Gravitational Interactions of Higher-Spin Fermions

Rakibur Rahman

Université Libre de Bruxelles, Belgium

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November 18, 2013

Dept. of Physics, UC Davis

- Gravitational Interactions of Higher-Spin Fermions,
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- Higher-Spin Fermionic Gauge Fields and Their Electromagnetic Coupling,
  G. Lucena Gómez, M. Henneaux and RR,
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- Consistent Couplings between Fields with a Gauge Freedom and Deformations of the Master Equation,
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Motivations

- Fermions are required by supersymmetry and their interactions could be qualitatively different from those of bosons.

- We will consider the coupling of an arbitrary-spin massless fermion to a spin-2 gauge field, in flat spacetime with $D \geq 4$.

- The latter cannot be the ordinary graviton that obeys principle of equivalence, since no-go theorems prohibit, in flat spacetime, minimal coupling to gravity for $s \geq 5/2$.

- Non-minimal couplings containing more derivatives do exist.

- We view our flat space construction as a step towards that in AdS, where gravitational coupling of higher spins makes sense.

- The ultimate goal is a standard action for the Vasiliev system.

- We use the BRST-BV cohomological methods, which could also be employed for AdS via the ambient space formulation.

- Search for consistent interactions becomes very systematic.

- Any nontrivial consistent interaction must be noticed. Explicit off-shell covariant vertices are natural output.

- Higher-order consistency of vertices can be checked easily.

Results

- Cohomological elimination of minimal coupling for $s \geq 5/2$.

- Number of derivatives in a cubic ***2-s-s*** vertex is restricted.

- 5 allowed values: $2n\text{-}2$, $2n\text{-}1$, $2n$, $2n+1$, $2n+2$ for $s = n+1/2$.

- Explicit construction of off-shell cubic vertices for arbitrary spin and presenting them in a very neat form.

1.	Non-Abelian	$(2n\text{-}2)$-derivative	$D \geq 4$
2.	Non-Abelian	$(2n\text{-}1)$-derivative	$D \geq 5$
3.	Abelian	$2n$-derivative	$D \geq 5$
4.	Abelian	$(2n+1)$-derivative	$D \geq 5$
5.	Abelian	$(2n+2)$-derivative	$D \geq 4$

- Generic obstruction for the non-Abelian cubic vertices.

Table 1: Prototypical Example of $2-\frac{5}{2}-\frac{5}{2}$ Vertices with p Derivatives

p	Vertex
2	$i\bar{\psi}_{\mu\alpha}R^{+\mu\nu\alpha\beta}\psi_{\nu\beta}+\frac{i}{2}\bar{\not\psi}_{\mu}\not R^{\mu\nu}\not\psi_{\nu}+\frac{i}{4}h_{\mu\nu}\bar{\Psi}_{\rho\sigma\Vert\lambda}\,\gamma^{\mu\rho\sigma\alpha\beta,\,\nu\lambda\gamma}\,\Psi_{\alpha\beta\Vert\gamma}$
3	$i\bar{\Psi}_{\mu\nu\Vert}{}^{\rho}\left(\mathfrak{h}^{+}_{\rho\sigma\Vert\lambda}\,\gamma^{\lambda\mu\nu\alpha\beta}+\gamma^{\lambda\mu\nu\alpha\beta}\,\mathfrak{h}^{+}_{\rho\sigma\Vert\lambda}\right)\Psi_{\alpha\beta\Vert}{}^{\sigma}$
4	$ih_{\mu\nu}\bar{\Psi}_{\rho\sigma\vert\tau\lambda}\,\gamma^{\mu\rho\sigma\alpha\beta,\,\nu\tau\gamma}\,\Psi_{\alpha\beta\vert\gamma}{}^{\lambda}$
5	$i\mathfrak{h}_{\mu\nu\Vert\lambda}\bar{\Psi}^{\mu\tau\vert}{}_{\rho\sigma}\,\gamma^{\lambda\rho\sigma\alpha\beta}\,\Psi^{\nu}{}_{\tau\vert\alpha\beta}$
6	$iR_{\mu\nu\rho\sigma}\bar{\Psi}^{\rho\sigma\vert\alpha\beta}\Psi_{\alpha\beta}{}^{\mu\nu}$

Outline

- Gravitational coupling of massless spin 5/2: nontrivial!

- Cohomological reformulation of the free gauge system in order to employ the BRST deformation scheme to construct consistent parity-preserving covariant cubic vertices.

- Generalization to arbitrary spin, $s = n+1/2$, coupled to gravity. This generalization is surprisingly easy!

- Second-order deformations and issues with locality.

- Concluding remarks.

Prototypical Example:

Massless Spin-5/2 Field

Coupled to Massless Spin 2

Step 0: Free Gauge Theory

- Free theory contains a graviton $\boldsymbol{h_{\mu\nu}}$ and a massless spin-5/2 field—symmetric rank-2 tensor-spinor $\boldsymbol{\psi_{\mu\nu}}$. The action reads:

$$S^{(0)}[h_{\mu\nu},\psi_{\mu\nu}]=\int d^Dx\left[G^{\mu\nu}h_{\mu\nu}+\tfrac{1}{2}\left(\bar{\mathcal{R}}^{\mu\nu}\psi_{\mu\nu}-\bar{\psi}_{\mu\nu}\mathcal{R}^{\mu\nu}\right)\right]$$

$$\mathcal{R}^{\mu\nu}=\mathcal{S}^{\mu\nu}-\gamma^{(\mu}\not{\mathcal{S}}^{\nu)}-\tfrac{1}{2}\eta^{\mu\nu}\mathcal{S}',\qquad \mathcal{S}_{\mu\nu}=i\left[\not{\partial}\,\psi_{\mu\nu}-2\partial_{(\mu}\not{\psi}_{\nu)}\right]$$

- It enjoys two Abelian the gauge invariances:

$$\delta_\lambda h_{\mu\nu}=2\partial_{(\mu}\lambda_{\nu)},\qquad \delta_\varepsilon\psi_{\mu\nu}=2\partial_{(\mu}\varepsilon_{\nu)},\quad \text{with}\ \not{\varepsilon}=0.$$

- Bosonic gauge parameter: $\boldsymbol{\lambda_\mu}$, fermionic gauge parameter: $\boldsymbol{\varepsilon_\mu}$

Step 1: Ghosts

- For each gauge parameter, we introduce a ghost field, with the same algebraic symmetries but opposite Grassmann parity:
  - Grassmann-odd bosonic ghost: C_μ
  - Grassmann-even fermionic ghost: ξ_μ
- The original fields and ghosts are collectively called fields:

$$\Phi^A = \{h_{\mu\nu}, C_\mu, \psi_{\mu\nu}, \xi_\mu\}.$$

- Introduce the grading: pure ghost number, ***pgh***, which is
  - 1 for the ghost fields
  - 0 for the original fields

Step 2: Antifields

- One introduces, for each field and ghost, an antifield Φ_A^*, with the same algebraic symmetries but opposite Grassmann parity. Each antifield has 0 pure ghost number: ***$pgh(\Phi_A^*)=0$***.

$$\Phi_A^* = \{h^{*\mu\nu}, C^{*\mu}, \bar{\psi}^{*\mu\nu}, \bar{\xi}^{*\mu}\}.$$

- Introduce the grading: antighost number, ***agh***, which is 0 for the fields and non-zero for the antifields:

$$agh(\Phi_A^*) = pgh(\Phi^A) + 1$$

Step 3: Antibracket

- On the space of fields and antifields, one defines an odd symplectic structure, called the antibracket:

$$(X, Y) \equiv \frac{\delta^R X}{\delta \Phi^A} \frac{\delta^L Y}{\delta \Phi^*_A} - \frac{\delta^R X}{\delta \Phi^*_A} \frac{\delta^L Y}{\delta \Phi^A}.$$

- Here *R* and *L* respectively mean right and left derivatives.
- The antibracket satisfies graded Jacobi identity.

Step 4: Master Action

- The master action S_0 is an extension of the original action; it includes terms involving ghosts and antifields.

$$S_0 = \int d^D x \left[G^{\mu\nu} h_{\mu\nu} + \tfrac{1}{2} \left(\bar{\mathcal{R}}^{\mu\nu} \psi_{\mu\nu} - \bar{\psi}_{\mu\nu} \mathcal{R}^{\mu\nu} \right) - 2h^{*\mu\nu} \partial_\mu C_\nu + \left(\bar{\psi}^{*\mu\nu} \partial_\mu \xi_\nu - \partial_\mu \bar{\xi}_\nu \psi^{*\mu\nu} \right) \right]$$

- Because of Noether identities, it solves the master equation:

$$(S_0, S_0) = 0$$

- Antifields are sources for the “gauge” variations.
- Antighosts source gauge-algebra deformation and are absent.

Step 5: BRST Differential

- S_0 is the generator of the BRST differential s of the free theory

$$sX = (S_0, X)$$

- Then the free master equation means: S_0 is BRST-closed.

- Graded Jacobi identity of the antibracket gives:

$$s^2 = 0$$

- The free master action S_0 is trivial in the cohomology of s, in the local functionals of the (anti)fields and their derivatives.

- The BRST differential decomposes into two differentials:

$$s = \Gamma + \Delta$$

- Δ is the Koszul-Tate differential.
- Γ is the longitudinal derivative along the gauge orbits.
- They obey: $\Gamma^2 = \Delta^2 = 0, \ \Gamma\Delta + \Delta\Gamma = 0.$
- Their action on the (anti)fields are explicitly given.
- All Γ, Δ, s increase the ghost number, ***gh***, by one unit, where

$$gh = pgh - agh$$

Step 6: Properties of Φ^A & Φ^*_A

Table 2: Properties of the Various Fields & Antifields ($n = 2$)

Z	$\Gamma(Z)$	$\Delta(Z)$	$pgh(Z)$	$agh(Z)$	$gh(Z)$	$\epsilon(Z)$
$h_{\mu\nu}$	$2\partial_{(\mu}C_{\nu)}$	0	0	0	0	0
C_μ	0	0	1	0	1	1
$h^{*\mu\nu}$	0	$G^{\mu\nu}$	0	1	-1	1
$C^{*\mu}$	0	$-2\partial_\nu h^{*\mu\nu}$	0	2	-2	0
$\psi_{\mu\nu}$	$2\partial_{(\mu}\xi_{\nu)}$	0	0	0	0	1
ξ_μ	0	0	1	0	1	0
$\bar{\psi}^{*\mu\nu}$	0	$\bar{\mathcal{R}}^{\mu\nu}$	0	1	-1	0
$\bar{\xi}^{*\mu}$	0	$2\partial_\nu\bar{\chi}^{*\mu\nu}$	0	2	-2	1

An Aside: BRST Deformation Scheme

- The solution of the master equation incorporates compactly all consistency conditions pertaining to the gauge transformations.

- Any consistent deformation of the theory corresponds to:

$$S = S_0 + gS_1 + g^2 S_2 + O(g^3)$$

where S also solves the master equation: $(S\,,\,S) = 0$.

- Coupling constant expansion gives, up to $O(g^2)$:

$$(S_0, S_0) = 0,$$

$$(S_0, S_1) = 0,$$

$$(S_1, S_1) = -2(S_0, S_2).$$

- The first equation is fulfilled by assumption.
- The second equation says S_1 is BRST-closed:

$$sS_1 = 0$$

- First order non-trivial consistent local deformations:

$$S_1 = \int a$$

are in 1-to-1 correspondence with elements of the cohomology of the ***s***, modulo total derivative ***d***, at ghost number 0.

- One has the cocycle condition:

$$sa \doteq 0.$$

- One can expand a cubic deformation in antighost number:

$$a = a_0 + a_1 + a_2, \qquad agh(a_i) = i$$

- a_0 is the deformation of the Lagrangian—the cubic vertex.
- a_1 gives the deformations of the gauge transformations.
- a_2 gives deformations of the the gauge algebra.
- The cocycle condition reduces, by $s = \Gamma + \Delta$, to a cascade

$$\begin{aligned} \Gamma a_2 &\doteq 0, \\ \Delta a_2 + \Gamma a_1 &\doteq 0, \\ \Delta a_1 + \Gamma a_0 &\doteq 0. \end{aligned}$$

- A cubic vertex will deform the gauge algebra (non-Abelian) if and only if a_2 is nontrivial in the cohomology of Γ modulo d.

- Otherwise, one can always choose $a_2 = 0$ and $a_1 = \Gamma$-closed modulo d. The vertex may deform the gauge transformations.

- If a_1 is trivial, the gauge symmetry remains intact, and the vertex a_0 is gauge invariant only up to a total derivative.

- Any Lagrangian deformation a_0 is Δ-closed.

- But trivial interaction terms are Δ-exact modulo d.

Step 7: Cohomology of *Γ*

- Cohomology of ***Γ*** consists of gauge-invariant objects that themselves are not gauge variation of something else.

- It is isomorphic to the space of functions of

- The undifferentiated ghosts $\{C_\mu, \xi_\mu\}$. Also the 1-curl of the bosonic ghost $\mathfrak{C}_{\mu\nu}$ as well as the γ-traceless part of the 1-curl of the fermionic ghost $\Xi_{\mu\nu}$,
- The antifields $\{h^{*\mu\nu}, C^{*\mu}, \bar{\psi}^{*\mu\nu}, \bar{\xi}^{*\mu}\}$ and their derivatives,
- The curvatures $\{R_{\mu\nu\rho\lambda}, \Psi_{\mu\nu|\rho\lambda}\}$ and their derivatives,
- The Fronsdal tensor $\mathcal{S}_{\mu\nu}$ and its symmetrized derivatives.

Step 8: Non-Abelian Vertices

- a_2 is Grassmann even, parity-even, Lorentz scalar satisfying:

$$\Gamma a_2 = 0, \qquad gh(a_2) = 0, \qquad agh(a_2) = 2 = pgh(a_2)$$

- The most general solutions are (for p derivatives in a_0)

$$a_2 = \begin{cases} p=1: & ig\, C^{*\mu} \bar{\xi}^{\alpha} \gamma_\mu \xi_\alpha \\ p=2: & ig\, C^{*\mu} \bar{\Xi}_{\mu\nu} \xi^\nu + \text{h.c.} \\ p=3: & ig\, C^{*\mu} \bar{\Xi}^{\alpha\beta} \gamma_\mu \Xi_{\alpha\beta}. \end{cases}$$

$$a_2 = \begin{cases} p=0: & g\, \bar{\xi}^{*\mu} \gamma^\alpha \xi_\mu C_\alpha + \text{h.c.} \\ p=1: & g\, \bar{\xi}^{*\mu} \left(\xi^\nu \mathfrak{C}_{\mu\nu} + \alpha_1 \Xi_{\mu\nu} C^\nu + \alpha_2 \gamma^{\alpha\beta} \xi_\mu \mathfrak{C}_{\alpha\beta} \right) + \text{h.c.} \\ p=2: & g\, \bar{\xi}^{*\mu} \gamma^\alpha \Xi^\beta{}_\mu \mathfrak{C}_{\alpha\beta} + \text{h.c.}\,, \end{cases}$$

- Half of these candidate $\boldsymbol{a_2}$'s are eliminated immediately since it is easy to see that they do get lifted to $\boldsymbol{a_1}$.

- The remaining possibilities are:

- Minimal Coupling (with $\boldsymbol{\alpha_1 = -1, \alpha_2 = 1/4}$):

$$a_2 = g\,\bar{\xi}^{*\mu}\left(\xi^\nu \mathfrak{C}_{\mu\nu} + \alpha_1 \Xi_{\mu\nu} C^\nu + \alpha_2 \gamma^{\rho\sigma}\xi_\mu \mathfrak{C}_{\rho\sigma}\right) + \text{h.c.}$$

- Gravitational Quadrupole (with $\boldsymbol{g}$ real)

$$a_2 = \left[ig\,C^{*\mu}\bar{\Xi}_{\mu\nu}\xi^\nu + \text{h.c.}\right] + \left[\tilde{g}\,\mathfrak{C}_{\mu\nu}\bar{\xi}^*_\rho \gamma^{\mu\nu\rho\alpha\beta}\Xi_{\alpha\beta} + \text{h.c.}\right]$$

- 3-Derivative Coupling

$$a_2 = -ig\,C^*_\lambda\,\bar{\Xi}_{\mu\nu}\gamma^{\lambda\mu\nu\alpha\beta}\Xi_{\alpha\beta}.$$

- Minimal coupling is ruled out, as the unambiguous term is

$$\hat{a}_1 = -2\left(g\,\bar{\chi}^{*\mu\rho}\psi_{\mu\nu\|\rho}C^{\nu} + \text{h.c.}\right) + \mathcal{Y}^{\mu\nu}\mathfrak{C}_{\mu\nu} + \dots$$

$$\hat{\beta}^{\mu} \equiv \frac{\delta}{\delta C_{\mu}}\Delta\hat{a}_1 = \left(2g\Delta\bar{\chi}^{*}_{\alpha\beta}\,\psi^{\mu\alpha\|\beta} + \text{h.c.}\right) + 2\Delta\partial_{\nu}\mathcal{Y}^{[\mu\nu]}.$$

- The ambiguity is in the cohomology of ***Γ*** : $\Gamma\tilde{a}_1 = 0$

- A lift to ***a₀*** happens if they fulfill

$$\Delta\hat{a}_1 + \Delta\tilde{a}_1 \doteq -\Gamma a_0 \doteq C_{\mu}\partial_{\nu}\mathcal{X}^{\mu\nu} + \dots$$

- But this cannot happen because it leads to a contradiction:

$$\Gamma\hat{\beta}^{\mu} = \partial^{\beta}\left[2g\,\Delta\bar{\chi}^{*}_{\alpha\beta}\,\xi^{\alpha\mu} + \text{h.c.}\right] + \partial_{\nu}\left(2\Gamma\Delta\mathcal{Y}^{[\mu\nu]}\right) = \partial_{\nu}\left(\Gamma\mathcal{X}^{\mu\nu}\right)$$

- For the gravitational quadrupole interaction, one has

$$a_1 = a_{1g} + a_{1\tilde{g}} + \tilde{a}_1,$$

$$\begin{aligned} a_{1g} &= ig\, h^{*\mu\nu}\left(\bar{\xi}_{\mu\lambda}\psi_\nu{}^\lambda + \bar{\psi}_\nu{}^\lambda\xi_{\mu\lambda} - 2\bar{\xi}^\lambda\psi_{\mu\lambda\|\nu} - 2\bar{\psi}_{\mu\lambda\|\nu}\xi^\lambda\right), \\ a_{1\tilde{g}} &= 2\tilde{g}\left(\mathfrak{C}_{\mu\nu}\bar{\chi}^*_{\rho\sigma}\gamma^{\mu\nu\rho\alpha\beta}\psi_{\alpha\beta\|}{}^\sigma - \mathfrak{h}_{\mu\nu\|}{}^\sigma\bar{\chi}^*_{\rho\sigma}\gamma^{\mu\nu\rho\alpha\beta}\xi_{\alpha\beta}\right) + \text{h.c.} \end{aligned}$$

- A lift to ***a*$_0$** exists only if the couplings are real, and satisfy

$$\tilde{g} = \tfrac{1}{8}g$$

- And the vertex is given by

$$a_0 = ig\left(\bar{\psi}_{\mu\alpha}R^{+\mu\nu\alpha\beta}\psi_{\nu\beta} + \tfrac{1}{2}\bar{\not{\psi}}_\mu \not{R}^{\mu\nu}\not{\psi}_\nu + \tfrac{1}{4}h_{\mu\nu}\bar{\psi}_{\rho\sigma\|\lambda}\gamma^{\mu\rho\sigma\alpha\beta,\nu\lambda\gamma}\psi_{\alpha\beta\|\gamma}\right)$$

- For the potential 3-derivative coupling we have

$$a_1 = -2igh^{*\sigma}_{\lambda}\left(\bar{\xi}_{\mu\nu}\gamma^{\lambda\mu\nu\alpha\beta}\psi_{\alpha\beta\|\sigma} - \text{h.c.}\right) + \tilde{a}_1$$

- This gives rise to the vertex:

$$a_0 = ig\,\bar{\psi}_{\mu\nu\|}{}^{\rho}\left(\mathfrak{h}^{+}_{\rho\sigma\|\lambda}\,\gamma^{\lambda\mu\nu\alpha\beta} + \gamma^{\lambda\mu\nu\alpha\beta}\,\mathfrak{h}^{+}_{\rho\sigma\|\lambda}\right)\psi_{\alpha\beta\|}{}^{\sigma}$$

- With the ambiguity given by

$$\Delta\tilde{a}_1 = -igR_{\mu\nu\rho\sigma}\bar{\xi}_{\lambda}\,\gamma^{\lambda\mu\nu\alpha\beta}\left(\tfrac{1}{2}\gamma^{\rho\sigma}\gamma^{\gamma\delta} - 2\gamma^{[\rho}\eta^{\sigma][\gamma}\gamma^{\delta]}\right)\Psi_{\alpha\beta|\gamma\delta}$$

- This vertex vanishes in ***D* = *4***.

To proceed, we note that we have exhausted all possible $\boldsymbol{a_2}$.

Any other possible vertex will not deform the gauge algebra.

Step 9: Abelian Vertices

- If a vertex comes from a_1 or a_0 itself, one can always choose:

$$a_0 = T^{\mu\nu} h_{\mu\nu}, \qquad \Gamma T^{\mu\nu} = 0,$$

$$\partial_\nu T^{\mu\nu} = \Delta M^\mu, \qquad \Gamma M^\mu = 0.$$

- The most generic form of the gauge-invariant current is:

$$T^{\mu\nu} = \bar{\Psi}^M \hat{\mathcal{O}}^{\mu\nu}{}_{MN} \Psi^N$$

- The current and the vertex contains at least 4 derivatives.

- If the current contains more than 6 derivatives, a_0 will always be Δ-exact modulo d . The vertex contains at most 6 derivatives.

- $T^{\mu\nu}$ can be chosen uniquely for given number of derivatives.

- The 4-derivative vertex comes with

$$T^{\mu\nu} = ig\left(\bar{\Psi}^{(\mu}{}_{\lambda|\alpha\beta}\Psi^{\nu)\lambda|\alpha\beta} + \alpha\eta^{\mu\nu}\bar{\Psi}_{\rho\sigma|\alpha\beta}\Psi^{\rho\sigma|\alpha\beta}\right)$$

- With fixed ***α*** fixed to the value of ***-1/4***.

- The vertex has the following forms

$$a_0 \;=\; ig\left(h_{\mu\nu} - \tfrac{1}{4}\eta_{\mu\nu}h'\right)\bar{\Psi}^{\mu}{}_{\lambda|\alpha\beta}\Psi^{\nu\lambda|\alpha\beta}$$

$$a_0 \approx -\tfrac{i}{8}g\,h_{\mu\nu}\bar{\Psi}_{\rho\sigma|\tau\lambda}\,\gamma^{\mu\rho\sigma\alpha\beta,\,\nu\tau\gamma}\,\Psi_{\alpha\beta|\gamma}{}^{\lambda}$$

- For ***D*** > ***4***, we have an Abelian 4-derivative vertex.

- This can be made gauge invariant up to a total derivative.

- The 5-derivative vertex corresponds to

$$T^{\mu\nu} = ig\,\bar{\Psi}^{\rho\sigma|\alpha\beta}\gamma^{(\mu}\overleftrightarrow{\partial}^{\nu)}\Psi_{\rho\sigma|\alpha\beta}$$

- The vertex also takes the following form

$$a_0 \approx \tfrac{i}{2} g\,\mathfrak{h}_{\mu\nu||\lambda}\bar{\Psi}^{\mu\tau|}{}_{\rho\sigma}\,\gamma^{\lambda\rho\sigma\alpha\beta}\,\Psi^{\nu}{}_{\tau|\alpha\beta}$$

- It is manifest that the vertex exists in ***D > 4***.

- Also, it does not deform the gauge transformations, since it is gauge invariant only up to a total derivative.

- The 6-derivative vertex has

$$T^{\mu\nu} = ig\,\bar{\Psi}^{\rho\sigma|\alpha\beta}\left(\overrightarrow{\partial}^{\mu}\overrightarrow{\partial}^{\nu} + \overleftarrow{\partial}^{\mu}\overleftarrow{\partial}^{\nu} - \eta^{\mu\nu}\overleftarrow{\partial}^{\lambda}\overrightarrow{\partial}_{\lambda}\right)\Psi_{\rho\sigma|\alpha\beta}$$

- The vertex has the following 3-curvature form

$$a_0 \approx igR_{\mu\nu\rho\sigma}\bar{\Psi}^{\rho\sigma|\alpha\beta}\Psi_{\alpha\beta}{}^{\mu\nu}$$

- This form of the vertex is strictly gauge invariant.

- The vertex exists for all $\boldsymbol{D \geq 4}$**.**

Arbitrary Spin: *s* = *n* + 1/2

- The sets of fields and antifields are:

$$\Phi^A = \{h_{\mu\nu}, C_\mu, \psi_{\mu_1\ldots\mu_n}, \xi_{\mu_1\ldots\mu_{n-1}}\}$$

$$\Phi^*_A = \{h^{*\mu\nu}, C^{*\mu}, \bar{\psi}^{*\mu_1\ldots\mu_n}, \bar{\xi}^{*\mu_1\ldots\mu_{n-1}}\}$$

- Grassmann odd bosonic ghost field C_μ .

- Grassmann-even rank-(***n-1***) fermionic ghost field $\xi_{\mu_1\ldots\mu_{n-1}}$

is *γ*-traceless. The original fermion is triply *γ*-traceless:

$$\not{\xi}_{\mu_1\ldots\mu_{n-2}} = 0, \qquad \not{\psi}^{\mu_2}{}_{\mu_2\mu_3\ldots\mu_n} = 0$$

- The vertices are exactly like the spin-5/2 case.
- The non-Abelian ones are

$$a_0 \; = \; ig\left[\bar{\psi}^{(n-2)}_{\cdots\|\,\mu\alpha} R^{+\mu\nu\alpha\beta} \psi^{(n-2)\cdots\|}{}_{\nu\beta} + \tfrac{1}{2}\,\bar{\not\psi}^{(n-2)}_{\cdots\|\,\mu}\, \not R^{\mu\nu}\, \not\psi^{(n-2)\cdots\|}{}_{\nu}\right] + \tfrac{i}{4} g\, h_{\mu\nu}\, \bar{\psi}^{(n-1)}_{\cdots\rho\sigma\|\,\lambda}\, \gamma^{\mu\rho\sigma\alpha\beta,\,\nu\lambda\gamma}\, \psi^{(n-1)\cdots}{}_{\alpha\beta\|\,\gamma}$$

$$a_0 = ig\,\bar{\psi}^{(n-1)}_{\cdots\,\mu\nu\,\|}{}^{\rho}\left(\mathfrak{h}^{+}_{\rho\sigma\|\lambda}\,\gamma^{\lambda\mu\nu\alpha\beta} + \gamma^{\lambda\mu\nu\alpha\beta}\,\mathfrak{h}^{+}_{\rho\sigma\|\lambda}\right)\psi^{(n-1)\cdots}{}_{\alpha\beta\|}{}^{\sigma}$$

- The Abelian ones are

$$p = 2n: \quad a_0 = ig\left(h_{\mu\nu} - \tfrac{1}{4}\eta_{\mu\nu}h'\right)\bar{\Psi}^{\mu}{}_{\ldots}\Psi^{\nu\,\cdots}\,,$$

$$p = 2n+1: \quad a_0 = ig\,h_{\mu\nu}\bar{\Psi}^{\cdots}\,\gamma^{(\mu}\overleftrightarrow{\partial}^{\nu)}\Psi_{\ldots}\,,$$

$$p = 2n+2: \quad a_0 = ig\,h_{\mu\nu}\bar{\Psi}^{\cdots}\left(\overrightarrow{\partial}^{\mu}\overrightarrow{\partial}^{\nu} + \overleftarrow{\partial}^{\mu}\overleftarrow{\partial}^{\nu} - \eta^{\mu\nu}\overleftarrow{\partial}^{\lambda}\overrightarrow{\partial}_{\lambda}\right)\Psi_{\ldots}$$

Second-Order Deformation

- Consistent 2nd-order deformation requires ***(S_1 , S_1)*** be ***s***-exact:

$$(S_1, S_1) = -2sS_2 = -2\Delta S_2 - 2\Gamma S_2.$$

- For Abelian vertices this antibracket is zero, so the first-order deformations always go unobstructed.

- Non-Abelian vertices are more interesting in this respect.

- The underlying assumptions are locality absence of other dynamical interacting degrees of freedom.

- The antibracket at zero antifields is required to satisfy:

$$[(S_1, S_1)]_{\Phi^*_A=0} = \Gamma N + \Delta M,$$

$$N \equiv -2\,[S_2]_{\Phi^*_A=0} \text{ and } M \equiv -2\,[S_2]_{\mathcal{C}^*_\alpha=0}.$$

- This antibracket is simple to compute:

$$[(S_1, S_1)]_{\Phi^*_A=0} = 2\left(\int a_0, \int a_1\right) \equiv \int b.$$

- Then ***b*** must satisfy:

$$b \doteq \Gamma\text{-exact} + \Delta\text{-exact}$$

- It is easy to compute ***b*** for 2-5/2-5/2 non-Abelian vertices
- ***p* = *2***:

$$b = 2ig\,T^{\mu}{}_{\nu}\left(\bar{\xi}_{\mu\lambda}\psi^{\nu\lambda} + \bar{\psi}^{\nu\lambda}\xi_{\mu\lambda} - 2\bar{\xi}^{\lambda}\psi_{\mu\lambda\|}{}^{\nu} - 2\bar{\psi}_{\mu\lambda\|}{}^{\nu}\xi^{\lambda} + \Gamma j_{\mu}{}^{\nu}\right) + \cdots$$

- ***p* = *3***:

$$b = -4ig\,T_{\mu}{}^{\nu}\left(\bar{\xi}_{\rho\sigma}\gamma^{\mu\rho\sigma\alpha\beta}\psi_{\alpha\beta\|\nu} - \bar{\psi}_{\rho\sigma\|\nu}\gamma^{\mu\rho\sigma\alpha\beta}\xi_{\alpha\beta}\right) + \cdots$$

- ***b***'s do not have the required property.
- Non-Abelian vertices are obstructed beyond the cubic order.
- Additional DoF and/or mild non-locality remove obstruction?

Remarks

- Matching with light-cone results (Metsaev '07) and those from tensionless limit of open string theory (Sagnotti-Taronna '10).
- Cubic coupling constants related by higher order consistency.
- Connection with "good" massive theory (Porrati et al '94).
- Bosonic *2-s-s* vertices (Boulanger, Leclercq, Sundell '08).
- Flat limit of Fradkin-Vasiliev vertices in (A)dS.
- Reduction of number of vertices in AdS, according to Metsaev hep-th/0612279, in contrast with the bosonic counterpart recently explored by Joung-Taronna arXiv:1311.0242 [hep-th].