Effective Theory of a Light Dilaton

Zackaria Chacko
University of Maryland, College Park

Roberto Franceschini
Rashmish Mishra
Daniel Stolarski
Introduction
Strong conformal dynamics plays an important role in many theories of electroweak symmetry breaking.

Some Examples

In theories of technicolor, and of the Higgs as a composite pseudo-Nambu-Goldstone boson (pNGB), strong conformal dynamics can help separate the electroweak scale from the flavor scale, allowing flavor constraints to be satisfied.

The AdS/CFT correspondence can be used to relate Randall-Sundrum models to theories where the hierarchy between the Planck and weak scales is realized through strong conformal dynamics.
In theories where an exact conformal symmetry is spontaneously broken, the low energy effective theory contains a massless scalar, the dilaton.

The dilaton can be thought of as the NGB associated with the breaking of conformal invariance. (Just 1 NGB, not 5, because conformal invariance is a space-time symmetry.)

The form of the dilaton couplings is fixed by the requirement that the symmetry be realized non-linearly.

AdS/CFT identifies radion in Randall-Sundrum setup with dilaton.

However, in the class of theories of interest for electroweak symmetry breaking, conformal symmetry is explicitly broken by nearly marginal operators that grow in the infrared to become large at the breaking scale. ➔ No reason to expect a light dilaton in the effective theory.
In this talk, I will show that in a specific class of theories, where the operator that breaks conformal symmetry remains close to marginal until the breaking scale, the dilaton mass can naturally lie below the scale of strong dynamics. (Rattazzi)

However, in general, this condition is not satisfied in the theories most relevant for electroweak symmetry breaking. Nevertheless, a light dilaton in these theories is only associated with mild tuning.

In this framework, corrections to the form of dilaton couplings from conformal symmetry violating effects are suppressed by the square of the ratio of the dilaton mass to the strong coupling scale, and are under good theoretical control if the dilaton is light.

I will show that the results for the radion in RS models match those of the dilaton, and provide a holographic interpretation.

Finally, I will consider the possibility that the 125 GeV resonance is a dilaton, rather than the SM Higgs.
The Mass of the Dilaton
Consider a theory where conformal symmetry is spontaneously broken. Then the low energy effective theory contains a dilaton field $\sigma(x)$.

Below the breaking scale the symmetry is realized non-linearly. Under scale transformations,

$$x^\mu \rightarrow x'^\mu = e^{-\omega} x^\mu$$

the dilaton transforms as

$$\sigma(x) \rightarrow \sigma'(x') = \sigma(x) + \omega f$$

where $f$ is the symmetry breaking scale.
It is convenient to define the object $\chi(x)$, which transforms linearly under scale transformations.

$$\chi(x) = f e^{\sigma(x)/f}$$

Under the scale transformation

$$x^\mu \rightarrow x'^\mu = e^{-\omega} x^\mu$$

$$\chi(x) \rightarrow \chi'(x') = e^\omega \chi(x)$$

The low energy effective theory will in general contain all terms consistent with this transformation.
What terms does the Lagrangian contain?

The symmetry allows derivative terms of the form

\[ \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + \frac{c}{\chi^4} (\partial_\mu \chi \partial^\mu \chi)^2 + \ldots \]

However, crucially, a non-derivative term is also allowed.

\[ V(\chi) = \kappa_0 \chi^4 \]

The effect of this term is to drive \( f \) to zero, corresponding to unbroken conformal symmetry, if the coefficient \( \kappa_0 \) is positive. If \( \kappa_0 \) is negative, \( f \) is driven to infinity, and conformal symmetry is never realized.

Only if \( \kappa_0 \) is identically zero is the symmetry spontaneously broken. The potential then vanishes and there is a massless dilaton. However, in general setting \( \kappa_0 \) to zero is associated with tuning, since there is no symmetry reason for it to vanish.
The situation changes if conformal symmetry violating effects are present. Add to the theory an operator $O(x)$ of dimension $\Delta$ close to 4.

$$\mathcal{L} = \mathcal{L}_{\text{CFT}} + \lambda_O \mathcal{O}(x)$$

Under scale transformations,

$$x^\mu \rightarrow x'^\mu = e^{-\omega} x^\mu$$

$$\mathcal{O}(x) \rightarrow \mathcal{O}'(x') = e^{\omega \Delta} \mathcal{O}(x)$$

Define a dimensionless coupling constant,

$$\hat{\lambda}_O = \lambda_O \mu^{\Delta - 4}$$

The operator $O(x)$ is normalized such that $\lambda \sim 1$ corresponds to strong coupling. For small $\lambda \ll 1$, it satisfies the RG equation,

$$\frac{d \log \hat{\lambda}_O}{d \log \mu} = -(4 - \Delta)$$
We can determine how $\lambda$ appears in the low energy theory by promoting it to a spurion. For small $\lambda$ the form of the UV theory is invariant under the following transformation.

\[
\begin{align*}
x'^\mu & \rightarrow x'^\mu = e^{-\omega} x'^\mu \\
\lambda' & \rightarrow \lambda' = e^{(4-\Delta)\omega} \lambda
\end{align*}
\]

To leading order in $\lambda$ the form of the potential now becomes

\[
V(\chi) = \kappa_0 \chi^4 - \kappa_1 \lambda \chi^\Delta
\]

(Rattazzi & Zaffaroni)

The dilaton potential admits a minimum at

\[
f(\Delta-4) = \frac{4\kappa_0}{\kappa_1 \lambda \Delta}
\]

From this, we find the dilaton mass at the minimum,

\[
m_\sigma^2 = \kappa_1 \lambda \Delta (4 - \Delta) f^{\Delta-2} = 4\kappa_0 (4 - \Delta) f^2
\]

The dilaton mass is suppressed if the operator that breaks conformal symmetry is marginal! (Goldberger, Grinstein & Skiba)
Since $\Delta$ is expected to be close to 4 in theories that address the flavor or hierarchy problems, potentially a very interesting result! ➔ A new light state below the strong coupling scale.

In technicolor frameworks, the new state observed by the LHC at 125 GeV could be the dilaton!

In theories where the Higgs is a pNGB, this result predicts the existence of an additional state below the strong coupling scale.

Unfortunately, the analysis that led up to this conclusion is only valid at small $\lambda$, corresponding to weak coupling. To validate this result, must establish it at strong coupling.

Our approach will be to assume small (perturbative) $\lambda$, but work to all orders in this parameter. Check if the result survives when $\lambda \to 1$, its strong coupling value.
Working to all orders in $\lambda$ involves incorporating 4 distinct effects.

- In writing down the Lagrangian, did not take into account the breaking of scale invariance by the regulator. Must include this.

- In determining the vacuum structure used the potential, not the effective potential. This needs to be accounted for.

- Need to include terms with all powers of $\lambda$ in the Lagrangian. Setting $\epsilon = 4 - \Delta$, the potential becomes

$$V(\chi) = \kappa_0 \chi^4 - \sum_{n=1}^{\infty} \kappa_n \lambda^n \chi^{(4-n\epsilon)}$$

- As $\lambda$ approaches strong coupling, its RG evolution is affected. The RG for $\lambda$ now takes the more general form

$$\frac{d \log \hat{\lambda}_\mathcal{O}}{d \log \mu} = -g(\hat{\lambda}_\mathcal{O})$$

where $g(\lambda)$ is a polynomial in $\lambda$. The constant term in this polynomial is $\epsilon = (4 - \Delta)$. 
Of these 4 effects, the first 3 do not alter the conclusions of the naive small $\lambda$ analysis. The underlying reason is that in these 3 cases, the leading effect is of order $\lambda$ while the corrections begin at order $\lambda^2$ and are therefore at most of the same size.

The 4th effect is qualitatively different. Consider again the RGE

$$\frac{d \log \hat{\lambda}_\mathcal{O}}{d \log \mu} = -g(\hat{\lambda}_\mathcal{O}) \quad g(\hat{\lambda}_\mathcal{O}) = \sum_{n=0}^{\infty} c_n \hat{\lambda}_\mathcal{O}^n$$

The leading order term in the polynomial $g(\lambda)$ is $(4 - \Delta) = \epsilon \ll 1$, while the corrections begin at order $\lambda$. Even before strong coupling is reached the higher order terms dominate, and their effects can alter the conclusions of the naïve small $\lambda$ analysis.
The form of the UV theory is invariant if $\lambda$ is promoted to a spurion that transforms under scale transformations as

$$
\hat{\lambda}_\mathcal{O}(\mu) \rightarrow \hat{\lambda}'_\mathcal{O}(\mu) = \hat{\lambda}_\mathcal{O}(\mu e^{-\omega})
$$

Then the combination

$$
\overline{\Omega}(\hat{\lambda}_\mathcal{O}, \chi/\mu) = \hat{\lambda}_\mathcal{O} \left( \frac{\chi}{\mu} \right)^{-g(\hat{\lambda}_\mathcal{O})}
$$

is invariant under infinitesimal scale and RG transformations.

By requiring invariance under spurious scale transformations, we can obtain the low energy effective theory for the dilaton. To leading order in $\Omega$, the potential takes the form

$$
V(\chi) = \frac{\chi^4}{4!} \left( \kappa_0 - \kappa_1 \overline{\Omega} \right)
$$

Exactly the same form as before, but with $\epsilon$ replaced by $g(\lambda)$. 
The dilaton mass is given by

\[ m^2_\sigma = 4 \frac{\kappa_0}{4!} g(\hat{\lambda}_\phi) f^2 \]

It is the scaling dimension of \( O(x) \) at the breaking scale that determines dilaton mass, not scaling dimension at fixed point.

\[ g(\hat{\lambda}_\phi) = \sum_{n=0}^{\infty} c_n \hat{\lambda}^n_\phi \]

To obtain a light dilaton, it is not sufficient that \( c_0 = \epsilon \ll 1 \). Require \( g(\lambda) \ll 1 \) at the breaking scale.

This is equivalent to requiring that not just \( c_0 \) but all the \( c_n \ll 1 \).

Although this can happen naturally in some cases, for example in theories with fixed lines, this criterion is not expected to be satisfied in most theories of interest for EWSB.

The presence of a light dilaton is associated with tuning!
However all is not lost. Consider again the dilaton potential,

\[ V(\chi) = \frac{\chi^4}{4!} (\kappa_0 - \kappa_1 \Omega) \]

If the conformal field theory is such that the parameter \( \kappa_0 \) is smaller than its natural strong coupling value by some factor `\( Q \)`, \( Q > 1 \), then the potential is minimized for \( \lambda \sim 1/Q \).

\[ m^2 = \frac{4 \kappa_0}{4!} g(\hat{\lambda} \phi) f^2 \]

For \( \epsilon > 1/Q \), \( g(\lambda) \) is dominated by the \( c_0 \) term and is of order \( \epsilon \). Then the dilaton mass is suppressed by a factor \( (\sqrt{\epsilon} / Q) \).

For \( \epsilon < 1/Q \), \( g(\lambda) \) is of order \( 1/Q \) at the breaking scale and it can be seen that the dilaton mass is suppressed by the same factor.

Now small values of \( \kappa_0 \) are associated with tuning (coincidence problem), the tuning scaling as \( Q \). This analysis shows that a dilaton mass a factor of 5 below the strong coupling scale is only tuned at the level of 1/5 (20%). The tuning is mild!
This analysis assumed $\lambda \ll 1$. To validate this conclusion, we must relax this, and incorporate the effects we have neglected.

Consider first the theory in the absence of conformal symmetry violation, $\lambda = 0$. The only term allowed in the potential is

$$V(\chi) = \frac{Z^2 \kappa_0}{4!} \chi^4$$

However, this term does not respect conformal symmetry at the quantum level. It is however, conformally invariant if we work in a scheme where renormalization scale depends on $\chi, \mu \to \mu \chi / f$.

This is equivalent to replacing $\mu$ with $\mu \chi / f$ in the effective action.

We can go over to a more conventional renormalization scheme. At one loop, this corresponds to RG evolution from $\mu \chi / f$ to $\mu$.

$$V(\chi) = \left\{ Z^2 \kappa_0 - \frac{d(Z^2 \kappa_0)}{d \log \mu} \log \left( \frac{\chi}{f} \right) \right\} \frac{\chi^4}{4!}$$
From this, we can obtain the effective action at one loop order,

\[ V_{\text{eff}}(\chi_{cl}) = \left\{ \kappa_0 - \frac{3\kappa_0^2}{32\pi^2} \left[ \log \left( \frac{\mu^2}{\frac{1}{2}\kappa_0 f^2} \right) - \frac{1}{2} \right] \right\} \frac{\chi_{cl}^4}{4!} \]

The coefficient of the \( \chi^4 \) term is RG invariant. This result confirms that there is no stable vacuum in the absence of tuning.

Since the theory is strongly coupled, it is preferable to work beyond one loop. Going from \( \mu \chi/f \) to \( \mu \), the potential becomes

\[ V(\chi) = \left\{ \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{d^n(Z^2\kappa_0)}{d \log \mu^n} \left[ \log \left( \frac{\chi}{f} \right) \right]^n \right\} \frac{\chi^4}{4!} \]

Rather than work with this directly, employ the Callan-Symanzik equation for the effective potential,

\[ \left\{ \mu \frac{\partial}{\partial \mu} + \beta_i \frac{\partial}{\partial \bar{g}_i} - \gamma \chi_{cl} \frac{\partial}{\partial \chi_{cl}} \right\} V_{\text{eff}}(\chi_{cl}, \bar{g}_i, \mu) = 0 \]

For a conformal theory, \( \beta = 0 \). We also have \( \gamma = 0 \). Then, again

\[ V_{\text{eff}}(\chi_{cl}) = \frac{\hat{\kappa}_0}{4!} \chi_{cl}^4 \]
The next step is to incorporate conformal symmetry violation.

\[ \mathcal{L} = \mathcal{L}_{\text{CFT}} + \lambda O(\chi) \]

By integrating the RG, we can construct an object \( \Omega(\lambda, \chi) \) which is invariant under (spurious) scale transformations, and also under changes in the renormalization scale \( \mu \).

\[ g(\hat{\lambda}_0) = \sum_{n=0}^{\infty} c_n \hat{\lambda}_0^n \]

\( \Omega(\lambda, \chi) \) takes on different forms in various limits.

When the \( c_0 = \epsilon \) term dominates the RG,

\[ \Omega(\overline{\lambda}_0, \chi) = \overline{\lambda}_0 \chi^{-\epsilon} \]

When the \( c_1 \) term dominates the RG,

\[ \Omega(\overline{\lambda}_0, \chi) = \frac{\overline{\lambda}_0}{1 + c_1 \overline{\lambda}_0 \log \chi} \]

RG invariant that depends on \( \lambda, \mu \)
In the scheme with renormalization scale $\mu \chi / f$, the potential is

$$V(\chi) = \frac{Z^2 \chi^4}{4!} \left[ \kappa_0 - \sum_{n=1}^{\infty} \kappa_n \Omega^n (\bar{\lambda}_0, \sqrt{Z} \chi) \right]$$

Again, rather than work with this directly, use Callan-Symanzik equation for the effective potential.

$$\left\{ \mu \frac{\partial}{\partial \mu} + \beta_i \frac{\partial}{\partial \bar{g}_i} - \gamma_{\phi_\alpha} \phi_\alpha \frac{\partial}{\partial \phi_\alpha} \right\} V_{\text{eff}}(\phi_\alpha, \bar{g}_i, \mu) = 0$$

The solution is of the form

$$V_{\text{eff}}(\chi_{cl}) = \frac{1}{4!} \chi_{cl}^4 \left\{ \hat{\kappa}_0 - \mathcal{F}[\Omega(\bar{\lambda}_0, \chi_{cl})] \right\}$$

The functional form of $\mathcal{F}(\Omega)$ cannot be obtained from symmetry considerations. However for $\lambda < 1$, $\mathcal{F}(\Omega)$ can be calculated in perturbation theory from potential. A self-consistent minimum can be found if $\kappa_0$ lies below its natural strong coupling value.

The dilaton mass at the minimum scales like $\kappa_0$, just as expected from our simplified analysis. Tuning is linear in the dilaton mass.
Corrections to the Dilaton Couplings
Consider a `conformal SM’, where all the SM fields are composites of a strongly interacting conformal sector. Neglect dilaton-Higgs mixing (suppressed if the Higgs is a pNGB).

Since UV theory is conformally invariant, dilaton couples so as to restore this symmetry to the interactions in the low energy effective field theory.

The form of the dilaton interactions with the SM fields is then completely predicted. (Goldberger, Grinstein & Skiba)

How do conformal symmetry violating effects alter the form of the dilaton couplings?
Consider the dilaton coupling to the W gauge bosons. Since the leading effect which breaks conformal invariance is the gauge boson mass term, the dilaton couples to compensate for this.

\[
\left( \frac{\chi}{f} \right)^2 m_W^2 W_\mu^+ W_\mu^- \quad \longrightarrow \quad 2\sigma \frac{m_W^2}{f} W_\mu^+ W_\mu^-
\]

When conformal symmetry violating effects are incorporated, the dilaton coupling generalizes to

\[
\left( \frac{\chi}{f} \right)^2 \left[ 1 + \sum_{n=1}^{\infty} \alpha_{W,n} \lambda^n \chi(-n\epsilon) \right] m_W^2 W_\mu^+ W_\mu^- \quad \text{more generally} \quad \Omega^n(\lambda, \chi)
\]

Expanding this out we get

\[
2\sigma \frac{m_W^2}{f} \left[ 1 + c_W \epsilon + \ldots \right] W_\mu^+ W_\mu^-
\]

where \(c_W\) is of order \(\lambda\). Then correction to the dilaton coupling is of order \(\epsilon \lambda\), which is the square of the dilaton mass over the strong coupling scale. Other effects scale the same way.
Next consider the dilaton coupling to the photon. Now the leading effect which breaks conformal invariance is the running of the electromagnetic gauge coupling

\[ \frac{\frac{d}{d \log \mu} \frac{1}{g^2}}{b_<} = \frac{b_<}{8\pi^2} \]

This leads to the dilaton coupling

\[ \frac{b_<}{32\pi^2} \frac{\chi}{f} F_{\mu\nu} F^{\mu\nu} \rightarrow \frac{b_<}{32\pi^2} \frac{\sigma}{f} F_{\mu\nu} F^{\mu\nu} \]

This is loop suppressed and small. Incorporating symmetry violation

\[ - \frac{1}{4\hat{g}^2} \left[ 1 + \sum_{n=1}^{\infty} \alpha_{A,n} \lambda_{O}^{n} \chi^{(-n\epsilon)} \right] F_{\mu\nu} F^{\mu\nu} \rightarrow \frac{\sigma}{f} \left[ \frac{b_<}{32\pi^2} + \frac{c_A}{4g^2} \epsilon + \ldots \right] F_{\mu\nu} F^{\mu\nu} \]

where \( c_A \) is of order \( \lambda \). Then correction to the dilaton coupling again suppressed by the square of the dilaton mass over the strong coupling scale. Nevertheless, for realistic values of parameters the conformal symmetry violating contribution can dominate!
The Holographic Viewpoint
The AdS/CFT correspondence allows theories where an exact or approximate conformal symmetry is spontaneously broken to be realized as Randall-Sundrum models in warped extra dimensions.

The presence of an IR brane in the RS scenario is associated with the spontaneous breaking of conformal symmetry at the IR scale.

In this framework, the dilaton is identified with the dynamical scalar field associated with fluctuations in the spacing between the two branes, the radion.

We would like to understand how our results for the dilaton emerge from the holographic viewpoint. (Seem to contradict the conventional wisdom that the radion in RS is naturally light.)
The first step is to obtain the effective potential for the radion in the absence of a mechanism that stabilizes the brane spacing.

The action for the RS model is of the form,

\[ S_{GR}^{5D} = \int d^4 x \int_{-\pi}^{\pi} d\theta \left[ \sqrt{G} \left( -2M_5^3 R[G] - \Lambda_b \right) - \sqrt{-G_h} \delta(\theta) T_h - \sqrt{-G_v} \delta(\theta - \pi) T_v \right] \]

- bulk cosmological constant
- brane tensions

The RS metric can be written as,

\[ ds^2 = e^{-2kr_c|\theta|} \eta_{\mu\nu}(x) dx^\mu dx^\nu - r_c^2 d\theta^2, \quad -\pi \leq \theta \leq \pi \]

To parametrize the radion we promote the undetermined constant \( r_c \) which represents the brane spacing to a field, \( r_c \rightarrow r(x) \).
Substituting this into the action we obtain the four dimensional effective theory for the radion field,

\[ \int d^4x \sqrt{-g} \left( \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) \right) \]

The canonically normalized radion field \( \varphi(x) \) is related to \( r(x) \) as,

\[ \varphi(x) = F e^{-k \pi r(x)} \quad F = \sqrt{24M_5^3/k}. \]

The potential \( V(\varphi) \) has the form,

\[ V_{GR}(\varphi) = \frac{\varphi^4}{F^4} \left( T\nu - \frac{\Lambda_b}{k} \right) \]

This has exactly the same form as the potential for the dilaton in the absence of conformal symmetry violating effects. As in the dilaton case, tuning is required to obtain a stable minimum. Here the brane and bulk cosmological constants must be balanced.
In order to stabilize the brane spacing, we introduce a scalar field sourced on the two branes. (Goldberger & Wise)

The action for the Goldberger-Wise (GW) scalar $\Phi$ has the form,

$$S_{GW} = \int d^4x \, d\theta \left[ \sqrt{G} \left( \frac{1}{2} G^{AB} \partial_A \Phi \partial_B \Phi - V_b(\Phi) \right) - \sum_{i=v,h} \delta(\theta - \theta_i) \sqrt{-G_i} V_i(\Phi) \right]$$

The bulk potential chosen for the GW field generally consists only of a mass term, $m^2 \Phi^2$. The mass is kept slightly small in units of $k$, the inverse curvature, in order to generate a large hierarchy between the UV and IR scales.

However, in general there is no symmetry that forbids higher powers of the scalar field in the potential $\Phi^3$, $\Phi^4$ etc. If the detuning of the IR brane is significant, these terms will dominate over $m^2 \Phi^2$ in the IR since the mass parameter is small.
Since the radion wave function is localized towards the infrared, to get the correct physics it is necessary to solve the system keeping higher powers of $\Phi$ in the bulk potential.

However, the problem is then non-linear, even in the limit that the gravitational back-reaction is neglected.

Nevertheless, an approximate analytical solution can be found, using the methods of singular perturbation theory (boundary layer theory).

The solution for $\Phi$ is characterized by the formation of a boundary layer near the location of the IR brane.
Consider a differential equation, with a small parameter $\varepsilon$.

$$\varepsilon \frac{d^2 y}{dx^2} - \frac{dy}{dx} + y = 0$$

This has a regular solution near $y = A \, e^x$, and also a singular solution $y = B \, e^{x/\varepsilon} + C$. The singular solution is needed to satisfy the two boundary conditions.

For general boundary conditions, $y(x)$ will take the form of the regular solution, except in a narrow strip of thickness $\varepsilon$ near one of the boundaries where the singular solution dominates.

To obtain $y(x)$ combine the regular and boundary layer solutions.
Consider the differential equation for the GW scalar.

\[ \partial^2_\theta \Phi - 4kr_c \partial_\theta \Phi - r^2_c V'_b(\Phi) = 0 \]

In the case of a large hierarchy, \(1/(kr_c) \ll 1\) is a small parameter. The regular solution is valid everywhere, except in a narrow strip of width \(1/(kr_c)\) near the IR brane where a boundary layer forms.

**OR:**

\[ \frac{d\Phi}{d\theta} = -\frac{r_c}{4k} V'_b(\Phi) \quad (0 \leq \theta \lesssim \pi - \epsilon) \]

**BR:**

\[ \frac{d^2\Phi}{d\theta^2} = 4kr_c \frac{d\Phi}{d\theta} \quad (\pi - \epsilon \lesssim \theta \leq \pi) \]

The form of the boundary layer solution is independent of the bulk potential \(V_b(\theta)\).

\[ \Phi_{BR}(\theta) = -\frac{k^{3/2}}{4} e^{4kr_c(\theta-\pi)} + C \]

The integration constant is used to match to the regular solution at the interface.
The potential for the GW scalar in the bulk has the general form,

\[ V_b(\Phi) = \frac{1}{2} m^2 \Phi^2 + \frac{1}{3!} \eta \Phi^3 + \frac{1}{4!} \zeta \Phi^4 + \ldots \]

Then, away from the IR brane the differential equation for \( \Phi \) is

\[ \frac{d \log \Phi}{d (kr_c \theta)} = - \frac{m^2}{4k^2} - \frac{\eta}{8\sqrt{k}} \frac{\Phi}{k^{3/2}} - \frac{\zeta}{24} \frac{\Phi^2}{k^3} + \ldots \]

In the AdS/CFT correspondence, introducing the GW scalar is equivalent to adding to the CFT an operator \( O(x) \) which grows in the IR and breaks the CFT.

In the semi-classical approximation, the value of the scalar field \( \Phi(\theta) \) is identified with the coefficient \( \lambda(\mu) \) of the operator \( O(x) \).

Comparing the differential equation for \( \Phi(\theta) \) with RGE for \( \lambda(\mu) \), we have perfect agreement. Notice that the self-interaction terms for \( \Phi \) are required to match the anomalous dimension of \( O(x) \).

\[ \frac{d \log \hat{\lambda}}{d \log \mu} = -(4 - \Delta) - c_1 \hat{\lambda} - c_2 \hat{\lambda}^2 \]
To understand the effect of the self-interaction terms, keeping only the $\Phi^3$ term in the potential, we solve for the GW scalar,

$$\Phi(\theta) = -\frac{k^{3/2}}{4} \alpha e^{4kr_c(\theta-\pi)} + \frac{k^{3/2}v}{1 + \xi kr_c \theta}$$

Then, by integrating out the GW field in the radion background, we obtain the potential for the radion

$$V(\varphi) \approx \varphi^4 \left[ \tau + \frac{z}{1 + \xi kr_c \pi} \left( \frac{\varphi}{ke^{-kr_c \pi}} \right)^{\frac{\xi}{1 + \xi kr_c \pi}} \right]$$

This is of the same form as the result from the CFT side of the correspondence.

$$V(\chi) = \frac{\chi^4}{4!} \left[ \kappa_0 - \kappa_1 \hat{\lambda} \left( \frac{\chi}{f} \right)^{-c_1 \hat{\lambda}} \right]$$

The mass of the dilaton scales as the detuning of the IR brane tension. Its natural size is of order the Kaluza-Klein mass scale.
Stabilizing the brane spacing leads to corrections to the radion couplings to SM fields.

These emerge from direct couplings of the GW scalar to the SM.

Focus on the case when all SM fields are on IR brane. For W and Z

\[
\mathcal{L} \supset \delta(\theta - \pi) \sqrt{-G_v} \left[ \left( 1 + \beta_W \frac{\Phi}{k^{3/2}} \right) G^\mu_v D_\mu H^\dagger D_v H \right]
\]

Since the VEV of the GW field is a function of the brane spacing,

\[
\Phi(\pi) \rightarrow \Phi(\pi) \left( 1 + \frac{\partial_\varphi \Phi(\pi)}{\Phi(\pi)} \tilde{\varphi} \right)
\]

This leads to a correction to the coupling of the radion to W and Z. This correction is suppressed by the square of the ratio of the radion mass to the Kaluza-Klein scale.
Is the 125 GeV Resonance a Dilaton, Rather than a Higgs?
Why is it easy to mistake the dilaton for a Higgs?

Consider the SM in the **classical** limit, with the parameters in the Higgs potential set to **zero**.

In this limit the SM exhibits conformal symmetry (at the classical level). Also, any value of the Higgs VEV constitutes a minimum.

For any non-zero value of the Higgs VEV, the conformal symmetry is spontaneously broken. Therefore, in this limit, the SM Higgs is itself a dilaton, its couplings determined by the non-linearly realized conformal symmetry!
It follows from this that the interactions of the Higgs that differ at lowest order from those of a dilaton are

- the couplings to gluons and photons, which are generated at loop level in the SM (quantum effect)

- the Higgs trilinear and quartic self-interaction terms, which depend on the form of the potential for the Higgs doublet

A determination of these couplings would be particularly useful in distinguishing the SM Higgs from a dilaton.

Complication is that in many realistic models where EWSB arises from strong conformal dynamics, the corresponding couplings of the dilaton also arise from conformal symmetry violating effects.
Focus on the case when (3\textsuperscript{rd} generation) fermions are composite.

The SM gauge bosons could be elementary or composite.

This class of theories can be realized as RS models.
Introduce the ratio $\eta$, which parametrizes the strength of the dilaton couplings relative to those of the SM Higgs.

$$\eta_{XX} \equiv \left( \frac{g_{\sigma XX}}{g_{hXX}} \right)^2$$
The dilaton couplings to the SM fields in the case of composite fermions are parametrized by the three parameters $\xi, \psi, \phi$.

$\psi$ controls dilaton couplings to gluons, and $\phi$ to photons.

In the case when SM gauge bosons are composite, corresponding to all the SM fields on the IR brane in RS, $\psi$ and $\phi$ are predicted:

$$\psi \simeq 132, \quad \phi \simeq 2.4$$

However, prediction does not survive symmetry violating effects.
The dilaton is a good fit to the data, comparable to SM Higgs.

However, given the range over which $\xi$, $\psi$, and $\phi$ can vary, the best fit point is suspiciously close to the SM.
Why are the parameters forced towards the SM?

The Tevatron is seeing associated production, $\xi \approx 1$.

Then to match the number of ZZ events at LHC, $\psi \approx 1$.

Then, for agreement with number of $\gamma\gamma$ events at LHC, $\phi \approx 1$.

We could have reached the same conclusion without Tevatron data, by using $\gamma\gamma +$ dijet data from the LHC.

All these conspire to shrink the dilaton parameter space towards that of the SM Higgs.
Conclusions
In theories where the operator that breaks conformal symmetry remains close to marginal until the breaking scale, the dilaton mass can naturally lie below the scale of strong dynamics.

However, in general, this condition is not satisfied in the theories most relevant for electroweak symmetry breaking.

Nevertheless, a light dilaton in these theories is only associated with modest tuning.

In this framework, corrections to the form of dilaton couplings from conformal symmetry violating effects are suppressed by the square of the dilaton mass over the strong coupling scale, and are under good theoretical control (if the dilaton is light).

In the case of dilaton couplings to marginal operators, conformal symmetry violating effects can sometimes dominate.
The couplings of the radion in RS models match those of the dilaton, as expected from holography.

The dilaton is a particularly dangerous Higgs impostor, because many of their couplings have exactly the same form.

At present the dilaton is a good fit to the data, comparable to the SM Higgs. More data is needed to distinguish between them.

However the best-fit parameters in the dilaton case are already quite close to the SM values (coincidence problem).