

GOLDSTONE FERMION DARK MATTER

JHEP 1109:035,2011 [arXiv:1106.2162]
& work in progress

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The WIMP Miracle

Contains factors of $M_{\text{Pl}}, s_0, \dots$

$$\Omega_{\text{DM}} h^2 \approx 0.1 \left(\frac{x_f}{20} \right) \left(\frac{g_*}{80} \right)^{-\frac{1}{2}} \left(\frac{\langle \sigma v \rangle_0}{3 \times 10^{-26} \text{ cm}^3/\text{s}} \right)^{-1}$$

$$\sim \left\langle \frac{\alpha^2 v}{(100 \text{ GeV})^2} \right\rangle$$

Logarithmic miracle: Within orders of magnitude!

Abundance vs direct detection

$$\sigma_{\text{ann.}} \sim 0.1 \text{ pb}$$

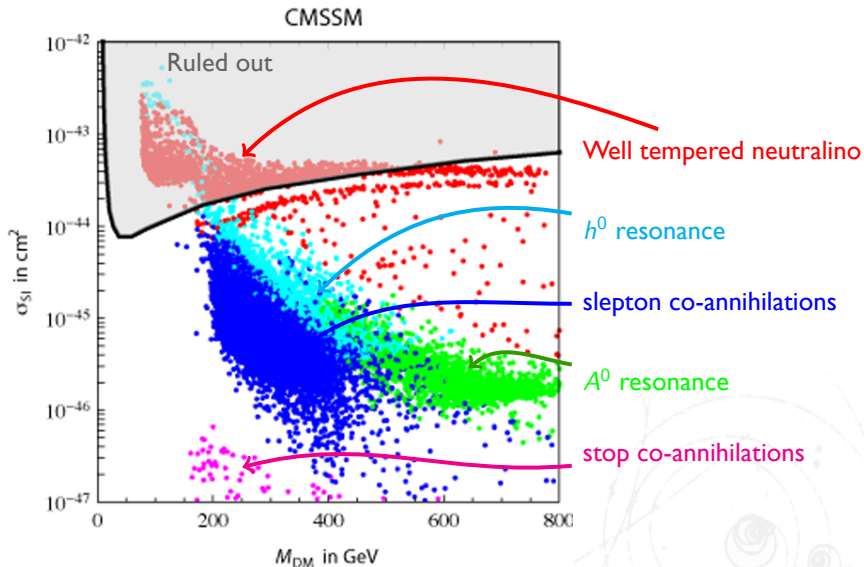
$$\sigma_{\text{SI}} \sim 7.0 \times 10^{-9} \text{ pb}$$

50 GeV WIMP

Typical strategy: pick parameters such that σ_{SI} is **suppressed**, then use tricks to **enhance** $\sigma_{\text{ann.}}$.

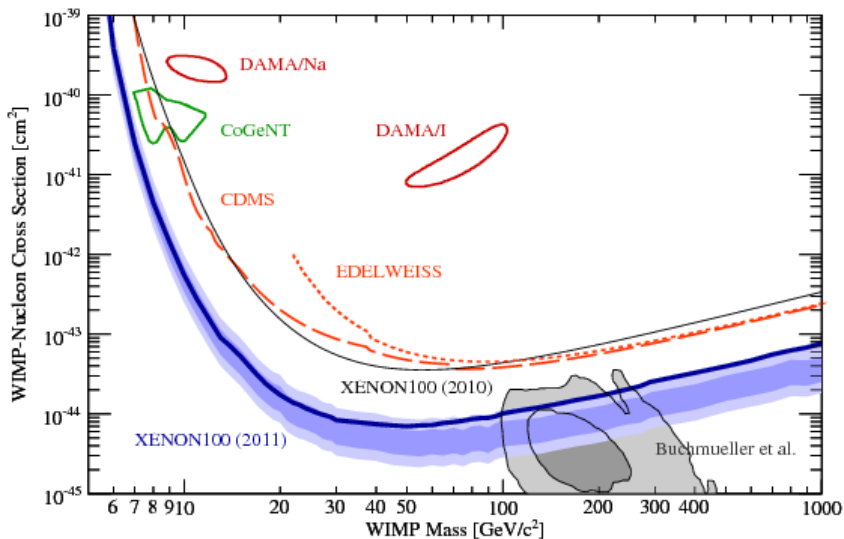
- Tune the neutralino composition (\tilde{B} vs. \tilde{W}, \tilde{H})
- Coannihilations (accidental slepton degeneracy)
- Resonant annihilation

Abundance vs direct detection



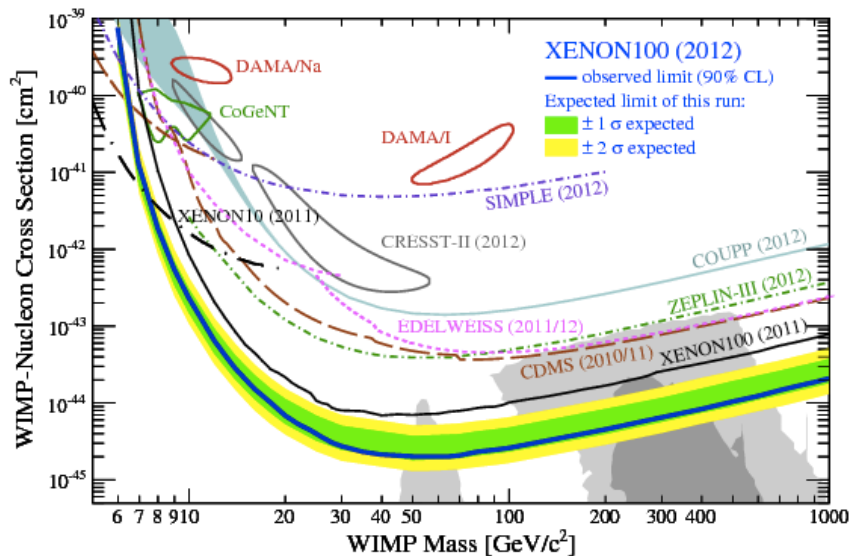
Farina, Kadastik, Raidal, Pappadopulo, Pata, Strumia [1104.3572]

The timid amoeba



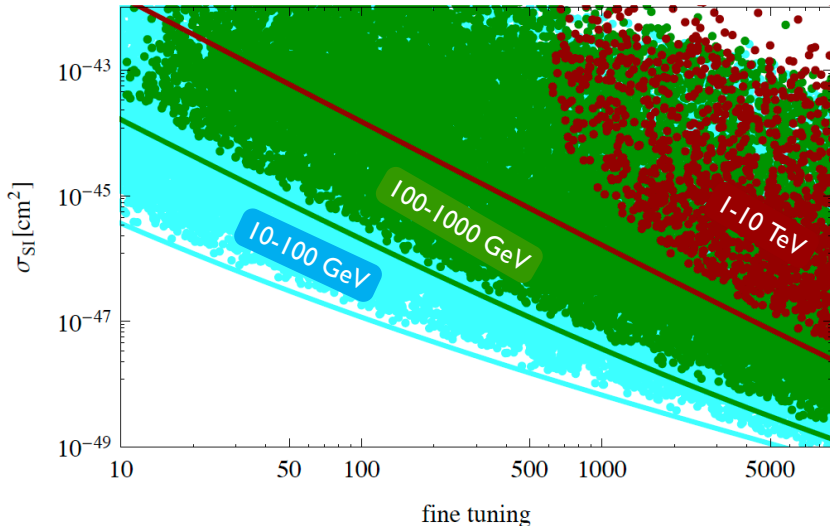
1104.2549

The timid amoeba



1207.5988

MSSM Dark Matter and EWSB Tuning



Perelstein and Shakya [1107.5048]

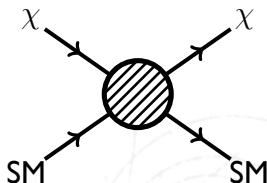
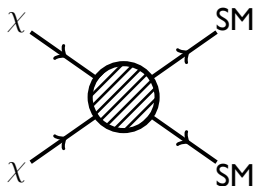
Abundance vs direct detection

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50 GeV WIMP

Assumed that these come from the same effective operator:



Can we separate into two different sectors?

One way to do this is with a Goldstone supermultiplet.

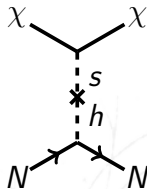
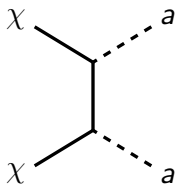
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50 GeV WIMP

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Can we separate into two different sectors?

One way to do this is with a Goldstone supermultiplet.

Motivation: natural WIMP

Typical MSSM WIMP: σ_{SI} **too large**

Want to naturally suppress direct detection while maintaining 'miracle' of successful abundance.

If LSP is part of a *Goldstone multiplet*, $(s + ia, \chi)$, additional suppression from derivative coupling.

- Like a weak scale axino, but unrelated to CP
- Like singlino DM, but global symmetry broken in SUSY limit

Goldstone Fermion Dark Matter

Parameterized class of models with a hidden sector and a spontaneously broken $U(1)$

Motivation: a natural WIMP

Annihilation: *p-wave* decay to Goldstones

$$\frac{1}{f} \bar{\chi} \gamma^\mu \gamma^5 \chi \partial_\mu a \Rightarrow \langle \sigma v \rangle \approx \left(\frac{m_\chi^2}{f^4} \right) \left(\frac{T_f}{m_\chi} \right) \approx \mathbf{1 \text{ pb}}$$

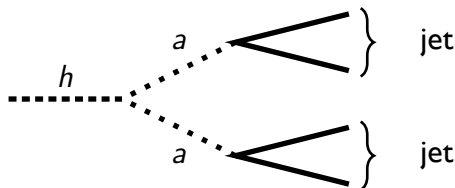
Direct detection: CP-even Goldstone *mixing* with Higgs

$$\frac{m_\chi v}{f^2} \sim 0.01 \Rightarrow \sigma_{\text{SI}} = \left(\frac{m_\chi v}{f^2} \right)^2 \sigma_{\text{SI}}^{\text{MSSM}} \approx \mathbf{\mathcal{O}(10^{-45} \text{ cm}^2)}$$

'Historical' Motivation: Buried Higgs

Idea: Light Higgs buried in QCD background

Global symmetry at $f \sim 500$ GeV with coupling $\frac{1}{f^2} h^2 (\partial a)^2$

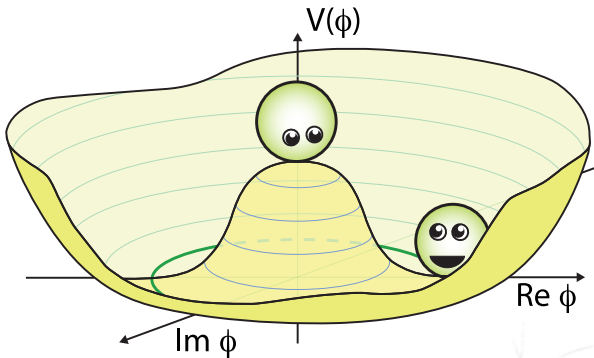


Bellazzini, Csáki, Falkowski, Hubisz, Shao, Weiler:
0906.3026, 1012.1316; Luty, Phalen, Pierce: 1012.1347

Can we **bury** the Higgs through a decays,
but **dig up** dark matter in χ ?



Goldstone Boson Review



Global $U(1)$ \Rightarrow massless pseudoscalar
Shift symmetry \Rightarrow derivative coupling

Nonlinear Σ Model (NL Σ M)

e.g. chiral perturbation theory

QCD is a theory of $\begin{cases} \text{quarks, gluons} & (E \gg \Lambda_{\text{QCD}}) \\ \text{pseudoscalar mesons } (\pi\text{s}) & (E \ll \Lambda_{\text{QCD}}) \end{cases}$

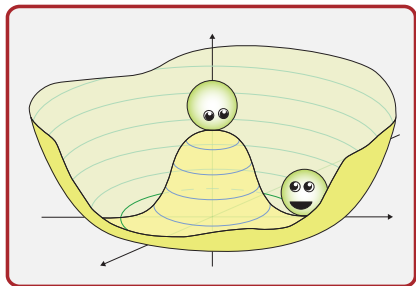
$$\langle \bar{q}q \rangle : SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$$

Nonlinear realization

$$U(x) = \exp(2i\pi^a(x)T^a/f) \quad \mathcal{L} = \frac{f^2}{4} \text{Tr} |\partial U|^2$$

Δm_{q_i} explicitly breaks flavor symmetry $\Rightarrow m_\pi \neq 0$

Couple NL Σ to the [low energy] SM



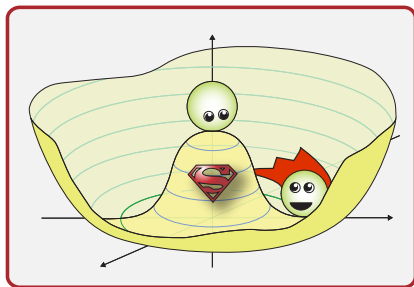
NL Σ ($\pi(x)$ theory)



Gauge bosons, e^\pm, \dots

SU(2) x U(1)

Our construction



Goldstone Supermultiplet

q, g, γ, h

MSSM

The Goldstone Supermultiplet

sGoldstone Goldstone boson Goldstone fermion

$$A = \frac{1}{\sqrt{2}} (s + i a) + \sqrt{2}\theta \chi + \theta^2 F$$

Carries the low-energy degrees of freedom of the UV fields,

$$\Phi_i = f_i e^{q_i A/f} \qquad f^2 = \sum_i q_i^2 f_i^2$$

SUSY \Rightarrow explicit s mass, $m_\chi \approx q_i \langle F_i \rangle / f$, a massless

a mass through small supersymmetric explicit $U(1)$ terms

A simple example of a $U(1)$ sector

UV theory

$$K = \bar{N}^\dagger \bar{N} + N^\dagger N + S^\dagger S \quad W = S (\bar{N} N - \mu^2)$$

$$N \sim f e^{+A/f}$$

$$\bar{N} \sim f e^{-A/f}$$

Effective theory

$$K = \cosh(A + A^\dagger) \quad W = 0$$

SUSY Breaking and the χ mass

Tamvakis-Wyler Thm. Phys. Lett B 112 (1982) 451; Phys. Rev. D 33 (1986) 1762

Global symmetry: $W[\Phi_i] = W[e^{i\alpha q_i} \Phi_i]$ so that

$$0 = \frac{\partial W[e^{i\alpha q_i} \Phi_i]}{\partial \alpha} = \sum_j W_j q_j \Phi_j,$$

Taking a derivative $\partial/\partial\Phi_i$ gives:

$$0 = \frac{\partial}{\partial\Phi_i} \left(\sum_j W_j q_j \Phi_j \right) \Big|_{\langle\Phi\rangle} = \sum_j W_{ij} q_j f_j + W_i q_i$$

$\chi = \sum_i q_i f_i \psi_i / f$ mass depends on the vevs of U(1)-charged F -terms in the presence of soft SUSY terms

Assuming no D -term mixing with gauginos

SUSY Breaking and the χ mass

If R symmetry unbroken: $R[\chi] = -1$ & no Majorana mass

- Soft scalar masses preserve R
- **A-terms** are holomorphic and generally break R symmetry

Assuming $A_i, m_i < f_i$, generic size is $|F_i| \approx A_i f_i$

$$m_\chi \sim A_i q_i$$

Often the A -terms are suppressed relative to other soft terms, so it's reasonable to expect χ to be the LSP.

SUSY Breaking and the χ mass

Contribution from Planck 'sloperators'

But one might worry (1104.0692) about Planck-scale operators giving an irreducible contribution to m_χ ,

$$\int d^4\theta \frac{(A + A^\dagger)^2 (X + X^\dagger)}{M_{\text{Pl}}} \sim m_{3/2} \chi \chi$$

However...

The A -term contribution to m_χ is equivalent to F -term mixing between U(1) charged fields and the SUSY spurion, X .

SUSY Breaking and the χ mass

Contribution from Planck 'sloperators'

For concreteness, consider gravity mediation with $m_{\text{soft}} \sim F/M_{\text{Pl}}$.

$$K = \sum_i Z(X, X^\dagger) \Phi_i^\dagger \Phi_i$$

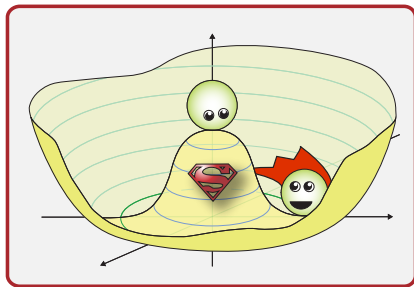
Analytically continue into superspace (hep-ph/9706540)

$$\Phi \rightarrow \Phi' \equiv Z^{1/2} \left(1 + \frac{\partial \ln Z}{\partial X} F \theta^2 \right) \Phi$$

Canonical normalization generates A-terms:

$$\Delta \mathcal{L}_{\text{soft}} = \left. \frac{\partial W}{\partial \Phi} \right|_{\Phi=\phi} Z^{-1/2} \left(-\frac{\partial \ln Z}{\partial \ln X} \frac{F}{M} \right) \Phi$$

Parameterize couplings to the MSSM



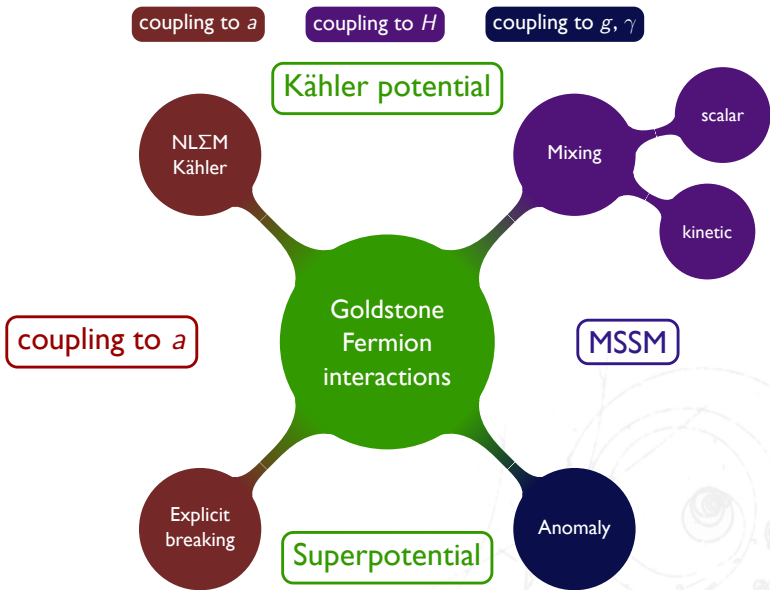
Goldstone Supermultiplet



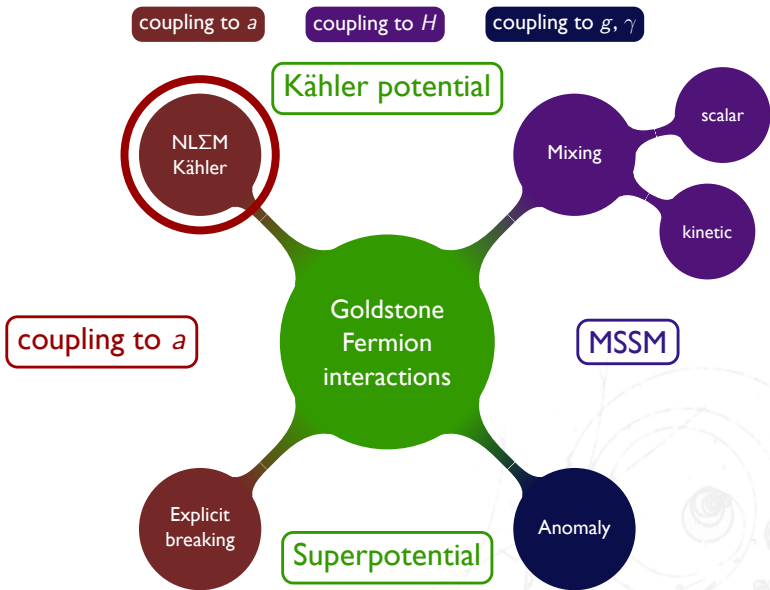
q, g, γ, h

MSSM

Interactions: Overview



Interactions: Overview



Interactions: NLSM Kähler potential

Non-linearly realized global U(1) leads to interactions of the Goldstone fields in through the Kähler terms:

$$\frac{\partial^2 K}{\partial A \partial A^\dagger} = 1 + b_1 \frac{q}{f} (A + A^\dagger) + \dots \quad b_1 = \frac{1}{qf^2} \sum_i q_i^3 f_i^2$$

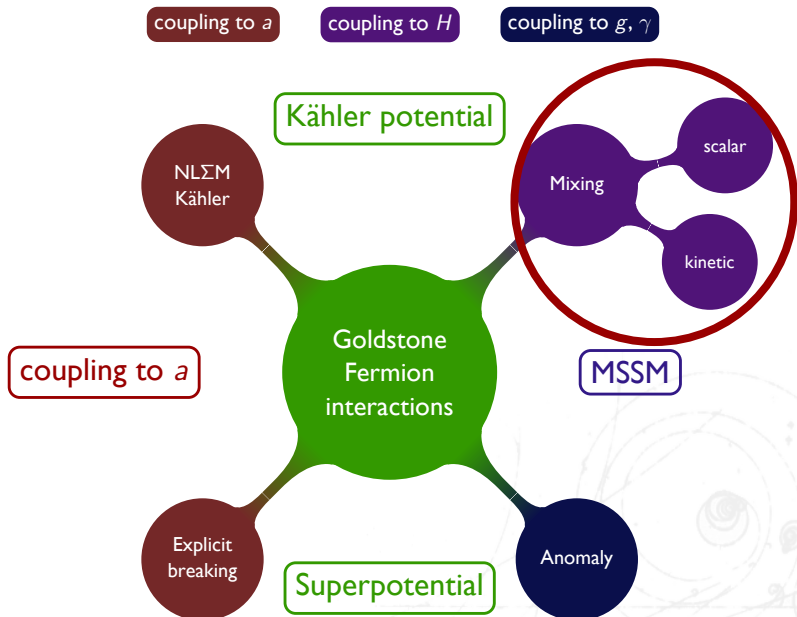
Note the manifest shift-invariance. This leads to:

$$\mathcal{L} = (\text{usual kinetic terms}) \left(1 + b_1 \frac{\sqrt{2}}{f} s + \dots \right) + \frac{1}{2\sqrt{2}} \left(b_1 \frac{1}{f} + b_2 \frac{\sqrt{2}}{f^2} s + \dots \right) (\bar{\chi} \gamma^\mu \gamma^5 \chi) \partial_\mu a + \dots$$

b_1 controls the annihilation cross section.

Zumino, Phys. Lett. B 87 (1979) 203

Interactions: Overview



Interactions: scalar mixing

MSSM fields are uncharged under the global $U(1)$, but may mix with the Goldstone multiplet through higher-order terms in K :

$$K = \frac{1}{f} (A + A^\dagger) (c_1 H_u H_d + \dots) + \frac{1}{2f^2} (A + A^\dagger)^2 (c_2 H_u H_d + \dots)$$

The new scalar interactions take the form

$$\mathcal{L} \supset \left[\frac{1}{2} (\partial a)^2 + \frac{1}{2} \bar{\chi} \not{\partial} \chi \right] \left(1 + c_h \frac{v}{f} h + \dots \right)$$

c_h depends on c_i and the Higgs mixing angles.

c_h controls direct detection

$c_h \rightarrow (m_h/m_s)^2$ in the large m_s limit.

We neglect mixing with the heavy higgses.

Interactions: other mixing

The higher order terms in K also induce kinetic $\tilde{H}-\chi$ mixing.

$$\mathcal{L} \supset i\epsilon_u \bar{\chi} \gamma^\mu \partial_\mu \tilde{H}_u^0 + i\epsilon_d \bar{\chi} \gamma^\mu \partial_\mu \tilde{H}_d^0 + \text{h.c.}$$

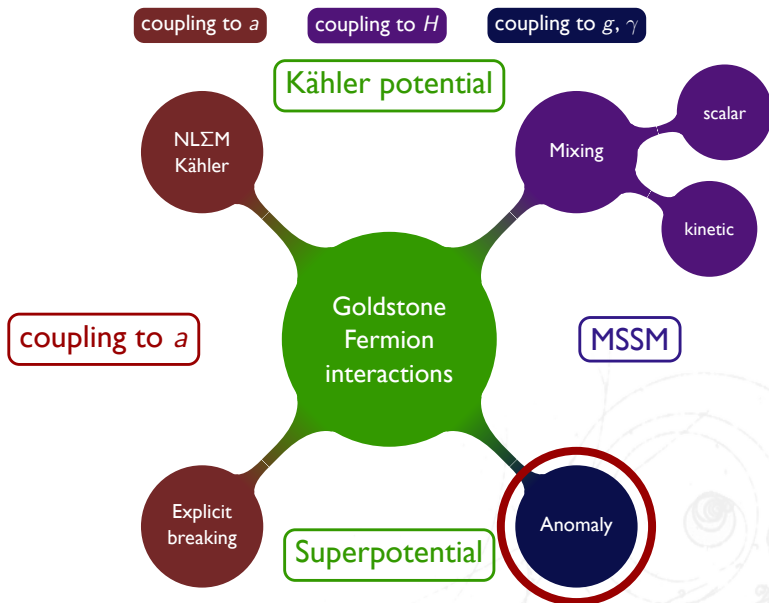
where $\epsilon \sim v/f$. For large μ : χ has a small \tilde{H} component of $\mathcal{O}(vm_\chi/f\mu)$.

Mixing with other MSSM fields is suppressed. Assuming MFV,

$$K = \frac{1}{f} (A + A^\dagger) \left(\frac{Y_u}{M_u} \bar{Q} H_u U + \dots \right)$$

where the scales $M_{u,d,\ell}$ are unrelated to f or v and can be large and dependent on the UV completion

Interactions: Overview

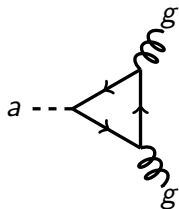


Interactions: anomaly

Fermions Ψ charged under global $U(1)$ and Standard Model

$$\mathcal{L}_{\text{an}} \supset \frac{c_{\text{an}}}{f\sqrt{2}} \left(a G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a + 2\bar{\chi} G_{\mu\nu}^a \sigma^{\mu\nu} \gamma^5 \lambda^a \right)$$

$$c_{\text{an}} = \frac{\alpha}{8\pi} q_{\Psi} N_{\Psi}$$



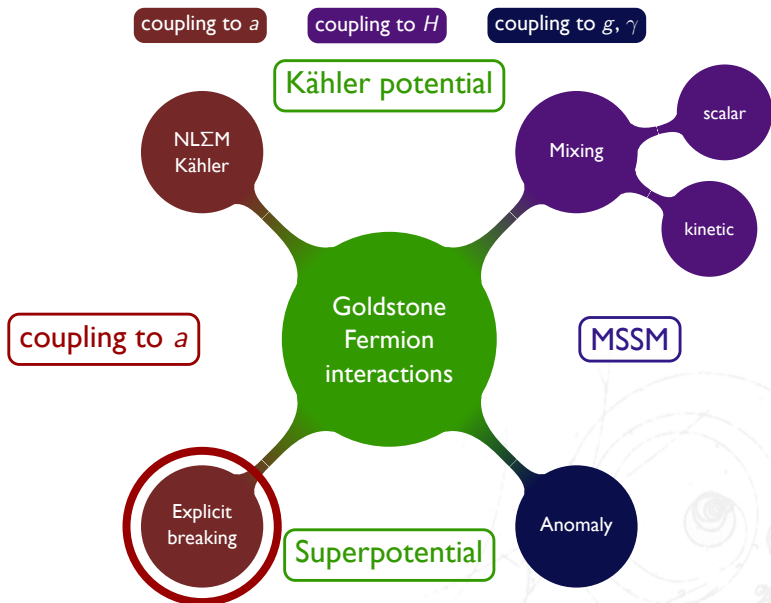
$U(1) SU(3)_c^2$
 $U(1) U(1)_{\text{QED}}^2$

Integrating out λ^a generates χ couplings to gluons, photons

$$\mathcal{L} \supset - \left(\frac{c_{\text{an}}^2}{2M_{\lambda} f^2} \right) \bar{\chi} \chi G G - i \left(\frac{c_{\text{an}}^2}{2M_{\lambda} f^2} \right) \bar{\chi} \gamma^5 \chi G \tilde{G}$$

These contribute to collider and astro operators.

Interactions: Overview



Interactions: explicit breaking

Include explicit $U(1)$ spurion $R_\alpha = \lambda_\alpha f$ with $\lambda_\alpha \ll 1$

$$W_{U(1)} = f^2 \sum_\alpha R_{-\alpha} e^{aA/f}$$

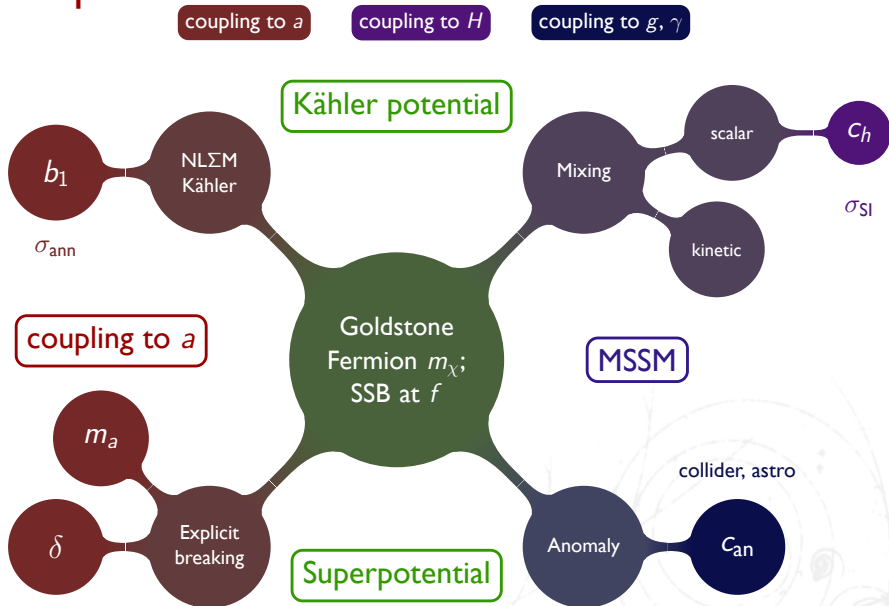
Preserve SUSY \Rightarrow at least two spurions with opposite charge.

This generates $m_a = m_\chi = m_s$ and couplings

$$\mathcal{L} \supset - \underbrace{\frac{m_a}{2\sqrt{2}f}(\alpha + \beta)}_\delta i a \bar{\chi} \gamma^5 \chi + \underbrace{\frac{m_a}{8f^2}(\alpha^2 + \alpha\beta + \beta^2)}_\rho a^2 \bar{\chi} \chi$$

By integration by parts this is equivalent to a shift in the b_1 coefficient from the Kähler potential

Main parameters



Parameter space scan

Abundance: $\langle\sigma v\rangle \approx \frac{b_1^4}{8\pi} \frac{T_f}{m_\chi} \frac{m_\chi^2}{f^4} \approx 1 \text{ pb}$

p-wave: $b_1 \gtrsim 1$, all other parameters take natural values

Parameter	Description	Scan Range
f	Global symmetry breaking scale	500 GeV – 1.2 TeV
m_χ	Goldstone fermion mass	50 – 150 GeV
m_a	Goldstone boson mass	8 GeV – $f/10$
b_1	$\chi\chi a$ coupling	[0, 2]
c_{an}	Anomaly coefficient	0.06
c_h	Higgs coupling	[-1, 1]
δ	Explicit breaking $ia\bar{\chi}\gamma^5\chi$ coupling	3/2

$$\mathcal{L} \supset \left[\frac{1}{2}(\partial a)^2 + \frac{1}{2}\bar{\chi}\not{\partial}\chi \right] c_h \frac{v}{f} h + \frac{b_1}{2\sqrt{2}f} (\bar{\chi}\gamma^\mu\gamma^5\chi) \partial_\mu a + \frac{c_{\text{an}}}{f\sqrt{2}} a G\tilde{G} + i\delta a\bar{\chi}\gamma^5\chi$$

Main Interactions summary

$$\frac{b_1}{f} \times \left(\begin{array}{c} \chi \\ \chi \end{array} \right) \rightarrow a, s \quad \begin{array}{c} a, s \\ a, s \end{array} \rightarrow s, a$$

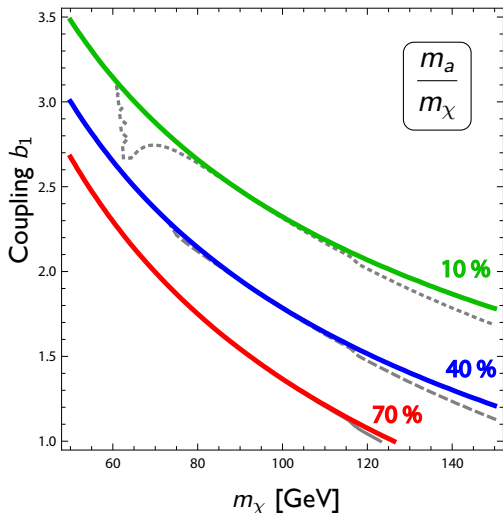
$$\frac{v}{f^2} \times \left(\begin{array}{c} \chi \\ \chi \end{array} \right) \rightarrow h \quad \begin{array}{c} a \\ a \end{array} \rightarrow h \quad s \rightarrow \times \rightarrow h$$

$$\frac{\alpha_S N}{8\pi f} \times \left(a \rightarrow \text{triangle} \rightarrow g \right) \quad \left(\frac{\alpha_S N}{8\pi f} \right)^2 \frac{1}{M_\lambda} \times \left(a \rightarrow \text{triangle} \rightarrow g \right)$$

$$m_a < m_\chi \ll m_s$$

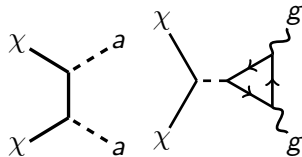
Annihilation: Contours of fixed Ω

$$\Omega h^2 = 0.11$$



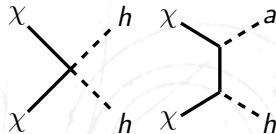
Dominant contribution

Kähler, anomaly, $U(1)$



Subleading

Mixing with Higgs



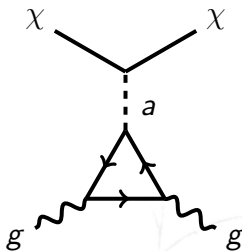
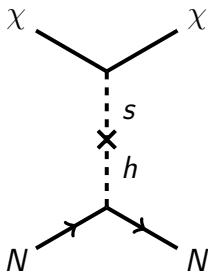
Negligible

$$\chi\chi \rightarrow s \rightarrow aa, \chi\chi \xrightarrow{t,u} hh$$

Direct Detection

Relevant couplings from EWSB and anomaly:

$$\mathcal{L} \supset \frac{c_h^V}{2f} \bar{\chi} \not{\partial} \chi h - \frac{c_{an}^2}{2M_\lambda f^2} \bar{\chi} \chi GG - \frac{i c_{an}^2}{2M_\lambda f^2} \bar{\chi} \gamma^5 \chi G \tilde{G}$$



Effective coupling to nucleons: $\mathcal{L} = G_{\text{nuc}} \bar{N} N \bar{\chi} \chi$,

$$G_{\text{nuc}} = c_h \frac{\lambda_N}{2\sqrt{2}} \left(\frac{m_\chi m_N}{m_h^2 f^2} \right) + \frac{4\pi c_{an}^2}{9\alpha_s} \frac{m_N}{M_\lambda f} \left(1 - \sum_{i=u,d,s} f_i^{(N)} \right)$$

Direct Detection

Higgs exchange typically dominates by a factor of $\mathcal{O}(10^3)$.

$$\sigma_{\text{SI}}^{\text{H}} \approx 2 \cdot 10^{-45} \text{ cm}^2 c_h^2 \left(\frac{125 \text{ GeV}}{m_h} \cdot \frac{700 \text{ GeV}}{f} \right)^4 \left(\frac{m_\chi}{100 \text{ GeV}} \cdot \frac{\mu_\chi}{\text{GeV}} \cdot \frac{\lambda_N}{0.5} \right)^2$$

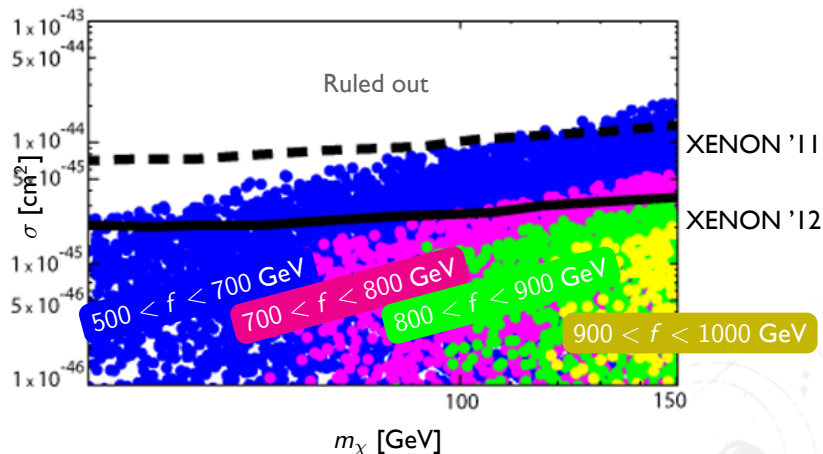
Compare this to the MSSM Higgs with $\mathcal{L} = \frac{1}{2}cg\bar{\chi}\chi h$:

$$\sigma_{\text{SI}}^{\text{MSSM}} \sim \frac{c^2 g^2}{2\pi} \frac{\lambda_N^2 \mu^2 m_N^2}{m_h^2 v^2} \approx c^2 \times 10^{-42} \text{ cm}^2$$

Natural suppression: $(m_\chi v/f^2)^2$

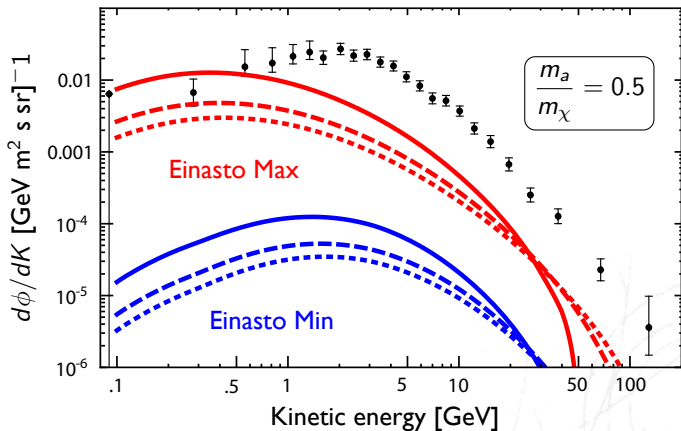
Is it enough to avoid current direct detection bounds?

Parameter space scan & direct detection



Indirect detection: \bar{p} flux vs. PAMELA

$$f = 700 \text{ GeV}, Q_\psi = 2, \delta = \frac{3}{2}, N_\psi = 5$$



Dotted: $m_\chi = 150 \text{ GeV}, b_1 = 1$
Dashed: $m_\chi = 100 \text{ GeV}, b_1 = 1.5$
Solid: $m_\chi = 50 \text{ GeV}, b_1 = 3$

Using Einasto DM Halo profile in 1012.4515, 1009.0224

Indirect detection: Fermi-LAT

Annihilation is p -wave, but this is suppressed at late times.
Indirect detection from anomaly diagrams:



γ -ray line search: 30 – 200 GeV

- $\chi\chi \rightarrow a \rightarrow \gamma\gamma$ via anomaly
- $\mathcal{O}(10)$ smaller than bound even for extreme parameters

Diffuse γ -ray spectrum: 20 – 100 GeV

- $\chi\chi \rightarrow a \rightarrow gg \rightarrow \pi^0s$
- $\mathcal{O}(10)$ smaller than bound

<http://fermi.gsfc.nasa.gov/science/symposium/2011/program>

Goldstone fermions at the LHC

Collider production through gluons.

ISR monojets: sensitive to $\sigma_{SI}^N \sim 10^{-46} \text{ cm}^2$ with 100 fb^{-1} .

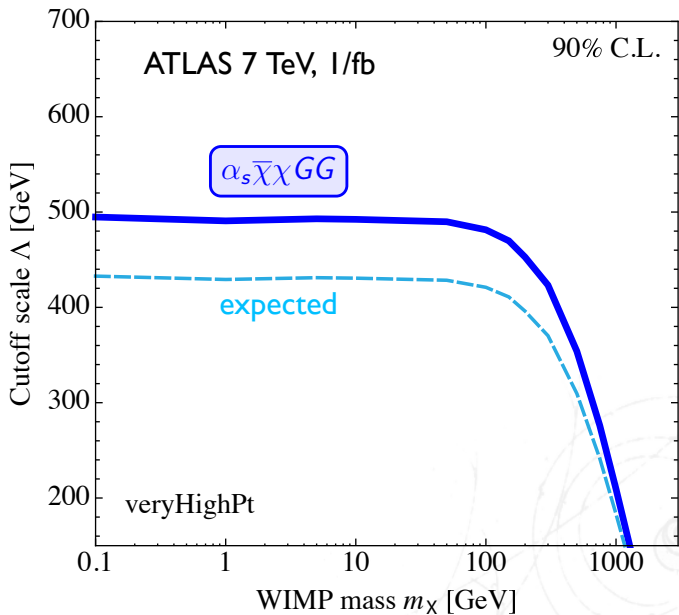
The dim-7 anomaly operators are too small:

$$\mathcal{L} \supset -\frac{c_{an}^2}{2M_\lambda f^2} \bar{\chi}\chi GG - \frac{ic_{an}^2}{2M_\lambda f^2} \bar{\chi}\gamma^5\chi G\tilde{G}$$

$gg \rightarrow a^* \rightarrow \chi\chi$ may be within 5σ reach with 100 fb^{-1}

1005.1286, 1005.3797, 1008.1783, 1103.0240, 1108.1196, 1109.4398

Goldstone fermions at the LHC



Fox, Harnik, Kopp, Tsai: 1109.4398

The $\gamma\gamma$ games

Experimental progress, model-building directions:

1. $m_h = 125$ GeV, possible enhancement in $h \rightarrow \gamma\gamma$?
2. FERMI line at 130 GeV, possibly $\chi\chi \rightarrow \gamma\gamma$?

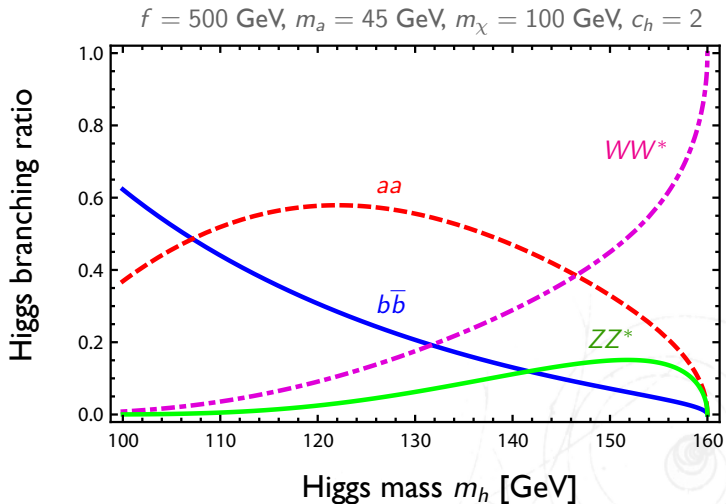
Can we realize this in Goldstone Fermion DM? **Maybe.**

Work in progress with B. Bellazzini, M. Cliche

1. Goldstone boson decays modify Higgs branching ratios
2. Anomaly coupling, SUSY framework to control spectrum for narrow box-shaped γ -ray spectrum.

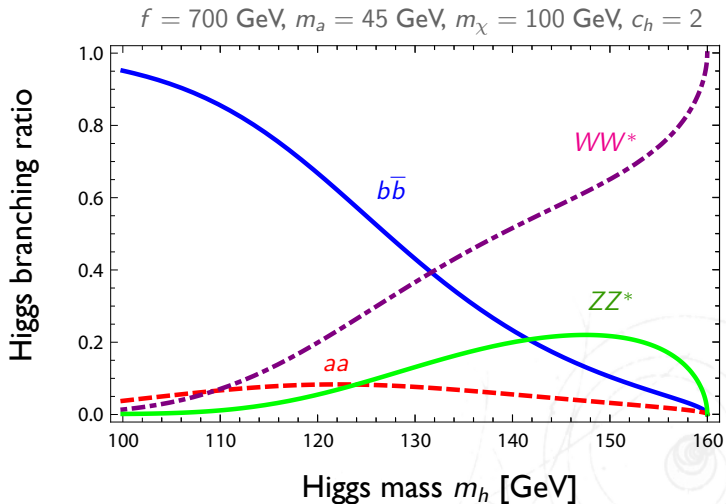
Non-standard Higgs decays

Hard to completely bury the Higgs. LEP: $\text{Br}(\text{SM}) \gtrsim 20\% \Rightarrow m_h \gtrsim 110 \text{ GeV}$



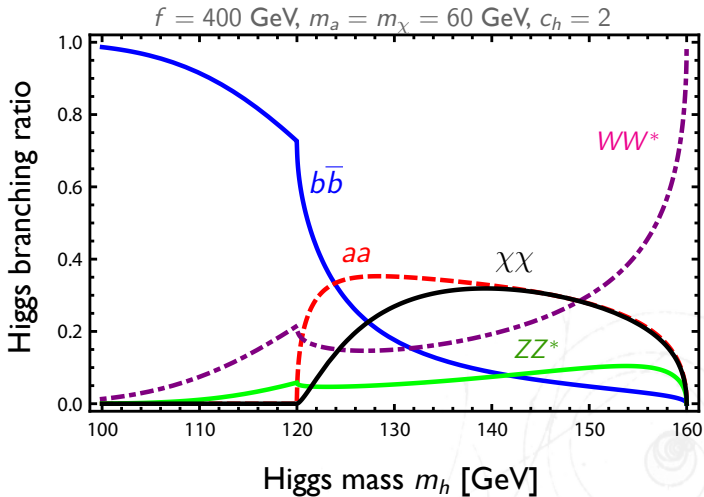
Non-standard Higgs decays

For larger f , can suppress $h \rightarrow aa$

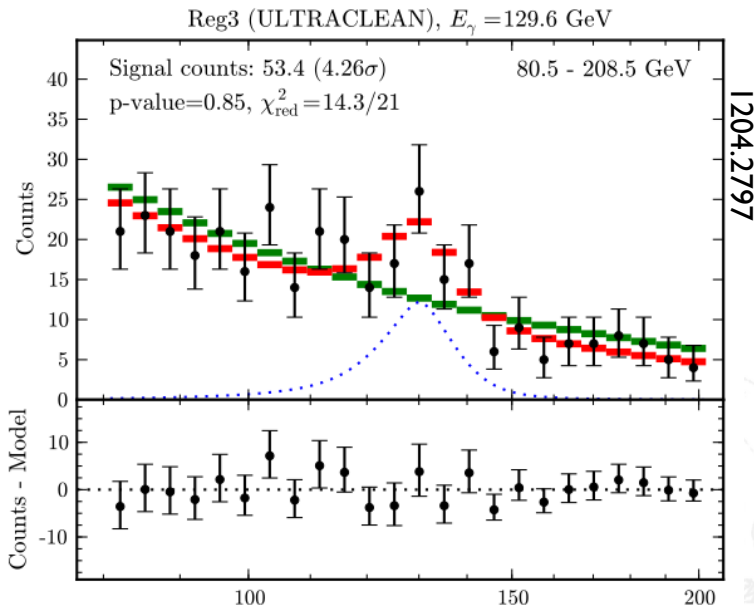


Non-standard Higgs decays

Partially buried & invisible: Suppressed SM channels, MET, $\Gamma_{\text{tot}} < 1$

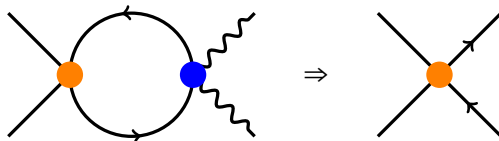


The 130 GeV Weniger Line

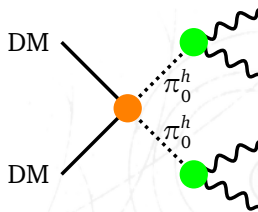
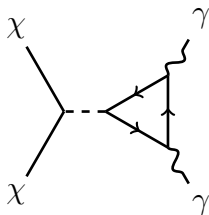


Model building for the Weniger Line

Problem: photon continuum Cohen, Lisanti, Slatyer, Wacker (1207.0800)

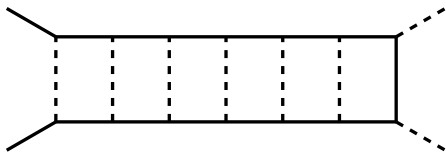


Tuned pseudoscalar processes Fan & Reece (1209.1097)



Sommerfeld enhancement for singular potentials

$\mathcal{O}(\text{few})$ sufficient to open up parameter space.
Larger enhancement may be used for $\gamma\gamma$ signal

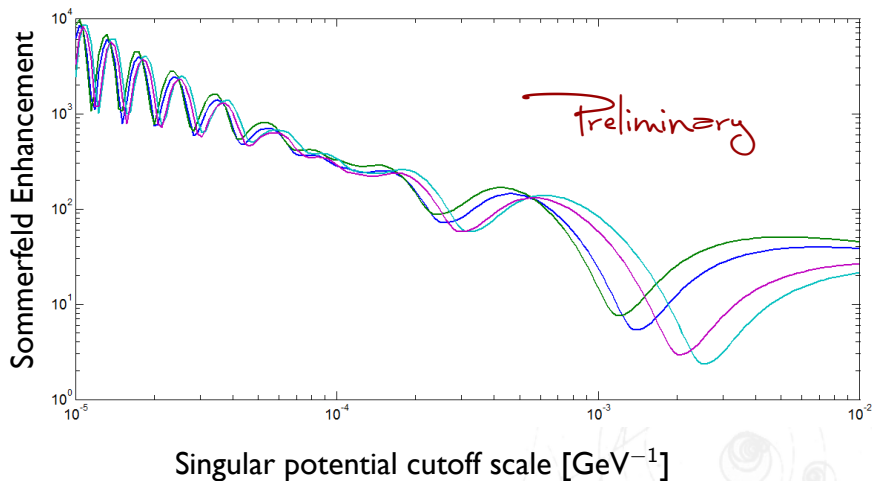


Pseudoscalar exchange $\Rightarrow 1/r^3$ potential, need to **regulate and renormalize** effective non-relativistic theory.

Need to clarify UV sensitivity, viability of matching.

0810.0713, 0902.0688, 0907.0235

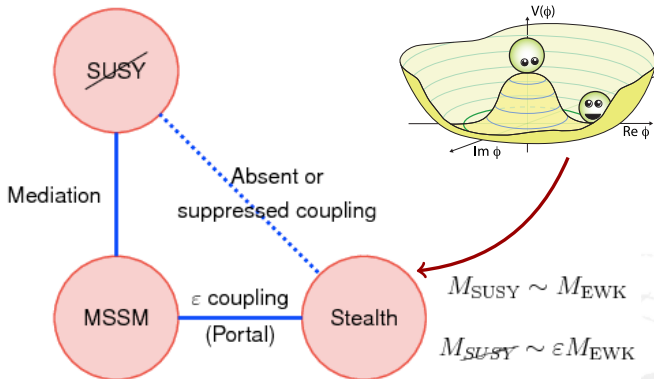
Pseudoscalar Yukawa enhancement



Other applications: Stealth SUSY limit

$$10^5 \text{ GeV} \lesssim f < 10^8 \text{ GeV}$$

$$m_\chi \rightarrow 0$$



Fan, Reece, Ruderman: 1201.4875

Conclusions

Executive summary: Goldstone Fermion dark matter

- SSB: global $U(1) \Rightarrow$ Goldstone boson a and fermion χ
- χ is LSP and DM, a can modify Higgs branching ratios

Simple extension of MSSM with natural WIMP dark matter

- Kähler $\chi\chi a$ interaction controls abundance
- Higgs mixing, anomaly controls direct detection

Further directions:

- p -wave Sommerfeld enhancement
- Non-abelian generalization
- $h, \chi\chi \rightarrow \gamma\gamma$ hints

Extra Slides



Examples of Linear Models

Simplest example:

$$W = yS (\overline{N}N - \mu^2) + \underbrace{N\overline{\phi}\phi}_{\text{anomaly}} + \underbrace{SH_u H_d}_{\text{mixing}} + \underbrace{W_{\text{explicit}}}_{\text{explicit } U(1)}$$

Example with $|b_1| \geq 1$:

$$W = \lambda XYZ - \mu^2 Z + \frac{\tilde{\lambda}}{2} Y^2 N - \tilde{\mu} \overline{N} N$$

$q_Z = 0$, $q_N = -q_{\overline{N}} = -2q_Y = 2q_X$. Goldstone multiplet:

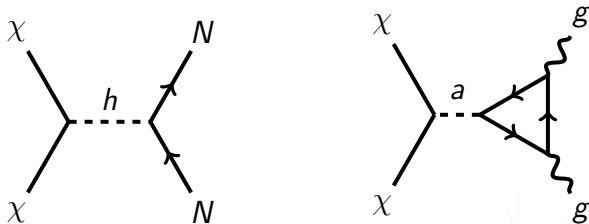
$$A = \sum_i \frac{q_i f_i \psi_i}{f} = \frac{q_Y}{f} (Y f_Y - X f_X + 2\overline{N} F_{\overline{N}})$$

$$b_1 = \frac{-f_X^2 + f_Y^2 + 8f_N^2}{f_X^2 + f_Y^2 + 4f_N^2}$$

Direct Detection

Relevant couplings from EWSB and anomaly:

$$\mathcal{L} \supset \frac{c_h^V}{2f} \bar{\chi} \not{\partial} \chi h - \frac{c_{an}^2}{2M_\lambda f^2} \bar{\chi} \chi GG - \frac{i c_{an}^2}{2M_\lambda f^2} \bar{\chi} \gamma^5 \chi G \tilde{G}$$



Effective coupling to nucleons: $\mathcal{L} = G_{\text{nuc}} \bar{N} N \bar{\chi} \chi$,

$$G_{\text{nuc}} = c_h \frac{\lambda_N}{2\sqrt{2}} \left(\frac{m_\chi m_N}{m_h^2 f^2} \right) + \frac{4\pi c_{an}^2}{9\alpha_s} \frac{m_N}{M_\lambda f} \left(1 - \sum_{i=u,d,s} f_i^{(N)} \right)$$

Direct Detection

Some details:

$$G_{\chi N} = c_h \frac{\lambda_N}{2\sqrt{2}} \left(\frac{m_\chi m_N}{m_h^2 f^2} \right) + \frac{4\pi c_{an}^2}{9\alpha_s} \frac{m_N}{M_\lambda f} \left(1 - \sum_{i=u,d,s} f_i^{(N)} \right)$$

For reduced mass $\mu_\chi = (m_\chi^{-1} + m_N^{-1})^{-1}$,

$$\sigma_{SI}^{\text{Higgs}} = \frac{4\mu_\chi^2}{A^2\pi} [G_{\chi p} Z + G_{\chi n} (A - Z)]$$

$$\sigma_{SI}^H \approx 3 \cdot 10^{-45} \text{ cm}^2 c_h^2 \left(\frac{115 \text{ GeV}}{m_h} \right)^4 \left(\frac{700 \text{ GeV}}{f} \right)^4 \left(\frac{m_\chi}{100 \text{ GeV}} \right)^2 \left(\frac{\mu_\chi}{1 \text{ GeV}} \right)^2 \left(\frac{\lambda_N}{0.5} \right)^2$$

$$\sigma_{SI}^{\text{glue}} \approx 2 \cdot 10^{-48} \text{ cm}^2 \left(\frac{700 \text{ GeV}}{M_\lambda} \right)^2 \left(\frac{700 \text{ GeV}}{f} \right)^4 \left(\frac{N_\Psi}{5} \right)^4 \left(\frac{q_\Psi}{2} \right)^4 \left(\frac{\mu}{1 \text{ GeV}} \right)^2$$

using $c_{an} = \alpha_s q_\Psi N_\Psi / 8\pi$

Why are the $\chi\chi \rightarrow aa$ annihilations p -wave?

If the initial state is a particle-antiparticle pair with zero total angular momentum and the final state is CP even, then the process must vanish when $v = 0$.

Under CP a particle/antiparticle pair picks up a phase $(-)^{L+1}$. When $v = 0$ momenta are invariant and thus the initial state gets an overall minus sign. Since final state is CP even, the amplitude must vanish in this limit. For Dirac particles P is sufficient, but for Majorana particles CP is the well-defined operation.

This is why $\chi\chi \rightarrow G\tilde{G}$ is s -wave while $\chi\chi \rightarrow aa$ is p -wave.

Nuclear matrix element and matching

The nucleon matrix element at vanishing momentum transfer:

$$M_N = \langle \Theta_\mu^\mu \rangle = \langle N | \sum_{i=u,d,s} m_i \bar{q}_i q_i + \frac{\beta(\alpha)}{4\alpha} G_{\alpha\beta}^a G_{\alpha\beta}^a | N \rangle$$

from: Shifman, Vainshtein, Zakharov. Phys. Lett 78B (1978)

$\beta = -9\alpha^2/2\pi + \dots$ contains only the light quark contribution, M_N is the nucleon mass. The GG matches onto the nucleon operator $\bar{N}N$.

$$M_N f_{i=u,d,s}^{(N)} = \langle N | m_i \bar{q}_i q_i | N \rangle \quad f_g^{(N)} = 1 - \sum_{i=u,d,s} f_i^{(N)}$$

Nuclear matrix element and matching

$$\frac{\beta(\alpha)}{4\alpha} G_{\alpha\beta}^a G_{\alpha\beta}^a \longrightarrow M_N \left(1 - \sum_{i=u,d,s} f_i^{(N)} \right) \bar{N}N$$

Where $f_{u,d}^{(N)} \ll f_s^{(N)} \approx 0.25$. For a detailed discussion, see 0801.3656 and 0803.2360.

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