SUSY Bulk Matter RS Model and Its Signatures

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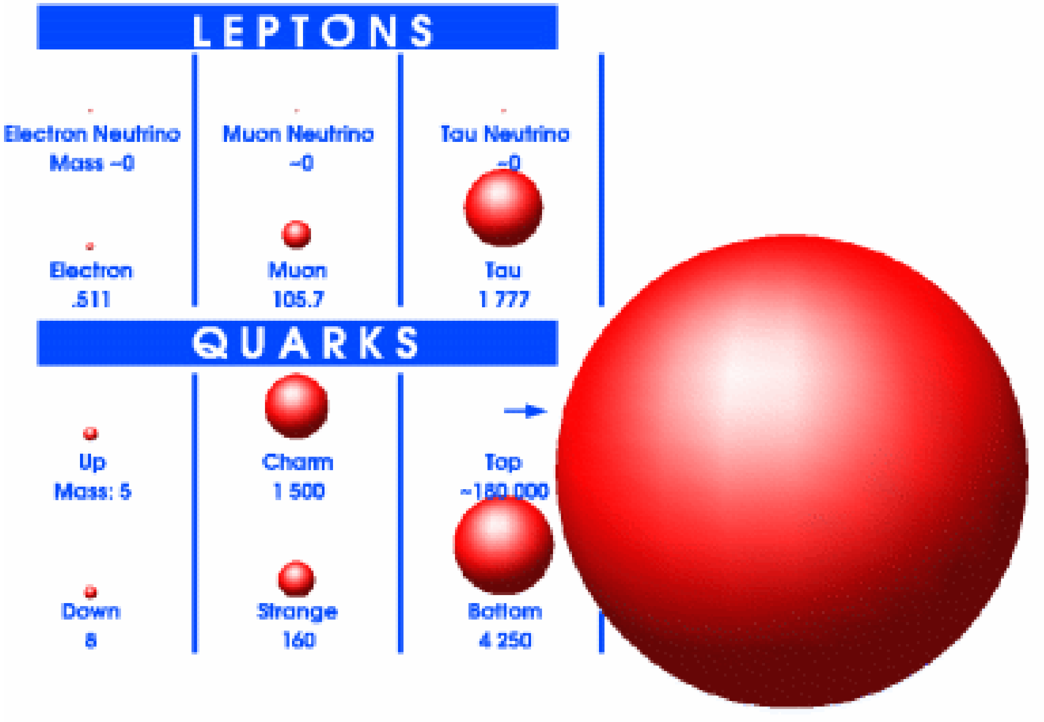
Based on TY, Phys.Rev.D85,016007

Motivation

Origin of the Yukawa coupling hierarchy ?



$$m_e \sim 511 \text{ keV}$$

$$m_t \sim 175 \text{ GeV}$$

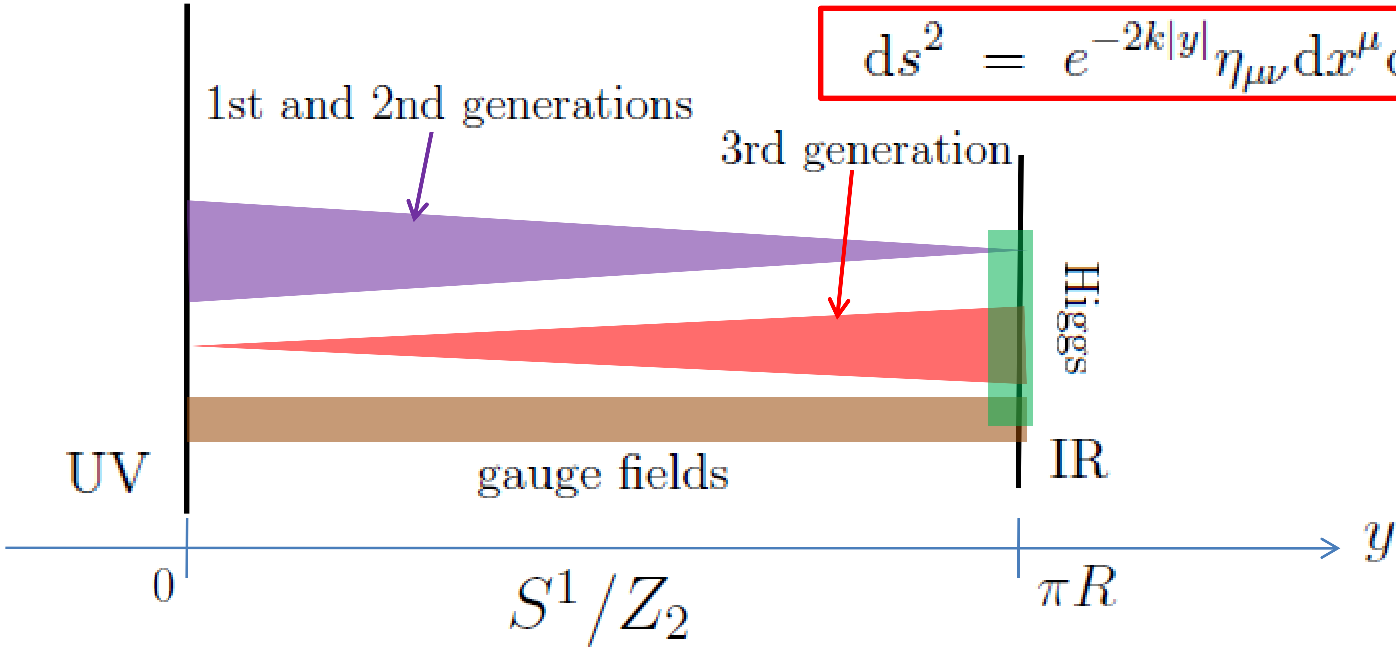
10^5 hierarchy

Want to derive the hierarchical Yukawas from O(1) fundamental parameters.

Bulk matter RS model is a viable solution.

Bulk Matter RS Model

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$



Each matter field has a O(1) (in unit of AdS curvature) 5D Dirac mass c_i .

Overlap btwn matter fields and IR brane depends **exponentially** on c_i :

$$\sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}}\sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}}$$

Exponential hierarchy of the Yukawa couplings

On Kaluza-Klein Scale

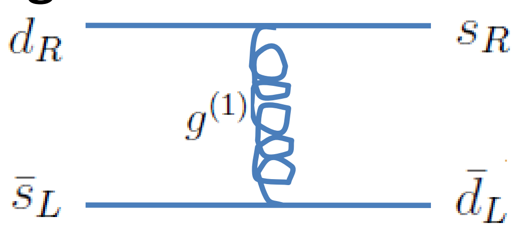
- Original bulk matter RS model

  Solution to the Yukawa & "**gauge hierarchy**" problem

- However,

1. The KK scale is already severely constrained :

   e.g. KK gluons induce FCNCs, such as



   Tree level contribution to K^0-$\bar{K}^0$ mixing.

   From ϵ_K measurement, 1st KK gluon mass : $\boxed{M_{g^{(1)}} > 21 \text{ TeV}}$.

   C.Csaki, A.Falkowski, A.Weiler (2008)

2. The KK scale need not be at *TeV* scale to explain only the Yukawa coupling hierarchy.

TeV scale New Physics + Bulk matter RS

- The ``gauge hierarchy problem" (i.e. $\delta m_h^2 \propto \frac{1}{16\pi^2}\Lambda^2$) must be solved by some TeV-scale new physics.

  Bulk matter RS may not serve for this purpose.

⇒ Other new physics **at TeV scale +** Bulk matter RS **at higher scale**.

Signatures of the bulk matter RS model ?

- What if the new physics at TeV-scale has its own **flavor structure** ?
- Correlation btwn the flavor structure of the TeV-scale new physics and the SM Yukawa couplings.

MSSM + Bulk Matter RS

- Take MSSM as an example of new physics at TeV.
- Flavor-violating gravity mediation contributions :

$$\int \mathrm{d}^4\theta \; c_{ij} \, \frac{X^\dagger X}{M_{Pl}^2} Q_i^\dagger Q_j \quad \text{with} \quad F_X \neq 0 \quad .$$

- c_{ij} has its own **flavor structure** which is supposed independent of the SM Yukawas, $(Y_u)_{ik}, \; (Y_d)_{il}, \; (Y_e)_{mn}$.
- However, if bulk matter RS emerges at a higher scale, there should be a correlation btwn the hierarchical structures of c_{ij} and $(Y_u)_{ik}, \; (Y_d)_{il}, \; (Y_e)_{mn}$.

⇒ **Observing signatures of bulk matter RS thru c_{ij} .**

Outline

- Motivation
- Setup

    Bulk Matter RS Model

    SUSY Bulk Matter RS Model

    SUSY Bulk Matter RS Model + Soft SUSY Breaking
- Soft SUSY Breaking Terms

    Gravity Mediation vs. Yukawa RG
- Signatures

    Observable quantities

    Predictions of bulk matter RS and other models
- Conclusion

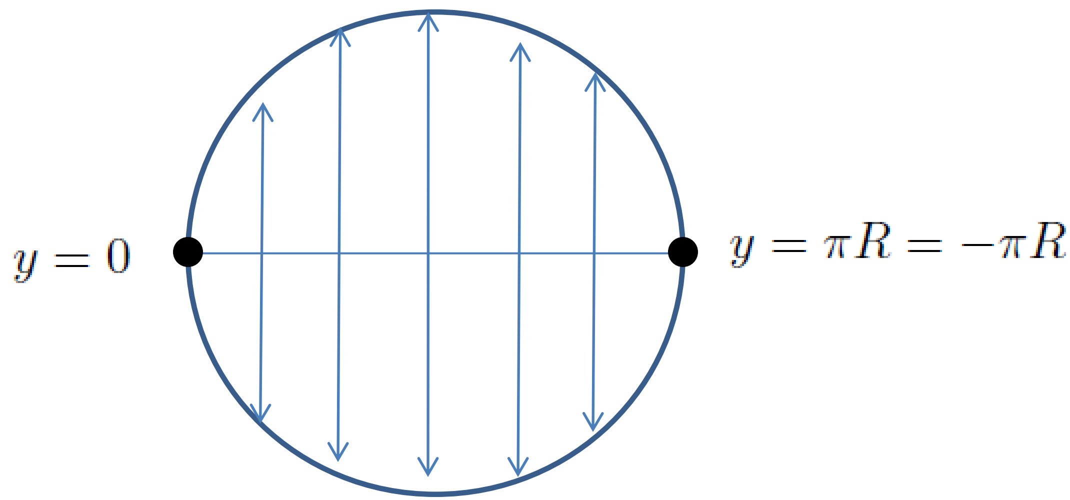
Setup -
Bulk Matter RS Model

5D Warped Spacetime

L.Randall, R.Sundrum (1999)

- Metric: $d^2s = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - d^2y$ (k : AdS curvature)

  The 5th dimension, y , is compactified on S^1/Z_2 .



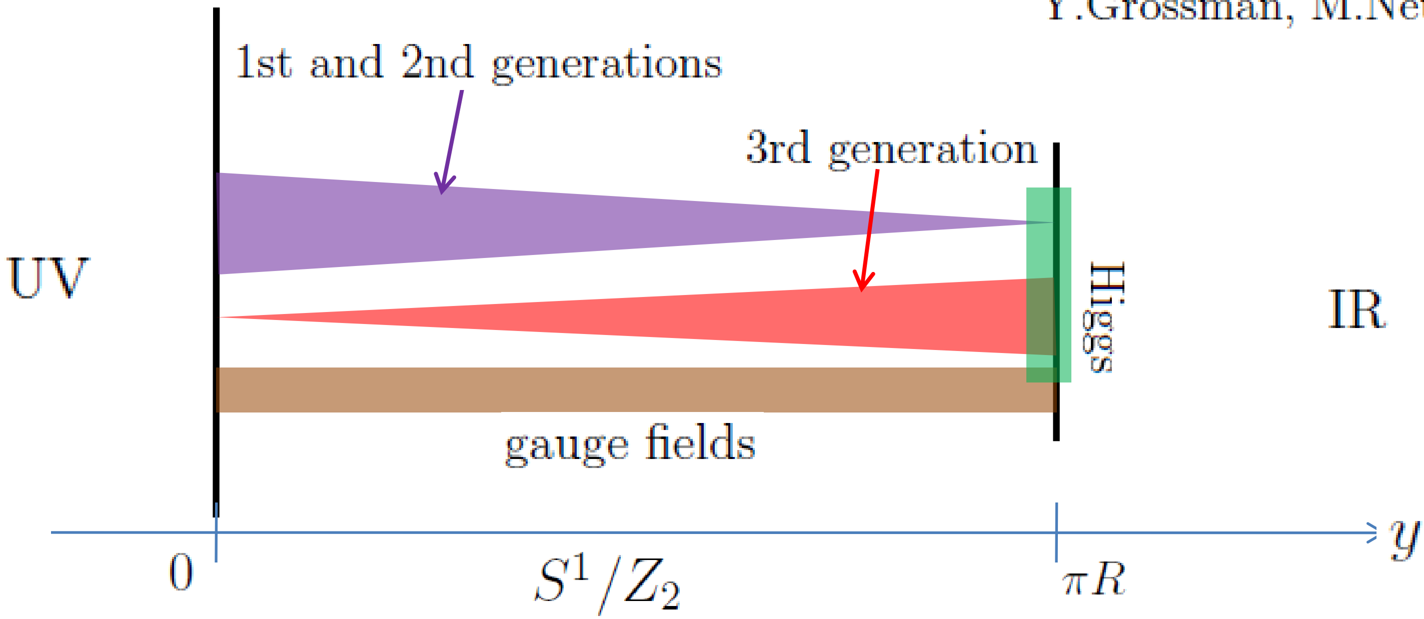
Identify $y \leftrightarrow -y$.

- In the following discussion, we take

$$\text{KK scale} \sim ke^{-kR\pi} \sim M_{Pl}e^{-kR\pi} >> \text{TeV}$$.

Bulk Matter RS Model

Y.Grossman, M.Neubert (2000)



- Thanks to Z_2 -parity, only one of the chiral fields has 0-mode.
- Each matter field has a 5D Dirac mass (c in unit of AdS curvature), which controls the field **localization** in 5D spacetime.

⇨ y -dependence of the 0-mode wavefunction $f_L^{(0)}(y) \propto e^{(2-c)k|y|}$.

- ``**Geometrical overlap**" btwn a matter field and IR-localized Higgs is proportional to

$$\sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}} \quad (c_i : \text{5D Dirac mass}, \quad i : \text{flavor index})$$

- Yukawa coupling in 4D effective theory is given by

$$Y_{ij} = y_{ij} \sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}} \sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}} \quad .$$

5D fundamental Yukawa coupling

- Now **assume** $y_{ij} \sim O(1)$ and attribute the Yukawa hierarchy solely to the geometrical overlap factors.
- Write $\alpha_i \equiv \sqrt{\frac{1-2c_{q\,i}}{2\{1-e^{-(1-2c_{q\,i})kR\pi}\}}}$ for SU(2) doublet quarks Q_i ,

β_i for SU(2) singlet up-type quarks U_i , γ_i for down-type quarks D_i ,
δ_i for SU(2) doublet leptons L_i , ϵ_i for charged leptons E_i .

On an **arbitrary** flavor basis,

Up-type Yukawa : $(Y_u)_{ij} \sim \beta_i \alpha_j$

Down-type Yukawa : $(Y_d)_{ij} \sim \gamma_i \alpha_j$

Charged lepton Yukawa : $(Y_e)_{ij} \sim \epsilon_i \delta_j$

Neutrino Majorana mass : $(M_\nu)_{ij} \propto \delta_i \delta_j$

We can determine $\alpha_i, \beta_i, \gamma_i, \delta_i, \epsilon_i$ from tables in PDG.

- From the fermion masses, CKM matrix components and nearly democratic structure of the neutrino mass matrix, we estimate (in MSSM)

$$
\begin{array}{llll}
\alpha_1 \sim \lambda^3 \,, & \alpha_2 \sim \lambda^2 \,, & \alpha_3 \sim 1 \\
\beta_1 \sim \lambda^{-3}\dfrac{m_u}{v} \,, & \beta_2 \sim \lambda^{-2}\dfrac{m_c}{v} \,, & \beta_3 \sim 1 \\
\gamma_1 \sim \lambda^{-3}\dfrac{m_d}{v}\tan\beta \,, & \gamma_2 \sim \lambda^{-2}\dfrac{m_s}{v}\tan\beta \,, & \gamma_3 \sim \dfrac{m_b}{v}\tan\beta \\
\delta_1 \sim \delta/3 \,, & \delta_2 \sim \delta \,, & \delta_3 \sim \delta \\
\epsilon_1 \sim \delta^{-1}\dfrac{m_e}{v}\tan\beta \,, & \epsilon_2 \sim \dfrac{1}{3}\delta^{-1}\dfrac{m_\mu}{v}\tan\beta \,, & \epsilon_3 \sim \dfrac{1}{3}\delta^{-1}\dfrac{m_\tau}{v}\tan\beta
\end{array}
$$

where $\lambda = 0.22$.

(Absolute scale of δ is undetermined because we do not know the seesaw scale.)

Setup -
SUSY Bulk Matter RS Model

5D N=1 SUSY

T.Gherghetta, A.Pomarol (2000)

- 5D N=1 SUSY can be expressed in terms of 4D N=2 SUSY.

- 5D N=1 gauge superfield ⇒

4D N=1 gauge supermultiplet + chiral supermultiplet + $\mathrm{SU}(2)_R$ btw (λ_1, λ_2)

$$V^a(x,\ \theta,\ \bar\theta,\ y) \ = \ -\theta\sigma^\mu\bar\theta A^a_\mu \ - \ i\bar\theta^2\theta\lambda^a_1 \ + \ i\theta^2\bar\theta\bar\lambda^a_1 \ + \ \frac{1}{2}\theta^2\bar\theta^2 D^a$$

$$\chi^a(x - i\theta\sigma\bar\theta,\ \theta,\ y) \ = \ \frac{1}{\sqrt{2}}(\ \Sigma^a \ + \ iA^a_5\ ) \ + \ \sqrt{2}\theta\lambda^a_2 \ + \ \theta^2 F$$

$\mathrm{SU}(2)_R$

- Action for 5D N=1 gauge superfield :

$$S_{5D\,gauge} \ = \ \int \mathrm{d}y \int \mathrm{d}^4x\ e^{-4k|y|}\ \frac{1}{(g_5)^2}\ \left[\ \frac{1}{4}\int \mathrm{d}^2\theta e^{k|y|}\ \mathrm{tr}\left\{\ (e^{\frac{3}{2}k|y|}W^\alpha)(e^{\frac{3}{2}k|y|}W_\alpha)\ \right\} \ + \ \mathrm{h.c.} \right.$$

$$\left. + \ \int \mathrm{d}^4\theta e^{2k|y|}\ \mathrm{tr}\left\{\ (\sqrt{2}\partial_y + \chi^\dagger)e^{-V}(-\sqrt{2}\partial_y + \chi)e^V \ - \ (\partial_y e^{-V})(\partial_y e^V)\right\}\right]$$

- Z_2-parity is assigned as $V(-y) = V(y)$, $\chi(-y) = -\chi(y)$.

- We want to know the y -dependence of the 0-modes. Thanks to SUSY, it is sufficient to calculate their fermionic components.
- For the fermionic components,

$$S_{5D\,gauge} \supset \int \mathrm{d}y \int \mathrm{d}^4x e^{-2k|y|} \frac{1}{g_5^2} \left(-i\ \bar{\lambda}_2 \partial_y \bar{\lambda}_1\ + i\ \lambda_2 \partial_y \lambda_1\right) \quad .$$

The 0-modes of λ_1 , λ_2 satisfy $\partial_y \lambda_1^{(0)} = 0$, $\partial_y \lambda_2^{(0)} = 0$.

From Z_2 -parity, we have $\lambda_1^0(x,y) = \lambda_1^0(x), \quad \lambda_2^0(x,y) = 0$.

⇒ **Only V has 0-mode, which is independent of y .**

- 5D N=1 matter superfield $\Rightarrow$
  Two 4D N=1 chiral supermultiplets + $\mathrm{SU}(2)_R$ btw (λ_1, λ_2) .

$$\Phi(x - i\theta\sigma\bar{\theta},\ \theta,\ y) \ = \ \phi \ + \ \sqrt{2}\theta\lambda_1 \ + \ \theta^2 F$$

$$\Phi^c(x - i\theta\sigma\bar{\theta},\ \theta,\ y) \ = \ \phi^c \ + \ \sqrt{2}\theta\lambda_2 \ + \ \theta^2 F^c$$

$\mathrm{SU}(2)_R$

Gauge transformation : $\Phi \ \to \ e^{\Lambda}\ \Phi$, $\Phi^c \ \to \ e^{-\Lambda^T}\ \Phi^c$.

- Action for 5D N=1 matter superfield :

$$S_{5D\,matter} \ = \ \int \mathrm{d}y \int \mathrm{d}^4x \ e^{-4k|y|} \left[\int \mathrm{d}^4\theta e^{2k|y|} \ (\Phi^\dagger e^{-V}\Phi \ + \ \Phi^c e^{V}\Phi^{c\,\dagger}) \right.$$
$$\left. + \ \int \mathrm{d}^2\theta e^{k|y|} \ \Phi^c\{\partial_y - \chi/\sqrt{2} - (3/2 - c)k\}\Phi \ + \ \mathrm{h.c.} \right]$$

5D Dirac mass

- Z_2 -parity is assigned as $\Phi(-y) = \Phi(y)$, $\Phi^c(-y) = -\Phi^c(y)$.

- For the fermionic components,

$$S_{5D\,matter} \supset$$

$$\int \mathrm{d}y \int \mathrm{d}^4 x e^{-2k|y|} \left[-i\, \bar{\lambda}_2 \{\partial_y - (3/2-c)k\,\mathrm{sgn}(y)\} \bar{\lambda}_1 + i\, \lambda_2 \{\partial_y - (3/2-c)k\,\mathrm{sgn}(y)\} \lambda_1 \right].$$

The 0-modes of λ_1, λ_2 satisfy

$$\{\partial_y - (3/2-c)\}\lambda_1^{(0)} = 0\,, \qquad \{\partial_y - (3/2-c)\}\lambda_2^{(0)} = 0 \quad .$$

From Z_2 -parity, we have $\lambda_1^0(x,y) = e^{(3/2-c)k|y|}\, \tilde{\lambda}_1^0(x), \quad \lambda_2^0(x,y) = 0$.

➡ Only Φ has 0-mode, which is prop. to $e^{(3/2-c)k|y|}$.

Radion

M.Luty, R.Sundrum (2000)

- So far, we have worked on Global SUSY in rigid RS spacetime.
- Solving 5D SUGRA, we find another Z_2-even superfield,
  Radion $T = g_{55} + iB_5 + \text{other components}$,
  which couples in the following way :

$$S_5 \supset \int \mathrm{d}y \int \mathrm{d}^4x \left[\int \mathrm{d}^4\theta \, \frac{T+T^\dagger}{2} e^{-(T+T^\dagger)k|y|} \left(\Phi^\dagger \Phi + \Phi^{c\dagger} \Phi^c \right) \right.$$

$$\left. + \int \mathrm{d}^2\theta \, e^{-3Tk|y|} \left\{ \Phi^c (\partial_y - (3/2 - c)k) \Phi + W_{IR} \delta(|y| - \pi R) + W_{UV} \delta(y) \right\} + h.c. \right]$$

- It is easy to stabilize the radius with large radion mass in SUSY limit.

  Soft SUSY breaking terms never destabilize the radius.

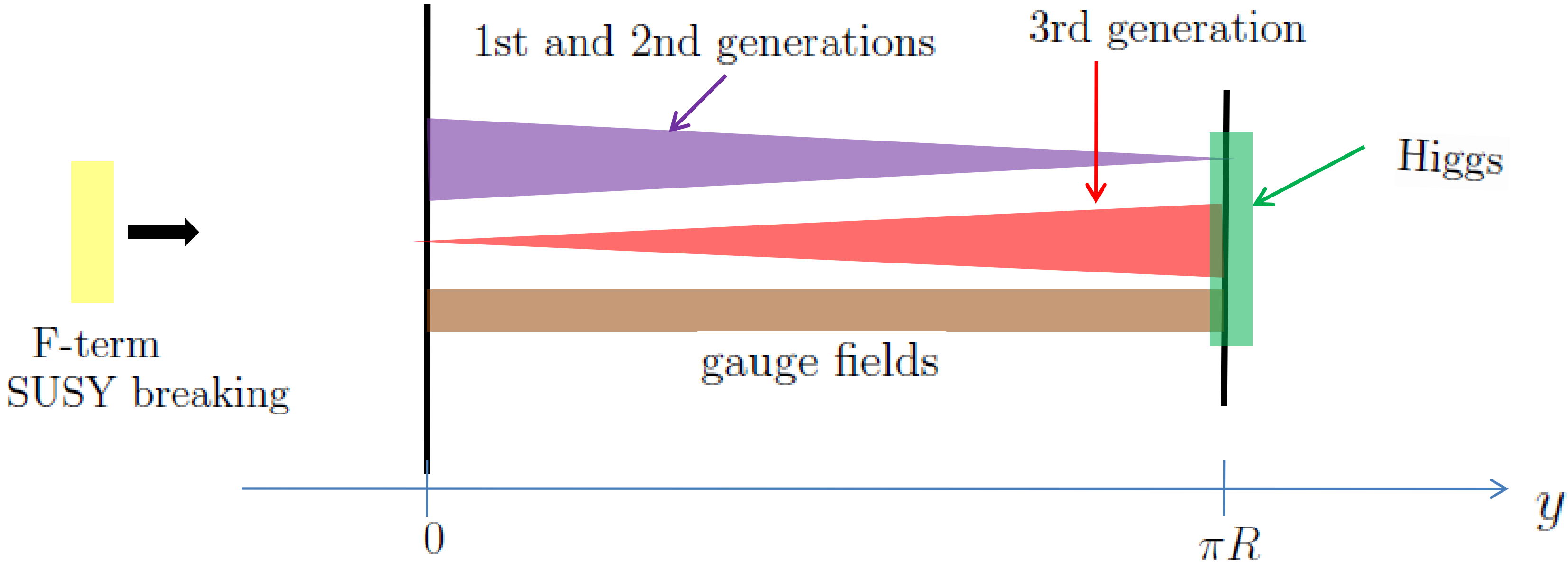
N.Maru, N.Okada (2003)

Just introduce a hypermultiplet : $(H,\ H^c)$ and
constant source terms on both branes :

$$W = HJ_0\delta(y) - HJ_\pi\delta(|y| - \pi) .$$

Setup -
SUSY Bulk Matter RS Model with Soft SUSY Breaking

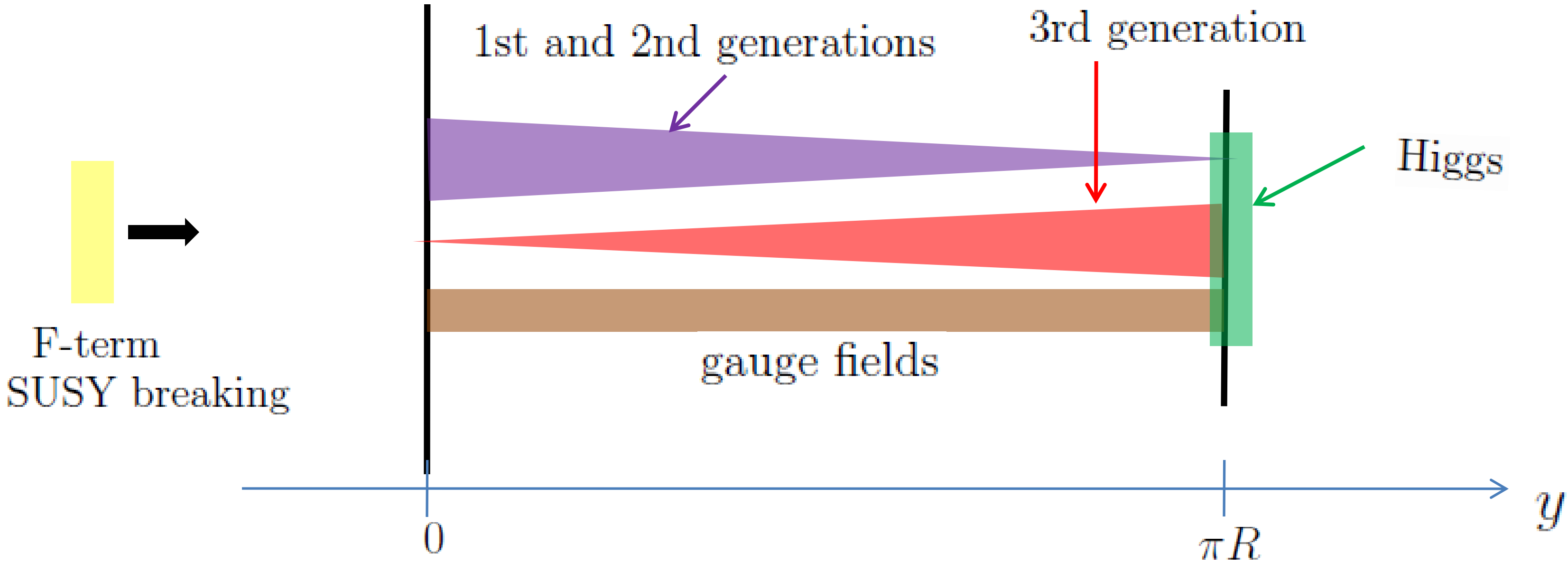
Where to Put SUSY Breaking Terms



- Three choices :

1. On the IR brane ⇒ Flavor-violating gravity mediation terms for 1st and 2nd generations are automatically suppressed.
2. On the UV brane ⇒ They are not suppressed; need additional setup like gauge mediation; the model reduces to conventional 4D one.
3. Radion F-term ⇒ The F-term of radion may have non-zero VEV. Discuss some other time.

Where to Put SUSY Breaking Terms



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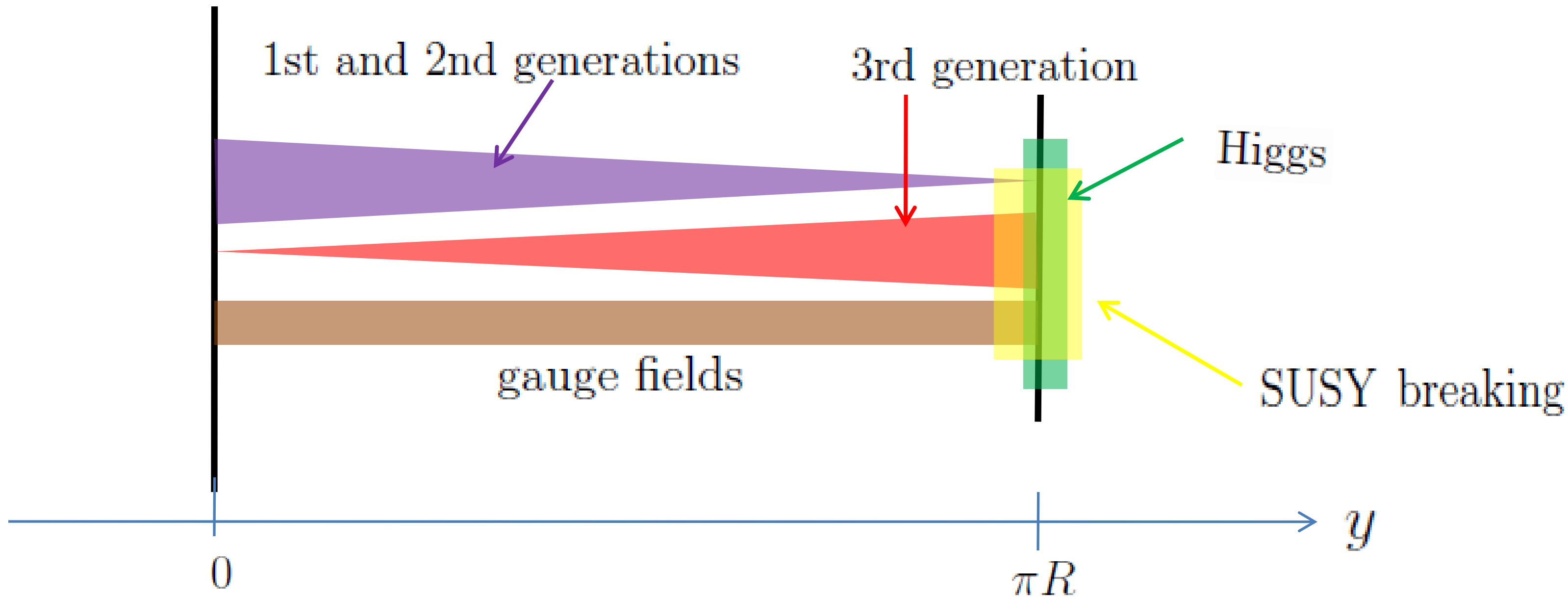
MSSM + Bulk Matter RS Model

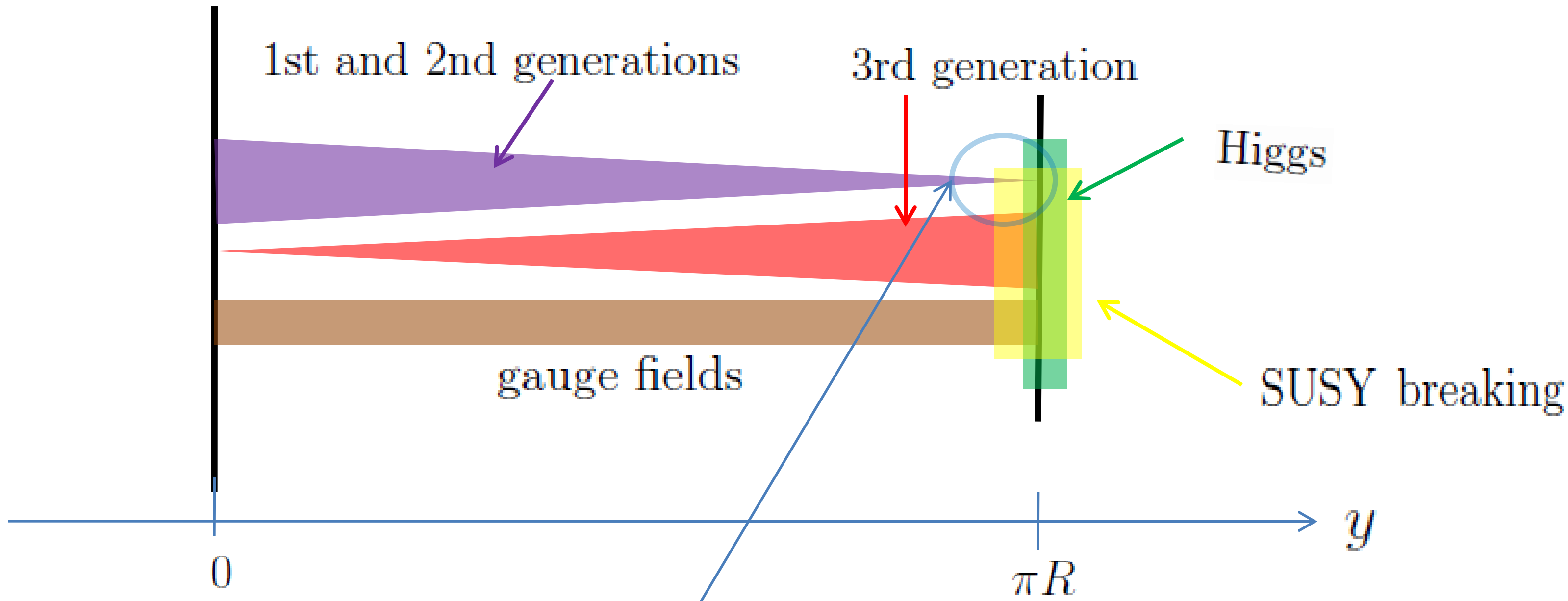


- Gravity mediation for 1st and 2nd generation matter superfields is geometrically suppressed.

  (“More than IR-scale suppression”)

  ⇨ Gaugino mediation works.

MSSM + Bulk Matter RS Model



- Gravity mediation for 1st and 2nd generation matter superfields is geometrically suppressed.

  ("More than IR-scale suppression")

  ⇨ Gaugino mediation works.

- We may add messenger fields to make gauge mediation work.

Hybrid of Gravity mediation on the IR brane, Gaugino mediation and (optional) gauge mediation.

(Anomaly mediation contributions are suppressed by warp factor and loop factor.)

Soft SUSY Breaking Terms

Two Scales of Soft Terms

I. Gravity mediation contributions

Gaugino mediation contributions

(Gaugino masses arise from contact terms, then their RG effects give rise to matter soft masses)

➡ The scale of these contributions is given by:

$$\sim \frac{|<F>|}{M_5 e^{-kR\pi}} \equiv M_X$$

II . Gauge mediation contributions

➡ The typical scale of messenger mass determines the scale of soft terms from these contributions :

$$\sim \frac{1}{16\pi^2}\frac{|<F>|}{M_{mess}} \equiv M_G$$

Flavor Structure of Soft Terms

- Gravity mediation violates flavor.
- Gravity mediation contributions arise from contact terms on IR brane:

$$\int \mathrm{d}^4\theta\, c_{ij} \frac{X^\dagger X}{(M_5 e^{-k\pi R})^2} \sqrt{\frac{1-2c_i}{2\{1-e^{-(1-2c_i)kR\pi}\}}} \sqrt{\frac{1-2c_j}{2\{1-e^{-(1-2c_j)kR\pi}\}}}\, Q_i^\dagger Q_j \ , \quad c_{ij} \sim O(1) \ .$$

They also depend on the overlap factors, $\alpha_i, \beta_i, \gamma_i, \delta_i, \epsilon_i$.

$$(m_Q^2) \sim \alpha_i\alpha_j M_X^2, \quad (m_U^2) \sim \beta_i\beta_j M_X^2, \quad (m_D^2) \sim \gamma_i\gamma_j M_X^2,$$
$$(m_L^2) \sim \delta_i\delta_j M_X^2, \quad (m_E^2) \sim \epsilon_i\epsilon_j M_X^2;$$
$$(A_u)_{ij} \sim \beta_i\alpha_j M_X, \quad (A_d)_{ij} \sim \gamma_i\alpha_j M_X, \quad (A_e)_{ij} \sim \epsilon_i\delta_j M_X$$

- Gaugino mediation and gauge mediation generate only flavor-universal terms.

Flavor-non-universal soft terms are the key to observe signatures of Bulk Matter RS Model !

Gravity Mediation vs. Yukawa RG

- Unfortunately, RG of Yukawa couplings also generate flavor-non-universal soft terms, as in ``Minimal Flavor Violation" scenario.
- Want to distinguish Gravity mediation contributions from Yukawa RG contributions.

Magnitudes of Yukawa RG Contributions

- Take the basis where $(Y_u)_{ij}$ is diagonal.
- MFV contributions to $(m_U^2)_{ij},\ (A_u)_{ij}$ in this basis can be expressed as

$$\Delta(m_U^2)_{ij} \sim (Y_u)_{ik}(Y_d^\dagger)_{kl}(Y_d)_{lm}(Y_u^\dagger)_{mj}\ \max\{M_X^2, M_G^2\}$$
$$\sim \beta_i(\alpha_i)^2(\gamma_3)^2(\alpha_j)^2\beta_j\ \max\{M_X^2, M_G^2\} \qquad (i \neq j)$$

$$\Delta(A_u)_{ij} \sim (Y_u)_{ik}(Y_d^\dagger)_{kl}(Y_d)_{lj}\ M_X^2$$
$$\sim \beta_i(\alpha_i)^2(\gamma_3)^2\alpha_j\ M_X^2 \qquad (i \neq j)$$

⇒ MFV contributions to $(m_U^2)_{ij},\ (A_u)_{ij}\ \ (i \neq j)$ is much smaller than Gravity mediation contributions due to extra small geometrical factors, unless $M_G >> M_X$.

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$$\Delta(A_u)_{ij} \sim (Y_u)_{ik}(Y_d^\dagger)_{kl}(Y_d)_{lj}\ M_X^2 \qquad (i \neq j)$$
$$\sim \beta_i(\alpha_i)^2(\gamma_3)^2\alpha_j\ M_X^2$$

⇒ MFV contributions to $(m_U^2)_{ij},\ (A_u)_{ij}\quad (i \neq j)$ can be much smaller than Gravity mediation contributions due to extra small geometrical factors, unless $M_G >> M_X$.

- Diagonal flavor-non-universal terms follow a slightly different formula.

MFV contributions :

$$\Delta(m_U^2)_{ii} \sim (Y_u)_{ii}(Y_u^\dagger)_{ii} \sim (\beta_i)^2(\alpha_i)^2 \max\{M_X, M_G\}^2 .$$

➡ For 1st and 2nd generations, gravity mediation contributions may surpass MFV ones.

- Similar arguments for $(m_D^2)_{ij},\ (A_d)_{ij},\ (m_E^2)_{ij},\ (A_e)_{ij}$.

- MFV contributions to $(m_Q^2)_{ij}$ can be expressed as

$$\Delta(m_Q^2)_{ij} \sim (Y_d^\dagger)_{ik}(Y_d)_{kj} \max\{M_X^2, M_G^2\}$$
$$\sim \alpha_i(\gamma_3)^2\alpha_j \max\{M_X^2, M_G^2\}$$

⇒ MFV contribution to $(m_Q^2)_{ij}$ can be of the same order as Gravity mediation contribution because $\gamma_3 \sim (m_b/v) \tan\beta$.

- Similar arguments for $(m_L^2)_{ij}$ if we assume seesaw.

Short Summary:

- Gravity mediation contributions may appear as a deviation from MFV in the following soft terms :

$$(m_U^2)_{ij},\ (A_u)_{ij},\ (m_D^2)_{ij},\ (A_d)_{ij},\ (m_E^2)_{ij},\ (A_e)_{ij}$$

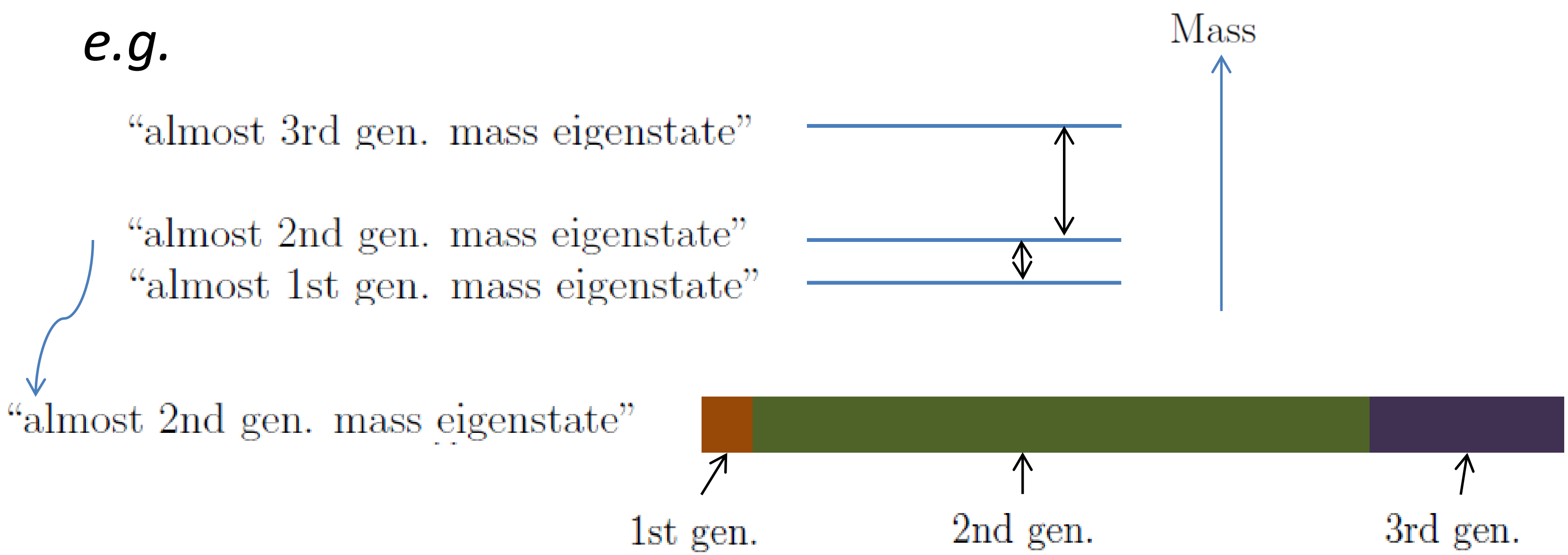
unless $M_G >> M_X$.

Signatures

Observable Quantities

- Flavor -non-universal soft terms give rise to **mass-splittings** of and **flavor-mixings** in SUSY particle mass eigenstates.

e.g.



- Gravity mediation in bulk matter RS model, MFV and other models are distinguishable through

Ratio of mass splittings btwn 1^{st}&2^{nd} gen. and 2^{nd} & 3^{rd} gen.

Flavor-mixing ratio of each mass eigenstate.

- When a mass matrix in the flavor-diagonal basis takes the form : $\begin{pmatrix} m_a^2 & \Delta m^2 \\ \Delta m^2 & m_b^2 \end{pmatrix}$ with $|m_a^2 - m_b^2| >> 2|\Delta m^2|$,

Mass splittings ➡ $|m_a^2 - m_b^2|$

Flavor-mixing ratios in the two mass eigenstates

➡ $|m_a^2 - m_b^2| \; : \; |\Delta m^2|$, $|\Delta m^2| \; : \; |m_a^2 - m_b^2|$.

e.g. When $M_X \gtrsim M_G$ is the case, the mass matrix for 2nd & 3rd gen. takes the form :

$$\begin{pmatrix} M_0^2 + (\beta_2)^2 M_X^2 & \beta_2 \beta_3 M_X^2 \\ \beta_2 \beta_3 M_X^2 & M_0^2 + (\beta_3)^2 M_X^2 \end{pmatrix} ,$$

the mass splitting is given by $\{(\beta_3)^2 - (\beta_2)^2\} M_X^2 \sim (\beta_3)^2 M_X^2$,

the ratio of U_3 (RH stop) component in ``almost U_2 (RH scharm) mass eigenstate" is given by β_2 / β_3 .

Predictions of the Model

Predictions of Bulk Matter RS Model

- Take SU(2) singlet up-type squarks, U_i , as an example.

Their mass matrix is given, up to $O(1)$ factor, by

$$\max\{M_X, M_G\}^2\ I_3 \ +\ \max\{M_X, M_G\}^2 \begin{pmatrix} (\beta_1)^2(\alpha_1)^2 & \beta_1(\alpha_1)^2(\alpha_2)^2\beta_2 & \beta_1(\alpha_1)^2(\alpha_3)^2\beta_3 \\ & (\beta_2)^2(\alpha_2)^2 & \beta_2(\alpha_2)^2(\alpha_3)^2\beta_3 \\ & & (\beta_3)^2(\alpha_3)^2 \end{pmatrix}$$

$$+\ M_X^2 \begin{pmatrix} (\beta_1)^2 & \beta_1\beta_2 & \beta_1\beta_3 \\ & (\beta_2)^2 & \beta_2\beta_3 \\ & & (\beta_3)^2 \end{pmatrix}$$

flavor-universal

Yukawa RG

gravity mediation
in bulk matter RS

- Focus on the **ratio of the two mass-splittings**,

$$\Delta_{12}/\Delta_{23}\ \equiv\ |m_2^2 - m_1^2|/|m_3^2 - m_2^2|$$, and

the **ratio of 3rd gen. component in ``almost 2nd gen. mass eigenstate”,**

r_{32} ,

as functions of M_X^2/M_G^2 .

- In the bulk matter RS model,

$$\Delta_{12}/\Delta_{23} = \frac{-(\beta_2)^2(\alpha_2)^2 M_G^2 + (\beta_2)^2 M_X^2}{-(\beta_3)^2(\alpha_3)^2 M_G^2 + (\beta_3)^2 M_X^2},$$

$$r_{32} = \frac{-\beta_2(\alpha_2)^2(\alpha_3)^2\beta_3 M_G^2 + \beta_2\beta_3 M_X^2}{-(\beta_3)^2(\alpha_3)^2 M_G^2 + (\beta_3)^2 M_X^2}.$$

How do they change with M_X^2/M_G^2 ?

$$\Delta_{12}/\Delta_{23} = \frac{(\beta_2)^2(\alpha_2)^2}{(\beta_3)^2(\alpha_3)^2} \qquad \frac{(\beta_2)^2 M_X^2}{(\beta_3)^2(\alpha_3)^2 M_G^2} \qquad \frac{(\beta_2)^2}{(\beta_3)^2}$$

$$\sim (\beta_2)^2(\alpha_2)^2 \qquad \sim (\beta_2)^2\frac{M_X^2}{M_G^2} \qquad \sim (\beta_2)^2$$

$$r_{32} = \frac{\beta_2(\alpha_2)^2(\alpha_3)^2\beta_3}{(\beta_3)^2(\alpha_3)^2} \qquad \frac{\beta_2\beta_3 M_X^2}{(\beta_3)^2(\alpha_3)^2 M_G^2} \qquad \frac{\beta_2\beta_3}{(\beta_3)^2}$$

$$\sim \beta_2(\alpha_2)^2 \qquad \sim \beta_2\frac{M_X^2}{M_G^2} \qquad \sim \beta_2$$

→ M_X^2/M_G^2

$M_X^2 << M_G^2$
MFV limit

$M_X^2 \gtrsim M_G^2$
Gaugino med. limit

Note : $(\alpha_2)^2 \sim 0.002 \qquad \beta_2 \sim 0.1$

- For SU(2) singlet charged leptons E_i, we find

$$\Delta_{12}/\Delta_{23} = \sim \left(\frac{\epsilon_2}{\epsilon_3}\right)^2 \qquad \sim \left(\frac{\epsilon_2}{\epsilon_3}\right)^2 \qquad \sim \left(\frac{\epsilon_2}{\epsilon_3}\right)^2$$

$$r_{32} = \sim \frac{\epsilon_2}{\epsilon_3}\delta^2 \qquad \sim \frac{\epsilon_2}{\epsilon_3}\frac{1}{\delta^2}\frac{M_X^2}{M_G^2} \qquad \sim \frac{\epsilon_2}{\epsilon_3}$$

$\longrightarrow$ M_X^2/M_G^2

$M_X^2 << M_G^2$ MFV limit	$M_X^2 \gtrsim M_G^2$ Gaugino med. limit

$$\text{Note}: \quad \epsilon_2/\epsilon_3 \sim 0.05$$

Although we do not know the scale of $\delta\ (\sim \delta_2 \sim \delta_3)$, it can be severely constrained by $\mu \to e\gamma$ experiment because we have $(m_L^2)_{12} \sim (\delta^2/3)\max\{M_X, M_G\}^2$.

Comparison with other Models

"4D Gravity Mediation"

- Consider a model where gravity mediation contributes to all flavor-non-universal terms with **equal strengths**.

  (In this case, gauge mediation is the dominant source of soft SUSY breaking terms, still gravity mediation has non-negligible effects.)

- The mass matrix of U_i is given, up to $O(1)$ factor, by

$$\max\{M_X, M_G\}^2\ I_3\ +\ \max\{M_X, M_G\}^2 \begin{pmatrix} (\beta_1)^2(\alpha_1)^2 & \beta_1(\alpha_1)^2(\alpha_2)^2\beta_2 & \beta_1(\alpha_1)^2(\alpha_3)^2\beta_3 \\ & (\beta_2)^2(\alpha_2)^2 & \beta_2(\alpha_2)^2(\alpha_3)^2\beta_3 \\ & & (\beta_3)^2(\alpha_3)^2 \end{pmatrix}$$

$$+\ M_X^2 \begin{pmatrix} O(1) & O(1) & O(1) \\ & O(1) & O(1) \\ & & O(1) \end{pmatrix}$$

4D gravity mediation

How do Δ_{12}/Δ_{23} and r_{32} change with M_X^2/M_G^2 ?

$\Delta_{12}/\Delta_{23} =$ $\sim (\beta_2)^2(\alpha_2)^2$ $\sim \frac{M_X^2}{M_G^2}$ $O(1)$

$r_{32} =$ $\sim \beta_2(\alpha_2)^2$ $\sim \frac{M_X^2}{M_G^2}$ $O(1)$

→ M_X^2/M_G^2

$M_X^2 << M_G^2$
MFV limit

$M_X^2 \gtrsim M_G^2$
Gaugino med. limit

For SU(2) singlet charged leptons E_i ,

$$\Delta_{12}/\Delta_{23} \;= \qquad \sim \left(\frac{\epsilon_2}{\epsilon_3}\right)^2 \qquad \sim \frac{1}{(\delta\epsilon_3)^2}\frac{M_X^2}{M_G^2} \qquad O(1)$$

$$r_{32} \;= \qquad \sim \frac{\epsilon_2}{\epsilon_3}\delta^2 \qquad \sim \frac{1}{(\delta\epsilon_3)^2}\frac{M_X^2}{M_G^2} \qquad O(1)$$

$\longrightarrow M_X^2/M_G^2$

$M_X^2 << M_G^2$ MFV limit	$M_X^2 \gtrsim M_G^2$ Gaugino med. limit

- Even if we do not know the value of M_X^2/M_G^2 , we can distinguish "gravity mediation in bulk matter RS" and "4D gravity mediation" through the ratio of Δ_{12}/Δ_{23} and r_{32} , unless $M_X^2 << M_G^2$.

"SUSY Froggatt-Nielsen"

- Each superfield has a **flavor-dependent U(1)** charge. The ratio of U(1) breaking VEV over the contact interaction scale gives rise to the Yukawa coupling hierarchy.

$$(<\phi>/\Lambda)^{a_i} \leftrightarrow \alpha_i \quad , \ldots$$

- The mass matrix of U_i is given, up to $O(1)$ factor, by

$$\max\{M_X, M_G\}^2\, I_3 + \max\{M_X, M_G\}^2 \begin{pmatrix} (\beta_1)^2(\alpha_1)^2 & \beta_1(\alpha_1)^2(\alpha_2)^2\beta_2 & \beta_1(\alpha_1)^2(\alpha_3)^2\beta_3 \\ & (\beta_2)^2(\alpha_2)^2 & \beta_2(\alpha_2)^2(\alpha_3)^2\beta_3 \\ & & (\beta_3)^2(\alpha_3)^2 \end{pmatrix}$$

$$+ M_X^2 \begin{pmatrix} O(1) & \beta_1/\beta_2 & \beta_1/\beta_3 \\ & O(1) & \beta_2/\beta_3 \\ & & O(1) \end{pmatrix}$$

gravity mediation in FN

How do Δ_{12}/Δ_{23} and r_{32} change with M_X^2/M_G^2 ?

$$\Delta_{12}/\Delta_{23} = \quad \sim (\beta_2)^2(\alpha_2)^2 \qquad \sim \frac{M_X^2}{M_G^2} \qquad O(1)$$

$$r_{32} = \quad \sim \beta_2(\alpha_2)^2 \qquad \sim \beta_2\frac{M_X^2}{M_G^2} \qquad \sim \beta_2$$

$\longrightarrow M_X^2/M_G^2$

$M_X^2 << M_G^2$ MFV limit	$M_X^2 \gtrsim M_G^2$ Gaugino med. limit

For SU(2) singlet charged leptons E_i ,

$$\Delta_{12}/\Delta_{23} = \quad \sim \left(\frac{\epsilon_2}{\epsilon_3}\right)^2 \qquad \sim \frac{1}{(\delta\epsilon_3)^2}\frac{M_X^2}{M_G^2} \qquad O(1)$$

$$r_{32} = \quad \sim \frac{\epsilon_2}{\epsilon_3}\delta^2 \qquad \sim \frac{\epsilon_2}{\epsilon_3}\frac{1}{(\delta\epsilon_3)^2}\frac{M_X^2}{M_G^2} \qquad \sim \frac{\epsilon_2}{\epsilon_3}$$

$\longrightarrow M_X^2/M_G^2$

$M_X^2 << M_G^2$
MFV limit

$M_X^2 \gtrsim M_G^2$
Gaugino med. limit

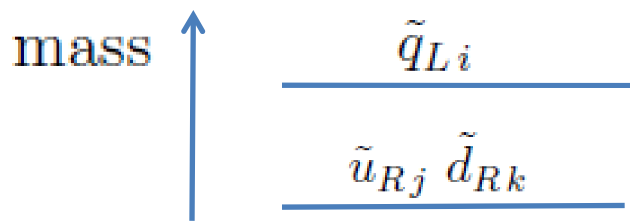
- Even if we do not know the value of M_X^2/M_G^2 , we can distinguish "gravity mediation in bulk matter RS" and "SUSY FN" through the ratio of Δ_{12}/Δ_{23} and r_{32} , unless $M_X^2 << M_G^2$.

Proposal for Experimental Study

Challenges for Collider Study

- Need to identify SUSY particle mass eigenstates.
- To study the flavor-mixing ratios of "**almost SU(2) singlet** mass eigenstates", we need to produce them selectively.

➡ Doable only at lepton colliders. – Use mass differences.



➡ "almost 3rd generation mass eigenstates" are not suitable, due to their large Left-Right mixings.

- Need to measure the ratios of their **main decay modes** and **rare decay modes**.

➡ Mis-ID rate of main decay products as rare decay products must be negligibly small.

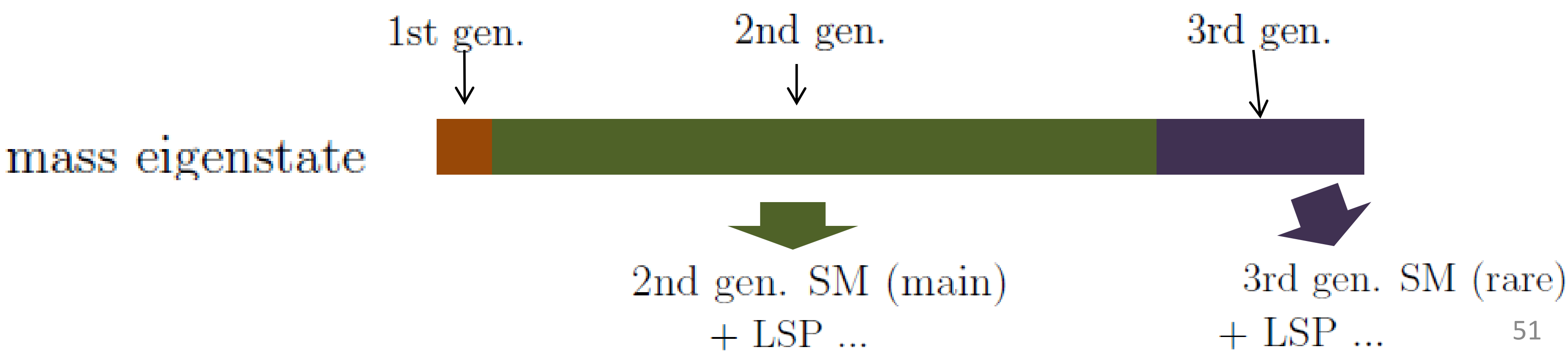
Channels for Studying Flavor-mixing Ratios of SUSY Particles

Measuring Flavor-mixing Ratios

- Measuring the flavor-mixing ratio of a SUSY particle mass eigenstate is a difficult task.

  Cascade decay, Mass degeneracy, Mis-ID, …..

- Consider a **lepton collider**.
- Tune the center-of-mass energy btwn the thresholds of SU(2) doublet and singlet sleptons (squarks),
  so that only SU(2) singlet sleptons (squarks) are produced on-shell.
- Study the pattern of the decay products.
- Measure the ratios of ``main decay mode” vs. ``rare decay modes”.



Benchmark Mass Spectrum

- Assume that Bulk Matter RS is the case.

  ➡ "almost 1st gen." and "almost 2nd gen." mass eigenstates are degenerate.

- Assume

$$\tilde{H}_u, \tilde{H}_d \;>\; \tilde{g} \;>\; \tilde{q}_L \;>\; \tilde{q}_R$$
$$>\; \chi_1^{\pm},\, \chi_2^0\, (\simeq \tilde{W}) \;>\; \chi_1^0\, (\simeq \tilde{B}) \;>\; \tilde{l}_L \;>\; \tilde{l}_R \;>\; \psi_{3/2} \;.$$

- Gravitino is always the lightest SUSY particle.
  The next-to-lightest SUSY particle is long-lived.

- The mass order of SUSY particles of different flavor is undetermined. We consider the following cases :

$$\tilde{\mu}_R \;\sim\; \tilde{e}_R \;>\; \tilde{\tau}_1$$
$$\tilde{c}_R \;\sim\; \tilde{u}_R \;>\; \tilde{t}_1$$

Channel 1 : Smuon Rare Decay

- Consider the case : $\tilde{\mu}_R \sim \tilde{e}_R > \tilde{\tau}_1$.
- Produce ``**almost SU(2) singlet smuon**” mass eigenstate and ``almost SU(2) singlet selectron” one.
- The former mainly decays into SM muon + tau + NLSP stau :

$$ee \rightarrow \tilde{\mu}_R \, \tilde{\mu}_R \rightarrow \mu \, \tau \, \tilde{\tau}_1 \, \mu \, \tau \, \tilde{\tau}_1$$

- Because of small **stau component**, it also decays into two SM taus + NLSP stau :

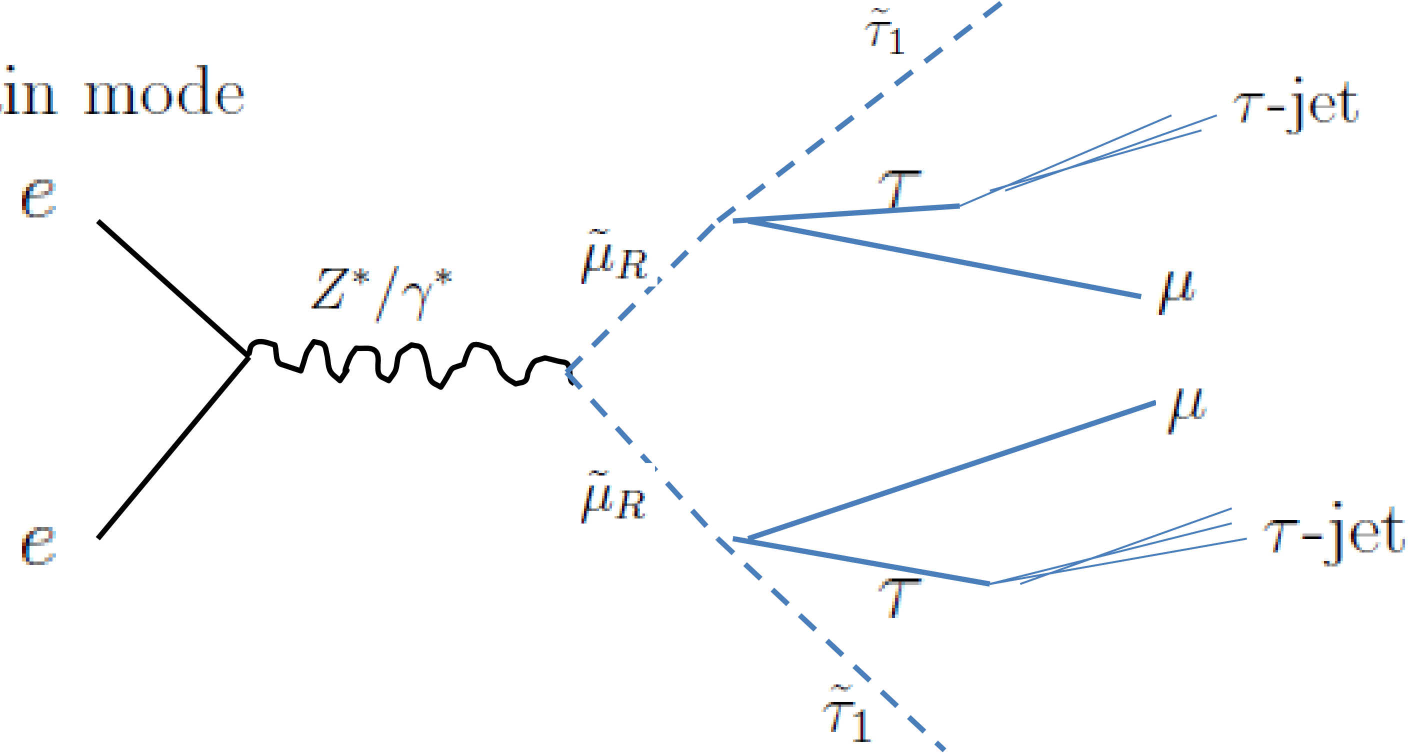
$$ee \rightarrow \tilde{\mu}_R \, \tilde{\mu}_R \rightarrow \tau \, \tau \, \tilde{\tau}_1 \, \mu \, \tau \, \tilde{\tau}_1$$

The branching ratio of the rare event can be as large as

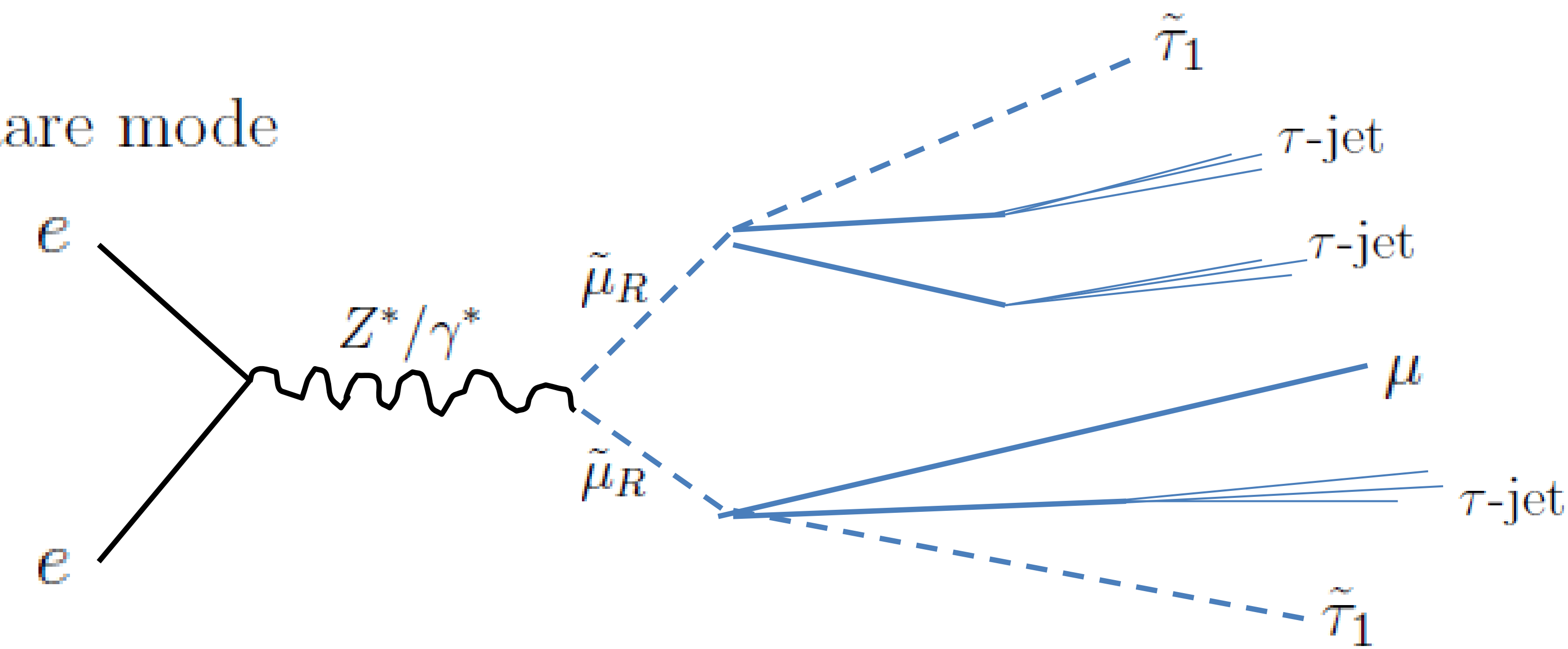
$$(\, \epsilon_2 \, / \, \epsilon_3 \,)^2 \sim (\, m_\mu \, / \, m_\tau \,)^2 \sim 0.004$$

in ``Gaugino mediation limit”.

Main mode



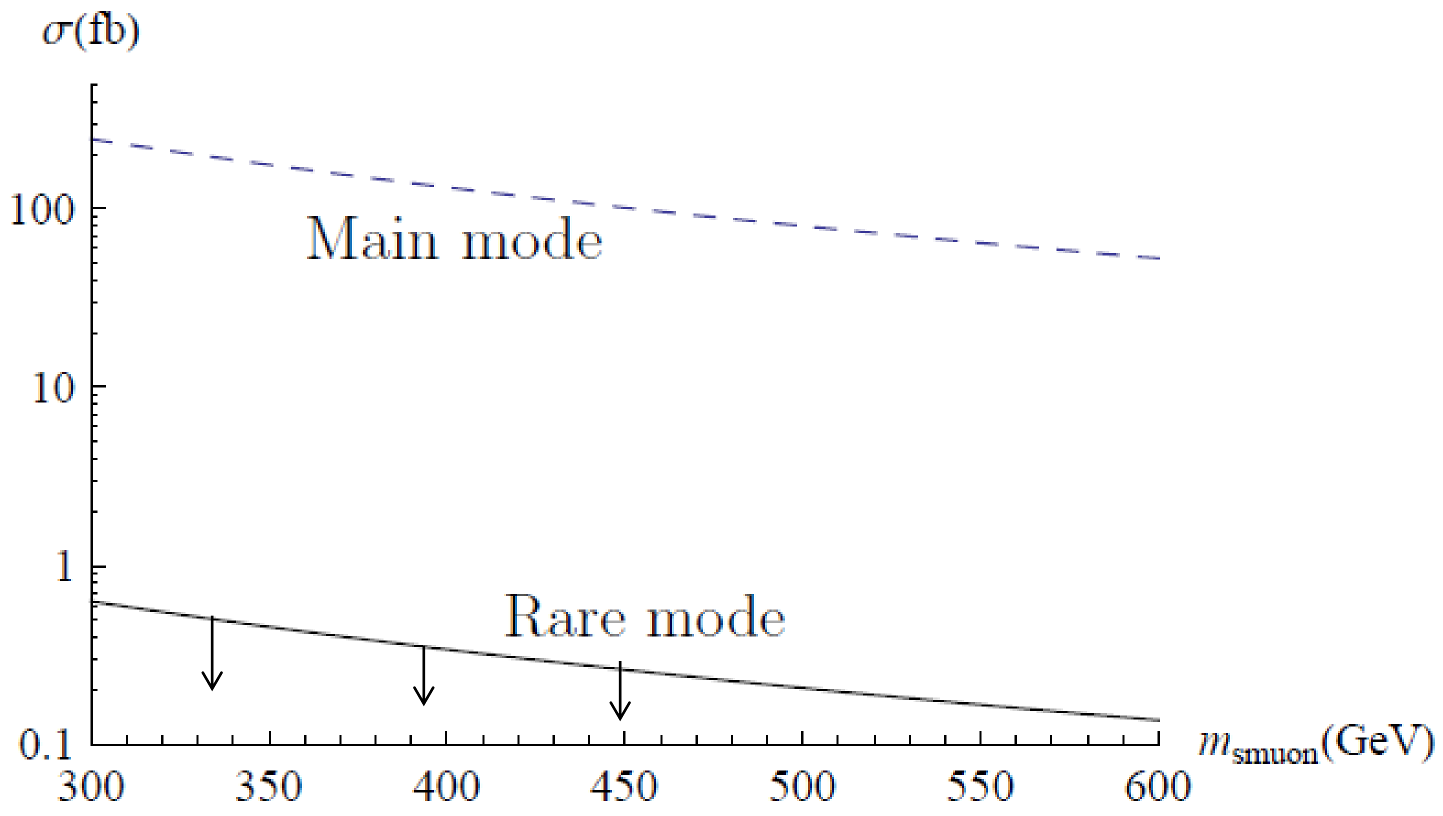
Rare mode



How Much Luminosity do we need ?

- Search for rare decay events ➡ Need high luminosity.
- For Channel 1, take $\sqrt{s} = 2m_{\tilde{\mu}_R} + 100\ \mathrm{GeV}$ to take advantage of threshold enhancement.

The cross sections for the main and rare modes are :



Channel 2 : Scharm Rare Decay

- Consider the case when $\tilde{c}_R \sim \tilde{u}_R > \tilde{t}_1$.
- Tune the center-of-mass energy and produce "**almost SU(2) singlet scharm**" mass eigenstate and "almost SU(2) singlet s-up" one almost at rest. (Boosted $\tilde{t}_1$ is also produced.)
- Main decay mode :

$$ee \rightarrow \tilde{c}_R\, \tilde{c}_R \rightarrow c\, \chi_1^0\, c\, \chi_1^0 \rightarrow (c\text{−jet})\ (c\text{−jet})\ (\text{leptons})\ \tilde{\tau}_1\, \tilde{\tau}_1$$

- Rare decay mode :

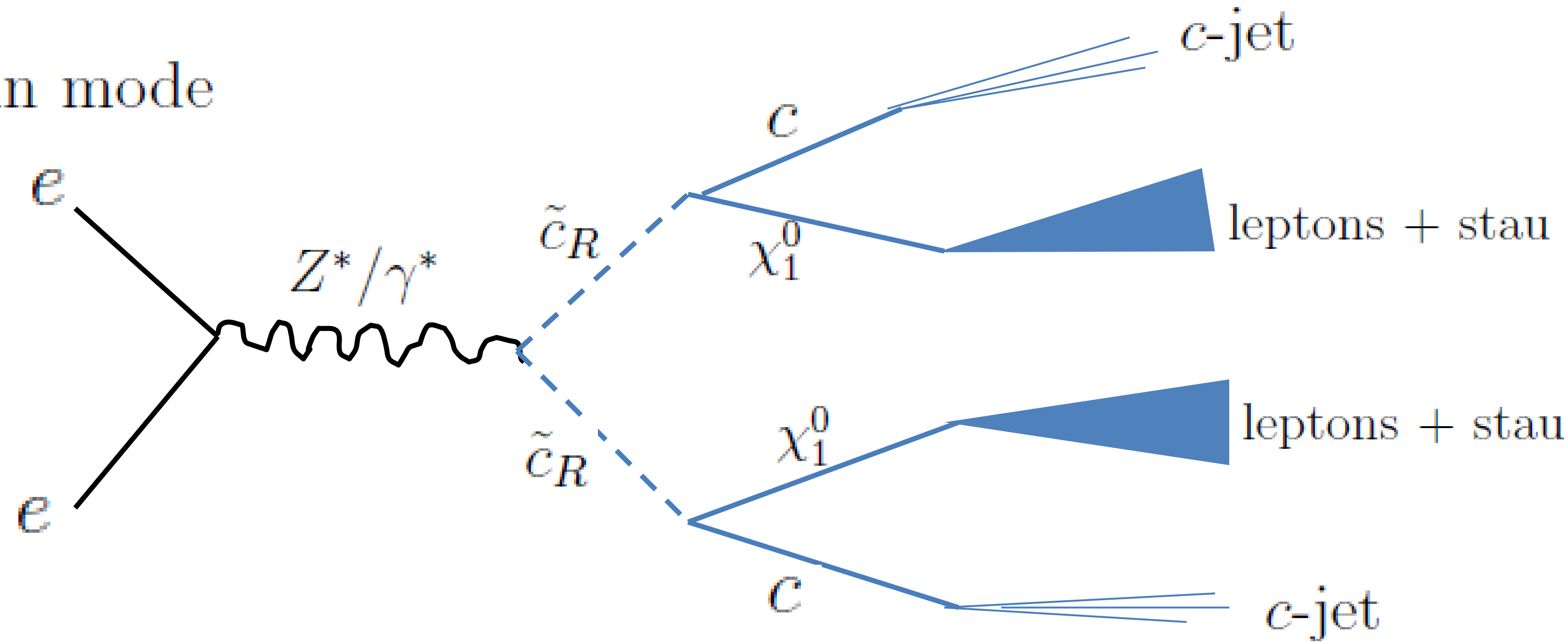
$$\begin{aligned} ee \rightarrow \tilde{c}_R\, \tilde{c}_R &\rightarrow t\, \chi_1^0\, c\, \chi_1^0 \\ &\rightarrow (\text{top decay products})\ (c\text{−jet})\ (\text{leptons})\ \tilde{\tau}_1\, \tilde{\tau}_1 \end{aligned}$$

The branching ratio can be as large as $(\beta_2 / \beta_3)^2 \sim 0.02$.

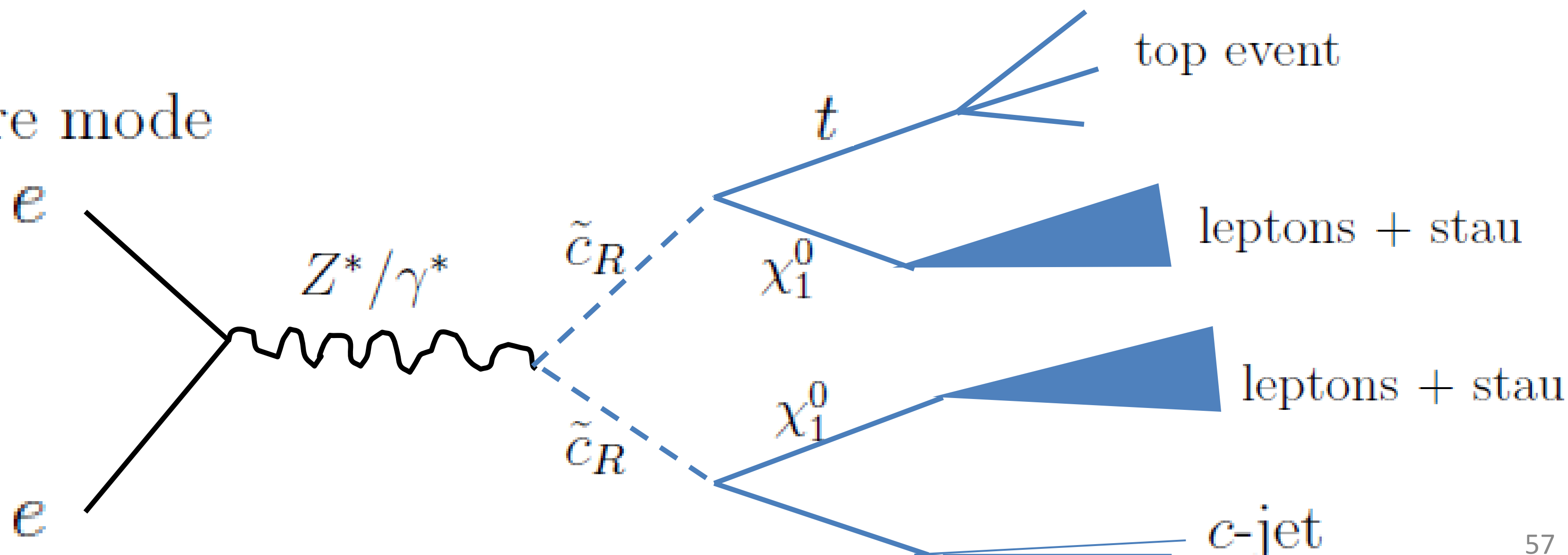
- Contamination from $ee \rightarrow \tilde{t}_1\, \tilde{t}_1 \rightarrow c\, \chi_1^0\, t\, \chi_1^0$ event can be reduced using the discriminants :

$$|\vec{p}_c| + \sqrt{|\vec{p}_c|^2 + m_\chi^2} \simeq m_{\tilde{c}_R} \quad , \quad \sqrt{|\vec{p}_t|^2 + m_t^2} + \sqrt{|\vec{p}_t|^2 + m_\chi^2} \simeq m_{\tilde{c}_R}$$

Main mode

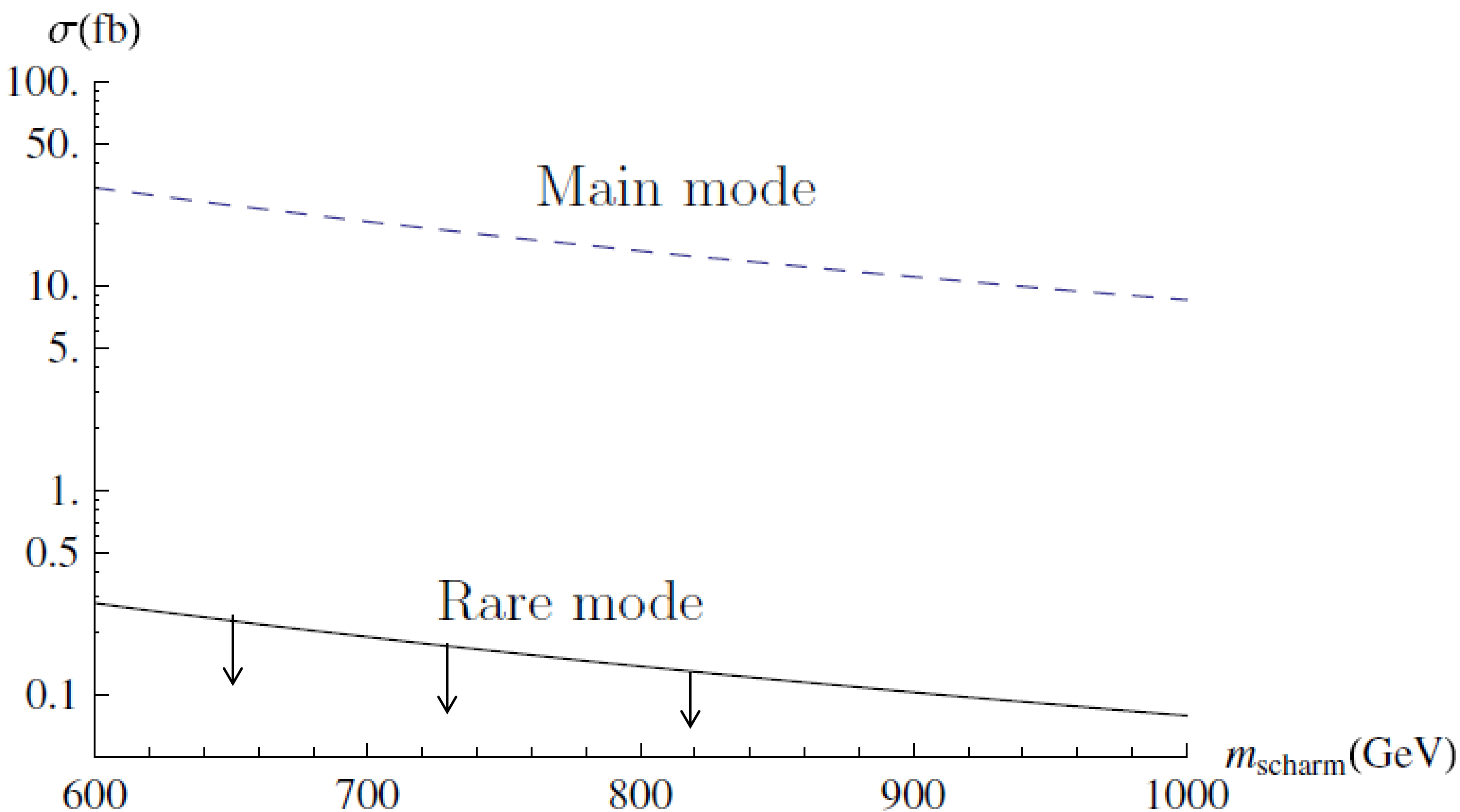


Rare mode



- For Channel 2, take $\sqrt{s} = 2m_{\tilde{c}_R} + 10\ \mathrm{GeV}$ so that we can distinguish the scharm event from the s-top event through the discriminants.

The cross sections for the main and rare modes are :



Conclusions

- If SUSY particles are discovered, it is possible to observe unique signatures of the bulk matter RS model, no matter how high the KK scale is.
- The bulk matter RS model predicts the flavor structure of gravity-mediation-origined soft SUSY breaking terms.
- This structure may be measured as a deviation from MFV through ``**ratio of mass-splittings**” and ``**flavor-mixing ratios in SUSY particle mass eigenstates**”.
- The effects of bulk matter RS setup do not follow the usual decoupling rule.

- More predictive framework : SUSY CFT
- Extension to “Other TeV scale new physics + Bulk Matter RS”.