METAPHOR FOR DARK ENERGY

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DAWN OF RELATIVITY

Newtons $\frac{1}{r^2}$ Law + Special Relativity
+ Equivalence Principle

Couple to Energy
$m=0$, spin-$2$

CURVED SPACETIME

$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}$

Einstein '15

Couple to Mass
$m=0$, spin-$0$

Nordstrom '13
DAWN OF RELATIVITY

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Couple to Energy
$m = 0$, spin -2

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$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}$

Einstein ’15

Couple to Mass
$m = 0$, spin -0

SPACETIME

Nordstrom ’13

$R = \text{Weyl} \ T_{\mu\nu}$

$C_{\mu\nu\rho\sigma} = 0$

Einstein, Fokker ’14
DAWN OF RELATIVITY
Newton's $\frac{1}{r^2}$ Law + Special Relativity
+ Equivalence Principle

Couple to Energy
$m = 0$, spin-2

CURVED

Couple to Mass
$m = 0$, spin-0

SPACETIME

$R_{\mu\nu} \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}$

Einstein '15

$\Gamma_{\mu\nu\rho} = 0$

Einstein, Fokker '14

Analog (quantum) gravity
+ Cosmological Const. Problem
+ Modified gravity

Sundrum '03
LEAVE NO STONE UNTURNED
BETWEEN US & DARK ENERGY
LEAVE NO STONE UNTURNED BETWEEN US & DARK ENERGY
Leave no stone unturned between us & dark energy.

IQ

It's a cosmological constant! Get over it!

Well, maybe it's something else. Let's do experiments.

SKA, ACT, APEX, Pan-STARRS, LSST, ALPACA, JDAM, Planck,…
Classical/Tree treatment

\[ Z = (\varphi \varphi)^2 + \sum_j \bar{\psi}_j (i \kappa - y_j \tilde{\phi} - m_j) \psi_j \]

\[ F_{ij} = -y_i y_j \frac{1}{r^2} \]

\[ \propto -m_i m_j \frac{1}{r^2} \Rightarrow m_i = y_i M \quad \text{for some } M \]

\[ \Rightarrow G_{\text{Newton}} = \frac{1}{M^2}, \quad M \text{ is Planck scale}. \]
\[ L = (\partial \phi)^2 + \sum_j \bar{\psi}_j (i \gamma^\mu \partial_\mu - m \phi) \psi_j \]

\[ \phi(x) = \tilde{\phi}(x) + M \]

\[ \Rightarrow \text{Spontaneously Broken scale (conformal) symmetry by } \langle \phi \rangle = M \]

\[ \tilde{\phi}(x) \text{ is Goldstone Boson} \]

\[ \equiv \text{“Dilaton”} \]
Dilaton NOT derivatively-coupled

Generally, \( S = \int d^4x \ J^\mu \) (other fields) \( \frac{\partial}{\partial f_{11}} \)

\[ = \int d^4x \ J_{\text{scale}}^\mu \frac{\partial \tilde{\phi}}{M} \]

**But** \( J_{\text{scale}}^\mu \sim T_{(x)}^{\mu\nu} \gamma_\nu \Rightarrow \)

\( S = \int d^4x \ T_{\mu\nu} \gamma^\nu \frac{\partial \tilde{\phi}}{M} \)

\[ = \int d^4x \ \frac{\tilde{\phi}}{M} \Sigma \overline{\psi}_j m_j \psi_j \]

\( S \text{ parts} \)
Light...

Scale invariance $\Rightarrow$

$$L = (\partial \phi)^2 + \sum_j \bar{\psi}_j (i \gamma^\mu \partial_\mu - m_j \gamma^0) \psi_j$$

$$- \frac{1}{4e^2} F_{\mu \nu}^2$$

...does not bend!

Dead as theory of real gravity...

...but great as an analogy
\[ M_{\text{grav}} = M_{\text{inertial}} \text{ for atoms} \]

\[ m_H = m_e + m_p - \frac{\alpha^2 m_e m_p}{m_e + m_p} = \left( y_e + y_p - \frac{\alpha^2 y_e y_p}{y_e + y_p} \right) M \]

But \( M \) only appears in \( \phi = \tilde{\phi} + M \) combo

\[ \Rightarrow m_H (1 + \tilde{\phi}_M) \]

Similarly, for planets, despite no \( \tilde{\phi} \).
Scalar Gravity = Curved Spacetime = Dilaton Chiral Lagrangian

Weyl transformation $\Rightarrow$

$$
\mathcal{L} = \sqrt{-g} \left\{ \mp \left( i \gamma^a e_a \Gamma^\mu_{\nu\lambda} \gamma_\mu - m \right) \psi - \frac{1}{4 e^2} g^{\mu\nu} g^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma} - M^2 R \right\}
$$

opposite sign to usual

generally covariantly

$g_{\mu\nu}(x) = \left( \frac{\phi(x)}{M} \right)^2 \eta_{\mu\nu}$

$e_a^\mu = \frac{\phi(x)}{M} \delta_a^\mu$, $R$ Ricci scalar

Dynamical spacetime, fixed mass scales, $m, M$

Flat spacetime, dynamical mass scales, $\phi(x), y, \phi(x)$

Einstein, Fokker '14

Isham, Salam, Strathdee '71
Weyl Anomaly

\[ L_{\phi + QCD} = (\partial \phi)^2 + \bar{\psi} i \gamma^{\mu} D_\mu \psi - \frac{1}{4 g^2 (\mu/\phi)} g_{\mu \nu} \]

\[ \frac{1}{4} \left( \frac{1}{g^2(\mu)} + \frac{b}{16 \pi^2} \ln \frac{\phi}{M} \right) \]

\[ \sim \frac{\phi}{M} \]

\[ \Rightarrow \text{Non-renormalizable} \]

Dilaton + light matter + radiation

Chiral Lagrangian \( \ll M \), symmetry scale

Equivalence Principle for Hadrons

\[ m_{\text{proton}} \sim \mu e^{-\frac{16\pi^2}{b g^2(\mu)}} \rightarrow \mu e^{-\frac{16\pi^2}{b g^2(\mu)} + \ln \frac{\phi}{M}} \]

\[ = m_{\text{proton}} \frac{\phi}{M} \]
Einstein loved it

Einstein hated it
Standard Cosmology in Conformal Coordinates

Co-moving coords. \( ds^2 = dp^2 - a^2(t) d\mathbf{x}^2 \)

\[
= \frac{\phi^2(t)}{M^2} \left( dt^2 - d\mathbf{x}^2 \right)
\]

\( \eta_{\mu\nu} dx^\mu dx^\nu \)

\( d\tau = \frac{\phi(t)}{M} dt \)

\( \alpha(\tau) = \frac{\phi(t)}{M} \)

Proper time

Conformal time
But \( \hat{E} \leftrightarrow t = \phi \frac{\dot{M}}{\rho} E_{\text{physical}} \leftrightarrow z \)

Dust co-moving density

Radiation co-moving \( \hat{E} \)-density \( \equiv \hat{n}_{\text{dust}} = \text{constant} \)

\( \hat{\rho}_{\text{rad}} = \text{constant} \)
I. Physical energy densities

\[ P_{\text{rad}} = \hat{P}_{\text{rad}} \left( \frac{M}{\phi(t)} \right)^4 = \hat{P}_{\text{rad}} / a^4(t) \]

\[ n_{\text{dust}} = \hat{n}_{\text{dust}} \left( \frac{M}{\phi(t)} \right)^3 \]

\[ P_{\text{dust}} = m \hat{n}_{\text{dust}} \left( \frac{M}{\phi(t)} \right)^3 = m \hat{n}_{\text{dust}} / a^3(t) \]
FRIEDMAN EQUATION

$$H = \frac{\dot{a}}{a} \quad \Rightarrow \quad H^2 = G_N \left( \frac{m \hat{\rho}_{\text{dust}}}{a^3} + \frac{\hat{\rho}_{\text{rad}}}{a^4} + \rho_{\text{vac}} \right)$$

$$\Rightarrow \quad \frac{\partial}{\partial t} \phi^2 = m \hat{\rho}_{\text{dust}} \phi + \hat{\rho}_{\text{rad}} + \frac{\rho_{\text{vac}} \phi^4}{M^4}$$

$$\frac{H}{M} = \frac{\phi}{\phi^2}$$

$$\Rightarrow \quad \mathcal{H} = m \frac{\hat{\rho}_{\text{dust}}}{M} \phi + \frac{\hat{\rho}_{\text{rad}}}{M^4} + \frac{\rho_{\text{vac}} \phi^4}{M^4} - \phi^2 = 0$$

"wrong-sign" conformity of standard GR

Hamiltonian constraint

energy density
Cosmology of Scalar Gravity

Conserved

\[ \mathcal{H} = \frac{M}{M_d} \dot{\phi}^2 + \dot{\phi}^2 + \frac{P_{\text{vac}}}{M^4} \phi^4 + \phi^2 \dot{\phi} \]

Dust

Compton Wavelength \( \propto \frac{1}{\phi(t)} \)

Early

Right-sign kinetic term

Conformally invariant \( \lambda \phi^4 \) coupling
Cosmology of Scalar Gravity

\[ \mathcal{H} = \frac{m}{M} \dot{\phi}_{\text{dust}} + \dot{\phi}_{\text{rad}} + \frac{P_\text{vac}}{M^4} \phi^4 + \phi^2 \neq 0 \]
GR Equivalences

Cosmological Constant $\equiv -\rho_{\text{vac}}$
$\geq 0$ in GR
$< 0$ scalar grav.

Radiation type energy $\equiv \mathcal{H} - \rho_{\text{rad}}$
$> 0$ in GR
$> 0$ scalar grav.

Dust energy $\equiv -\frac{M}{M_{\text{dust}}}\phi$
$> 0$ in GR
$< 0$ scalar grav.

Naively, no matter-dominated cosmology $\approx$ isotropic+homogeneous. BUT...
Newtonian Cosmology

Exploding ball with Newtonian (scalar) attraction

\[-\frac{d^2 \Phi}{dt^2} + \nabla^2 \Phi = \frac{P_{\text{dust}}}{M}\]

\(R(t) = R_0 \alpha(t)\), \(P_{\text{dust}}(t) = P_0 \left(\frac{a_0}{\alpha(t)}\right)^3\)

\(H = M_{\text{ball}}^2 R^2 - M_{\text{ball}}^2 \frac{\dot{a}}{M^2 R^2}\)

\[\Rightarrow \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{P}{M^2} + \frac{H}{M_{\text{ball}} R_0} \frac{1}{a^2}\]

\(\sim\) Galilean invariance makes physics inside ball homogeneous + isotropic
Vacuum Energy

Instead of general Coleman-Weinberg potentials $\phi^4 \ln \phi, \phi^2$, which are conformal, we can only get $P_{\text{vac}} \sqrt{-g} = \frac{P_{\text{vac}}}{M_4^4} \phi^4$

$\Rightarrow$ Cosmological Constant Problem of Scalar Gravity

Sundrum '03

Even Weinberg's No-Go Theorem for CCP '89 applies to scalar gravity
UV Completion
of Chiral Lagrangian of Spontaneous conformal
\( = \text{CFT} \quad \text{with} \quad \div \text{MODULI SPACE} \equiv \text{Dilaton} \)

MODIFIED GRAVITY \( \equiv \text{QFT} \quad \equiv \text{Equivalence Principle} \)

Running is above the “Planck scale” \( \langle \phi \rangle \equiv M \)!
CONTINUUM LIMIT
about FP with single IR-relevant coupling

\[ L^{(x)} = L_{\text{CFT}}^{(x)} + \sigma^\phi O^{(x)} \]

\[ \uparrow \text{relevant coupling,}\]
\[ \text{mass \ parameter}\]
\[ \text{scaling dimension} \ 4-\delta \]

Consider "critical exponent"

\[ \gamma \ll 1 \ \text{parametrically} \]

We will assume \( \sigma \ll \langle \phi \rangle \) & check
self-consistency, so we can expand perturbatively in
\( (\sigma/\phi)^\gamma < 1 \)

\( \sigma^\delta \equiv \text{explicit conformal spurion.} \ \delta > 0 \equiv \text{soft breaking} \)
Cosmological Constant Suppression!

Contino, Pomarol, Rattazzi

(talk by Rattazzi, “Planck 2010”)

\[ Z_{\text{eff}} = (2\phi)^2 - \lambda \phi^4 + \lambda' \phi^4 \left( \frac{\sigma}{\phi} \right)^\delta \]

\[ V(\phi) = \lambda \phi^4 \left( 1 - \left( \frac{e^{1/4} M}{\phi} \right)^\delta \right) \]

\[ \left( \frac{\sigma}{M} \right)^\delta = \frac{4}{4-\delta} \frac{\lambda}{\lambda'} \]

Normally only relative scale factors (\( \phi \)) physical, but now absolute scale factor (relative to \( \sigma \)) is.
Useful Approximation

\[ V \approx -\chi \lambda \varphi^4 \ln e^{\frac{1}{4} M} \frac{\ln M/\varphi}{\varphi} \]

if \( |\ln M/\varphi| \ll 1 \), \( M e^{-\frac{1}{8}} \ll \varphi \ll M e^{\frac{1}{8}} \)

Recalling

\[ H^2 = \frac{\dot{\varphi}^2}{M^2} \]

\[ \mathcal{H}_{\text{conserved}} = \dot{\varphi}^2 + V(\varphi) \]

\[ a = \frac{\varphi}{M} \]

\[ \Rightarrow \text{Friedman Eq.} \]

\[ H^2 = G_N \left( \mathcal{H}_{\text{conserved}} a_4 - \chi \lambda M^4 \ln e^{\frac{1}{4} M} \right) \]

fake "radiation" cosmo. const. \( \Lambda \) \( \rightarrow \chi \lambda \ln M/\varphi (\ll \lambda) \)

Dark Energy!

\[ \lambda = \text{Planck} \gg 10^{-60} \Rightarrow \chi \sim 10^{-60} ! \]
Equivalence Pr.

\[ L = (\partial \Phi)^2 - V(\Phi) + \sum_j \bar{\psi}_j \left[ i \gamma^5 D_j \Phi + i \Phi \gamma^5 D_j \right] \psi_j \]

\[ m_j = \left[ y_j + y_j' \left( \frac{\sigma}{M} \right)^\delta \right] M \]

\[ \Phi_j = y_j + y_j' \left( 1 - \delta \right) \left( \frac{\sigma}{M} \right)^\delta \]

\(~\text{fractional error in equivalence}\)

Effect on long-range gravity

\[ m_\phi^2 \sim 8 \lambda M^2 \sim \rho_{\text{vac}} / M^2 \]
\( \chi \ll 1 \) Naturalness

\[ \text{AdS/CFT Dual} \equiv \text{RS} | \text{Goldberger-Wise Stabilization} \]

\[ \text{AdS}_5 \text{ "slice"} \]

\[ \text{5D GR} \]

\[ \text{Radion} = g_{55}(x) = \varphi(x) = \text{Dilaton} \]

\[ \text{Randall, Sundrum '99} \]
\[ \text{Goldberger, Wise '99} \]
\[ \text{Arkani-Hamed, Porrati, Randall '00} \]
\[ \text{Rattazzi, Zaffaroni '00} \]
$\mathfrak{g} \ll 1$ Naturalness

AdS/CFT Dual $\equiv$ RS1 + Goldberger-Wise

AdS$_5$ “slice”

5D scalar $\Sigma$

dual of $Q_{\text{CFT}}^{(x)}$

\[
M_{\Sigma, 5D}^2 = d_0 (d_0 - 4) \frac{R_{\text{AdS}}^2}{R_{\text{AdS}}^2} = -\frac{\mathfrak{g} (4 - \mathfrak{g})}{R_{\text{AdS}}^2} \approx -\frac{4\mathfrak{g}}{R_{\text{AdS}}^2}
\]

IR boundary position $\equiv q(x)$

nearly massless “good” tachyon.

$\Sigma$ nearly massless, 5D naturally because $\approx 5$D Pseudo-Goldstone Boson

Rattazzi Planck 2010 talk
PGB protection from Quantum Gravity?

AdS$_5 \times S^1$ $\quad \Lambda m$

6D Gauge Field $\Sigma = A_6 \Rightarrow m^2_{\Sigma, 5D} \sim e^{-m_{\text{charge}}} R^6 \equiv \gamma$

via Itosatani mechanism

naturally extremely small

But can have light charges on IR brane

$\Rightarrow$ brane-localized $\Sigma$-potential
Modified Cosmology

Sundrum
(to appear)

$\phi$ suppression of cosmological constant
if $\ln M/\phi(\text{today}) \sim O(1)$.

Seems mild, but obviously tiny part of phase space of model.
Such tuning in phase space may be no better than the continuum
landscape story in standard GR.

but sensible field theory, while the GR continuum landscape Θ Super-Planck VEVs.
Phase transition in matter sector suddenly changes vacuum energy $\lambda \to \lambda'$

Consider $\eta \leq 1$, $\lambda'$ unchanged say:

Potential $V(\phi) = 2\phi^4 (1 - \left(\frac{e^{\eta/\lambda} M}{\phi}\right)^8 )$

$\rightarrow 2\phi^4 (1 - \left(\frac{e^{\eta/\lambda} M}{\phi^{1/8}}\right)^8 )$
Suppose originally $\phi \sim O(M)$ so cosmological constant cancelation in effect.

before

state @ phase transition
Suppose originally $\varphi \sim O(M)$ so cosmological constant cancelation in effect.

Now, $V$

before

$M$

state @ phase transition

after

$\frac{M}{\sqrt[4]{\varphi}}$

We thereby “start” new phase without $\gamma$-suppression. In fact $-2 \varphi^5 (\frac{\xi}{\varphi})^\gamma$ dominates $\equiv$ HIGH INFLATION!
Eventually inflation ends as $\Phi \sim 0(\frac{M}{\phi^{1/2}})$ & $\phi$-suppression returns.

But by then $\sim 1/8$ e-foldings have inflated away any original matter/radiation.
**REHEATING AFTER AUTO-INFLATION**

Phase-transition-robust initial conditions,

\[ \phi \sim 0^+, \; \dot{\phi} \sim 0^+, \; V = \lambda \phi^4 \left( 1 - \left( \frac{e^{1/4} M}{\phi} \right)^8 \right) \]

naively \( \Rightarrow \) Cyclic Cosmology

But this neglects Cosmological Particle Production

E.g. into (real) radiation \( T_{\text{rad}} \sim H \)

requires some mass scale in radiation, e.g. massless QED
REHEATING AFTER AUTO-INFLATION maximal (in absolute terms, not Planck units) when $\phi \sim O(M)$, $\text{Tr}_{\text{rad}} \sim \sqrt{N} \ M$

$\implies$ Energy in $\phi$ reduced from $\phi_+$ by $\frac{\text{Tr}_{\text{rad}}}{\phi_+}^{\frac{4}{9}} \sim \delta^2 \chi^2 \ M^4$

In Planck units, at first bounce $\frac{\text{Tr}_{\text{rad}}}{\phi_+} \sim \sqrt{N}$

but after 2nd bounce $\frac{\text{Tr}_{\text{rad}}}{\phi_+} \sim (\delta \chi)^{\frac{1}{4}} \equiv 3^\circ \text{K radiation}$

BUT NO "BARYONS"...
"BARYONS" ($m \gg (\rho_{\text{vac, today}})^{1/4} \sim 10^{-3} \text{eV}$)

Imagine particles with "$\nu$-like" masses so that radiation can annihilate into them.

> $\phi^*$:

Few $\nu$ re-annihilate back to radiation:

$\hat{n}_\nu \sim T_{\text{rad}}^3$ conserved

Suppose $\nu$ decays @ $\phi \sim \alpha(M)$, each to $N \gg 1$ "$\chi$'s" (Eg. $Z^0 \rightarrow$ many hadrons $\exists$ many $\pi^0 \rightarrow$ many $\chi$s.)

$\Rightarrow \hat{n}_\chi \sim N \hat{n}_\nu \sim N T_{\text{rad}}^3 \sim N M^3 (\gamma)_{\text{had}}^3/2$, $E_\gamma \sim y_\nu M / N \sim (\gamma)_{\text{had}}^{1/4} M / N$.
"BARYONS" \((M \gg (\rho_{\text{vac, today}})^{1/4} \sim 10^{-3}\text{eV})\)

Energy density thereby dumped into radiation & lost to \(\phi \sim \hat{\gamma}_8 E_8 \sim (\gamma_8)^{7/4} M^4\).

\(\Rightarrow 4^{\text{th}}\) bounce @ \(\phi_{***} : \gamma A \phi_{***}^4 \sim (\gamma A)^{7/4} M^4\)

\(\phi_{***} \sim (\gamma A)^{3/16} M\)

In Planck units, the radiation can be quite energetic \(\frac{E_8}{\phi_{***}} \sim (\gamma A)^{1/16} \frac{1}{N}\).
can be produced now
\[ M_{\text{baryon}} \sim E_\gamma \sim (\gamma \Delta)^{1/6} \]

\[ \frac{E_\gamma}{\Phi_{**}} \]

\[ \text{radiation} \sim \text{"baryons"} \]

\[ \hat{n}_\gamma^2 \sigma_{\gamma \gamma \rightarrow \text{baryons}} \sim \frac{1}{M_{\text{baryon}}^2} \]

\[ \hat{n}_\gamma^2 \sigma_{\gamma \gamma \rightarrow \text{baryons}} \sim N^2 M^6 (\gamma \Delta)^3 \frac{N^2}{(\gamma \Delta)^{1/2} M^2} \]

\[ \sim N^4 M^4 (\gamma \Delta)^{5/2} \]

\[ \text{Rate/volume} \sim N^4 M^4 (\gamma \Delta)^{5/2} \]

\[ \text{over Period} \quad H^{-1} \sim \frac{1}{(\gamma \Delta)^{1/2} \Phi_{**}} \]

\[ \Rightarrow \quad \rho_{\text{baryons}} \sim m_{\text{baryon}} H^{-1} \hat{n}_\gamma^2 \sigma_{\gamma \gamma \rightarrow \text{baryons}} \sim N^3 (\gamma \Delta)^{21/16} \]

\[ \leq \frac{\rho_{\text{rad}}}{\Phi_{**}} \sim \gamma \Delta \]

\[ \text{but} \quad \frac{\rho_{\text{rad}}}{\Phi_{**}} \sim \gamma \Delta \]

Saturating, \[ m_{\text{baryon}} \sim (\gamma \Delta)^{1/6} \Phi_{**} \equiv \text{"10 MeV"}, \quad N \sim 10^{12} \]
DARK ENERGY EQUATION OF STATE \( \omega = \frac{P}{\rho} \)

Generally, \( \rho \propto a^{-3(1 + \omega)} \propto \phi^{-3(1 + \omega)} \)

For Dark Energy \( P_{DE} \propto \frac{1 - 3(1 + \omega) \ln \phi}{\phi^{**}} \)

We have \( \frac{P}{M^4} = \alpha 1 + \frac{\ln \phi}{\phi^{**}} \frac{\ln \phi}{\phi^{**}} M \)

\( \omega + 1 \sim \frac{1}{\ln \phi^{**} / M} \sim 1\% \)
CENTRAL THORNY ISSUE for good analogy

Our scalar cosmo = "Cold" Big Bang \( \sim 3\,\text{K} \) while we have evidence in real world of "Hot" Big Bang (eg BBN).

Can hot follow cold?

Eg. Newtonian Matter Cosmology

\[ \text{v.dilute matter ball at rest} \rightarrow \text{Newtonian collapse stopped by short range matter repulsion} \rightarrow \text{Bounce outwards } \Rightarrow \text{illusion of high density matter dominated Cosmology} \]
MODIFIED spin-2 GR?

Egs. 
- Dvali - Gabadadze - Porrati '00
- Einstein - Aether Jacobson, Mattingly '01
- Ghost Condensate Arkani-Hamed, Cheng, Luty, Mukohyama '04
- Horava - Lifshitz '09

Moral I draw from scalar gravity:
- Deform holographic (non-redundant)
- Description harder for real GR!

Eg. deforming CFT \rightarrow \text{Modified Sundrum '07}
- AdS gravity
BACKUP SLIDES
Radiative Corrections

Scale anomaly of QED + classical φ

\[ \mathcal{L}_{4+\varepsilon_D} = \overline{\psi} (i \gamma \partial - g \phi) \psi - \frac{\mu^3}{4e^2} F_{\mu\nu}^2 \]

\[ \Pi(q) \sim \frac{1}{e^2(\mu)} + \frac{1}{16\pi^2} \frac{\ln \frac{\mu^2}{y^2\phi^2 + q^2}}{y^2\phi^2 + q^2} \]
V. Long distances

\[ L_{\text{eff}} = -\frac{1}{4} \left( \frac{1}{e^2} + \frac{1}{16\pi^2} \ln \frac{\mu^2}{\nu^2} \phi^2 \right) F_{\mu\nu}^2 \]

Gravity picture must maintain general covariance in 4+\(\epsilon\)

Graviton background \(\Rightarrow\) conf. symmetry

\[ L_{4+\epsilon} = \sqrt{-g} \left\{ \bar{\psi} \left( i\gamma^\mu \right) \gamma^\nu \psi - \mu^2 g_{\nu\rho} \gamma^\nu \gamma^\rho \left( \frac{\mu\phi}{M} \right)^3 F_{\mu\nu} \right\} + \text{grav} \]

Long Distances \( L_{\text{eff}} = -\frac{1}{4} \left( \frac{1}{e^2} + \frac{1}{16\pi^2} \ln \frac{\mu^2}{\nu^2} \phi^2 \right) F_{\mu\nu}^2 \)
MORAL Scale invariant theory:

\[ L = -\frac{1}{4e^2(M\Phi)} F_{\mu\nu}^2 + \Phi (i\slashed{D}m - y\Phi) \Phi \]

\[ = -\frac{1}{4} \left( \frac{1}{e^2 M} + \frac{1}{16\pi^2} \left( \frac{\Phi}{M^2} + \frac{\Phi^2}{M^4} + \ldots \right) \right) F_{\mu\nu}^2 + \Phi (i\slashed{D} - y\Phi) \Phi \]

price is non-renormalizability

Non-ren. Chiral Lagrangian for dilaton + light matter/radiation < M
MORAL Scale invariant theory:

\[ \mathcal{L} = - \frac{1}{4 e^2 (\mu \phi)} F_{\mu \nu}^2 + \bar{\psi} \left( i D_{\mu} - m - y \phi \right) \psi \]

\[ = - \frac{1}{4} \left( \frac{1}{e^2 \lambda} + \frac{1}{16 \pi^2} \left( \frac{\phi}{M} + \frac{\phi^2}{M^2} + \cdots \right) \right) F_{\mu \nu}^2 \]

\[ + \bar{\psi} \left( i D_{\mu} - m - y \phi \right) \psi \]

Price is non-renormalizability

Non-ren., Chiral Lagrangian for dilaton + light matter/radiation < M

UV-completed by CFT with \( \pm \) modulus \( \phi \) & other light (\( \ll \ll \phi \)) matter/radiation,
\[ m_{\text{grav}} = m_{\text{inertial}} \text{ for proton?} \]

\[
L_{\text{QCD}} = \sum_{m_{\text{quarks}}=0} + 0
\]

\[
= -\frac{1}{4} \left( \frac{1}{g^2 m} - \frac{1}{16\pi^2} \ln \phi \right) G_{\mu\nu}^a
\]

\[
+ 4 i \not{D} \not{a} \phi
\]

\[
= -\frac{1}{4g^2 (M^2)} G_{\mu\nu}^a
\]

Normally, \( m_{\text{proton}} \approx \mu e^{-\frac{16\pi^2}{g^2 m}} \)

\[ + \text{soft qQ} \text{x1} \]

\[ \approx \mu e^{-\frac{16\pi^2}{g^2 m}} + \ln \phi \frac{M}{M} \equiv m_{\text{proton}} \frac{\phi}{M} \]
**Discretum**

**Adjustment Mechanisms of Standard GR**

Continuum Landscape

- **v. gentle slope**
  - starting here, remains unstuck here
  - liveable

Abbott '85

- fall/tunnel
  - long-lived...
  - but matter/rad. inflated away

Can be borrowed. Anything new?

Garriga, Vilenkin '03; Dimopoulos, Thomas '03