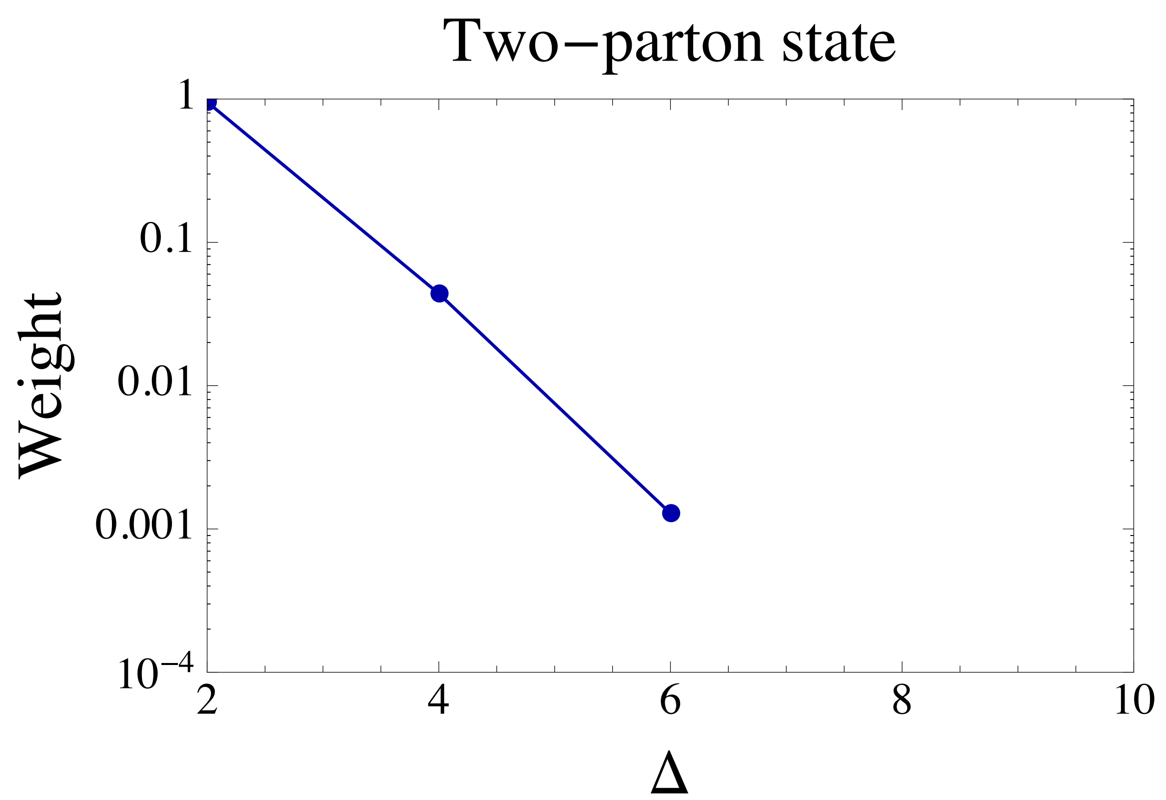
convergence: $M^2 - M_{asym}^2 = e^{-\Delta_{max}}$

perturbation in e^{-1} to power of Δ_{max}

Results - effective conformal dominance

$$|\Psi\rangle = c_2|\tilde{\mathcal{O}}_{\Delta=2}\rangle + c_3|\tilde{\mathcal{O}}_{\Delta=3}\rangle + c_4|\tilde{\mathcal{O}}_{\Delta=4}\rangle + ...$$

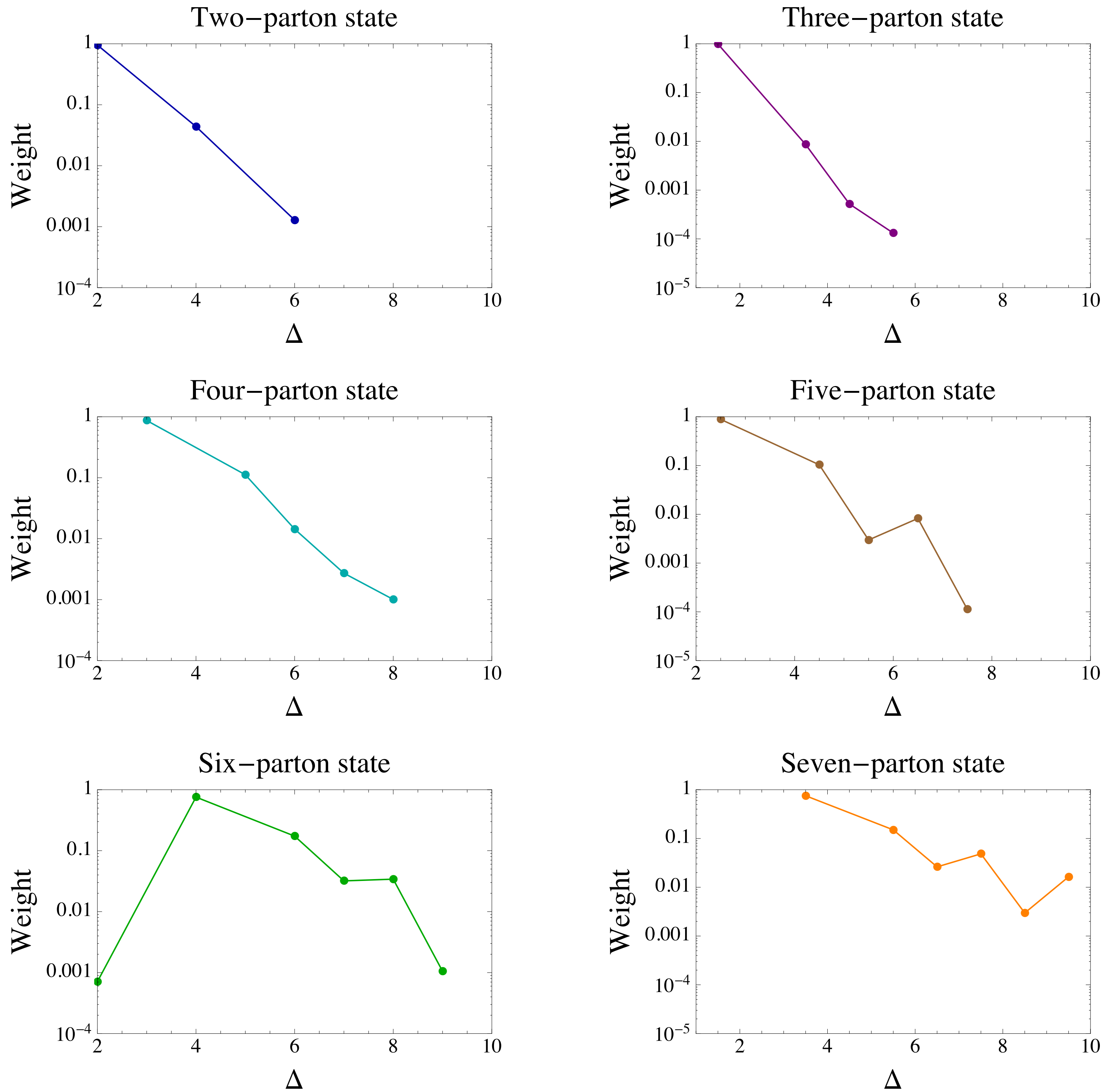
B1



$$\sum_i c_{\Delta,i}^2 \sim \exp(-\lambda\Delta)$$

Results - effective conformal dominance

$$\sum_i c_\Delta^2 \sim \exp(-\lambda\Delta)$$



B1 F1

B2 F2

B3 F3

Results

Analytic ground states parton wavefunction :

B1 $\mathrm{Tr}\left(\psi\overleftrightarrow{\partial_-}\psi\right)$

$$\begin{aligned}&\langle 2\text{ - parton}|M^2|2\text{ - parton}\rangle\\&=\frac{g^2N}{\pi}\int_0^1 dx_1dx_2\delta(x_1+x_2-1)\int_0^1 dy\frac{6\left((x_2-x_1)-(1-2y)\right)^2}{2(x_1-y)^2}\\&=12\times\frac{g^2N}{\pi}.\end{aligned}$$

F1 $\mathrm{Tr}\left(\psi\psi\psi\right)$

$$\begin{aligned}&\langle 3\text{ - parton}|M^2|3\text{ - parton}\rangle\\&=\frac{g^2N}{\pi}\int dx_1dx_2dx_3\delta\left(x_1+x_2+x_3-1\right)(\sqrt{6})^2\frac{1}{(x_1+x_2)^2}\int_0^{x_1+x_2}dy\\&=6\times\frac{g^2N}{\pi},\end{aligned}$$

The spectrum is approximated with the lowest operators to <15% accuracy.

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compared to $11.3\times\frac{g^2N}{\pi}$

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compared to $5.7\times\dfrac{g^2N}{\pi}$

The spectrum is approximated with the lowest operators to <15% accuracy.

Results - the continuum

Two-particle threshold

discovered by previous studies

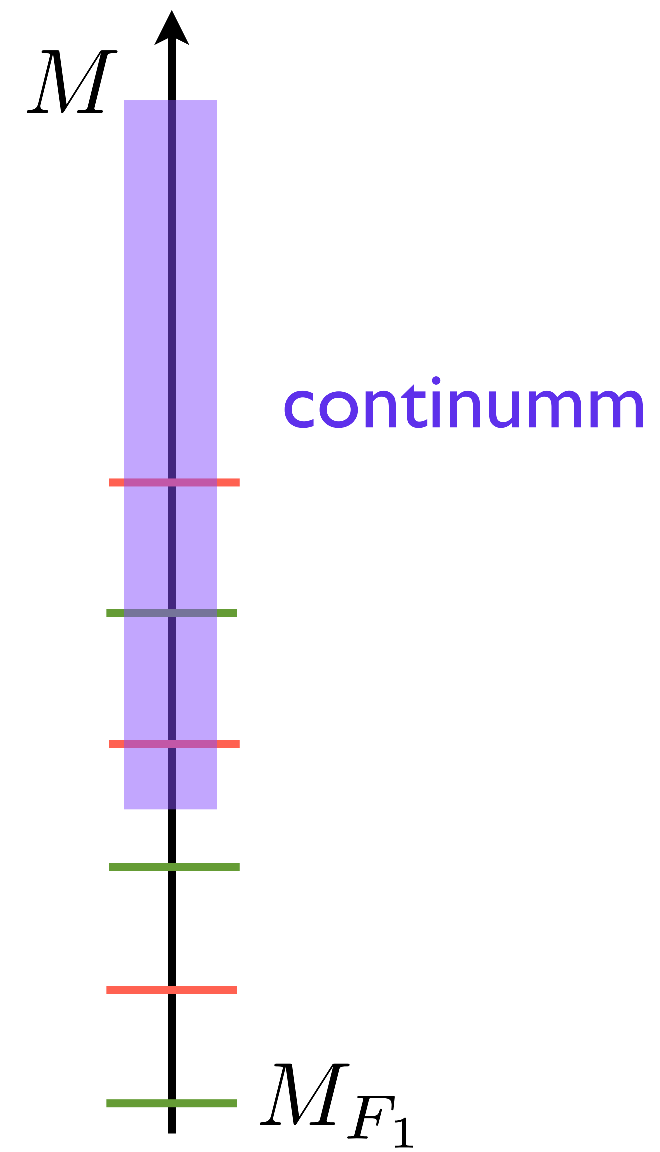
hep-th/9710240

hep-th/0110058

Two-free-particle states of single-particle

e.g. $|F_1\rangle \otimes |F_1\rangle$

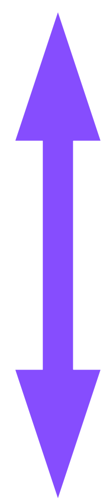
$|F_1\rangle \otimes |F_2\rangle$



Results - approximate the continuum

Using Δ_{max} as a matching parameter, we can approx. the continuum with the discrete spectrum in the truncated, finite Hilbert space.

Real theory - QCD w/ an adjoint fermion



Δ_{max}

Two-particle free theory

Results - the truncated free theory

Free two - particle state

$$M^2 = 2P^+P^-$$
$$= 2P^+\left(\frac{m_1^2}{2P_1^+} + \frac{m_2^2}{2P_2^+}\right)$$
$$= \frac{m_1^2}{x} + \frac{m_2^2}{1-x}$$

$$x \equiv \frac{P_1^+}{P^+}$$

Results - the truncated free theory

Free two - particle state

$$M^2 = 2P^+P^-$$
$$= 2P^+\left(\frac{m_1^2}{2P_1^+} + \frac{m_2^2}{2P_2^+}\right)$$
$$= \frac{m_1^2}{x} + \frac{m_2^2}{1-x}$$

$$x \equiv \frac{P_1^+}{P^+}$$

mass of the single particle states

Results - the truncated free theory

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mass of the single particle states

e.g. two-particle threshold, $|F_1\rangle \otimes |F_1\rangle, m_1^2 = m_2^2 = 5.7g^2N/\pi$

Results - the truncated free theory

Basis:

$$\mathcal{O}^{2-part}_{\Delta_n} = \begin{cases} \psi_1(x) P_n^{(0,0)}\left(\overleftarrow{\partial} - \overrightarrow{\partial}\right)\psi_2(x)\,, & \text{for 2 fermions,} \\ \partial\phi(x) P_n^{(1,0)}\left(\overleftarrow{\partial} - \overrightarrow{\partial}\right)\psi(x)\,, & \text{for a boson and a fermion} \\ \partial\phi_1(x) P_n^{(1,1)}\left(\overleftarrow{\partial} - \overrightarrow{\partial}\right)\partial\phi_2(x)\,, & \text{for 2 bosons.} \end{cases}$$

Mass matrix:

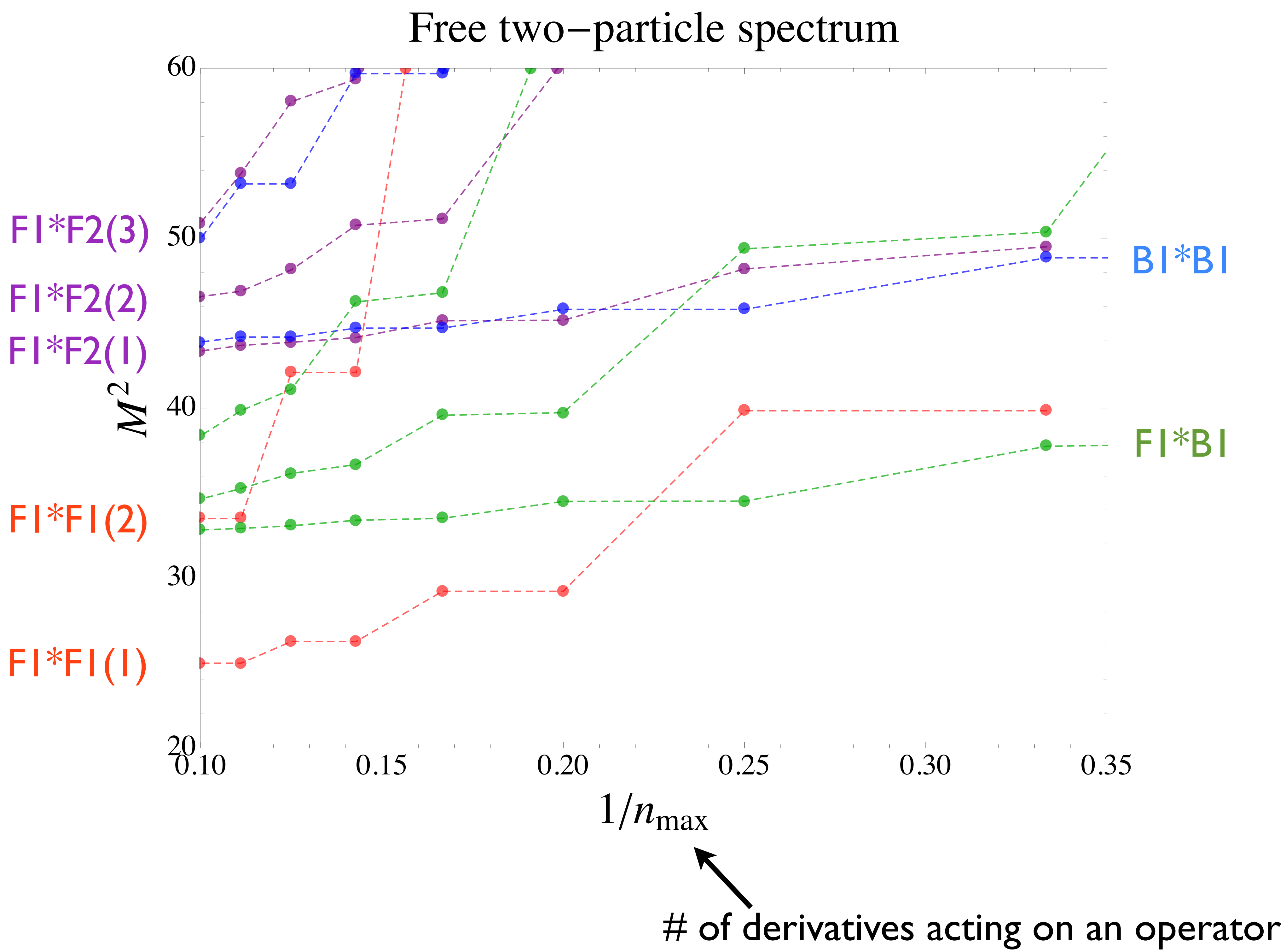
$$\delta(P-P^{'})M^2_{i,j} = \int dxdye^{iPx-iP^{'}y}\langle\mathcal{O}_i(x)|M^2|\mathcal{O}_j(y)\rangle$$

$$\phi_\Delta(x) \equiv \langle x, 1-x|\tilde{\mathcal{O}}^{2-part}_\Delta|\Omega\rangle$$

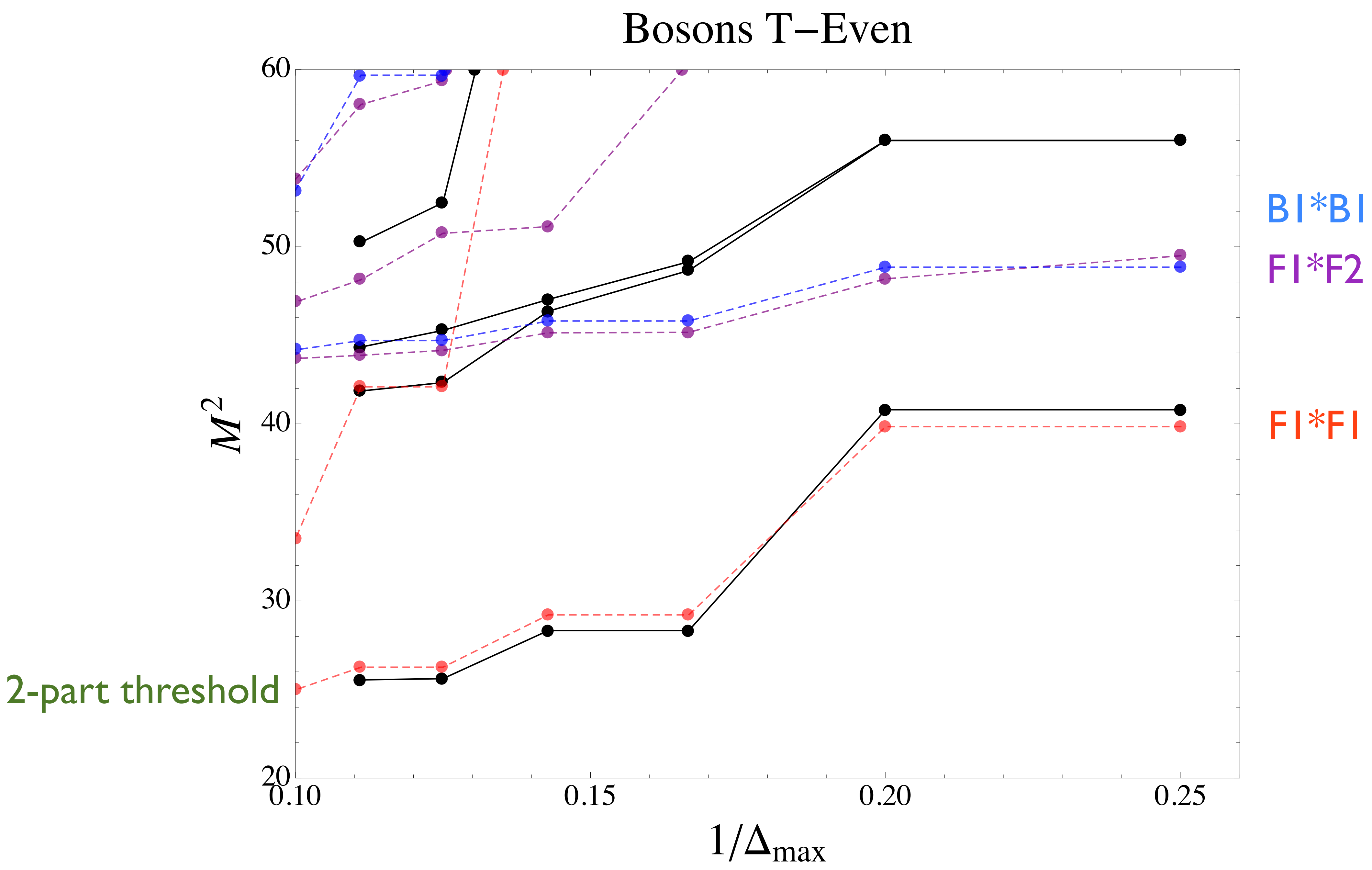
$$[M^2_{2part}]_{\Delta,\Delta'} = \int_0^1 dx\phi^*_\Delta(x)\left(\frac{m_1^2}{x}+\frac{m_2^2}{1-x}\right)\phi_{\Delta'}(x)$$

topological sector (fermions) - change op. dim by 1/2

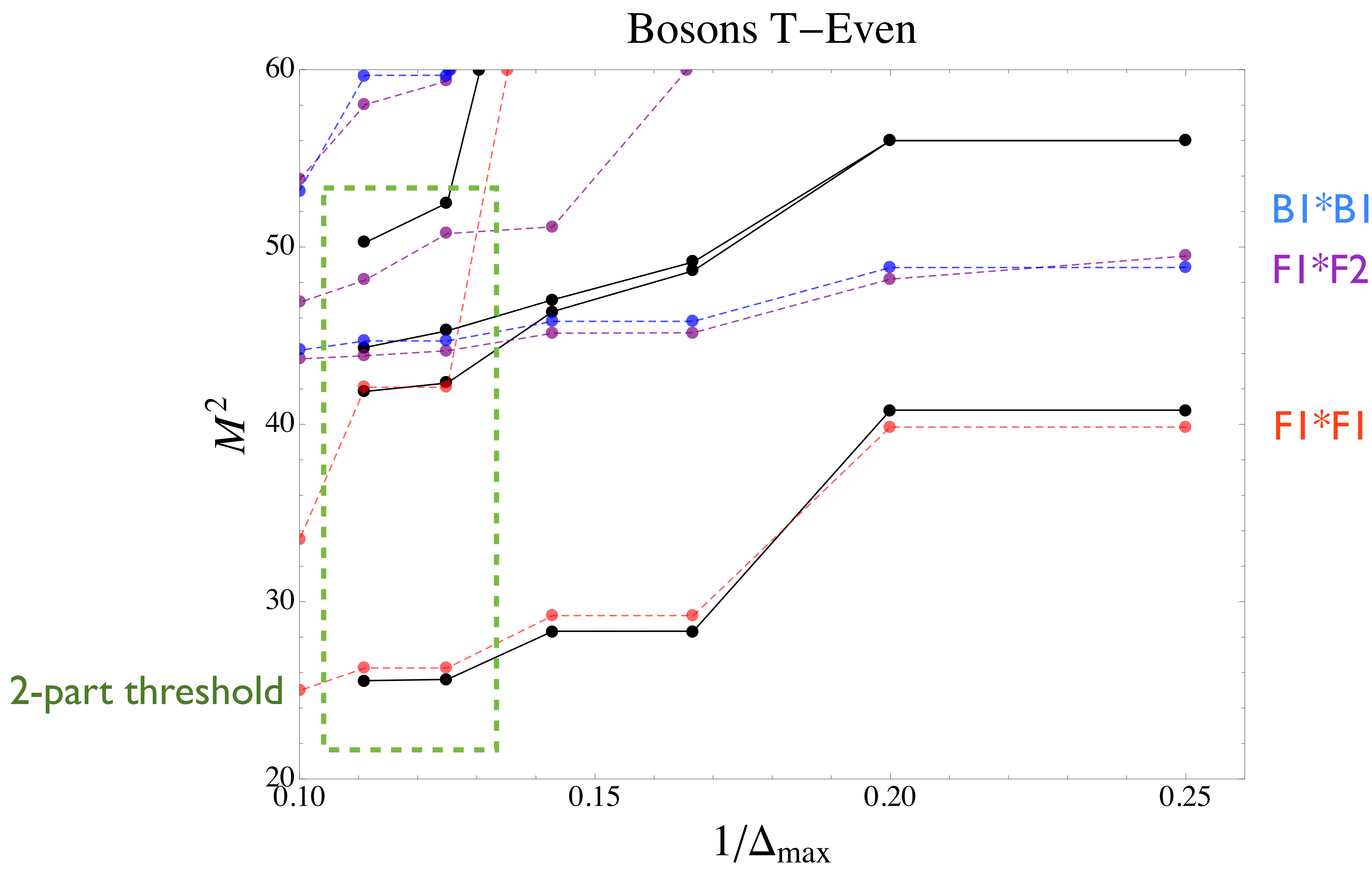
Results - the truncated free theory



Results - the real theory

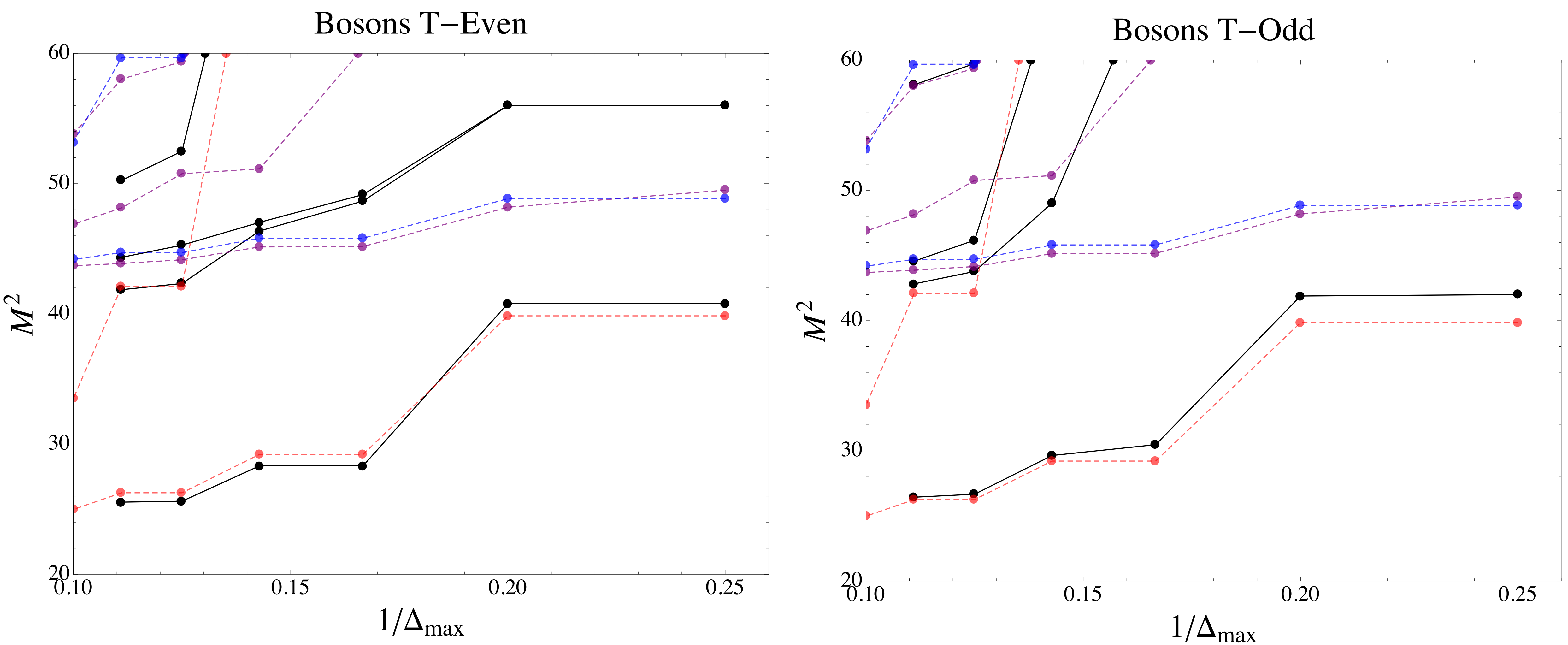


Results - the real theory



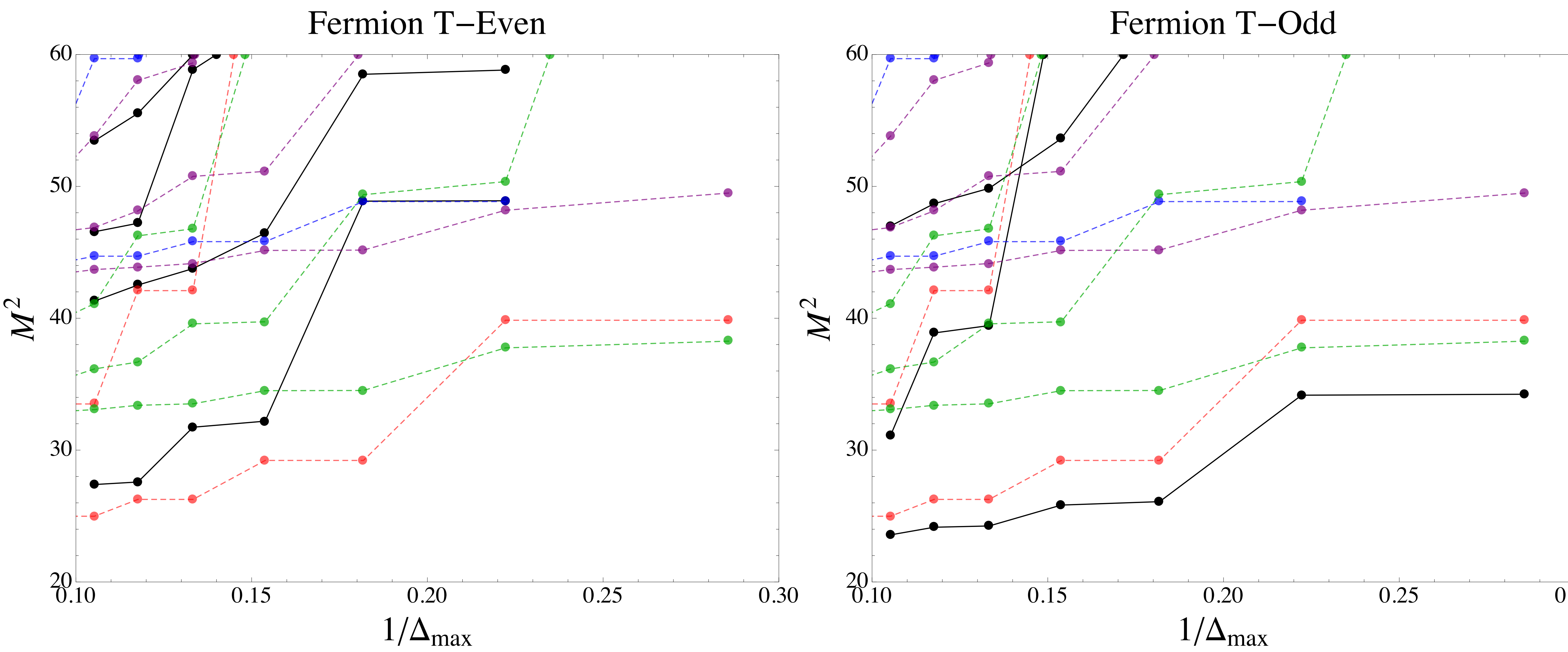
further evidence of effective conformal dominance

Results - bosonic sector



F1*F1 F1*F2 B1*B1

Results - fermionic sector



F1*F1 F1*F2 B1*B1 F1*B1

no evidence of bosonic bound states?

Conclusions

- Decoupling leads to a basis of quasi-primaries, with which the spectrum converges as $e^{-\Delta_{max}}$.
- Our method agrees with DLCQ method, with a much smaller basis.
- Effective conformal dominance suggests an expansion parameter $e^{-\Delta_{max}}$.
- Analytic parton wavefunctions.
- Test this method in other strongly interacting systems.
- Finite N.

Conclusions

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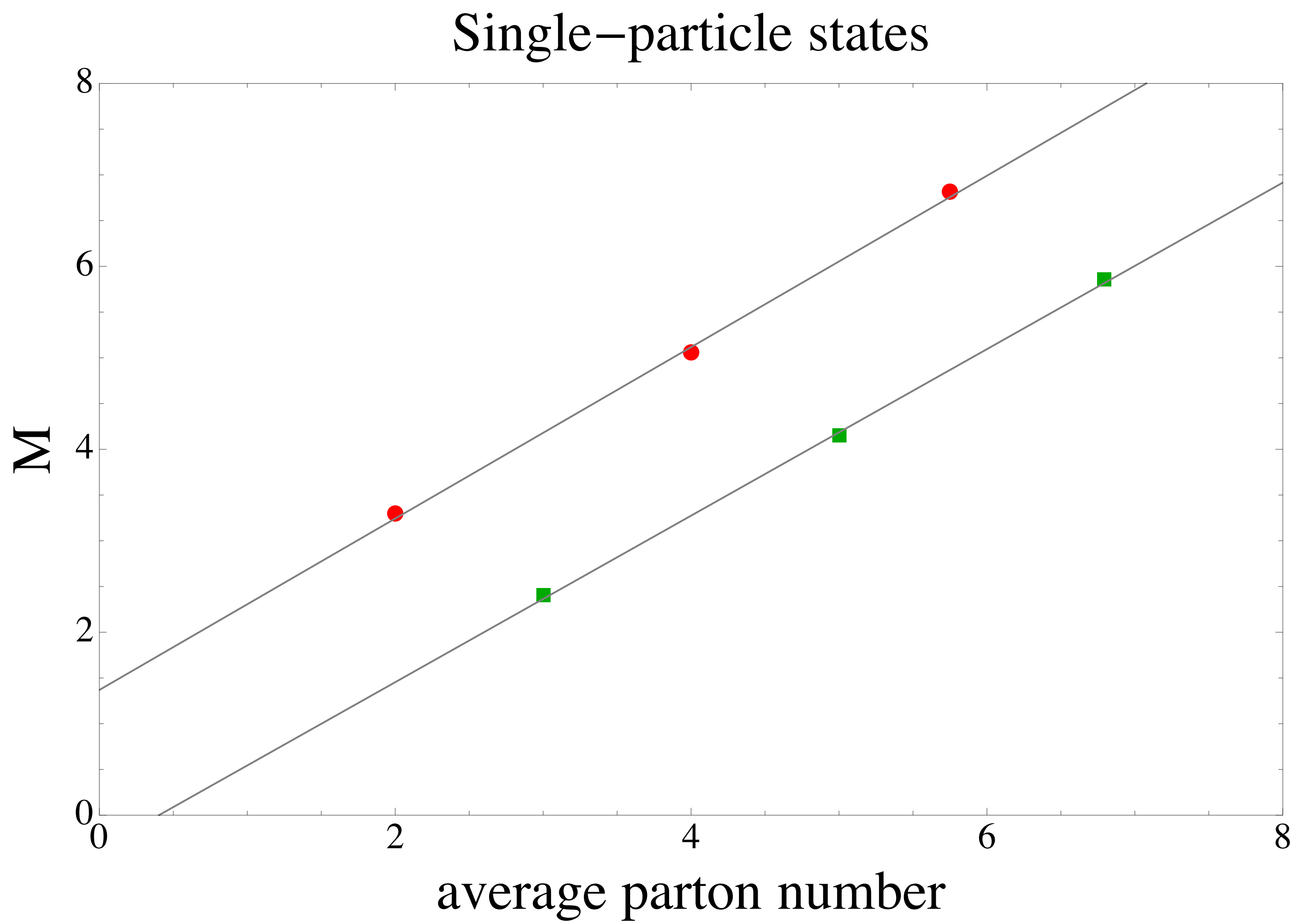
Thank you!

Backup slides

2d QCD with an adjoint fermion



red: bosons

green: fermions

Linear spectrum, not Regge.

Also contains a continuum, multi-particle states

The basis

Write the momenta on a simplex in angle variables:

$$\begin{aligned} p_k &= P\cos^2\theta_1, \\ p_{k-1} &= P\sin^2\theta_1\cos^2\theta_2, \\ &\dots \\ p_2 &= P\sin^2\theta_1\sin^2\theta_2 \dots \cos^2\theta_{k-1}, \\ p_1 &= P\sin^2\theta_1\sin^2\theta_2 \dots \sin^2\theta_{k-1}. \end{aligned}$$

Solutions to the Killing equation:

$$\begin{aligned} & f_{n,l_1,l_2,\dots,l_{k-2}}(P,\theta_1,\theta_2\dots,\theta_{k-1}) \\ & = P^n \sin^{2l_1}\theta_1 \sin^{2l_2}\theta_2 \dots \sin^{2l_{k-2}}\theta_{k-2} \\ & \times P^{(2l_1+k-2,0)}_{n-l_1}(\cos 2\theta_1)\, P^{(2l_2+k-3,0)}_{l_1-l_2}(\cos 2\theta_2) \dots P^{(2l_{k-2}+1,0)}_{l_{k-3}-l_{k-2}}(\cos 2\theta_{k-2})\, P_{l_{k-2}}(\cos 2\theta_{k-1}) \end{aligned}$$

Cyclicity and symmetries $\longrightarrow$ coefficients of $f_{n,l_1,l_2,\dots,l_{k-2}}$

T sym $\longrightarrow$ $\psi_{ij} \to \psi_{ji}$