Reconstructing Bulk from Boundary: clues and challenges

Ben Freivogel

GRAPPA and ITFA
Universiteit van Amsterdam

Need quantum gravity to address:

- Measure problem of eternal inflation
- Initial conditions for inflation

Need quantum gravity to address:

- Measure problem of eternal inflation
- Initial conditions for inflation
- Singular bounces

We have a theory of quantum gravity. But

Technical problems:

- Constructing landscape of vacua
- Constructing inflation in string theory
- Etc.

In principle problem:

- What quantity does string theory compute? Analog of S-matrix in asymptotically flat space, CFT correlators in AdS.
- What is the holographic description of cosmology?
- What are correct degrees of freedom for quantum gravity description of cosmology?

Two main strategies:

- Guess holographic description- dS/CFT, etc
- Learn from AdS/CFT. How is bulk information encoded in holographic description?

Which CFT degrees of freedom describe a given region of the bulk?

Understanding this would allow us to start changing asymptotics.

We have some hints.

First hint: smearing function. (Hamilton, Kabat, Lifschytz, Lowe) ...

Take simplest limit in bulk: $G_N \to 0$, classical. Model: free scalar in pure AdS spacetime.

$$(\Box + m^2)\phi = 0$$

Rules: fix theory $\to$ non-normalized modes in bulk, CFT sources turned off.

Don't know bulk state (classically, don't know which normalizable bulk solution).

Challenge: Given computer with complete CFT data, reconstruct value of bulk field at center.

In this case, we know the answer.

One entry we know in AdS/CFT dictionary: near boundary limit of bulk field $\leftrightarrow$ CFT expectation value.

Using r for radial coordinate, x^μ for boundary coordinates,

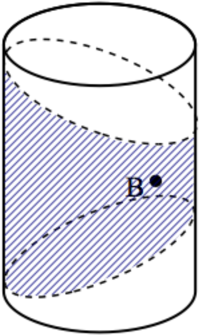
$$ds^2 = \frac{dr^2}{r^2} + r^2 dx_\mu dx^\mu$$

$$\phi_0(x^\mu) \equiv \lim_{r\to\infty} r^\Delta \phi(r, x^\mu) = \langle \mathcal{O}(x^\mu) \rangle_{\mathrm{CFT}}$$

This is the sense in which the CFT lives on the AdS boundary.

So CFT determines "boundary values" $\phi_0(x^\mu)$ of bulk field.
(Also other CFT data, but translation to bulk more difficult.)

Does $\phi_0(x^\mu)$ determine $\phi(r, x^\mu)$?
Nonstandard Cauchy problem.



Answer in this case: yes. By decomposing in modes, can explicitly find smearing function K:

$$\phi(r, x^\mu) = \int d^4y K(r, x^\mu; y^\mu)\phi_0(y^\mu)$$

Explicit formula for bulk field in terms of simple CFT quantities (integral over time and space).

If want to write the bulk field in terms of CFT operators at a single time, becomes complicated.

Challenge:

Apply more generally than vacuum AdS.

This simple procedure does not always work. (Bousso BF Leichenauer Rosenhaus Zukowski 2012)

E.g. eternal AdS-Schwarzschild black hole. Now have 2 computers, one with left CFT, one with right.
Suppose only interested in reconstructing bulk field outside horizon, in right asymptotic region- throw away left computer.

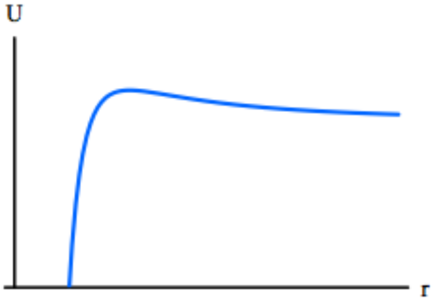
Seems same approach will work, but then sums diverge...

In fact, smearing function does not exist.

Null geodesics in bulk that never reach boundary $\rightarrow$
no continuous mapping from boundary data to bulk data.

Physical reason: can construct solutions arbitrarily well localized along any null geodesic $\rightarrow$
arbitrarily small boundary imprint.

Region with trapped null geodesics: $r < 3GM$ (in 4d).



Conclusion: Cannot reconstruct bulk physics inside $r = 3GM$ from ϕ_0

Local CFT operators $\langle \mathcal{O}(x^\mu) \rangle_{\rm CFT}$ do not determine bulk solution near horizon.

Need to work harder to reconstruct bulk: nonlocal CFT operators.

In fact, cannot even reconstruct bulk for $r > 3GM$ (Leichenauer Rosenhaus 13)

Problem can arise even for bulk solutions without horizons, as long as they have trapped null geodesics. (Leichenauer Rosenhaus 13)

Field theory question: Hiding information from local operators for all time?

If

$$\langle\psi_1|\mathcal{O}\mathcal{O}\ldots\mathcal{O}|\psi_1\rangle = \langle\psi_2|\mathcal{O}\mathcal{O}\ldots\mathcal{O}|\psi_2\rangle$$

for all local $\mathcal{O}$ for all time, must $|\psi_1\rangle = |\psi_2\rangle$?

Flat space black holes: knowing solution for large r, at all time, does not determine solution near horizon $\rightarrow$
Not possible to "evolve back" outgoing Hawking photons to find state of radiation field near horizon.

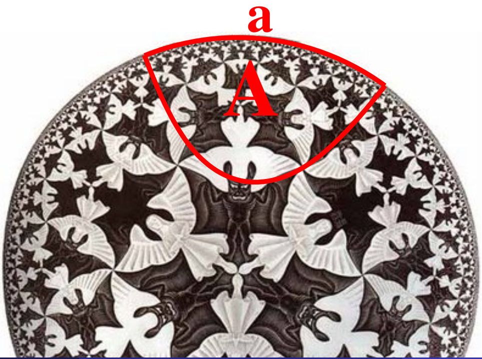
We have more to learn from this kind of simple reconstruction problem- apply to asymptotically de Sitter spacetime.

To fully understand what AdS/CFT has to say about firewalls and near horizon region, need to go beyond local operators.

2nd clue: hints that geometric subregions of CFT correspond to geometric subregions of bulk.

Simplest example: Take CFT on $S^3 \times t$ in vacuum state. At some time, divide CFT in half into Northern Hemisphere (region A) and Southern Hemisphere.

Surprisingly simple subregion correspondence:
Northern CFT $\leftrightarrow$ Northern half of bulk. (Casini Huerta Myers 2011)



More generally: Given spatial region Σ in CFT, what region of bulk does it describe?

Only two natural ways to associate a bulk region to Σ:

- Bulk region bounded by minimal area surface ending on $\partial\Sigma$.
- Bulk region causally connected to "boundary diamond" defined by Σ.

For a bulk region bounded by some surface, expect entanglement entropy

$$S_{\mathrm{ent}} = \frac{A}{4G} + \dots$$

(Myers Pourhasan Smolkin) (Bianchi Myers) and earlier work

Ryu-Takayanagi proposal, justified recently by (Lewkowycz Maldacena):

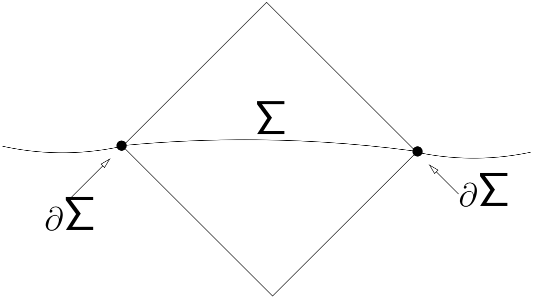
$$S_{\mathrm{ent}}(\Sigma) = \frac{1}{4G}\,(\text{Area of minimal surface ending on } \partial\Sigma)$$

Success of this proposal suggests:
CFT on $\Sigma \leftrightarrow$ Bulk region bounded by minimal surface.

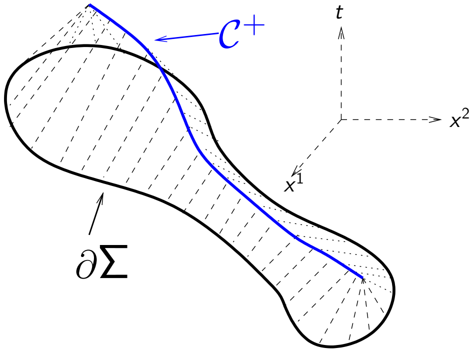
This conclusion may be too strong- perhaps minimal surface is just computing saddle point.

Another, smaller bulk region can be associated to boundary region: causally connected region.

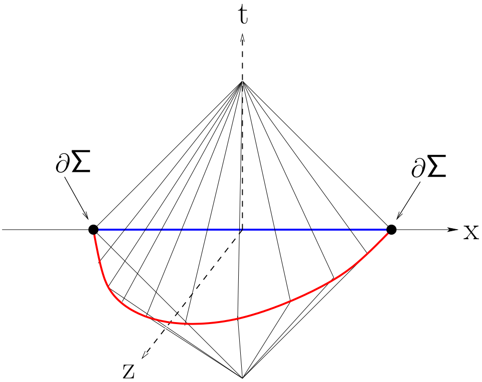
Data on spatial region Σ in CFT specifies a spacetime region in CFT. In 1+1 dimensions:



In 2+1 boundary dimensions:



Now extend into bulk: all bulk points causally connected to boundary region:



Identified by (Bousso Leichenauer Rosenhaus) as natural bulk region dual to CFT on Σ.

(Hubeny Rangamani) focused on "causal holographic information" χ,

$$\chi \equiv \frac{A}{4G}$$

Surely χ computes something about CFT on Σ. What CFT quantity is it?

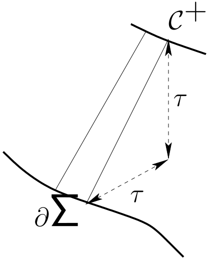
Clue: can compute χ and compare to S. Both are quadratically UV divergent.
Focus on universal logarithmically divergent term: independent of state and regulator.

$$S = \cdots + \log\frac{1}{\epsilon}\left[-\frac{a}{720\pi}\int_{\partial\Sigma} R_{\partial\Sigma} + \frac{c}{720\pi}\int_{\partial\Sigma}\left(-K^{a}_{\mu\nu}K^{a\mu\nu} + \frac{1}{2}K_a K^a\right)\right] + \ldots$$

$$\chi = \log\frac{1}{\epsilon}\left[-\frac{a}{720\pi}\int d^2\xi\sqrt{\tilde{g}}R_{\partial\Sigma} + \frac{c}{720\pi}\int d^2\xi\sqrt{\tilde{g}}\left(R_{\partial\Sigma} + 2\left(\frac{1}{\tau^2} + \frac{K}{\tau}\right)\right)\right.$$

(BF Mosk)

Formula for S depends on local geometric quantities on $\partial\Sigma$.
χ depends on quantity τ, which depends nonlocally on geometry of Σ:



τ is local "height" of causal diamond.

What does χ compute in CFT?
From bulk point of view, χ is entanglement entropy of causally connected region:
Bulk region that can be directly probed by local, stimulus-response measurements beginning and ending on boundary region.

Full density matrix ρ_Σ contains information inaccessible to any single CFT observer within boundary region.

Define $\tilde{\rho}$ by throwing away this information.

$$\chi = S(\tilde{\rho})$$

How precisely to define $\tilde{\rho}$?

Maximum entropy density matrix that correctly reproduces observations of any single CFT observer within boundary region. (Balasubramanian Chowdury Czech de Boer)

Phenomenology: even for simple boundary diamonds, generally χ differs from S. Natural way to throw away information:

$$\tilde{\rho} = \sum_i \mathcal{P}_i \rho \mathcal{P}_i$$

for $\mathcal{P}_i$ a set of projection operators with $\sum_i \mathcal{P}_i = \mathcal{I}$.

Erasing off-diagonal elements of ρ in some basis.

What basis?

Basis picked out by evolution operator U evolving from bottom to top of causal diamond.

In cases with symmetry, energy eigenstates of Rindler-type Hamiltonian in region.

Agrees with phenomenology: $S = \chi$ when density matrix is thermal.

Conclusions:

- To address important cosmological questions, need to understand quantum gravity in more general spacetimes.
- Fortunately, a number of clues from AdS/CFT to lead us in the right direction
- Tractable calculations that will help us understand holography better.