Wilsonian and Large N approaches to Non-Fermi Liquids

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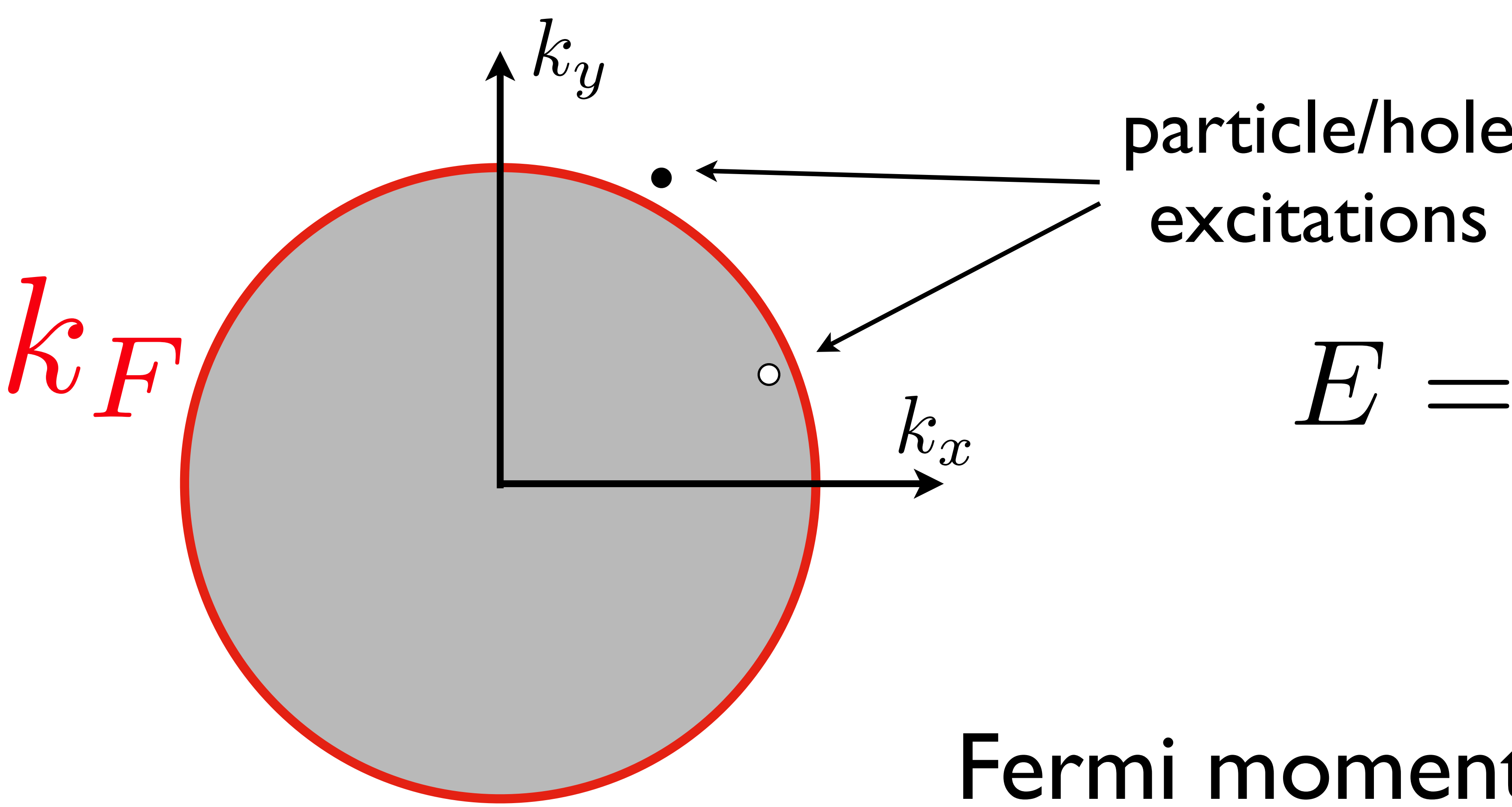
Introduction to Fermi Liquids

Fermions at finite density have a Fermi surface



Fermi energy

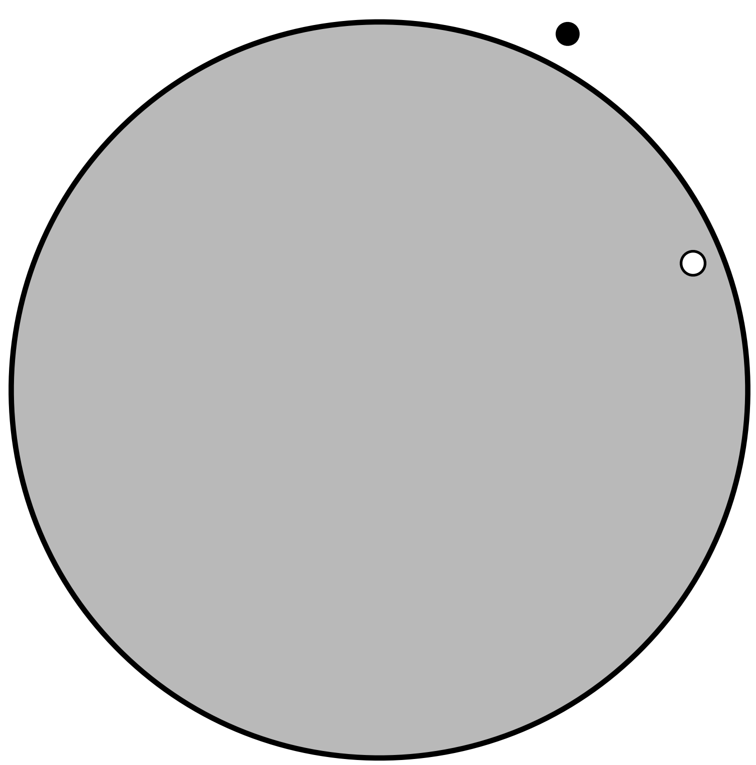
$$E = \frac{k^2}{2m} - \epsilon_F$$

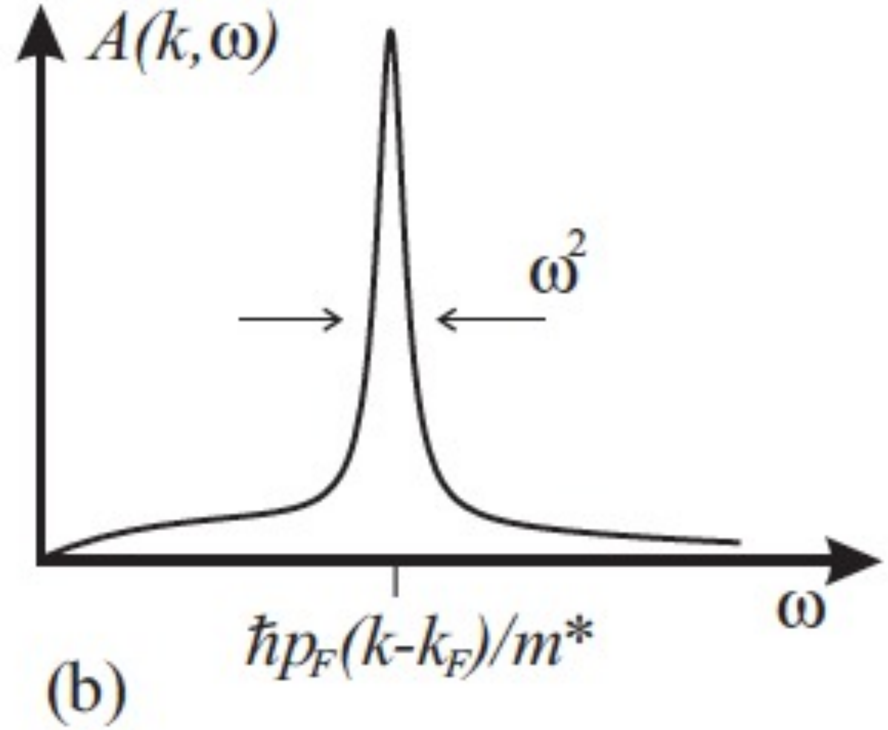
Fermi momentum: $\dfrac{k_F^2}{2m} = \epsilon_F$

Landau Fermi Liquids

In simple metals, excitations are weakly coupled quasi-particles

$$m^* \neq m$$





$$\frac{1}{\tau} = \mathrm{Im}(\Sigma) \sim \frac{\omega^2}{k_F}$$

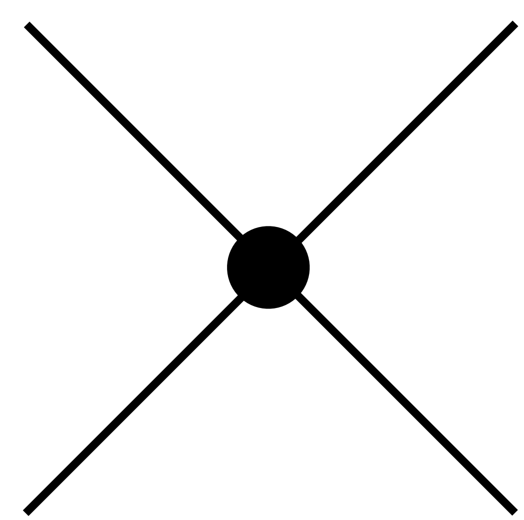
Landau Fermi Liquids

Why are emergent quasiparticles well-described by weak coupling?

Modern EFT description:
(almost) all interactions are irrelevant

Shankar
Polchinski

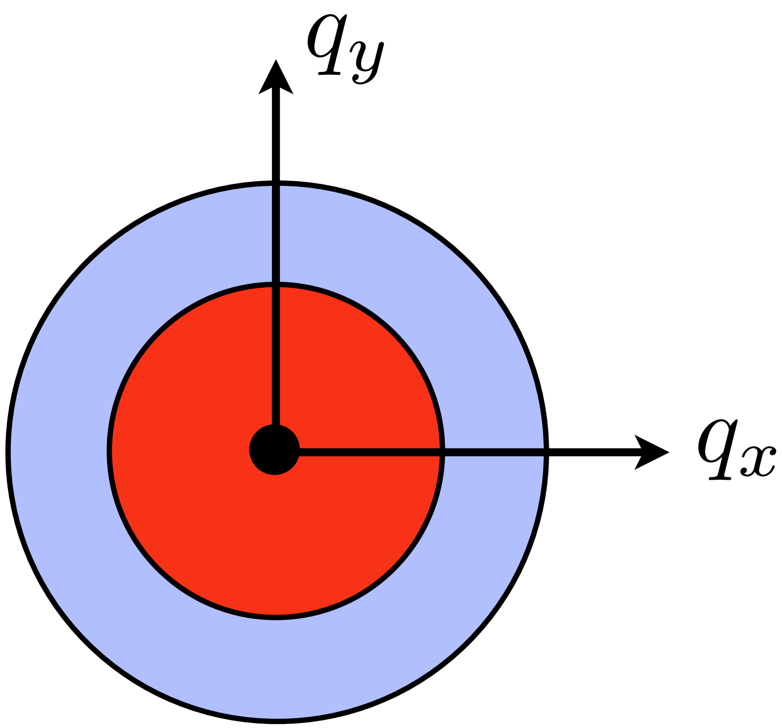
$$\frac{\psi^\dagger \psi \psi^\dagger \psi}{\Lambda}$$



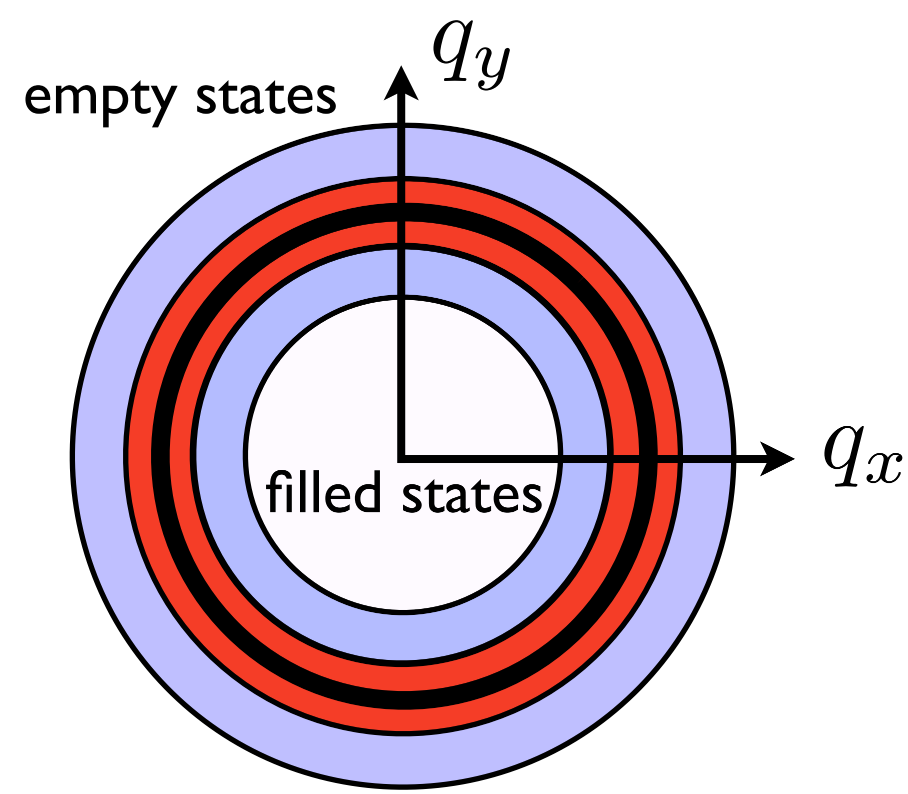
Landau Fermi Liquids

Scaling:

Standard:



Fermi Surface:



Fix angle and scale toward nearest point on Fermi surface:

$$\vec{q} = \hat{\theta}(k_F + \ell)$$

$$\omega \to e^{\lambda}\omega \qquad \ell \to e^{\lambda}\ell$$

Landau Fermi Liquids

$$S_2 = \int dS^{d-1} \left[\int d\omega d\ell \psi^\dagger (\omega - v_F \ell) \psi \right]$$

$$\ell \equiv |k| - k_F$$

$$\omega \to e^\lambda \omega$$

$$\ell \to e^\lambda \ell$$

So we see that the fermions should scale as

$$\psi \to e^{-\frac{3}{2}\lambda} \psi$$

Landau Fermi Liquids

First interaction is four-fermion interaction

$$S_4 = \int d^{d-1}S_1 d\omega_1 d\ell_1 \ldots d^{d-1}S_4 d\omega_4 d\ell_4 \, \delta(\omega_1 + \omega_2 + \omega_e + \omega_4)$$

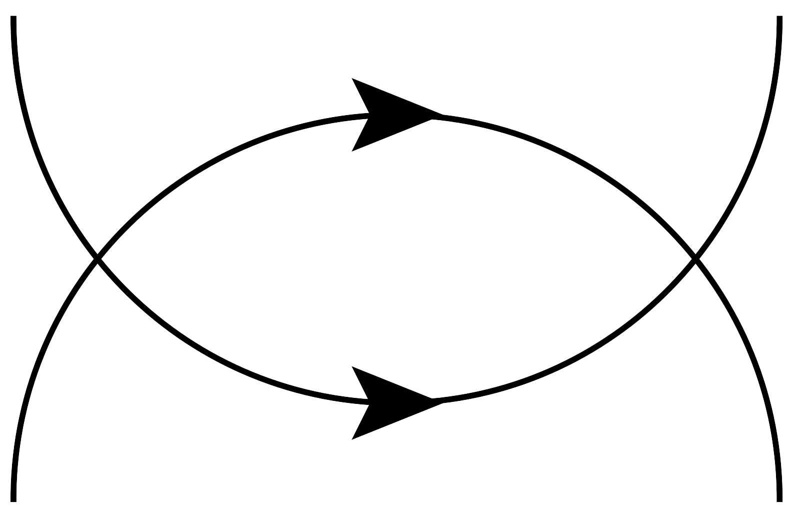
$$V(\theta_i)\psi_1^\dagger \psi_2^\dagger \psi_3 \psi_4 \;\; \delta^d(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)$$

It naively scales like e^λ and is irrelevant

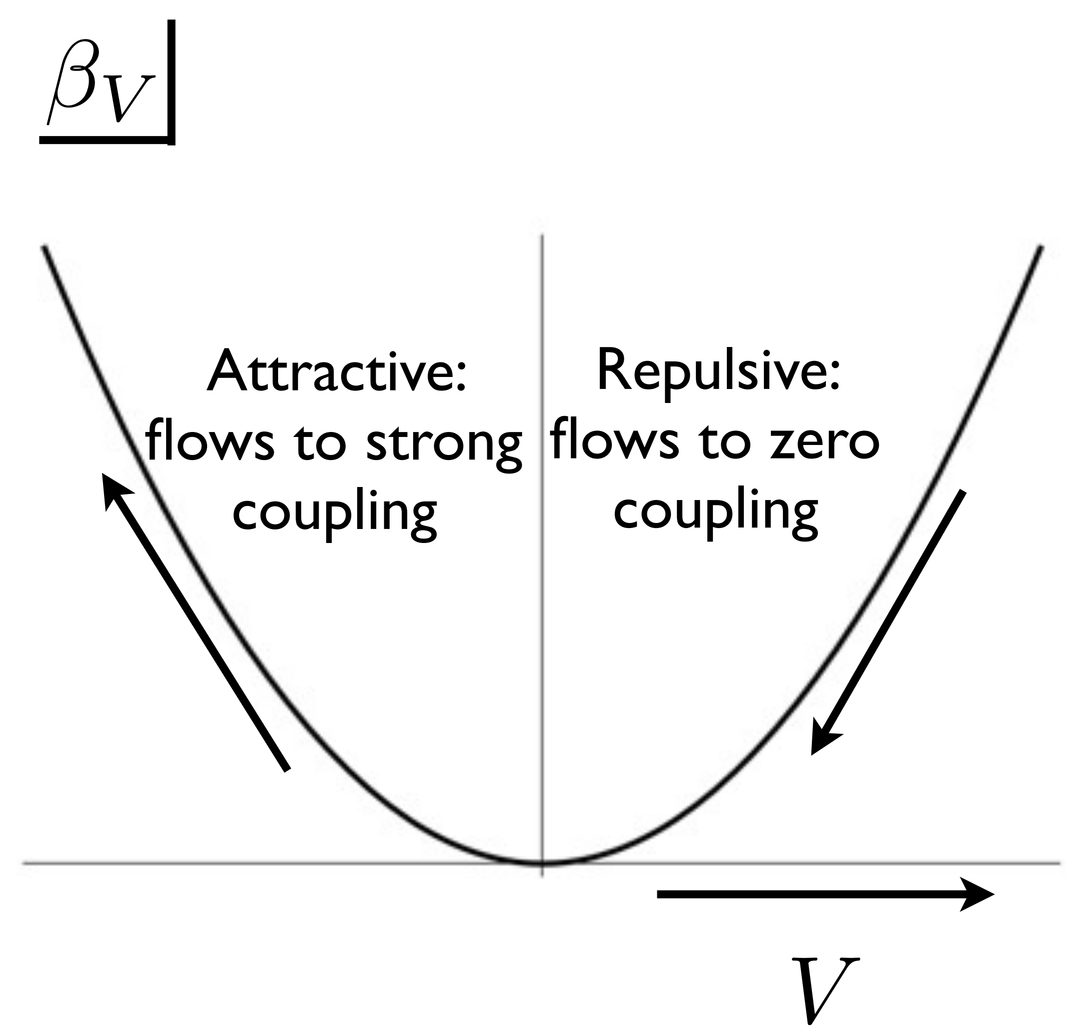
But for certain kinematic configurations, the delta function scales like $e^{-\lambda}$ and the interaction becomes *marginal*

Landau Fermi Liquids

BCS instability:
At one-loop, the interaction between antipodal points runs and becomes marginally relevant/irrelevant



$$\frac{dV}{d\log\mu} = V^2$$



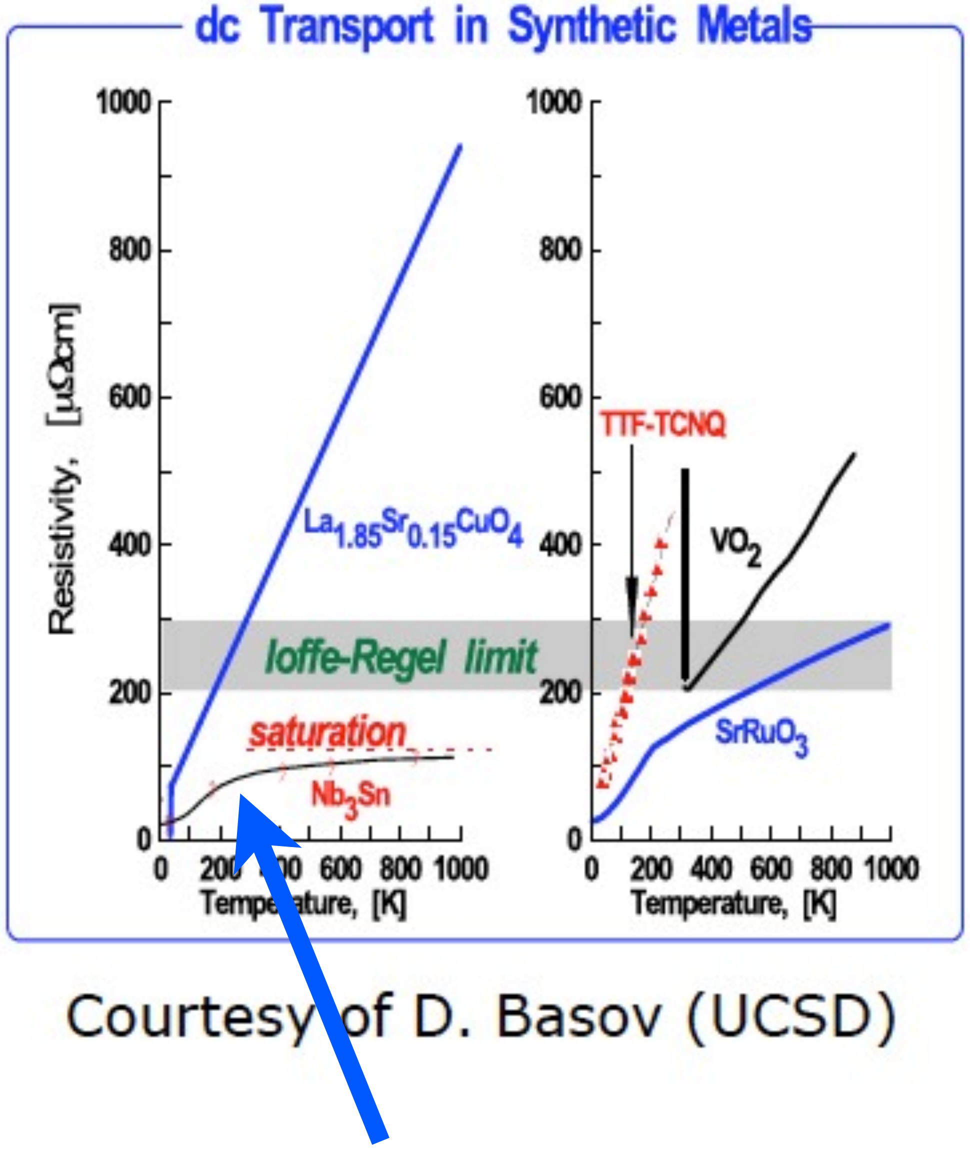
Landau Fermi Liquids

(Mott-)Ioffe-Regel Resistivity Limit

Drude Model based on quasi-particle transport:

$$\rho \sim \frac{m}{ne^2}\frac{1}{\tau}$$

If $\rho \gg \rho_{\mathrm{MIR}}$ then mean free path is shorter than wavelength, and quasi-particle description wouldn't make sense



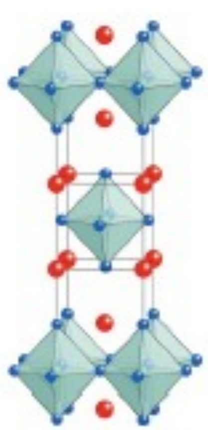
Courtesy of D. Basov (UCSD)

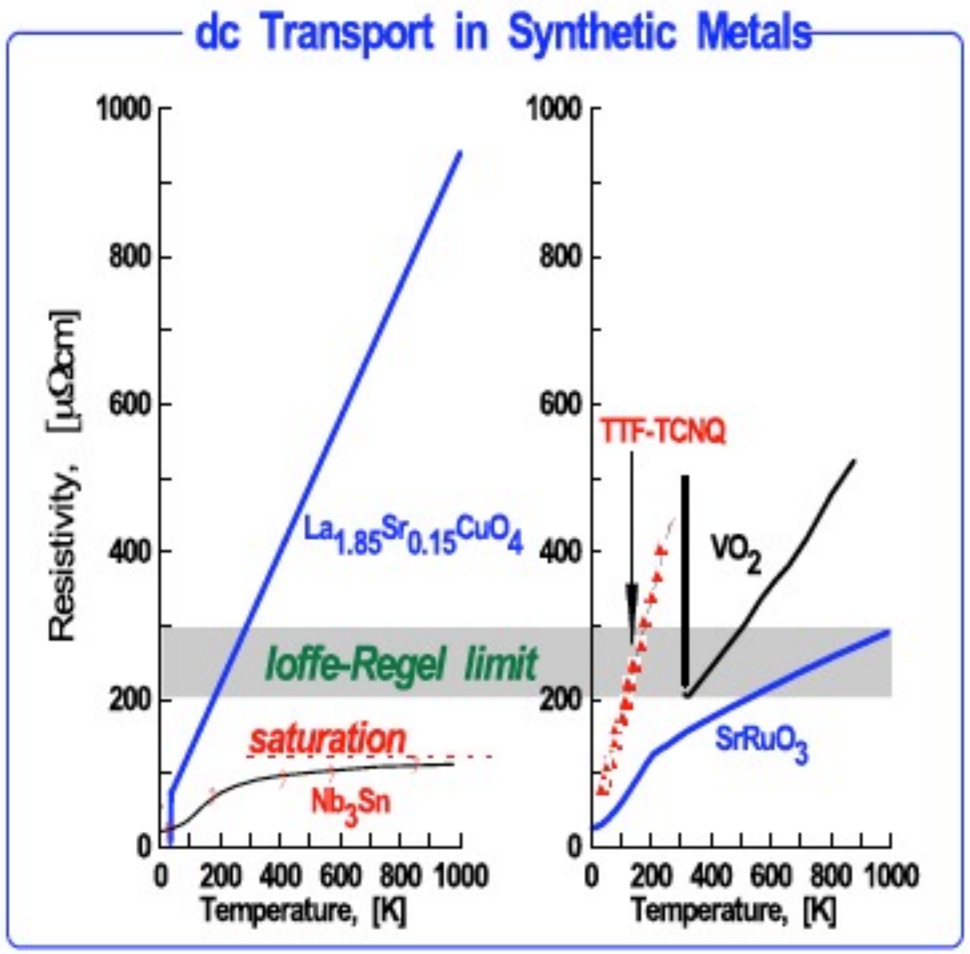
"Good" metal doesn't exceed bound and is typically significantly below bound at moderate T

Non-Fermi Liquids

Landau fermi liquid theory breaks down in examples with T-linear resistivity above Ioffe-Regel limit

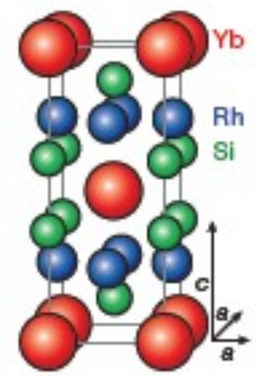
$La_{2-x}Sr_xCuO_4$

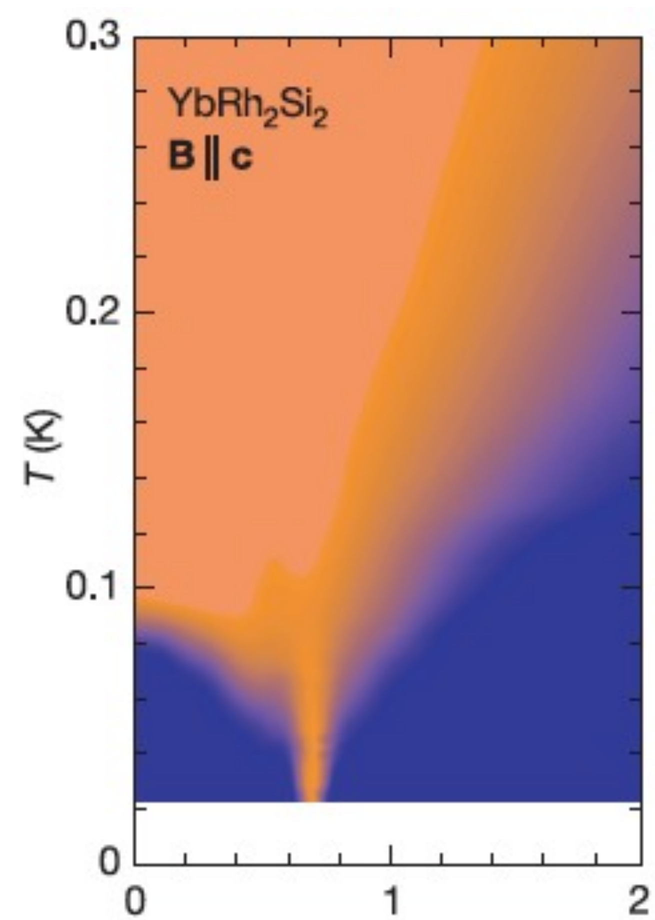




Courtesy of D. Basov (UCSD)

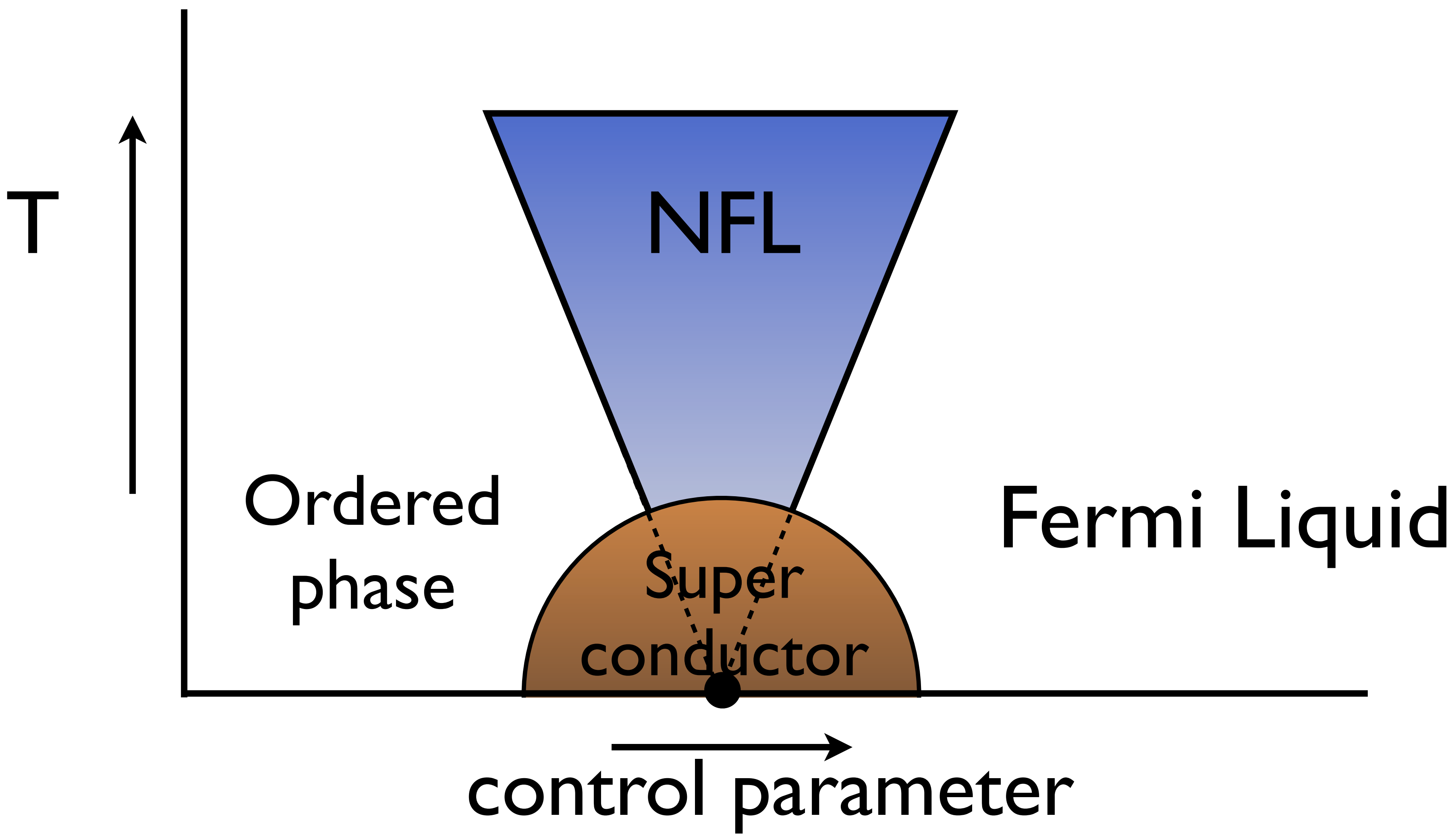
$YbRh_2Si_2$





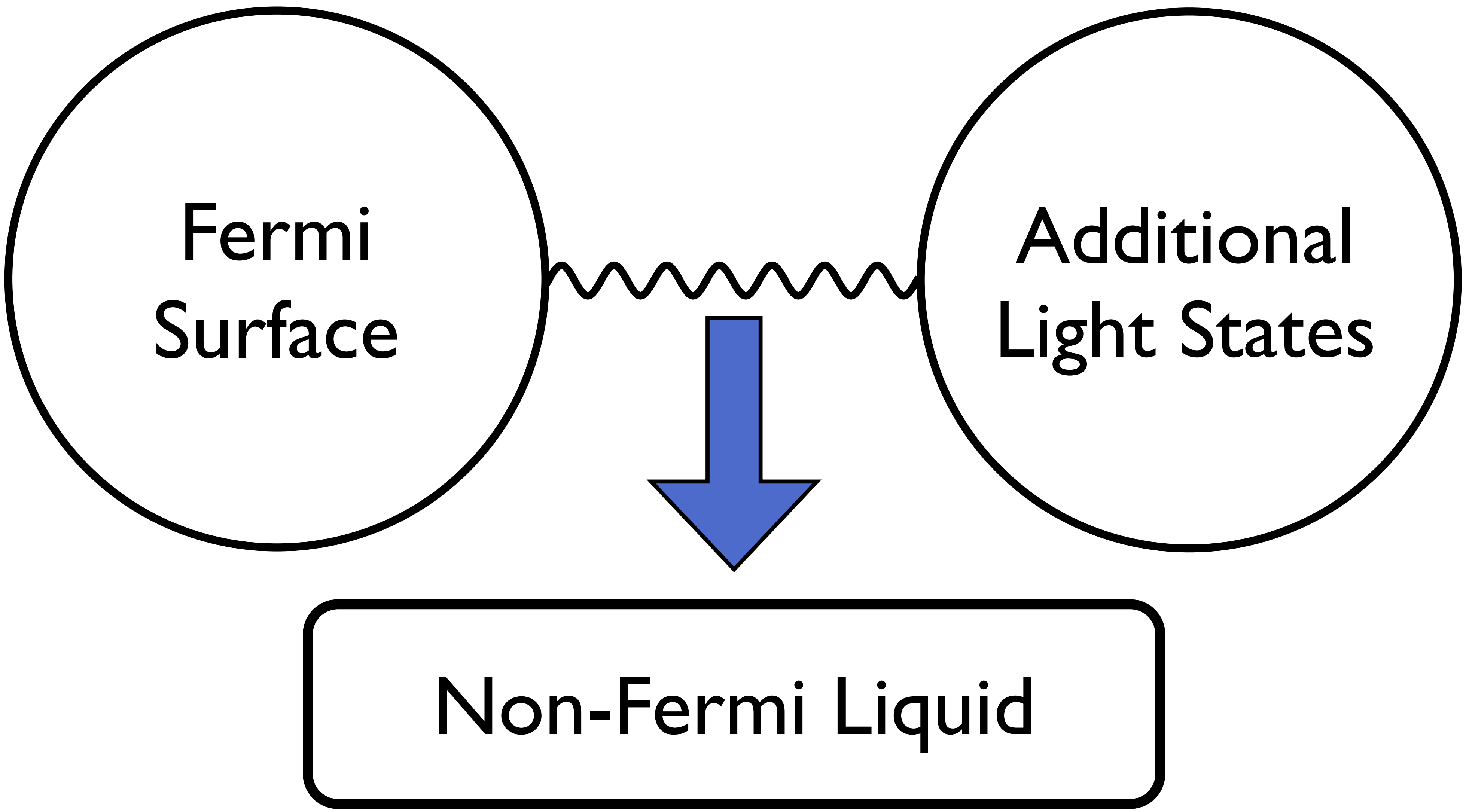
Quantum Critical Points

One Class of Non-fermi liquids Arises
Near Quantum Phase Transitions
Phase transition at zero temp



EFTs of Non-Fermi Liquids

Goal: Couple Fermi surface to new massless degrees of freedom to get interesting IR



EFTs of Non-Fermi Liquids

Wilsonian approach: start with *local* action in UV and integrate out high energy modes

We will not add *by hand* any terms like

$$(\psi^\dagger\psi)k^{2-x}(\psi^\dagger\psi) \quad \text{or} \quad \phi\frac{|\omega|}{|\ell|}\phi$$

EFTs of Non-Fermi Liquids

As a high energy physicist, I will take some lessons from the study of QCD:

1) It was hard to see *a priori* what QFTs (if any!) could explain deep inelastic scattering

The classification and study of local QFTs was wildly successful

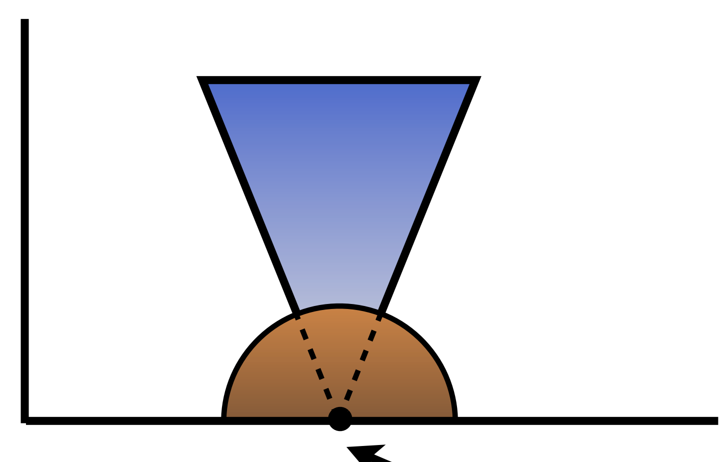
2) Confinement especially was hard to tackle directly, and simplifying special cases (2d, large N, SUSY) played a crucial role in our qualitative understanding

Quantum Critical Points

Additional Light States = Massless scalar field ϕ

Think of ϕ as order parameter, tuned to be massless at the QCP

$$S_\phi = \phi(\omega^2 - c_s^2 q^2)\phi + \lambda_\phi \phi^4$$

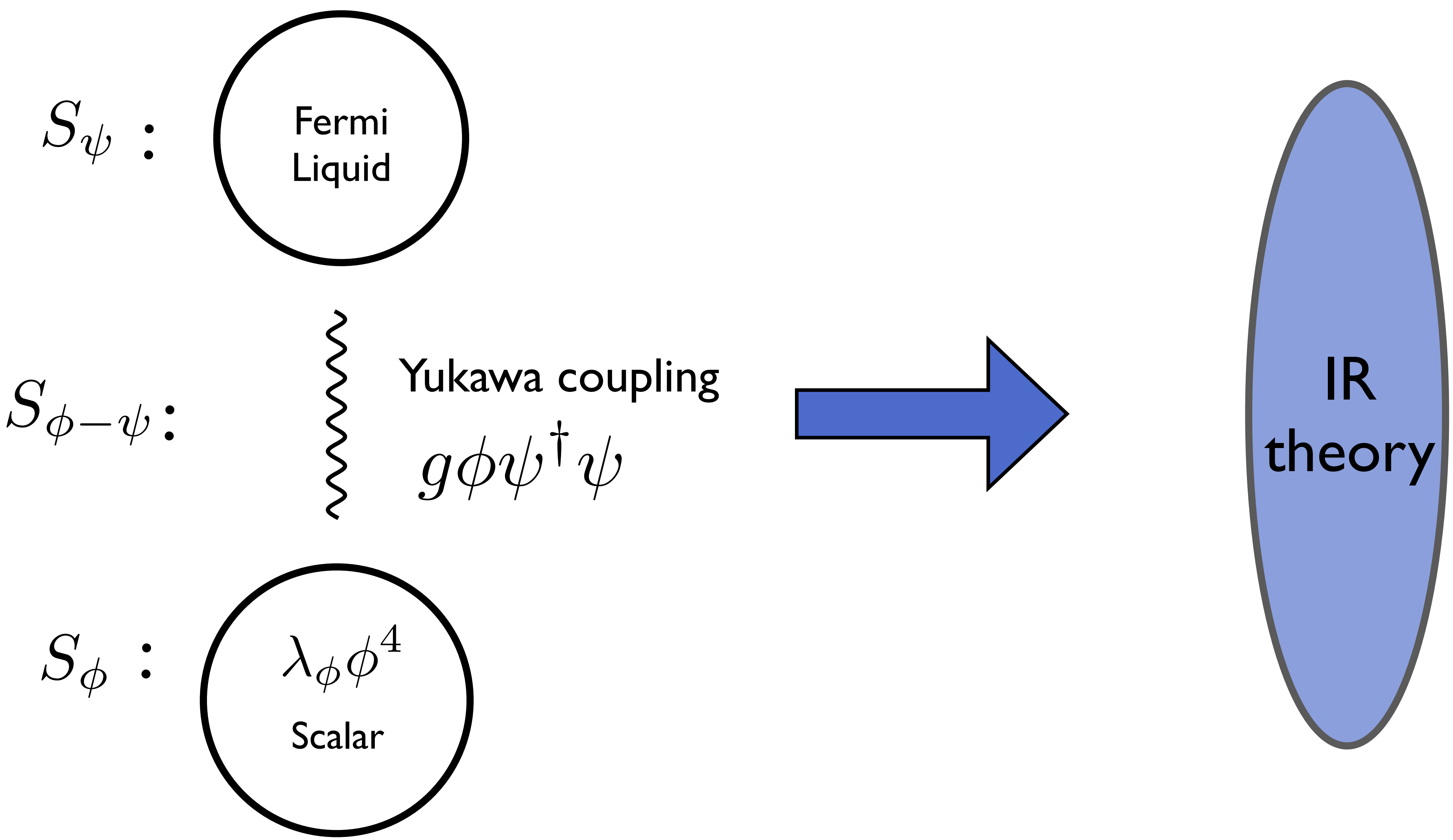


$m_\phi^2 = 0$

(For example $\phi \sim$ magnetization M_z in a ferromagnet)

Quantum Critical Points

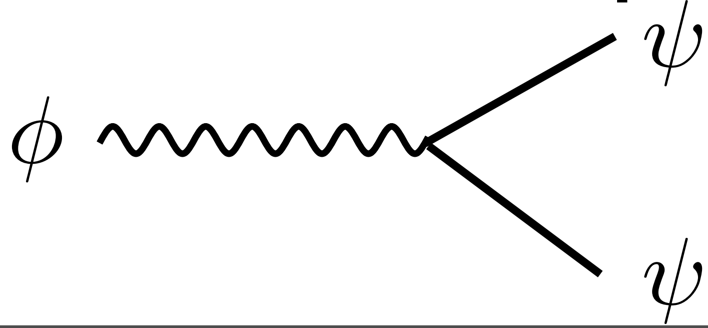
$$S = S_\psi + S_\phi + S_{\phi-\psi}$$



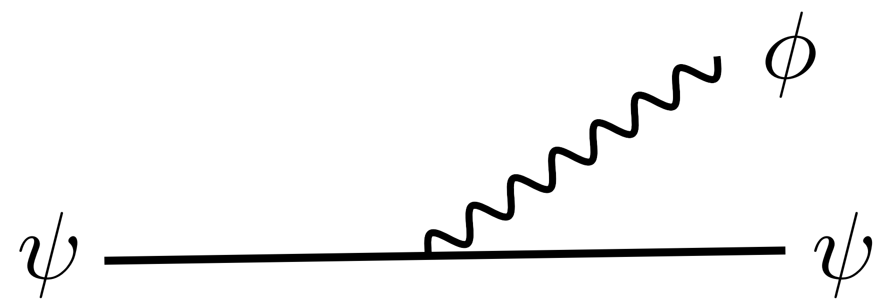
Tug-of-War

Fermions renormalize bosons and vice versa
Who wins?

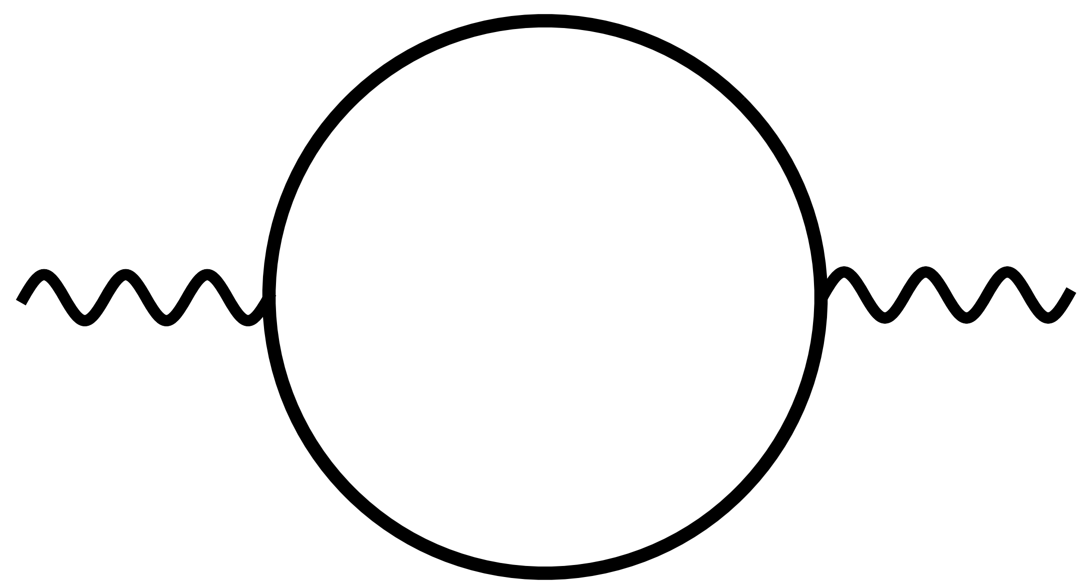
Bosons can decay to particle/hole pairs: "Landau damping"



Fermions can decay: Non-Fermi Liquid



Landau Damping



One-loop boson self-energy has non-analytic term

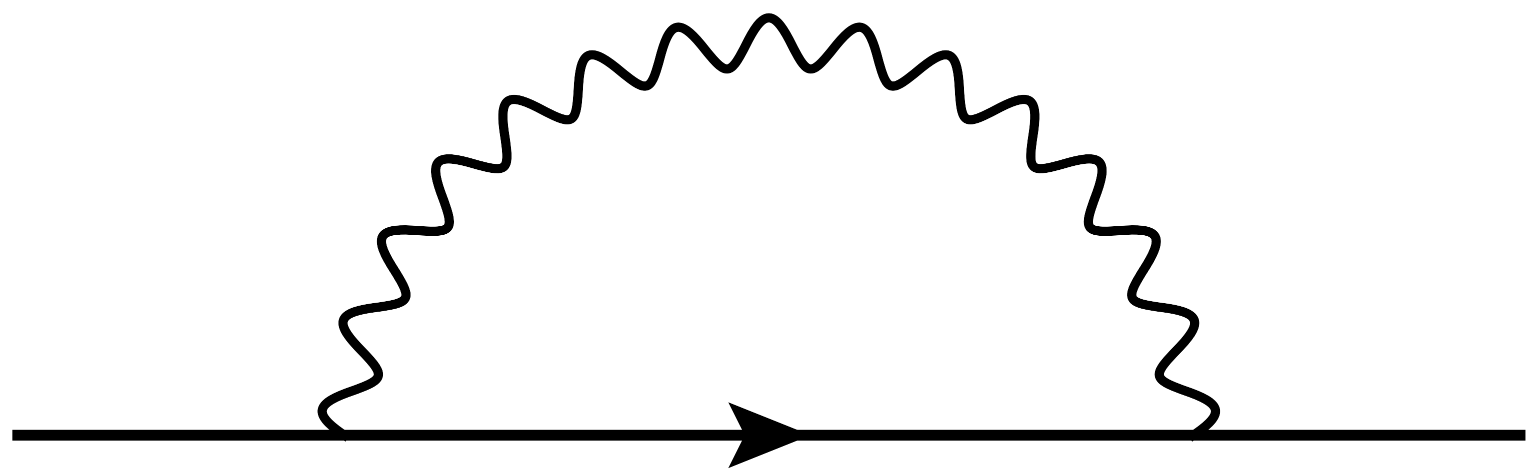
$$\Pi(q_0, q) \sim g^2 \frac{m^2 v}{2\pi} \frac{|q_0|}{\sqrt{q_0^2 + v^2 q^2}}$$

Strong coupling at IR scale:

$$\Pi(q_0, q) > q_0^2$$

at $q_0^2 \lesssim \omega_{\mathrm{LD}}^2 \equiv \frac{g^2 m^2}{2\pi}$

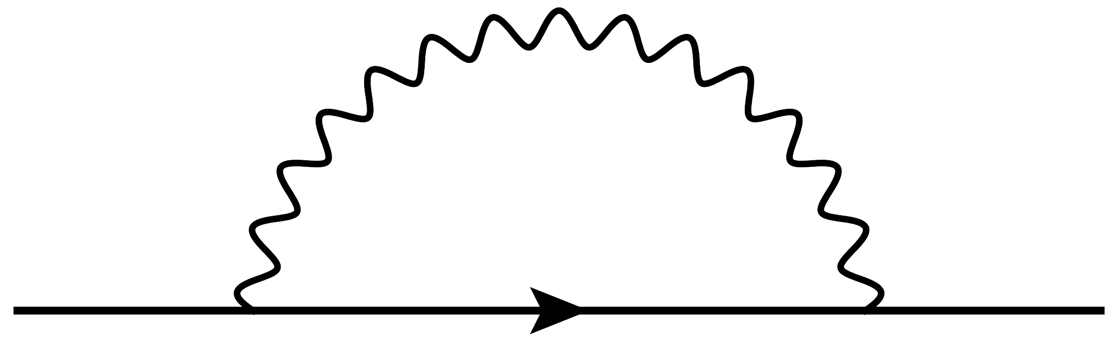
Anomalous Dimension



Wavefunction
renormalization

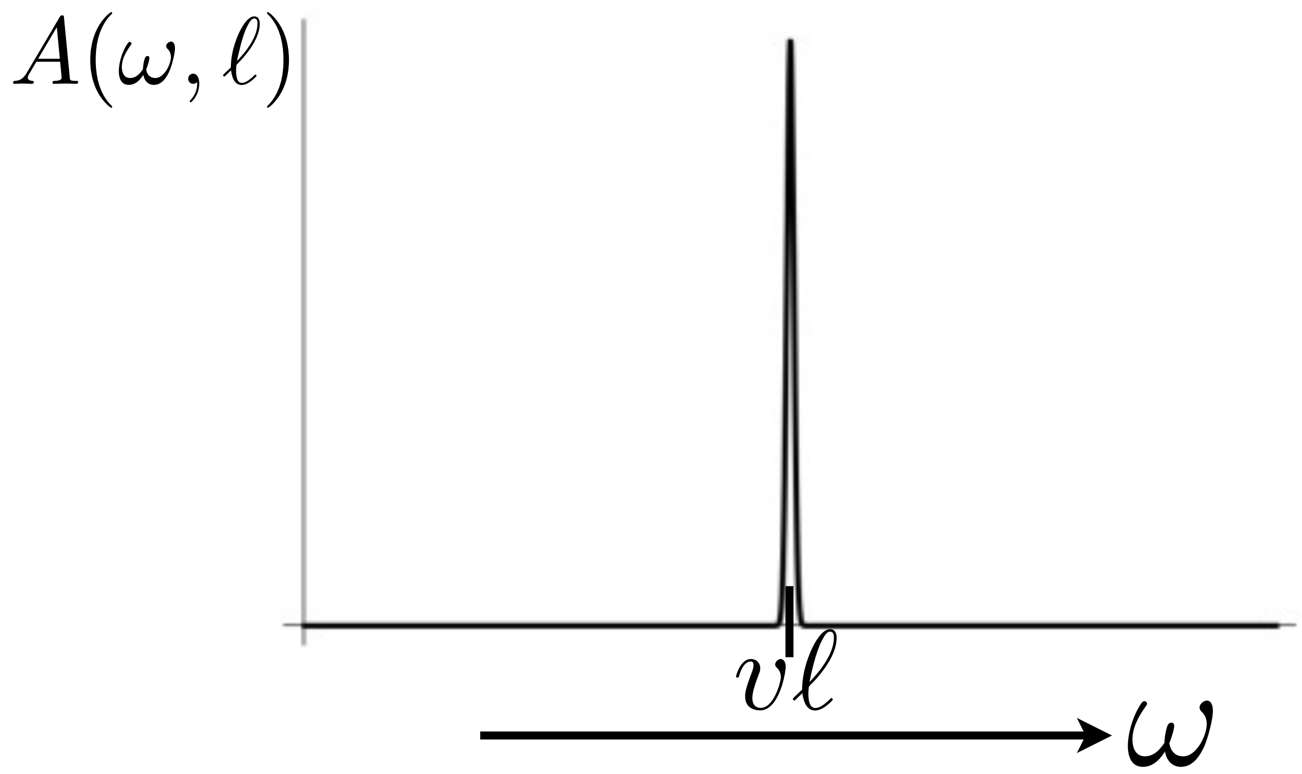
Anomalous dimension: $$2\gamma = -\frac{d\delta Z}{d\log\Lambda}$$

Anomalous Dimension

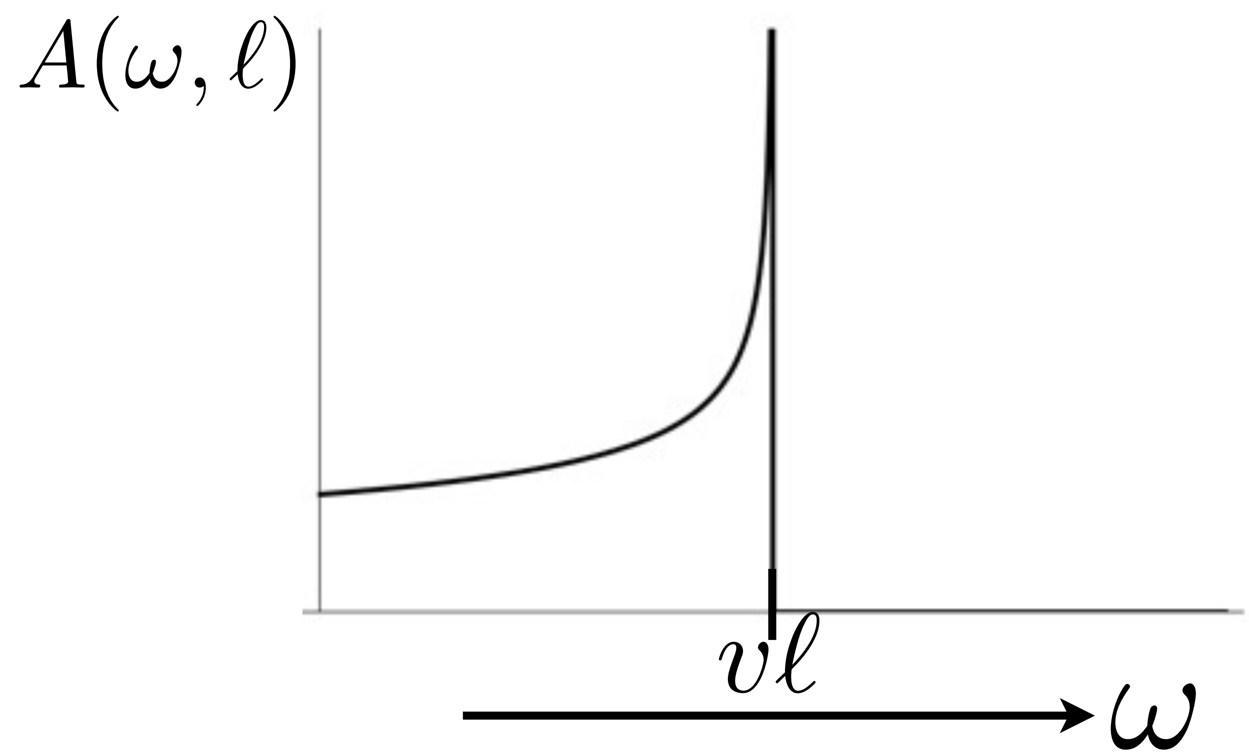


Anomalous dimension: destruction of quasi-particles

$$\mathrm{Im}\left(\frac{1}{\omega - v\ell + i\epsilon}\right) \sim \delta(\omega - v\ell)$$



$$\mathrm{Im}\left(\frac{1}{\omega - v\ell + i\epsilon}\right)^{1-2\gamma} \sim \frac{1}{(\omega - v\ell)^{1-2\gamma}}$$

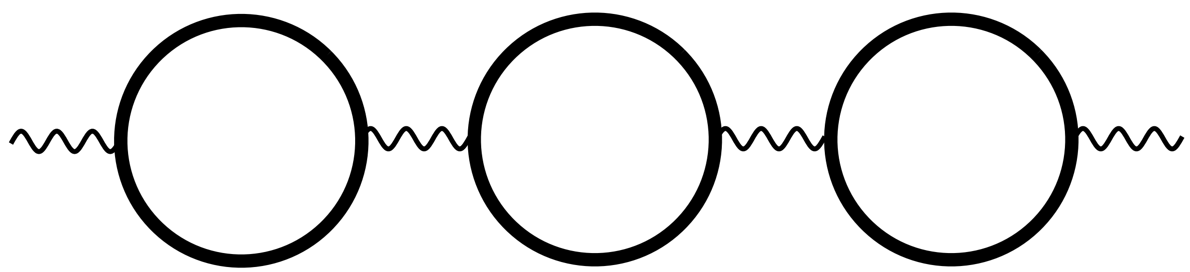


Landau Damping

Mainstream philosophy
Hertz (1976):

"Fermions Win"

"Keep 1PI diagrams but drop all others, resum to get new kinetic term"



$$S_{\mathrm{eff}} \sim \int \left[\omega^2 + q^2 + g^2 \frac{|\omega|}{\sqrt{\omega^2 + q^2}} \right] \phi^2$$

"Then feed this back into corrections to fermion"

Landau Damping

Mainstream philosophy
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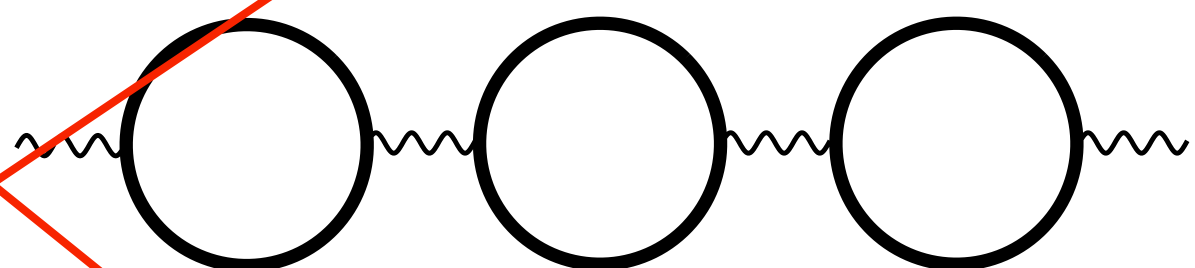

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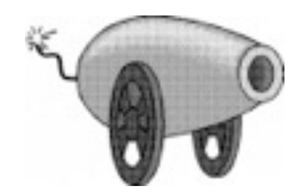
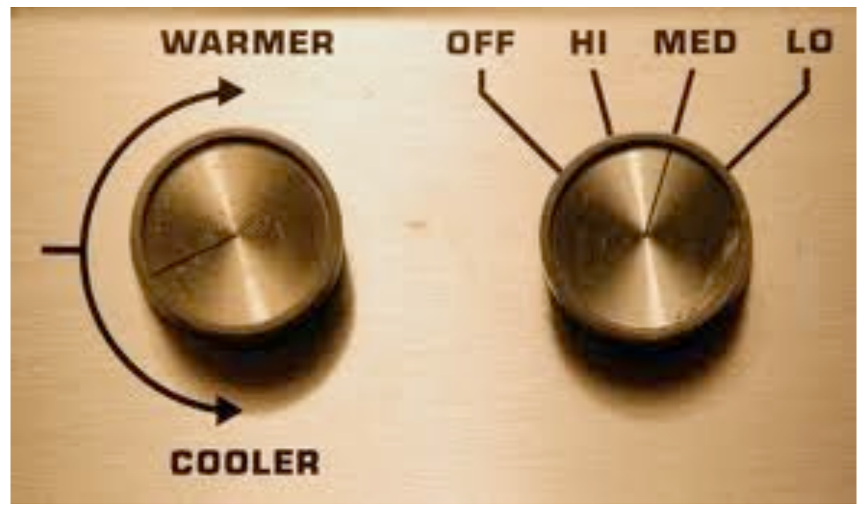
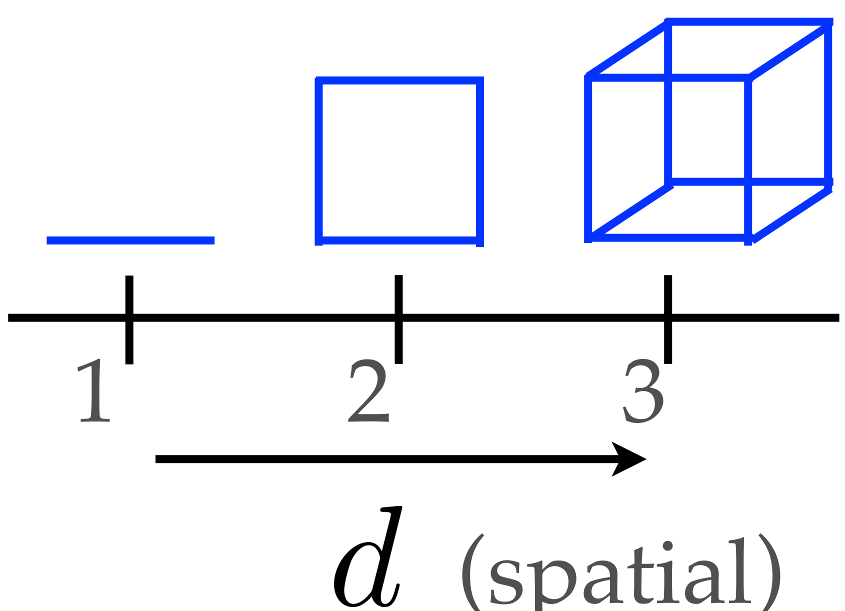


"Then feed this back into corrections to fermion"

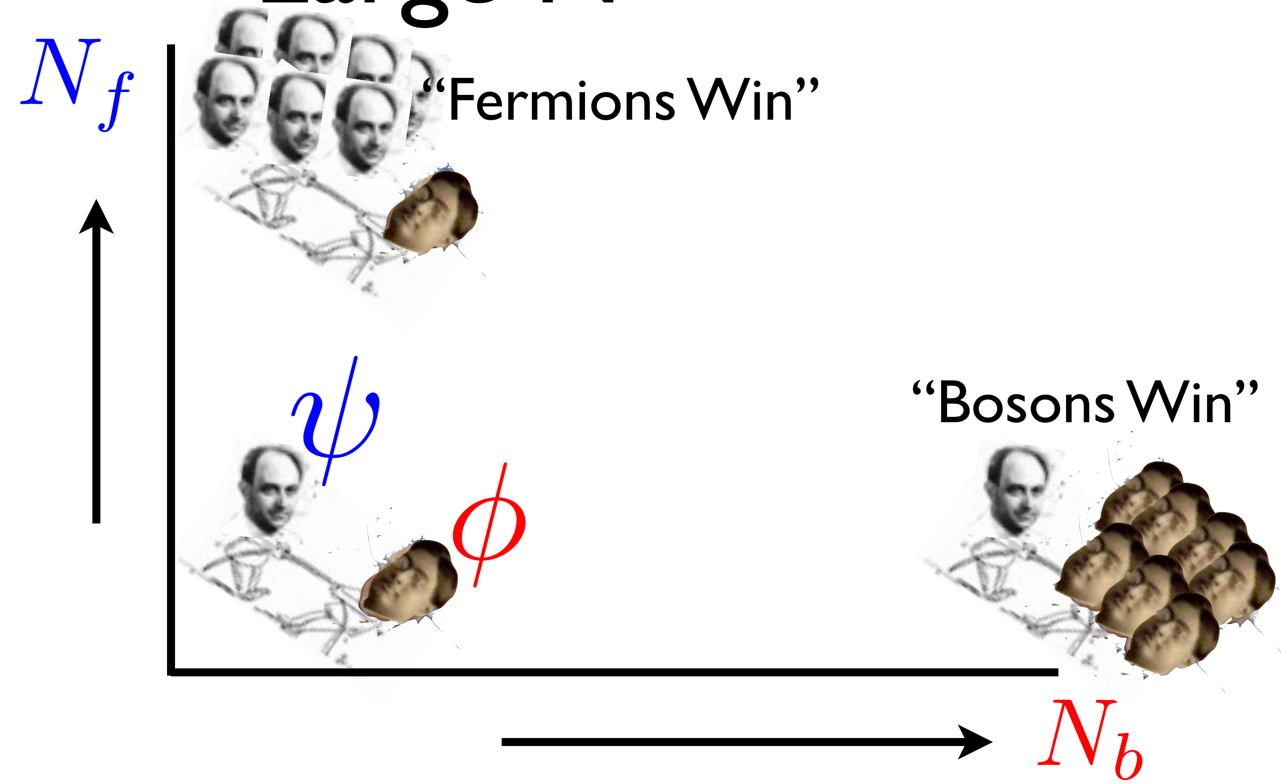
Dials



Dimension: small ϵ



Large N



Epsilon Expansion

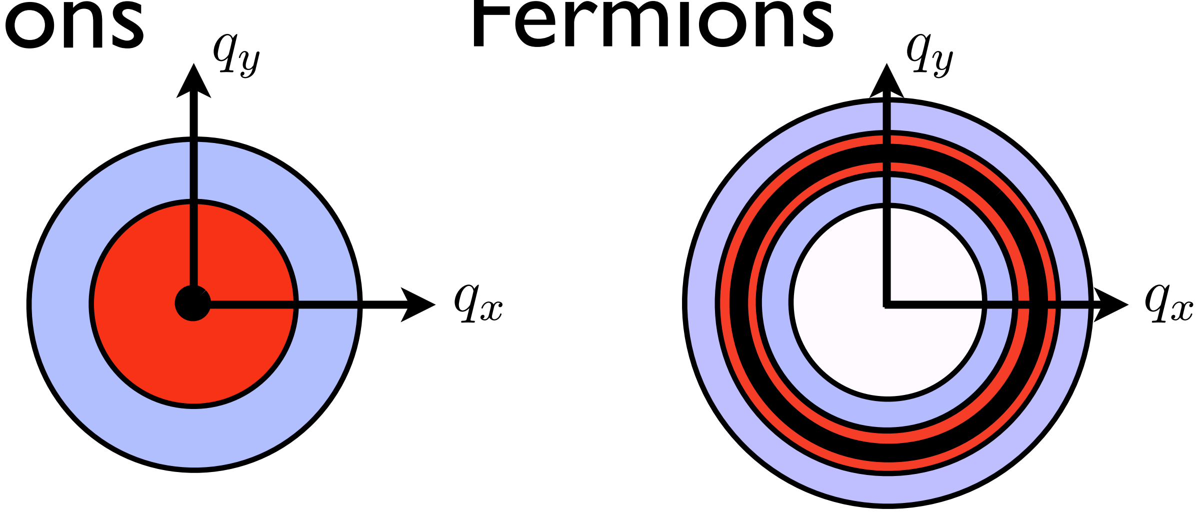
Work near upper critical dimension to find a scale-invariant fixed point at weak coupling

All three couplings are classically marginal in $d = 3$

$$g\phi\psi^\dagger\psi \qquad \lambda_\phi\phi^4$$
$$\lambda_\psi(\psi^\dagger\psi)^2$$

Bosons

Fermions



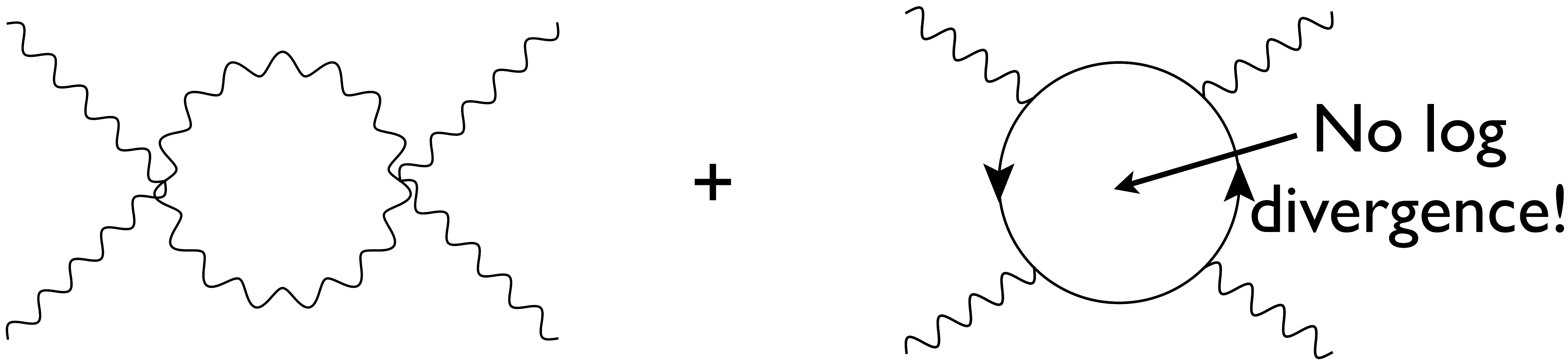
$$g \to e^{\frac{3-d}{2}\lambda} g$$

$$\lambda_\phi \to e^{(3-d)\lambda}\lambda_\phi$$

$$\lambda_\psi \to \lambda_\psi$$

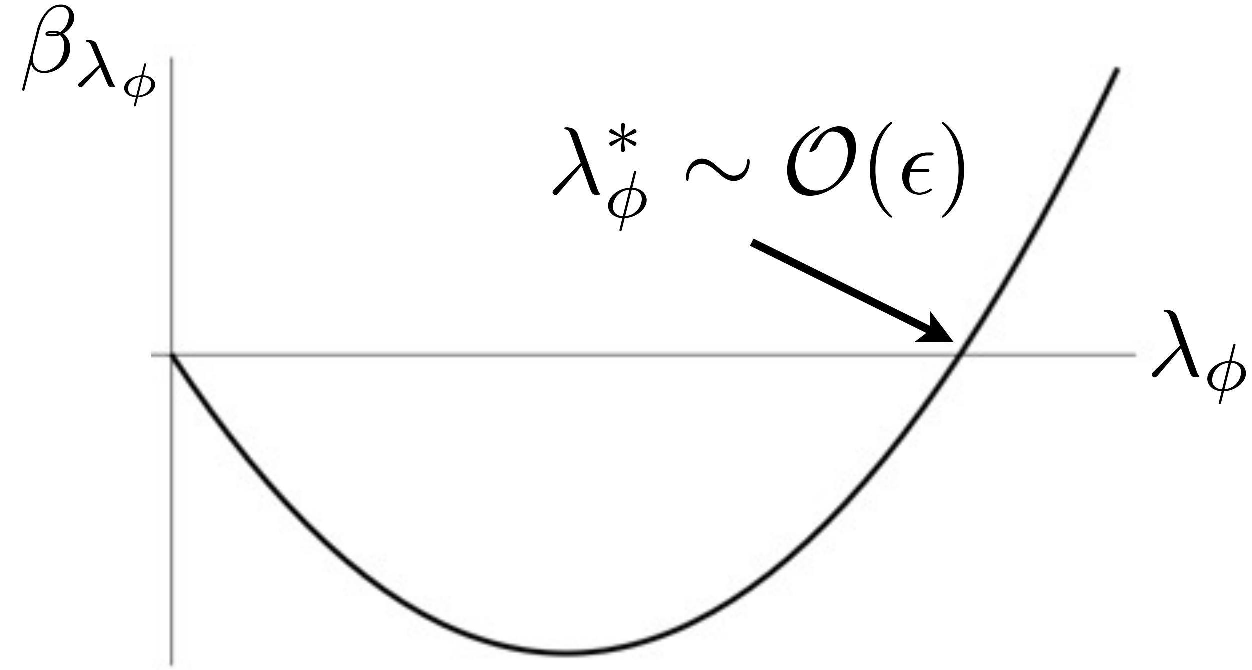
Epsilon Expansion

$$d = 3 - \epsilon$$



Scalar quartic running is the same as in Wilson Fisher

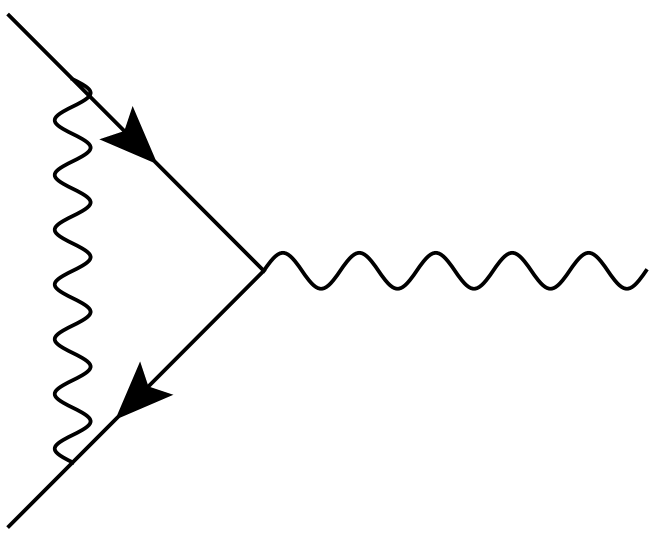
$$\frac{d}{d\log\mu}\lambda_\phi = -\epsilon\lambda_\phi + a_{\lambda_\phi}\lambda_\phi^2$$



Epsilon Expansion

$$d = 3 - \epsilon$$

γ_ψ from Wavefunction renormalization



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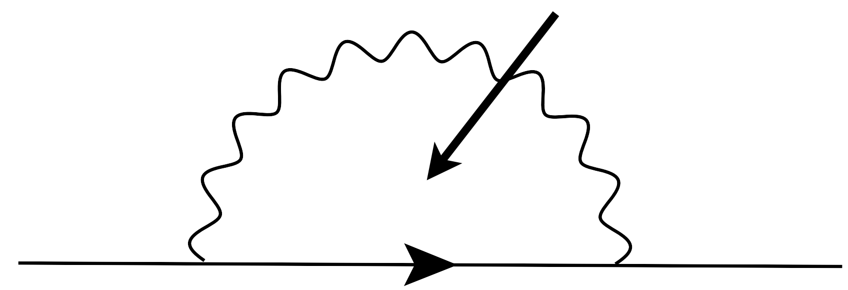
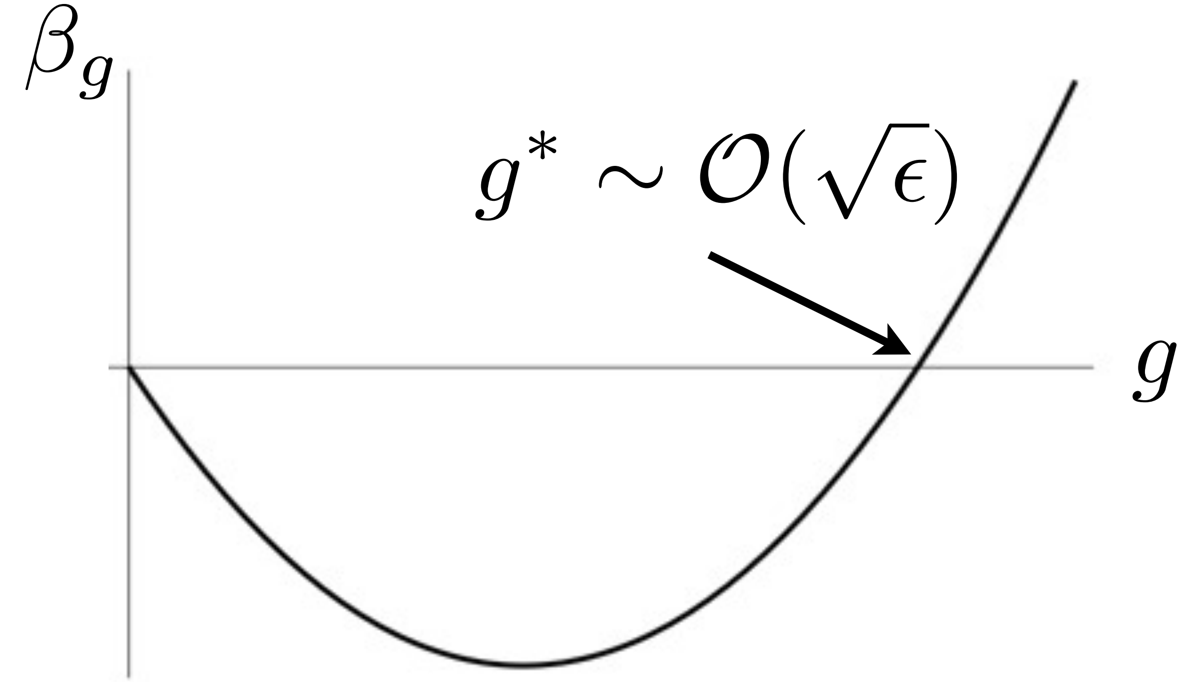


Yukawa runs to IR fixed point

$$\frac{d}{d\log\mu} g = -g\left(\frac{\epsilon}{2} - a_g g^2\right)$$

$$+\mathcal{O}(g^2\epsilon)$$

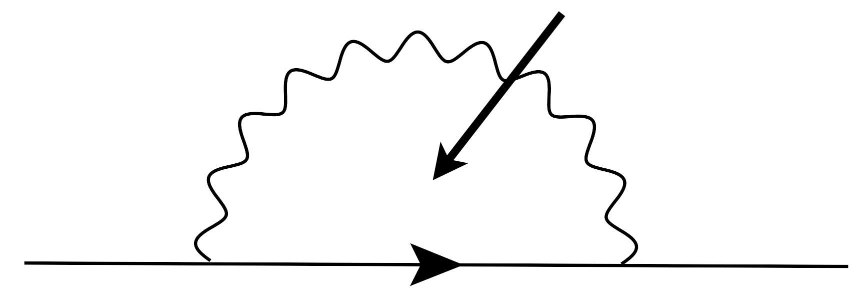


Epsilon Expansion

$$d = 3 - \epsilon$$

γ_ψ from Wavefunction renormalization

$$2\gamma_\psi \sim \frac{\epsilon}{4}$$



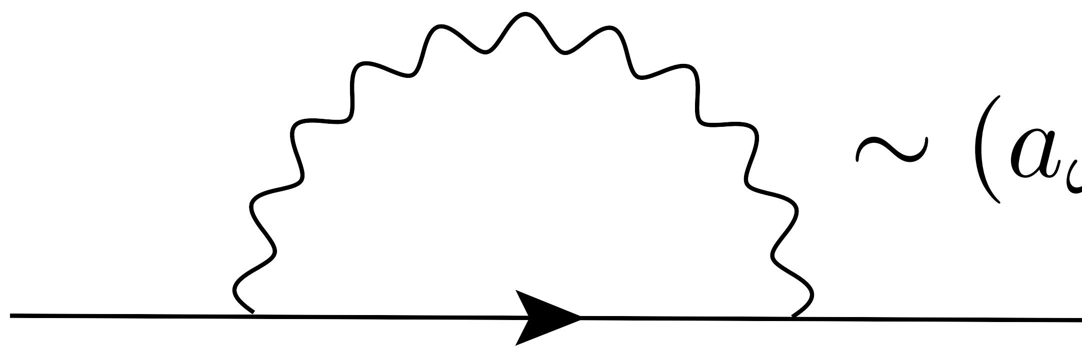
Scale-invariant fixed point with non-vanishing anomalous dimension

Fermion Green's function at fixed point must take the form

$$G(\omega, \ell) = \frac{1}{\omega^{1-2\gamma_\psi}} f(\frac{\omega}{\ell})$$

Epsilon Expansion

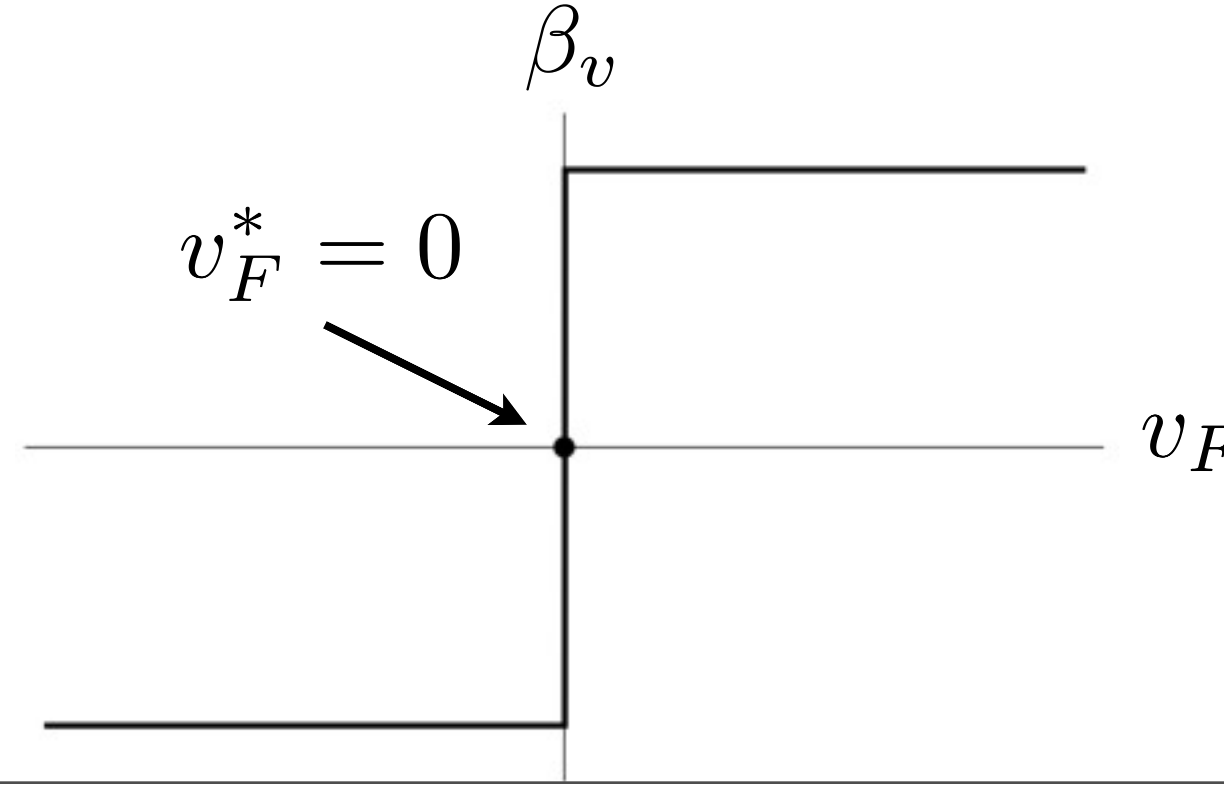
$$d = 3 - \epsilon$$



$$\sim (a_\omega \omega - a_\ell v_F \ell) \log \mu$$

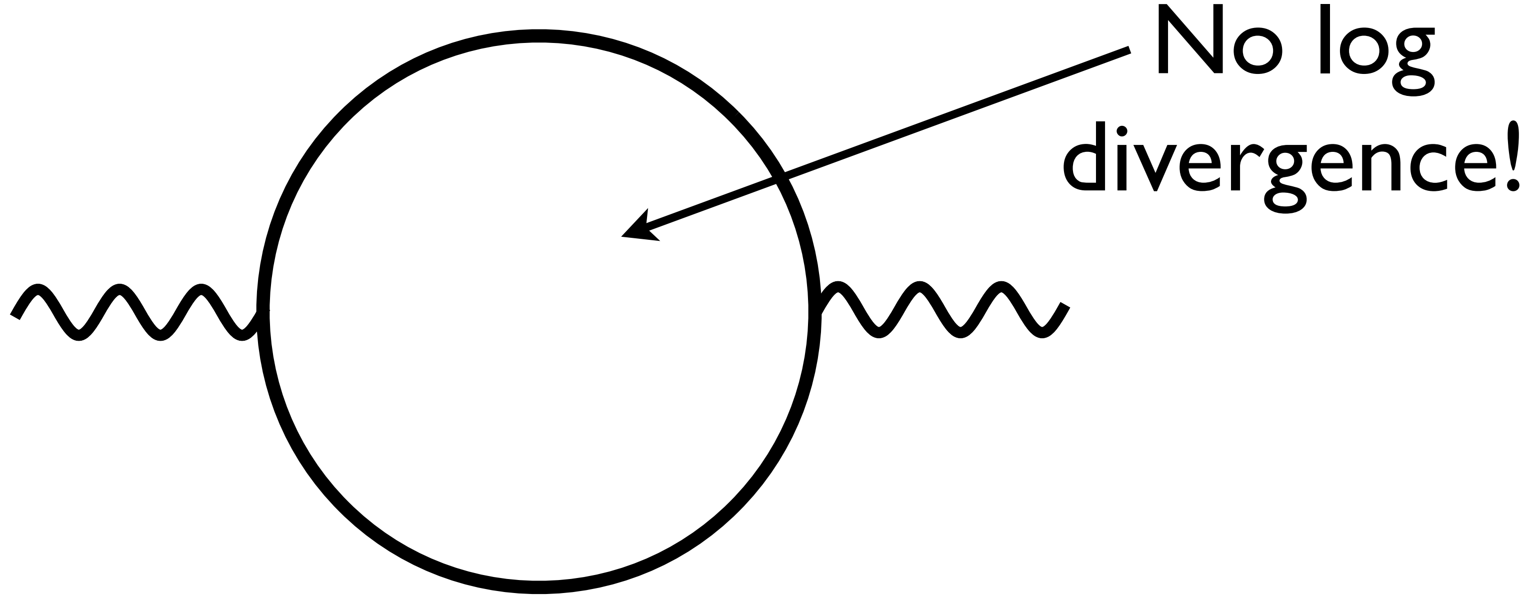
Fermion velocity runs!

$$\frac{d}{d \log \mu} v_F = a_v \mathrm{sign}(v_F)$$



Epsilon Expansion

Landau damping has no effect on RG

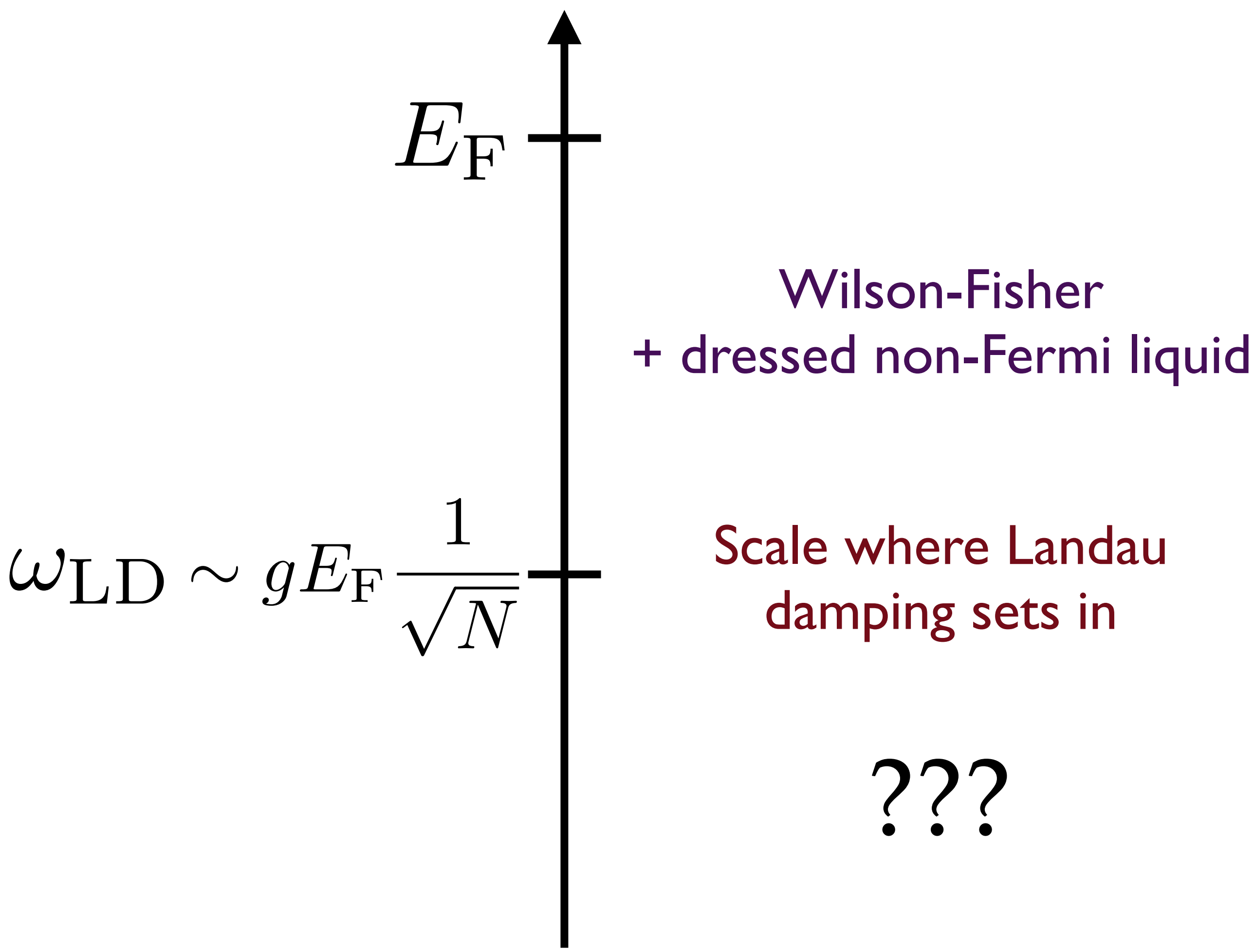


Furthermore, Landau damping pushed to very low scale

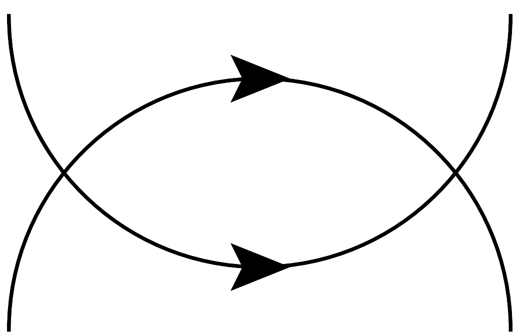
$$\Pi(q_0, q) \sim g^2 \frac{m^2 v}{2\pi} \frac{|q_0|}{\sqrt{q_0^2 + v^2 q^2}} \qquad \omega_{\mathrm{LD}}^2 = \frac{g^2 m^2}{2\pi} = \mathcal{O}(\epsilon m^2)$$

Epsilon Expansion

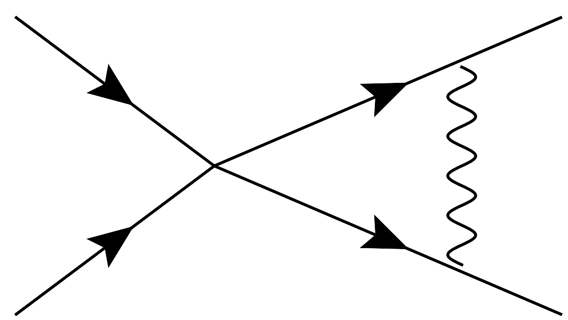
Landau damping pushed to very low scale



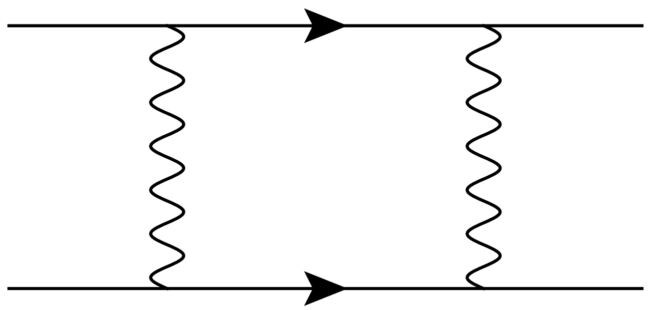
BCS Instability



λ_ψ^2



$\lambda_\psi g^2 = \lambda_\psi \mathcal{O}(\epsilon)$



$\mathcal{O}(g^4) = \mathcal{O}(\epsilon^2)$

BCS instability is a higher order effect and happens only at exponentially lower scales (if at all)

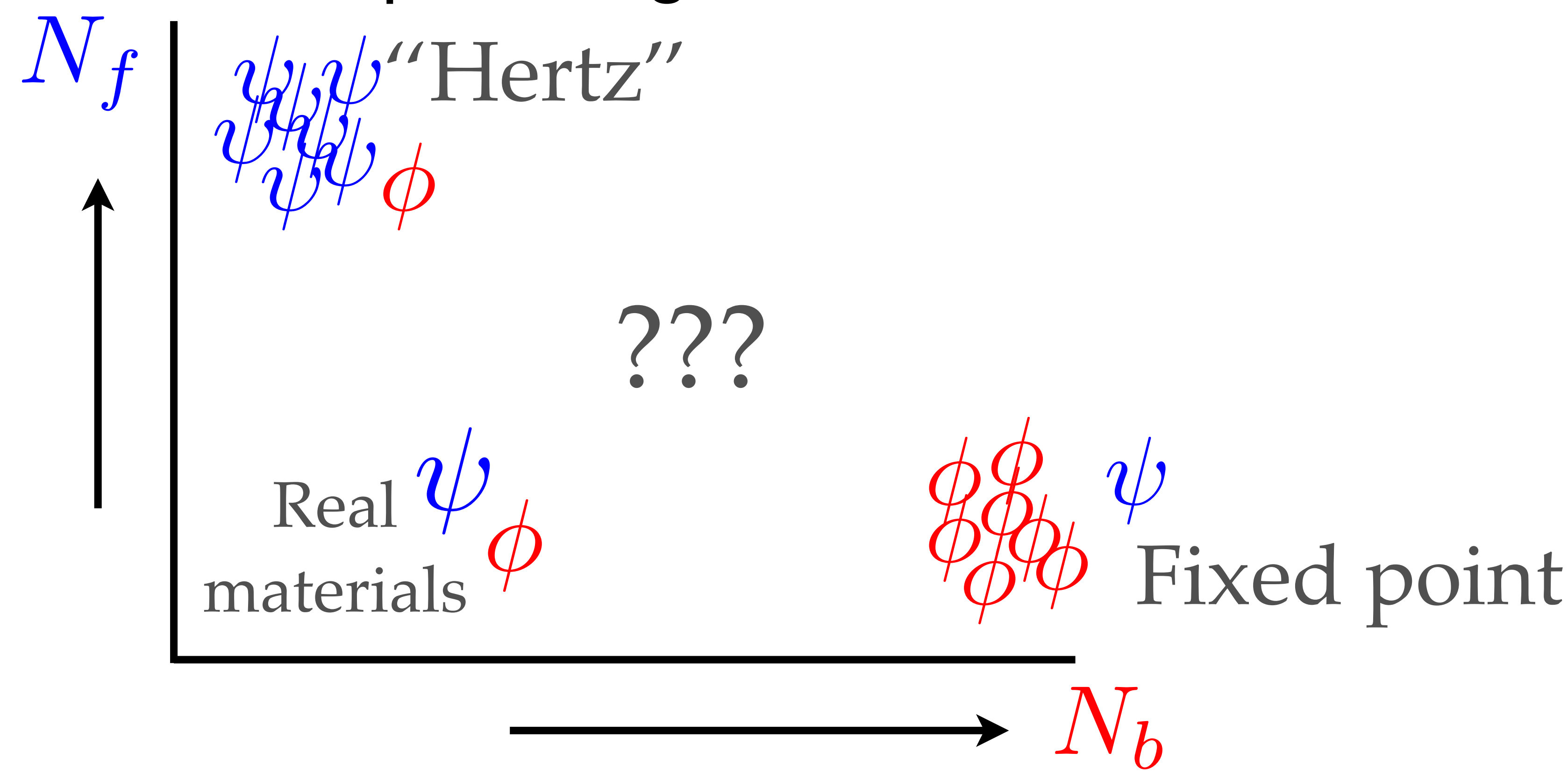
Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

Now we will look at simplifications in large N limits

We will find qualitatively different dependence at large N_b as compared with large N_f

This indicates a rich phase diagram of such theories

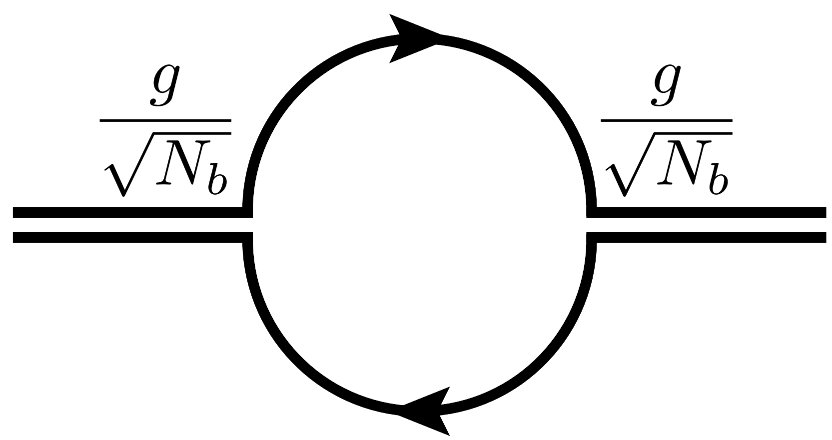


Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\Box$	$\bar{\Box}$

At $N_b \to \infty$ N_f fixed

"Bosons Win"



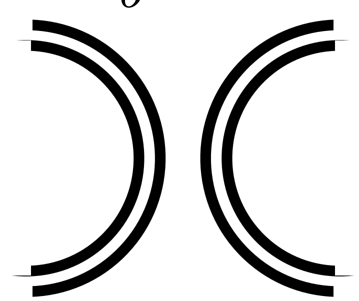
Landau Damping is a non-planar diagram and has no effect at infinite N_b

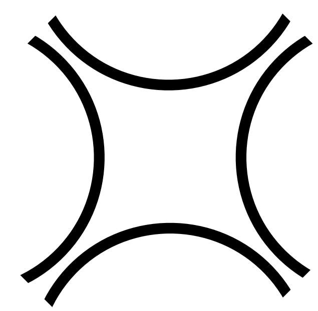
Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

At $N_b \to \infty$ N_f fixed

$$\frac{\lambda_\phi^{(2)}}{8N_b^2}(\mathrm{tr}[\phi^2])^2 \qquad \frac{\lambda_\phi^{(1)}}{8N_b}\mathrm{tr}[\phi^4]$$



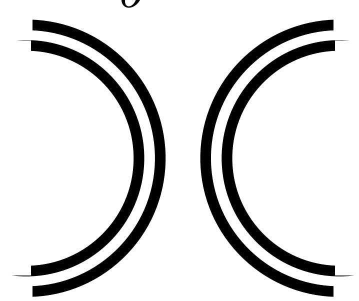


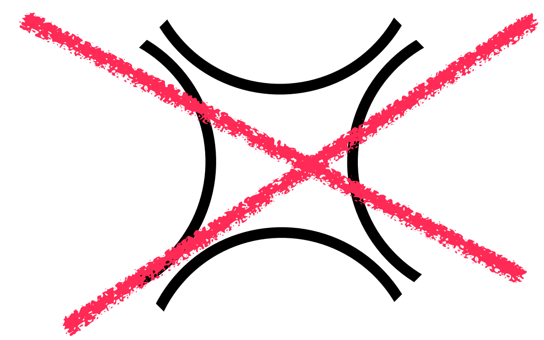
Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

At $N_b \to \infty$ N_f fixed

$\frac{\lambda_\phi^{(2)}}{8N_b^2}(\mathrm{tr}[\phi^2])^2$ $\frac{\lambda_\phi^{(1)}}{8N_b}\mathrm{tr}[\phi^4]$





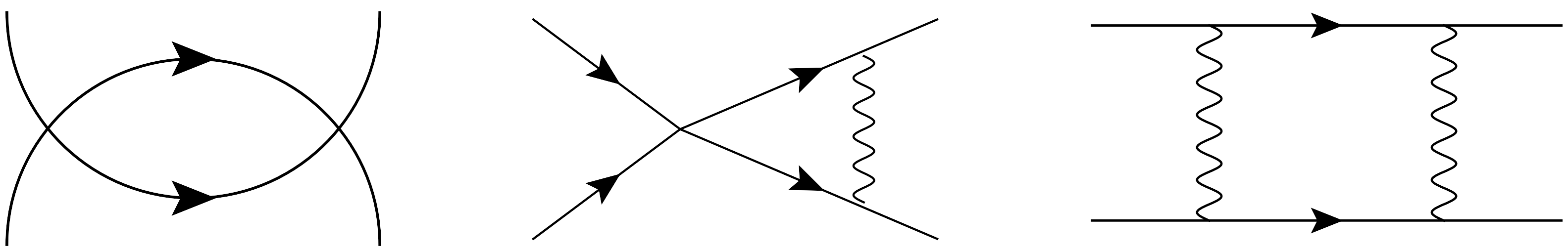
One can set $\lambda_\phi^{(1)} = 0$ naturally (in the ‘t Hooft sense)

Then the ϕ sector is isomorphic to the SO(N_b^2) Wilson-Fisher fixed point

Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\Box$	$\bar{\Box}$

At $N_b \to \infty$ N_f fixed



are all $\mathcal{O}(\frac{1}{N_b})$

The only contribution to four-fermi running is wavefunction renormalization

$$\frac{d\lambda_\psi}{d\log\mu} = 4\gamma_\psi \lambda_\psi$$

Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
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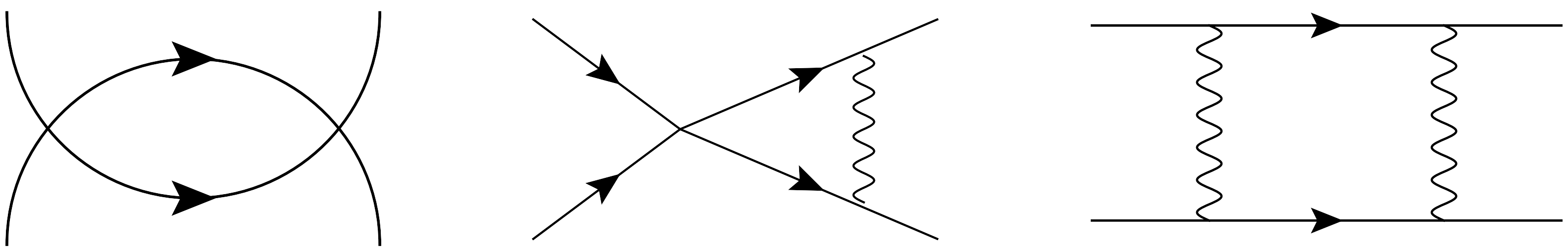
At $N_b \to \infty$ N_f fixed

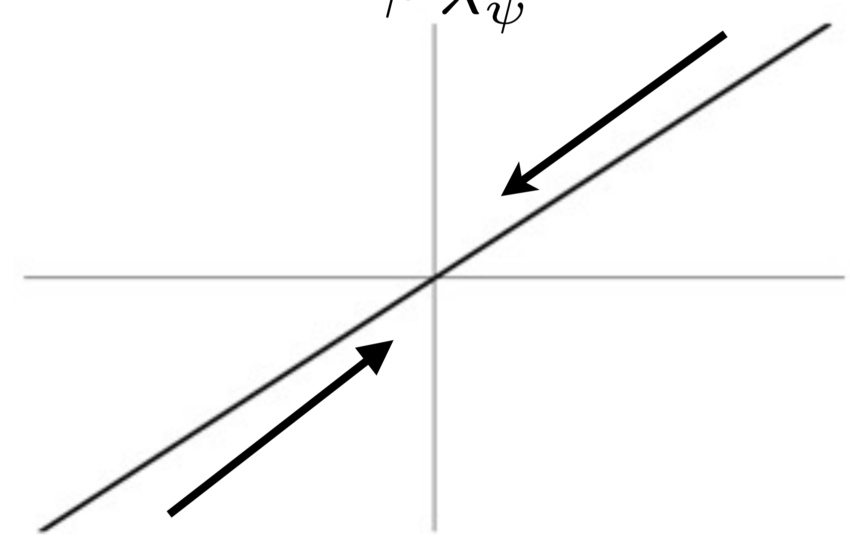


are all $\mathcal{O}(\frac{1}{N_b})$

The only contribution to four-fermi running is wavefunction renormalization

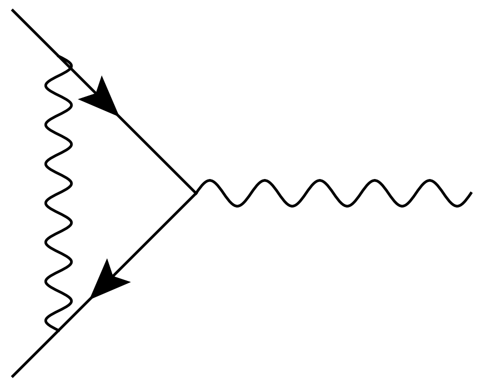
$$\frac{d\lambda_\psi}{d\log\mu} = 4\gamma_\psi \lambda_\psi$$

Stable against superconductivity



Large N Dials

At $N_b \to \infty \quad N_f$ fixed



is $\mathcal{O}(\frac{1}{\sqrt{N_b}})$

So all running of g is through wavefunction renormalization:

$$\frac{d}{d\log\mu} g = -g\left(\frac{\epsilon}{2} - 2\gamma_\psi(g)\right)$$

Scale-invariant fixed point even for $\epsilon \sim \mathcal{O}(1)$

$$2\gamma_\psi = \frac{\epsilon}{2}$$

The fermion Green's function therefore takes the form

$$G(\omega,\ell) = \frac{1}{\omega^{1-2\gamma_\psi}} f(\frac{\omega}{\ell})$$

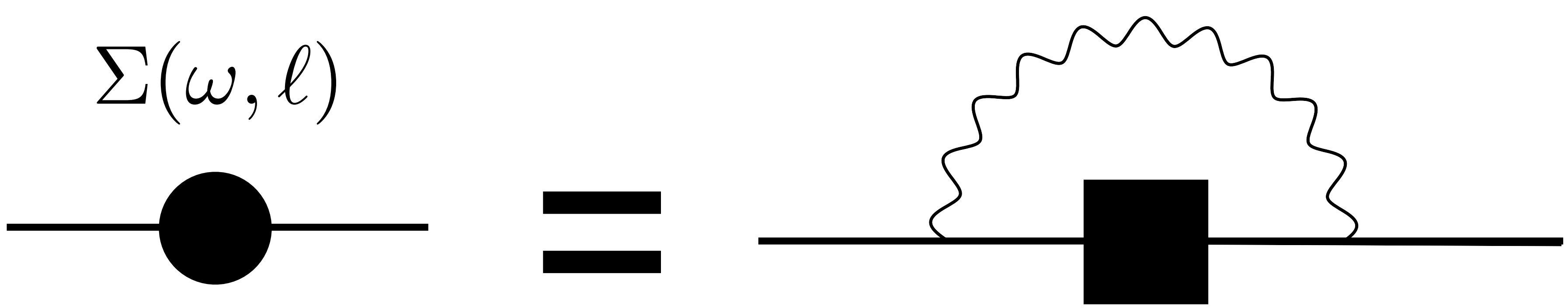
Large N Dials

At $N_b \to \infty$ N_f fixed

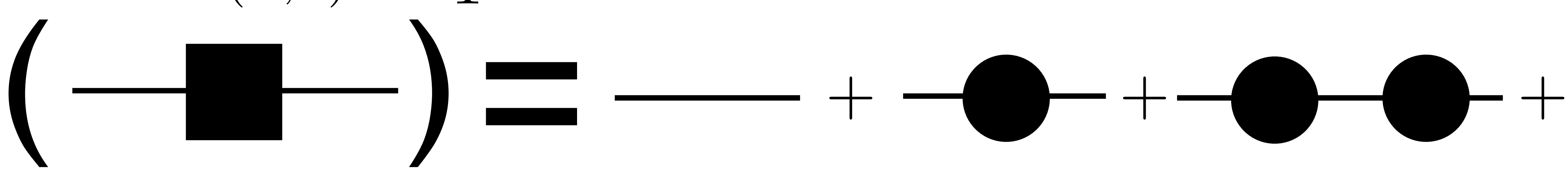
Actually, we can even calculate the scaling function $f\left(\frac{\omega}{\ell}\right)$

Gap equation for fermion Green's function

$\Sigma(\omega, \ell)$

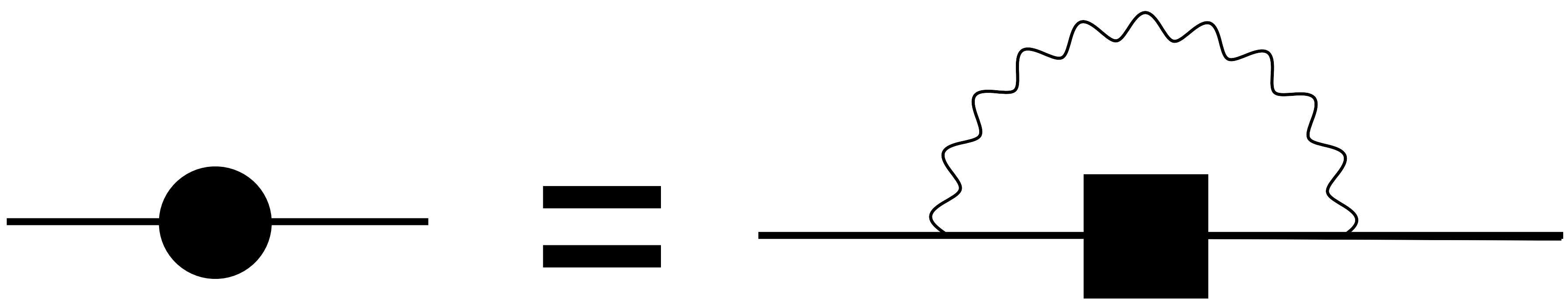


$G^{-1}(\omega, \ell)$ -1



Large N Dials

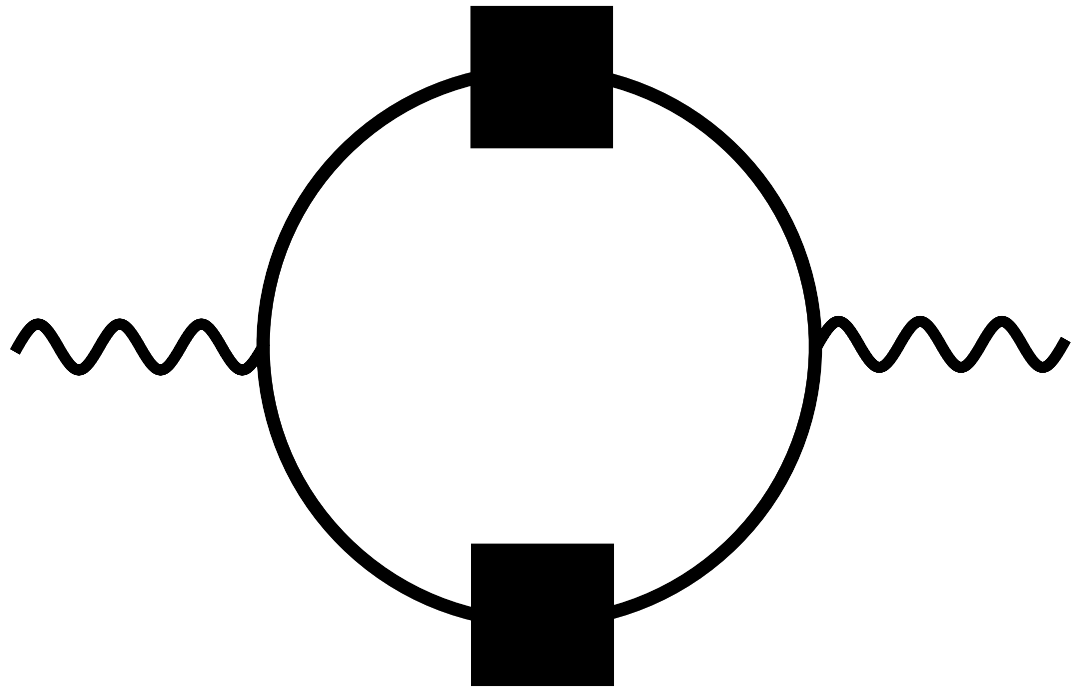
At $N_b \to \infty \quad N_f$ fixed



Solution: $G(\omega, \ell) = \frac{1}{\omega^{1-\frac{\epsilon}{2}}}$ $\qquad f\left(\frac{\omega}{\ell}\right) = 1$

Large N Landau Damping

Now we can look at 1/N correction to boson



$d = 2:$ $$\Pi(q_0, q) \sim \frac{g^2}{N_b} q k_F \log(q_0/\Lambda)$$

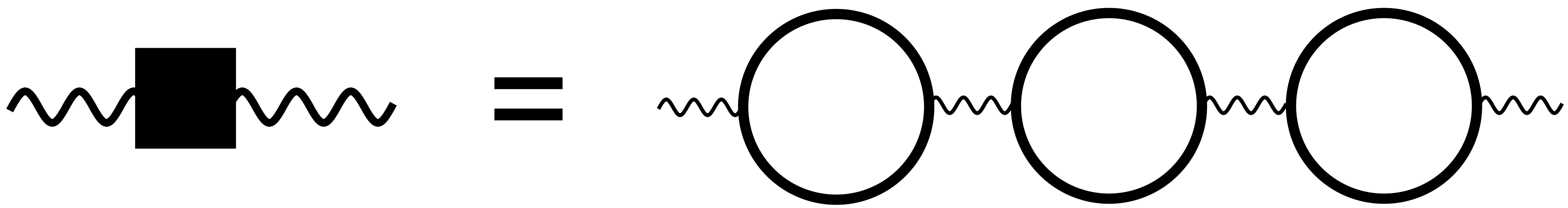
Very different from the boson self-energy in the original "Hertz" treatment!

Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\Box$	$\bar{\Box}$

At $N_f \to \infty \quad N_b$ fixed

"Fermions Win"



Hertz's theory is exact: $G_\phi(q_0, q) = \dfrac{1}{q_0^2 + c_s^2 q^2 + \Pi(q_0, q)}$

1/N Issues

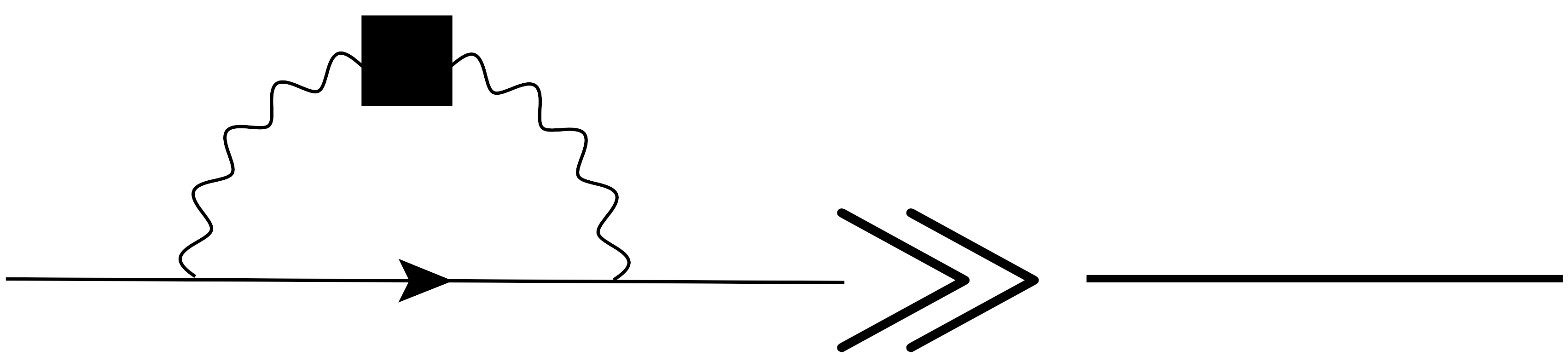
	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

If we look at subleading orders in 1/N, non-planar diagrams dominate deep in the IR

$$d = 2$$

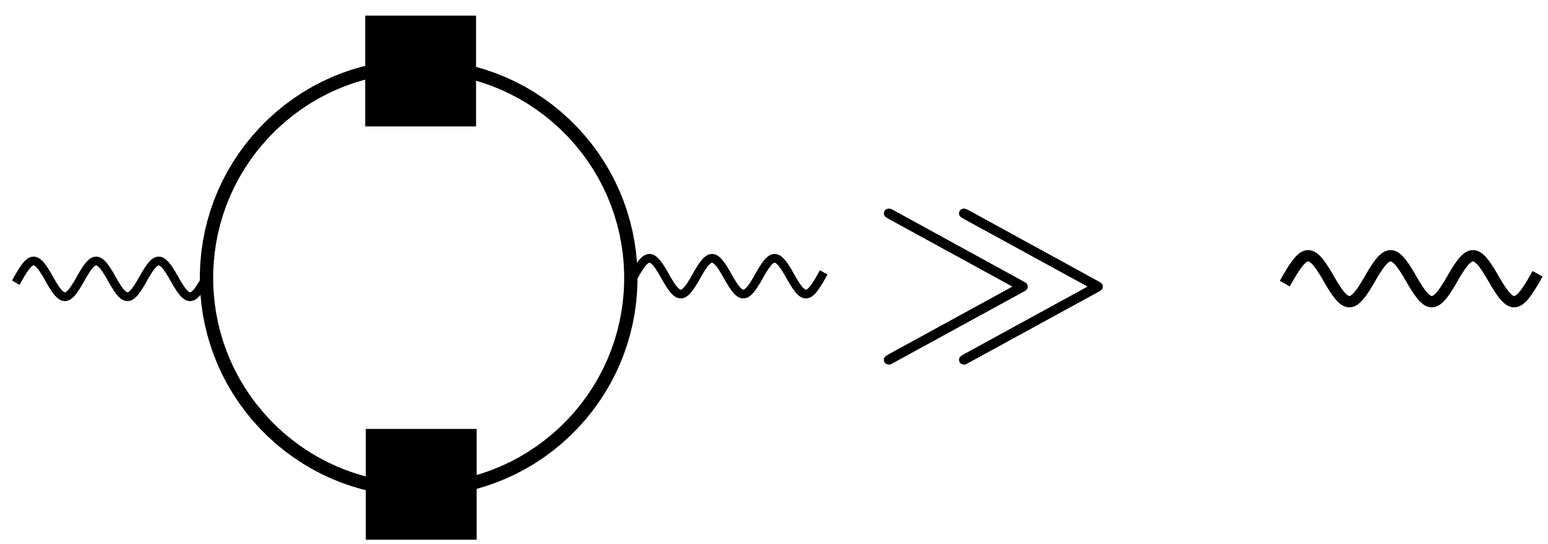
S.S. Lee

$$\frac{1}{N_f}$$



at $\omega \lesssim \dfrac{g^4}{mN_f^3}$

$$\frac{1}{N_b}$$



at $\omega \lesssim \dfrac{g^2 m}{N_b}$

1/N Issues

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

If we look at subleading orders in 1/N, non-planar diagrams dominate deep in the IR

$d = 2$

S.S. Lee

$$\frac{1}{N_f}$$

at $\omega \lesssim \frac{g^4}{m N_f^3}$

Complicated effects arise as we leave the regime of small parameters

$$\frac{1}{N_b}$$

at $\omega \lesssim \frac{g^2 m}{N_b}$

Conclusion

Non-Fermi liquids have new dynamics in need of a theoretical description

We are looking for local EFTs of the Fermi surface (plus light states) that exhibit similar dynamics

A rich structure of such theories exists depending on various parameters of the theory

In some limits (large N, small ϵ) the theory can be solved and leads to new fixed points

An enormous range of local EFTs remains to be explored!

The End