IMPERFECT DARK ENERGY OF KINETIC GRAVITY BRAIDING

Alexander Vikman
(CERN)

THIS TALK IS BASED ON WORK IN PROGRESS &

arXiv:1008.0048 [hep-th], **JCAP 1010:026, 2010**

IN COLLABORATION WITH

Cédric Deffayet, Oriol Pujolàs and Ignacy Sawicki

SUMMARY

- **Non-canonical scalar field ϕ which "acts" like imperfect fluid: on general (not exact FRW) background**

$$T_{\mu\nu} \neq \mathcal{E} u_\mu u_\nu - \perp_{\mu\nu} \mathcal{P}$$

- ϕ **kinetically mixes / "braids" with the metric**

$$(\partial\phi)^2 \, \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi\right) \quad \text{c.f.} \quad F^{(1)}_{\mu\nu}\left(A^\alpha\right) F^{(2)\mu\nu}\left(B_\beta\right)$$

SUMMARY

- **Manifestly stable (no ghosts and no gradient instabilities) and large violation of the Null Energy Condition (NEC) is possible even in *minimally coupled stable* theories: *stable Phantom*** $w < -1$

- **Vanishing shift-charge (charge with respect to** $\phi \to \phi + c$ **) corresponds to cosmological attractors similar to Ghost Condensate / "bad" k-Inflation. These attractors can be manifestly stable (no ghosts and no gradient instabilities) and their exact properties depend on external matter. These attractors generically evolve to de Sitter in late time asymptotic. Interesting for DE!**

BRAIDING METRIC WITH A SCALAR FIELD

WHAT IS KINETIC GRAVITY BRAIDING?

$$S_\phi = \int \mathrm{d}^4x\sqrt{-g}\left[K(\phi, X) + G(\phi, X)\Box\phi\right]$$

where $X \equiv \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$

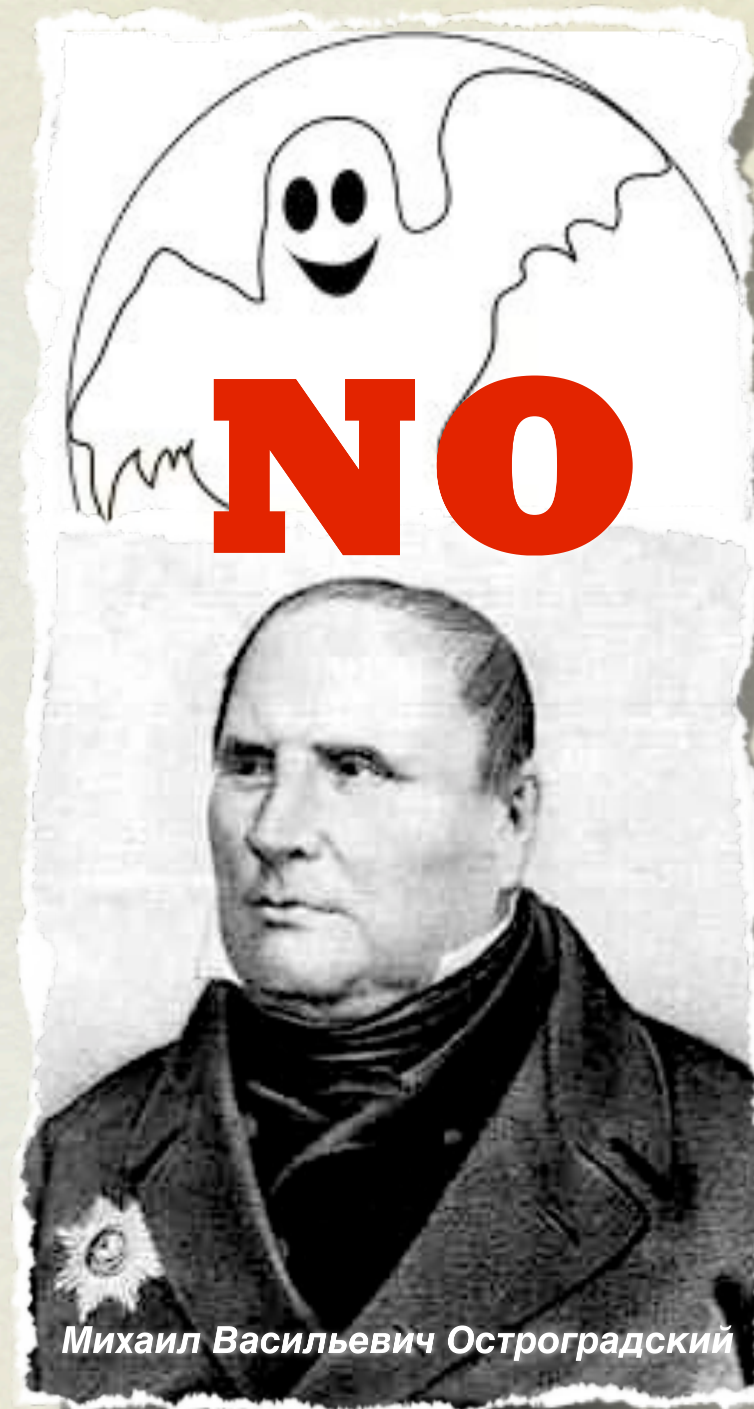
Minimal coupling to gravity $S_{\mathrm{tot}} = S_\phi + S_{\mathrm{EH}}$

However, derivatives of the metric are coupled

to the derivatives of the scalar, provided

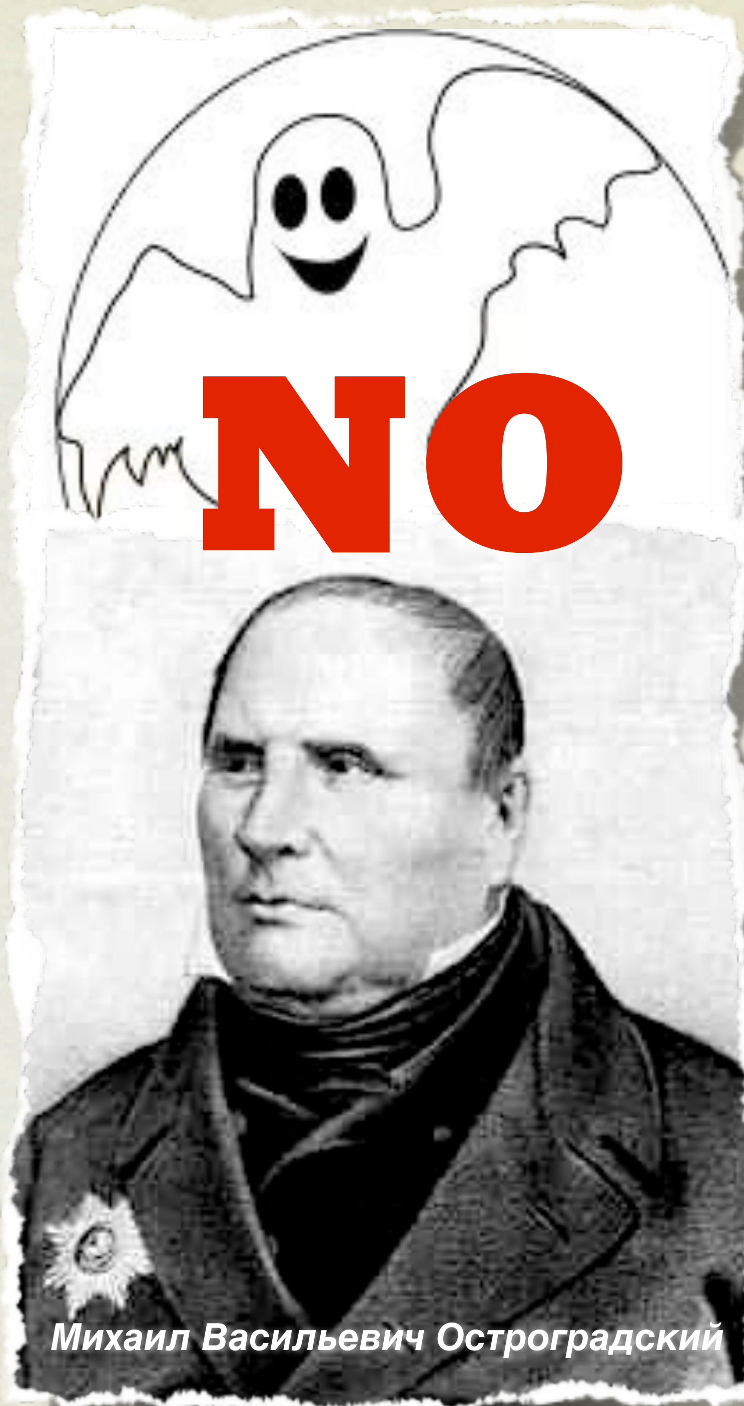
$$G_X \neq 0$$

ACTION FOR KINETIC GRAVITY BRAIDING IS SIMILAR TO EINSTEIN-HILBERT ACTION



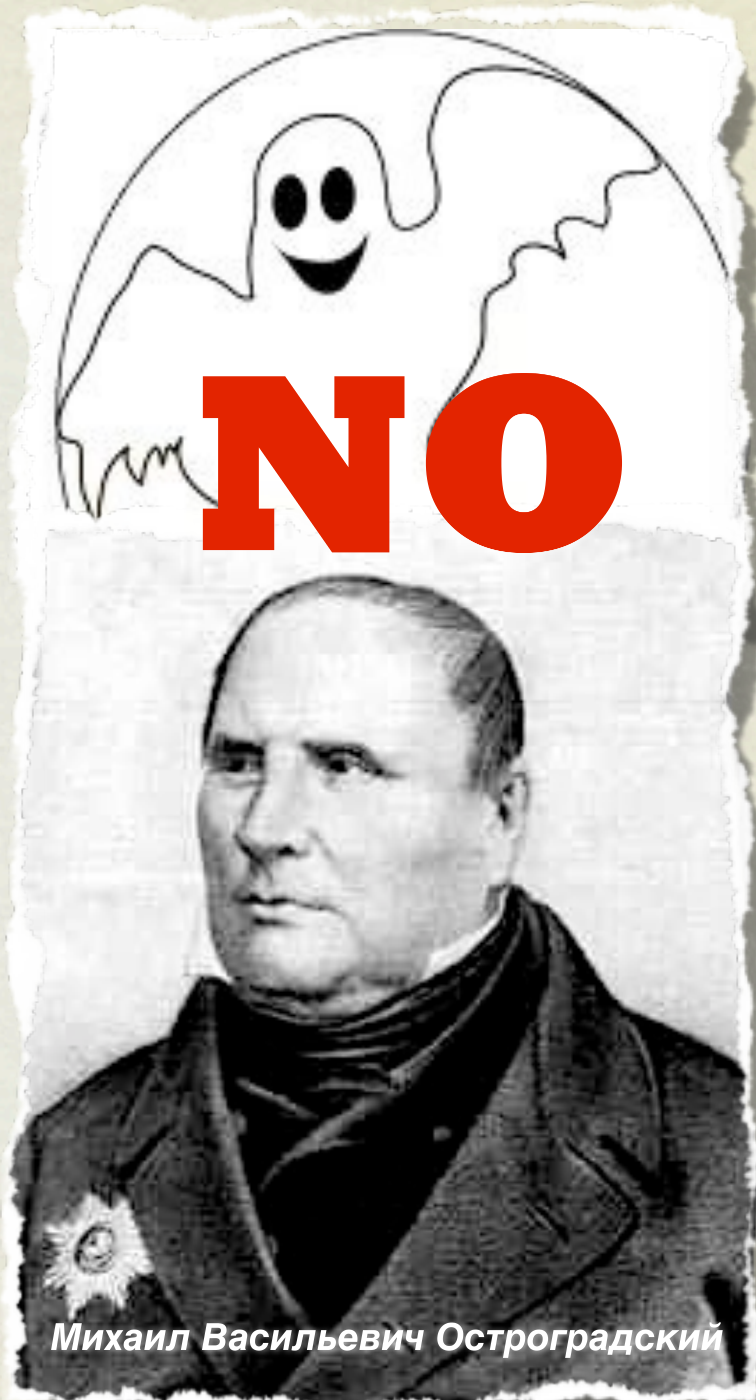
ACTION FOR KINETIC GRAVITY BRAIDING IS SIMILAR TO EINSTEIN-HILBERT ACTION

- The second derivatives (higher derivative -HD) enter the action but only **linearly**



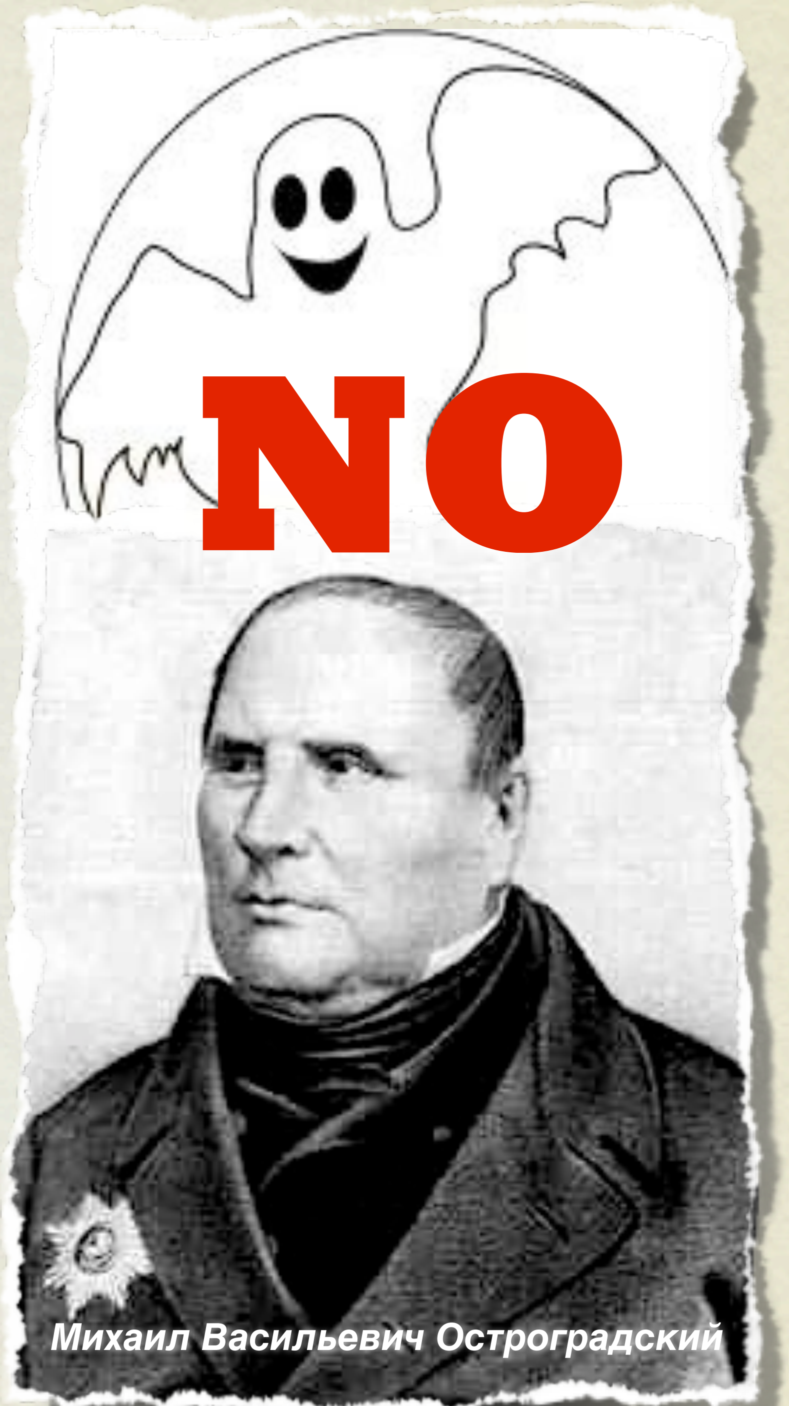
ACTION FOR KINETIC GRAVITY BRAIDING IS SIMILAR TO EINSTEIN-HILBERT ACTION

- The second derivatives (higher derivative -HD) enter the action but only **linearly**
- One can eliminate the HD only by breaking the Lorentz-invariant formulation of the theory.



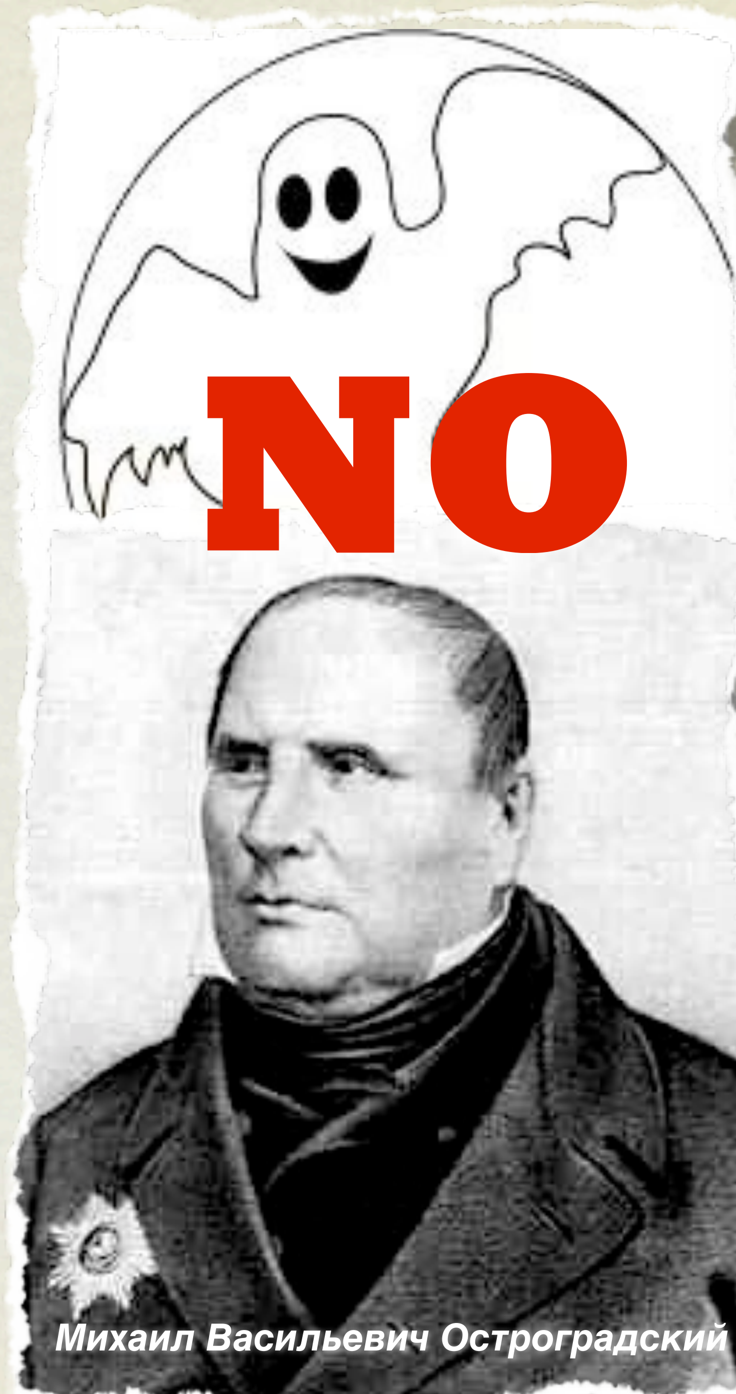
ACTION FOR KINETIC GRAVITY BRAIDING IS SIMILAR TO EINSTEIN-HILBERT ACTION

- The second derivatives (higher derivative -HD) enter the action but only **linearly**
- One can eliminate the HD only by breaking the Lorentz-invariant formulation of the theory.
- Boundary terms are required!



ACTION FOR KINETIC GRAVITY BRAIDING IS SIMILAR TO EINSTEIN-HILBERT ACTION

- The second derivatives (higher derivative -HD) enter the action but only **linearly**
- One can eliminate the HD only by breaking the Lorentz-invariant formulation of the theory.
- Boundary terms are required!
- Despite the HD in the action, the equations of motion are still of the 2nd order: **NO new degrees of freedom - NO Ostrogradsky's ghosts**



KINETIC GRAVITY BRAIDING IS SIMILAR TO GALILEON (©*Nicolis, Rattazzi, Trincherini 2008*) BUT

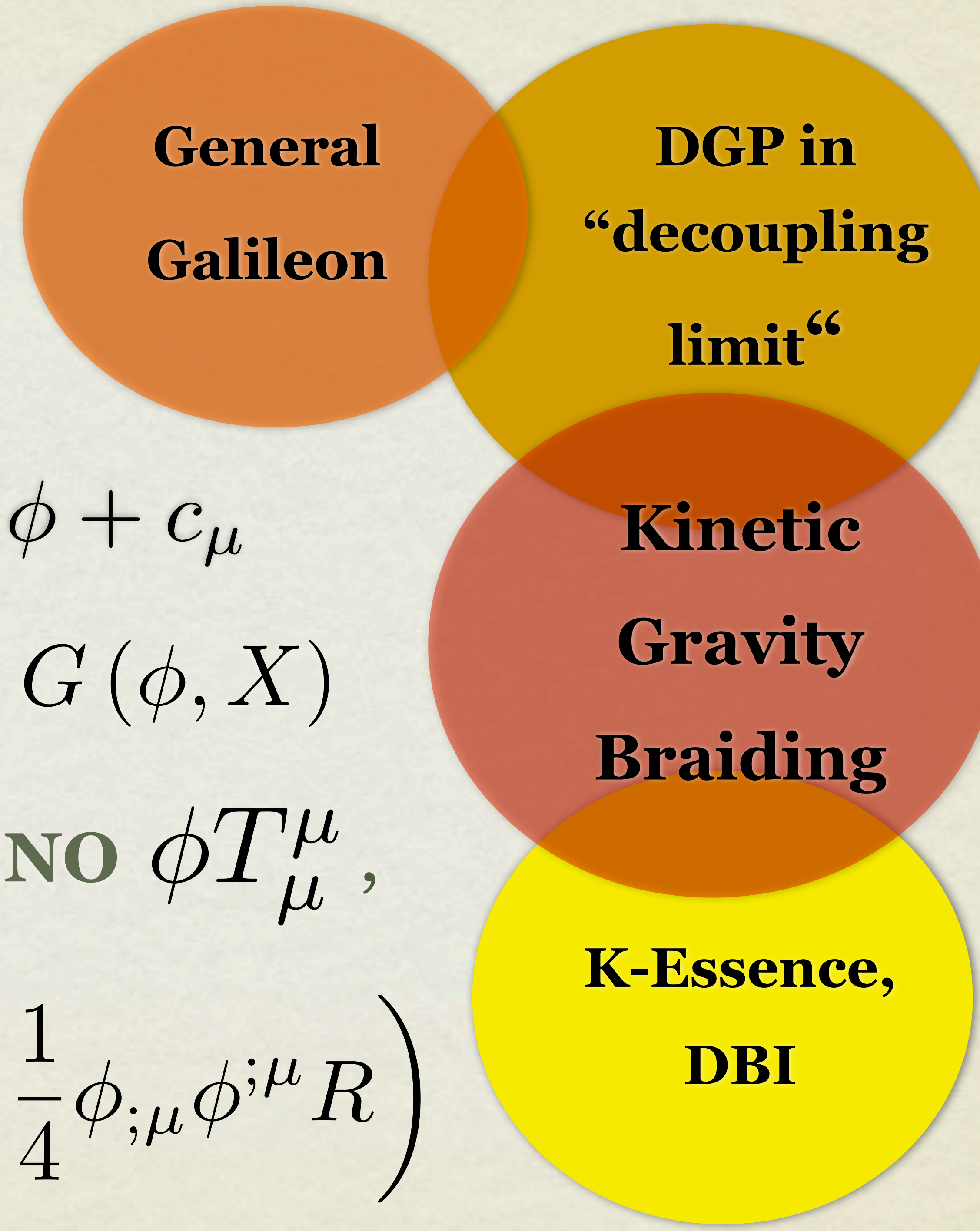
- Does **not require** the Galilean symmetry:

$$\phi \longrightarrow \phi + c \quad \text{and} \quad \partial_\mu \phi \longrightarrow \partial_\mu \phi + c_\mu$$

- General functions $K(\phi, X)$ and $G(\phi, X)$

- **Minimal coupling to gravity, NO** ϕT^μ_μ **, NO higher order therms like**

$$\phi_{;\lambda}\phi^{;\lambda}\left((\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu} - \frac{1}{4}\phi_{;\mu}\phi^{;\mu}R\right)$$



EXPANSIONS IN GRADIENT TERMS

- K-Essence, DBI etc

$$K(\phi, X) \sim X\left(1 + c_1(\phi) X + c_2(\phi) X^2 + ...\right)$$

- Kinetic Gravity Braiding – integrate the canonical kinetic energy by parts

$$G(\phi, X) \Box\phi \sim -\phi\Box\phi\left(1 + \tilde{c}_1(\phi) X + \tilde{c}_2(\phi) X^2 + ...\right)$$

EQUATION OF MOTION I

$$L^{\mu\nu}\nabla_\mu\nabla_\nu\phi + (\nabla_\alpha\nabla_\beta\phi)\,Q^{\alpha\beta\mu\nu}\,(\nabla_\mu\nabla_\nu\phi) + $$
$$+Z - G_X R^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi = 0$$

Braiding

EOM is of the second order: $L_{\mu\nu}\,, Q^{\alpha\beta\mu\nu},\ Z$

constructed from field and it's first derivatives

$Q^{\alpha\beta\mu\nu}$ **is such that EOM is a 4D Lorentzian generalization of the Monge-Ampère Equation, always *linear* in** $\ddot{\phi}$

EQUATION OF MOTION II

- **Shift-Charge Current:** J_μ
- **New Equivalent Lagrangian:** $\mathcal{P}$
- **Equation of motion is a "conservation law":**

EQUATION OF MOTION II

- **Shift-Charge Current:** J_μ
- **New Equivalent Lagrangian:** $\mathcal{P}$
- **Equation of motion is a “conservation law”:**

$$\boxed{\nabla_\mu J^\mu = \mathcal{P}_\phi}$$

EQUATION OF MOTION II

- **Shift-Charge Current:** J_μ

$$J_\mu = (\mathcal{L}_X - 2G_\phi)\,\nabla_\mu\phi - G_X\nabla_\mu X$$

- **New Equivalent Lagrangian:** $\mathcal{P}$

- **Equation of motion is a "conservation law":**

$$\boxed{\nabla_\mu J^\mu = \mathcal{P}_\phi}$$

EQUATION OF MOTION II

- **Shift-Charge Current:** J_μ

$$J_\mu = (\mathcal{L}_X - 2G_\phi)\,\nabla_\mu \phi - G_X \nabla_\mu X$$

- **New Equivalent Lagrangian:** $\mathcal{P}$

$$\mathcal{P} = K - 2XG_\phi - G_X \nabla^\lambda \phi \nabla_\lambda X$$

- **Equation of motion is a "conservation law":**

$$\boxed{\nabla_\mu J^\mu = \mathcal{P}_\phi}$$

BRAIDING

$$\text{Einstein Equations}\,(\phi, \partial\phi, \partial\partial\phi, g, \partial g, \partial\partial g) = 0$$

$$\phi\text{EoM}\,(\phi, \partial\phi, \partial\partial\phi, g, \partial g, \partial\partial g) = 0$$

Cannot solve separately !!!!

characteristics (cones of propagation)

depend on external matter

IMPERFECT FLUID FOR TIMELIKE GRADIENTS

- **Four velocity** $u_\mu \equiv \frac{\nabla_\mu \phi}{\sqrt{2X}}$ **projector:** $\perp_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$
- **Time derivative** $\dot{(\ )} \equiv \frac{\mathrm{d}}{\mathrm{d}\tau} \equiv u^\lambda \nabla_\lambda$
- **Acceleration** $a_\mu \equiv \dot{u}_\mu$
- **Expansion** $\theta \equiv \perp^\lambda_\mu \nabla_\lambda u^\mu = \dot{V}/V$

comoving volume

EFFECTIVE MASS & CHEMICAL POTENTIAL

- charge density: $n \equiv J^{\mu} u_{\mu} = n_0 + \kappa\theta$

"Braiding"

- energy density: $\mathcal{E} \equiv T_{\mu\nu} u^{\mu} u^{\nu} = \mathcal{E}_0 + \theta m \kappa$
- effective mass per shift-charge / chemical potential:

$$m \equiv \left(\frac{\partial \mathcal{E}}{\partial n}\right)_{V,\phi} = \sqrt{2X} = \dot{\phi}$$

SHIFT-CURRENT AND DIFFUSION

"Diffusion"

$$J_\mu = n u_\mu - \frac{\kappa}{m} \perp_\mu^\lambda \nabla_\lambda m$$

§ 59, L&L, vol. 6

$$\kappa \equiv 2 X G_X$$

Is a "diffusion"/ transport coefficient

IMPERFECT FLUID ENERGY-MOMENTUM TENSOR

- **Pressure**

$$\mathcal{P} \equiv -\frac{1}{3}T^{\mu\nu} \perp_{\mu\nu} = P_0 - \kappa\dot{m}$$

- **Energy Flow**

$$q_\mu \equiv \perp_{\mu\lambda} T^\lambda_\nu u^\nu = m \perp_{\mu\nu} J^\nu$$

$$q_\mu = -\kappa \perp^\nu_\mu \nabla_\nu m$$

No Heat Flux!

- **Energy Momentum Tensor**

$$T_{\mu\nu} = \mathcal{E}u_\mu u_\nu - \perp_{\mu\nu} \mathcal{P} + 2u_{(\mu}q_{\nu)}$$

Solving for $\dot{m}$ *for small gradients or small* κ *one obtains bulk viscosity*

ENERGY CONSERVATION IN COMOVING VOLUME

Energy conservation: $u_\nu \nabla_\mu T^{\mu\nu} = 0$

$$\mathrm{d}E = -\mathcal{P}\mathrm{d}V + m\mathrm{d}\mathcal{N}_{\mathrm{dif}}$$

Euler relation: $\mathcal{E} = mn - P_0$

Momentum conservation:

$$\perp_{\mu\nu} \nabla_\lambda T^{\lambda\nu} = 0$$

VACUUM-ATTRACTORS

Euler relation: $\mathcal{E} = mn - P_0$

↓

for no particles: $n_* = 0$

↓

$$\mathcal{E}_* = -\mathcal{P}_* - \kappa_* \dot{m}_*$$

almost dS!

COSMOLOGY

$$q_\mu = 0 \quad \textbf{and} \quad \theta = 3H$$

Friedmann Equation:

$$H^2 = \kappa m H + \frac{1}{3}\left(\mathcal{E}_0 + \rho_{\rm ext}\right)$$

$$r_{\rm c}^{-1} = \kappa m$$

"crossover" scale in DGP

CHARGE CONSERVATION

$$\dot{n} + 3Hn = \mathcal{P}_\phi$$

If there is shift-symmetry then

$$\mathcal{P}_\phi = 0$$

$$n \propto a^{-3}$$

INFLATION BRINGS THE SCALAR TO ATTRACTOR

$$n_* = 0$$

EXAMPLE:
SIMPLEST IMPERFECT DARK ENERGY

Only one free parameter μ

- Lagrangian

$$\mathcal{L} = X\left(-1 + \mu\Box\phi\right)$$

- shift-charge density

$$n = m\left(3\mu H m - 1\right)$$

NONTRIVIAL ATTRACTOR

No Particles: $n_* = 0$

$$m_* = (3\mu H)^{-1}$$

$$H_*^2 = \frac{1}{6}\rho_{\rm ext}\left(1 + \sqrt{1 + \frac{2}{3}\left(\mu\rho_{\rm ext}\right)^{-2}}\right)$$

STABLE

$$m_* = 0$$

$$H_*^2 = \frac{1}{3}\rho_{\rm ext}$$

GHOSTY

HIGH FREQUENCY STABILITY

Effective metric for perturbations

$$\mathcal{G}_{\mu\nu} = D u_\mu u_\nu + \Omega \perp_{\mu\nu} - \frac{2\kappa}{m} \mathcal{K}_{\mu\nu} - 2\kappa_m a_{(\mu} u_{\nu)}$$

Extrinsic curvature for $\phi = \text{const}$

Ghosts

$$D = \frac{\mathcal{E}_m - \kappa\theta}{m} + \frac{3}{2}\kappa^2$$

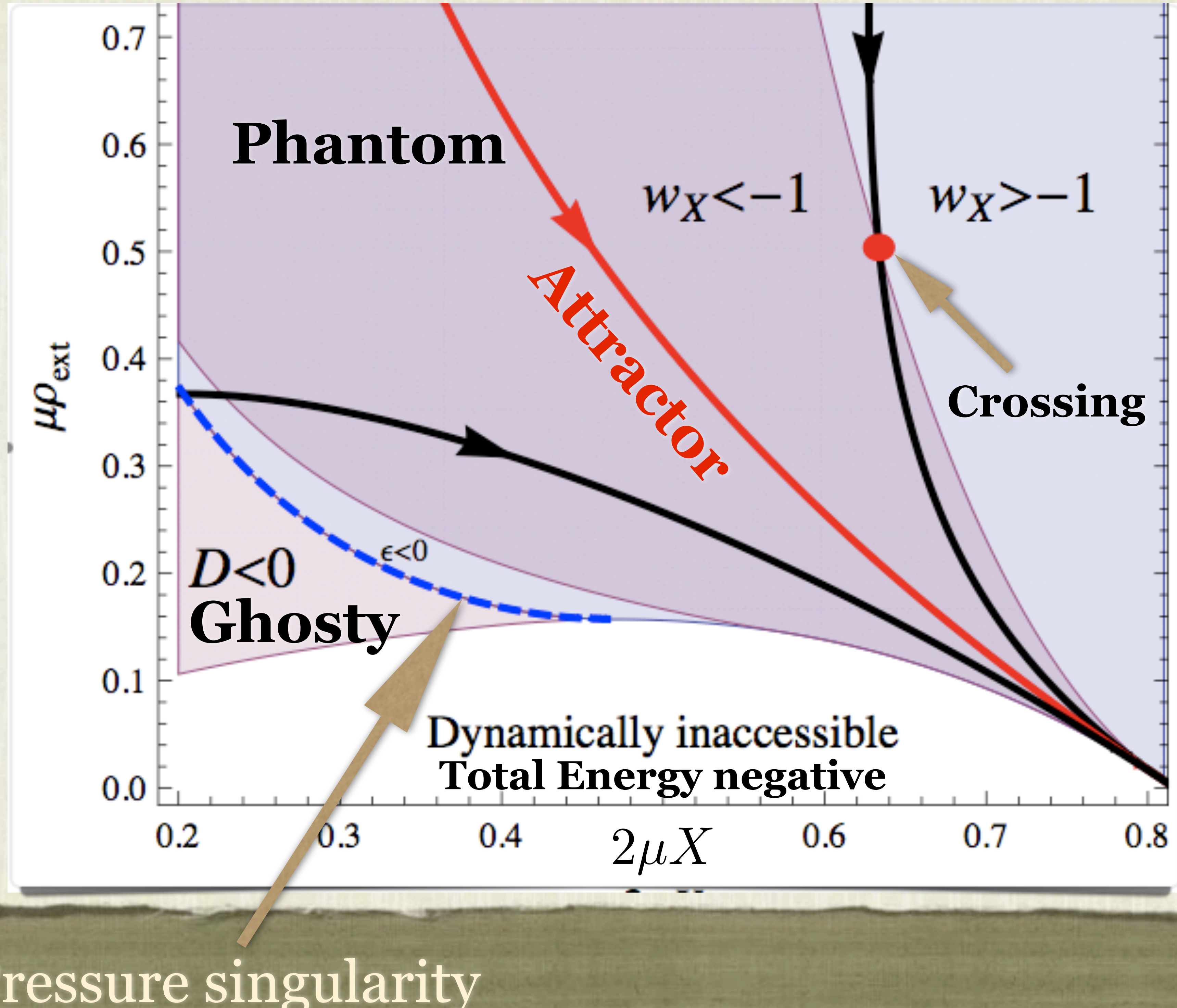
$$\Omega = \frac{n + \nabla_\lambda \left(\kappa u^\lambda\right)}{m} - \frac{1}{2}\kappa^2$$

In general propagation is anisotropic, but in cosmology:

$$c_\mathrm{s}^2 = \frac{\Omega m - 2\kappa H}{m D}$$

SOUND SPEED

$$c_{\mathrm{s}}^2 = \frac{\mathcal{P}_m + 2\dot{\kappa} + \kappa\left(4H - \kappa m/2\right)}{\mathcal{E}_m - 3\kappa\left(H - \kappa m/2\right)} \neq \frac{\dot{\mathcal{P}}}{\dot{\mathcal{E}}}$$



Phase portrait for scalar field & dust

DARK ENERGY

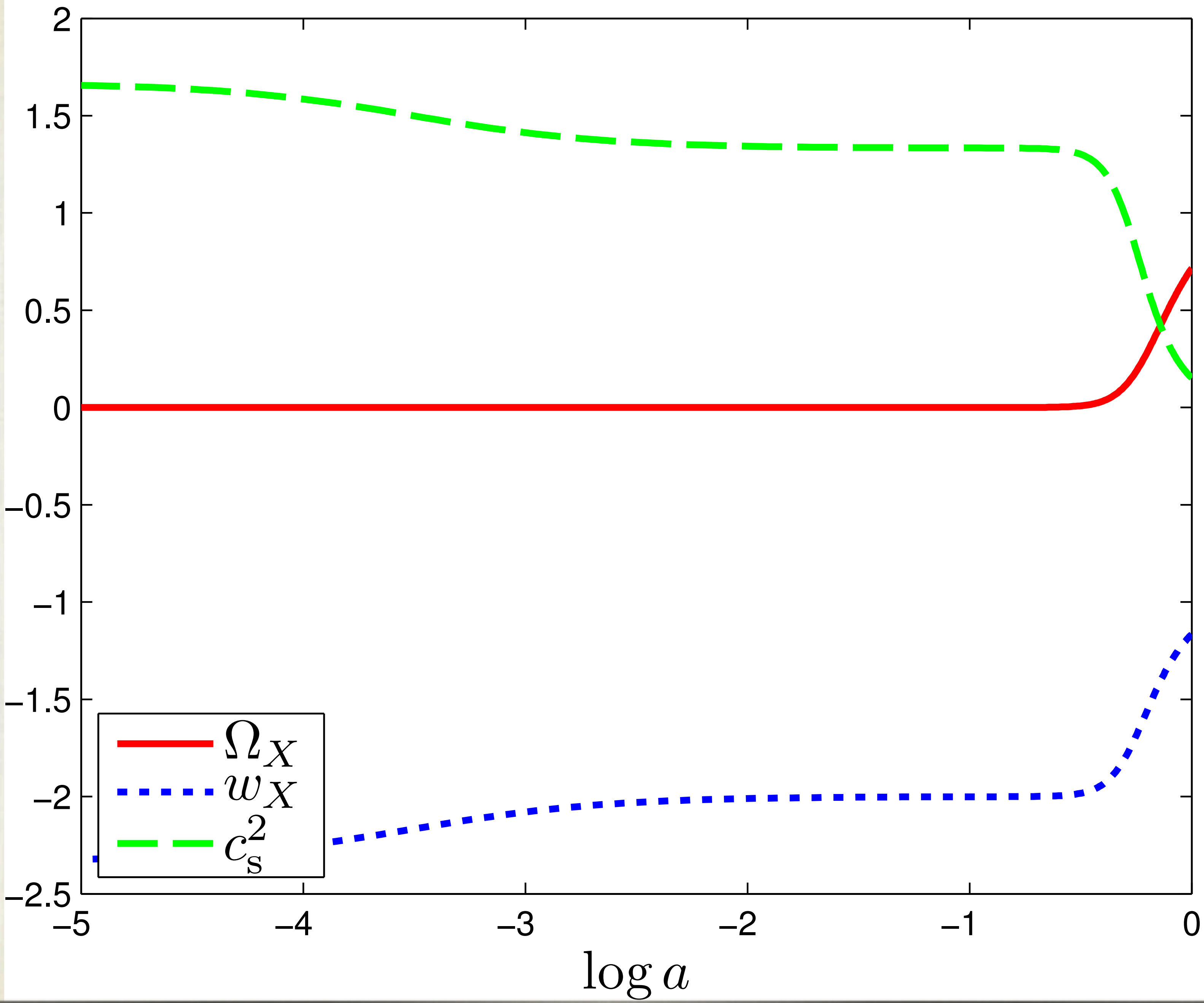
assume today $\sqrt{\frac{3}{2}}\mu\rho_{\rm ext} \ll 1$ ➡ $H_*^2 \simeq \frac{1}{6}\sqrt{\frac{2}{3}}\mu^{-1}$

$\Lambda_* \simeq \frac{1}{2}\sqrt{\frac{2}{3}}\mu^{-1} \simeq 3\rho_{\rm CDM}$ ➡ $\sqrt{\frac{3}{2}}\rho_{\rm CDM}\mu \simeq \frac{1}{6} \ll 1$

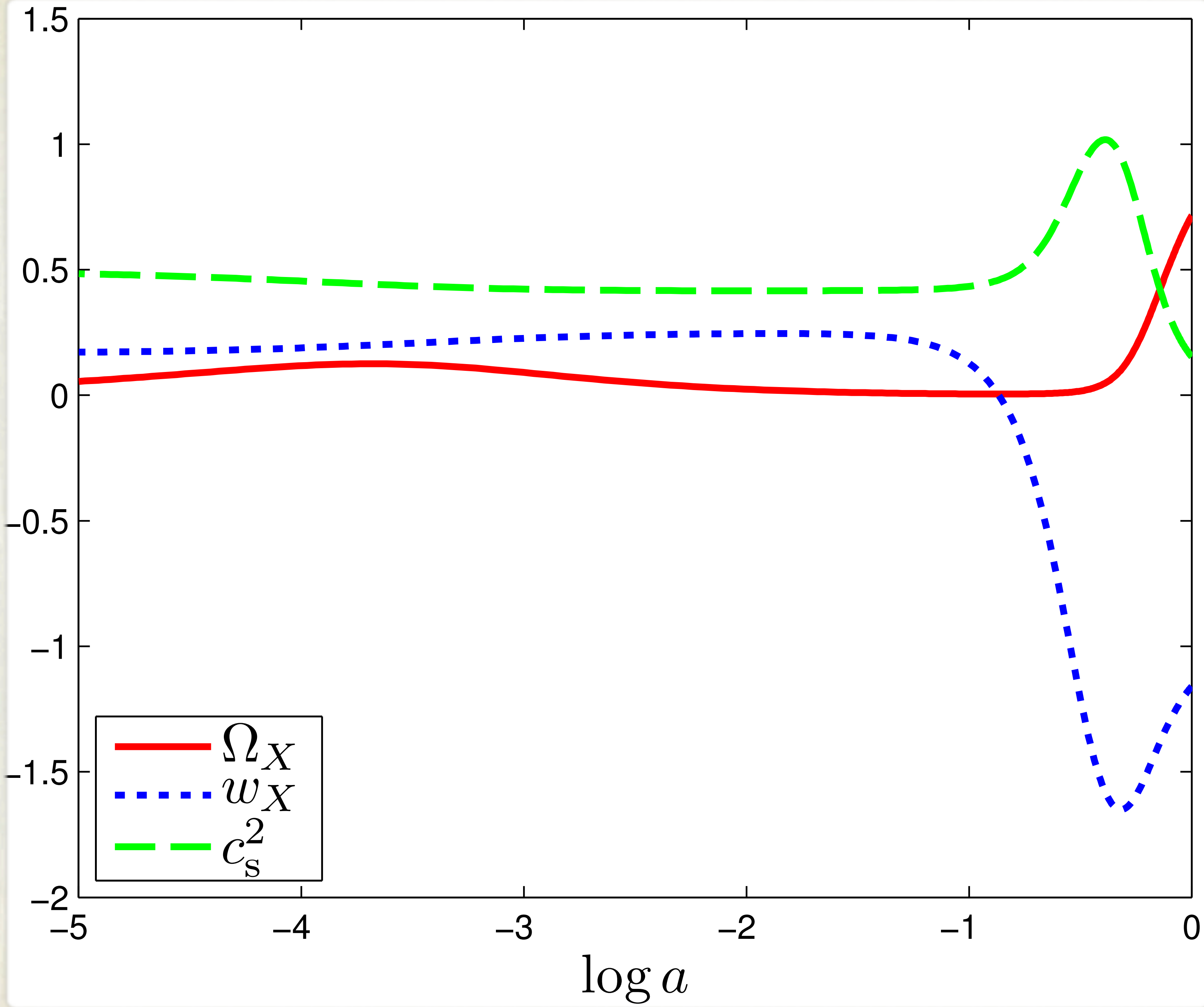
$$\text{Mass Scale} \sim \mu^{-1/3} \sim (H_0^2 M_{\rm Pl})^{1/3} \sim 10^{-13}\text{eV}$$

Length Scale: **1000 km**

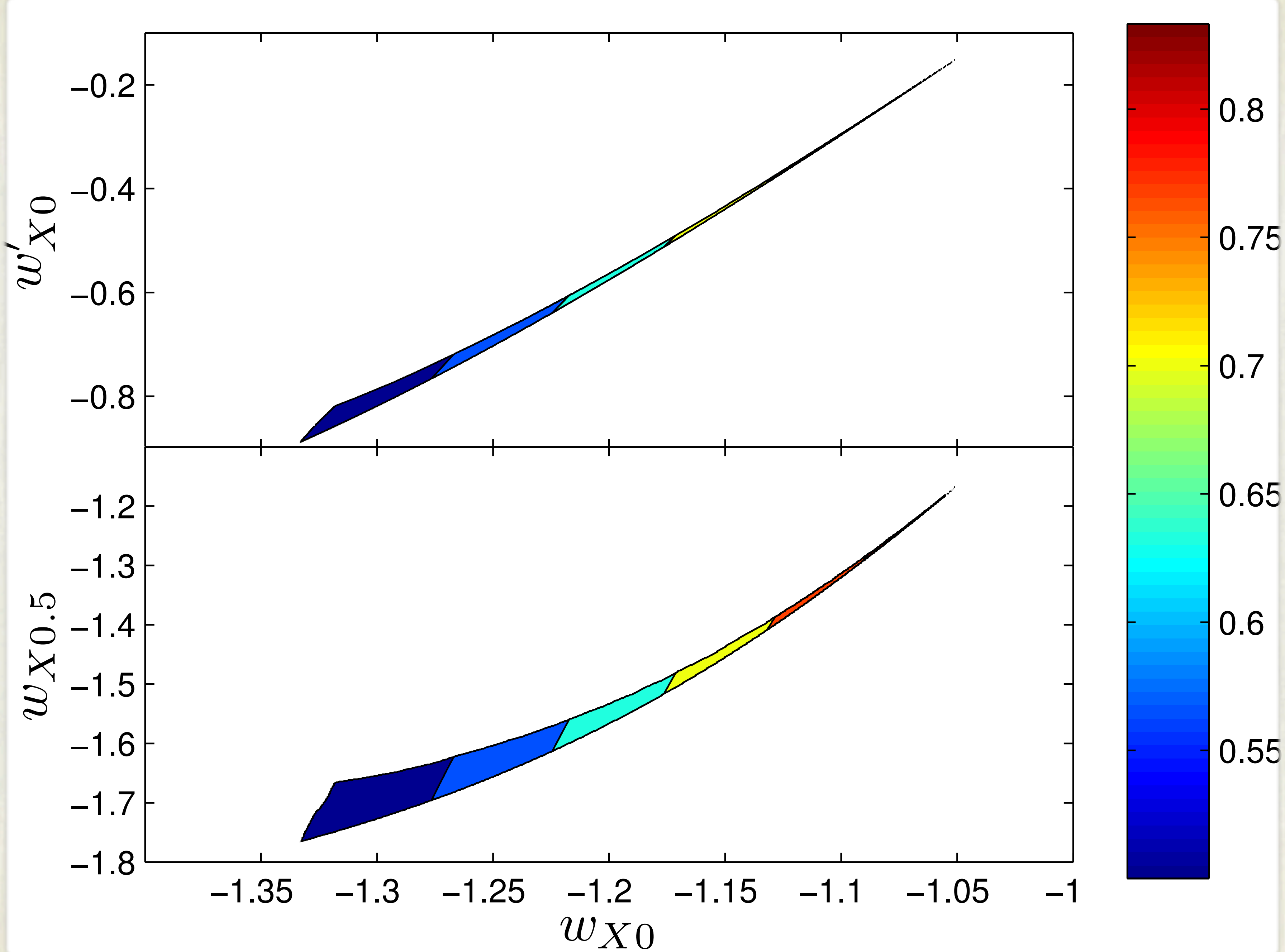
In Quintessence - the size of the universe



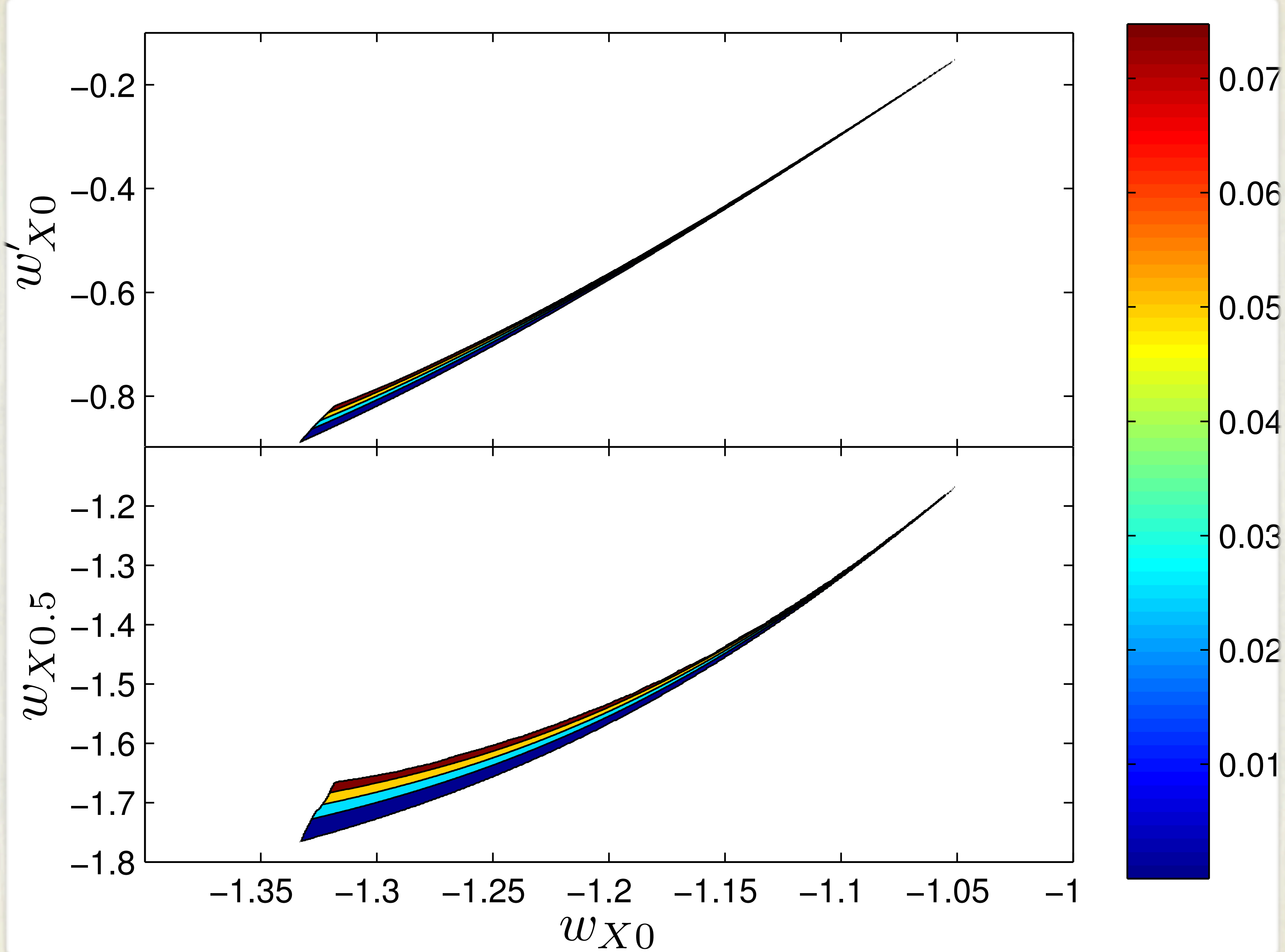
Evolution of dark energy properties in the Friedmann universe also containing dust and radiation. The scalar evolves on its attractor throughout the presented period. During matter domination $w_X = -2$, while $w_X = -7/3$ during radiation domination. The sound speed is superluminal when the scalar energy density is subdominant, becoming subluminal when $\Omega_X \approx 0.1$ and $w_X \approx -1.4$



Evolution of DE properties in the Friedmann universe which also contains dust and radiation. The energy density in the scalar is J-dominated (off attractor) until a transition during the matter domination epoch. This allows the scalar to increase its contribution to the total energy budget throughout radiation domination ($w_X = 1/6$) and provide an early DE peaked at matter-radiation equality, from whence it begins to decline with $w_X = 1/4$. The transition to the attractor behaviour is rapid. The equation of state crosses $w_X = -1$ and the scalar energy density begins to grow. The final stages of evolution are on the attractor and are similar to those presented in previous figure.



$0.1 < \Omega_m < 0.5$ and $\Omega_{X\mathrm{eq}} < 0.1$. The shading contours correspond to the energy density of DE today Ω_{X0}. Two parameterisations of DE behaviour are shown: w_X and w'_X evaluated today, and w_X evaluated today and at $z = 1/2$. The requirement that the energy density in DE at matter-radiation equality be small, $\Omega_X^{\mathrm{eq}} < 0.1$ forces the value of the shift charge to be small today $Q_0 < 10^{-2}$. This means that in the most recent history, the evolution has effectively been on attractor or very close to it and the permitted value of w_X is very restricted and determined to all intents and purposes by Ω_X^0.



The shading representing the contribution of DE to energy density at matter-radiation equality. We choose to cut the parameters such that the contribution to this early DE at that time is no larger than 10%. It can clearly be seen that values of w_X closer to -1 are obtained when the shift charge is larger, but this leads to more early DE, eventually disagreeing with current constraints

FURTHER DEVELOPMENT

Kinetic Gravity Braiding with $G \propto X^n$

arXiv:1011.2006v2 [astro-ph.CO], *Rampei Kimura, Kazuhiro Yamamoto*

5

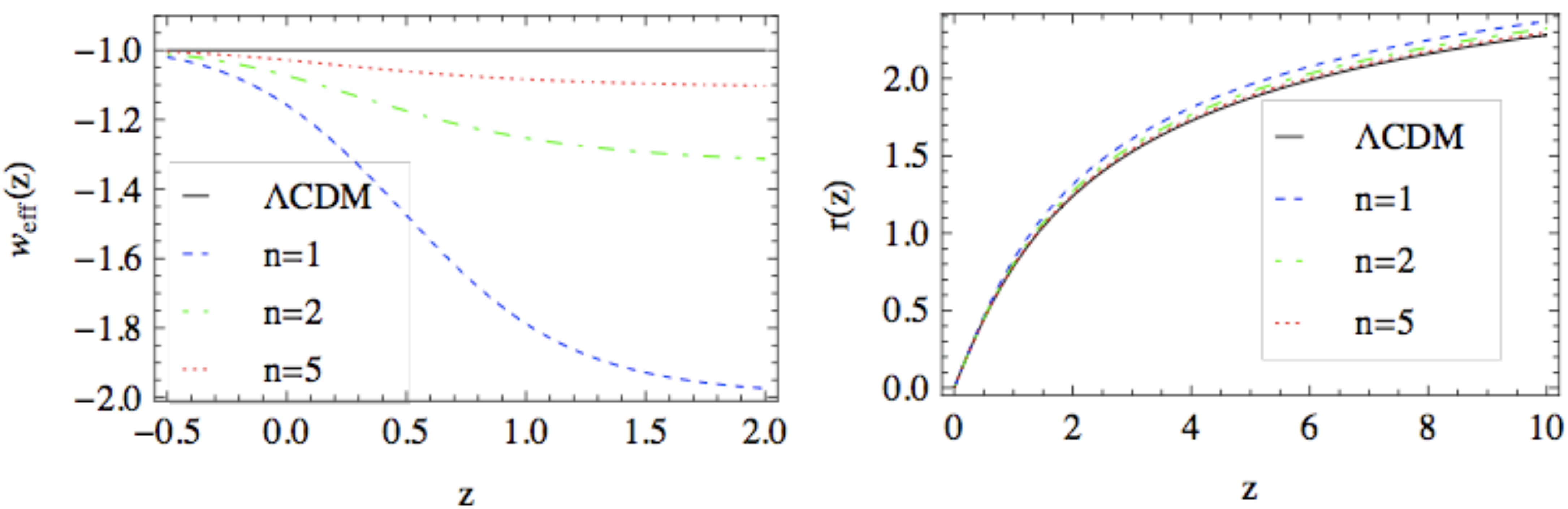


FIG. 1: Left panel: The effective equation of state $w_{\rm eff}$ as a function of redshift for ΛCDM (solid curve) and the kinetic braiding mode with $n = 1$ (dashed curve), $n = 2$ (dash-dotted curve), and $n = 5$ (dotted curve), respectively. Right panel: The comoving distance $r(z)$, normalised by H_0, as a function of redshift for ΛCDM and this model.

CONSTRAINTS FROM CMB AND SN IA

Kinetic Gravity Braiding with $G \propto X^n$

arXiv:1011.2006v2 [astro-ph.CO], *Rampei Kimura, Kazuhiro Yamamoto*

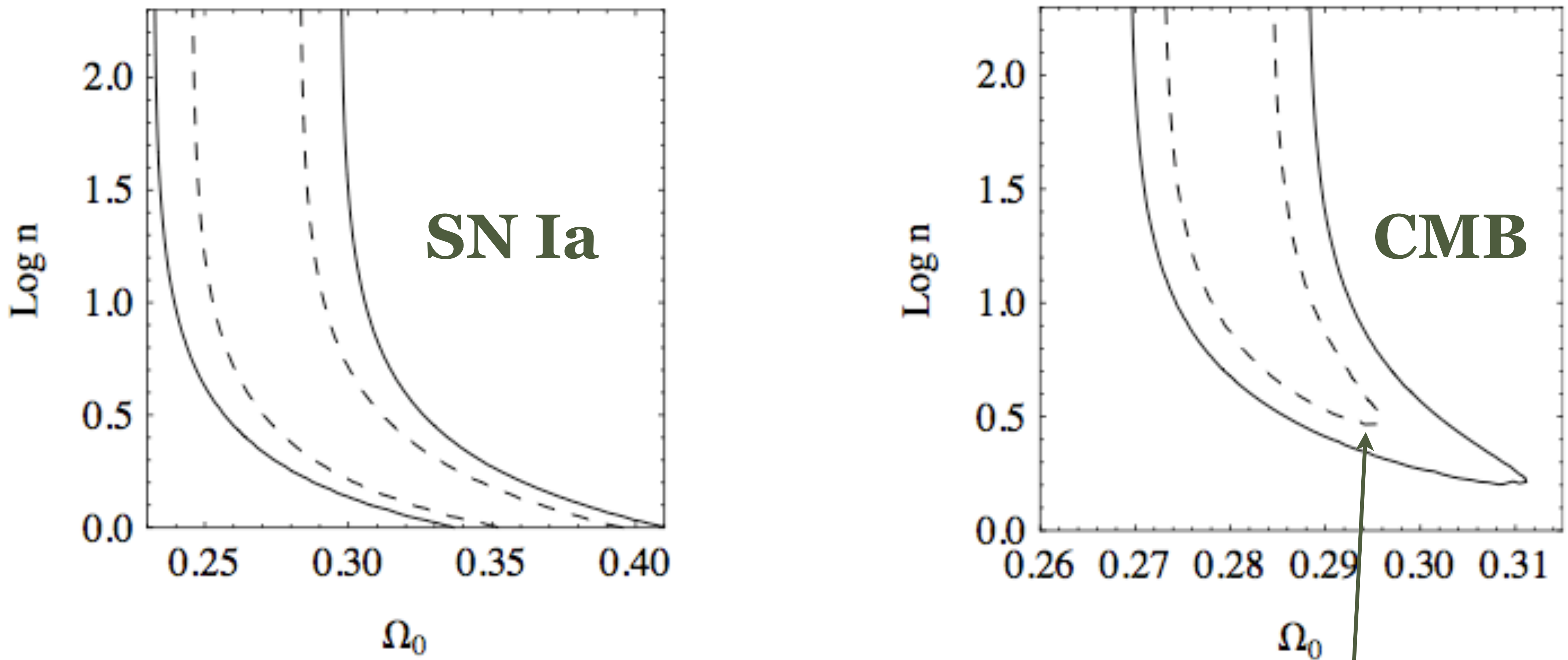


FIG. 3: The left panel is the contour of $\chi^2_{\rm SN}$ on the plane Ω_0 and n for the kinetic braiding model. The dashed curve and the solid curve are the 1 σ and 2 σ contours, respectively. The right panel is the same but of $\chi^2_{\rm CMB}$.

The SCP Union2 Compilation is a collection of 557 type Ia supernovae data whose range of the redshift is 0.015 < z < 1.4

Thus $n \gtrsim 3$ mass scale $\sim 10^{-3}$eV

Length Scale: **1/10 mm**

GROWTH FACTOR

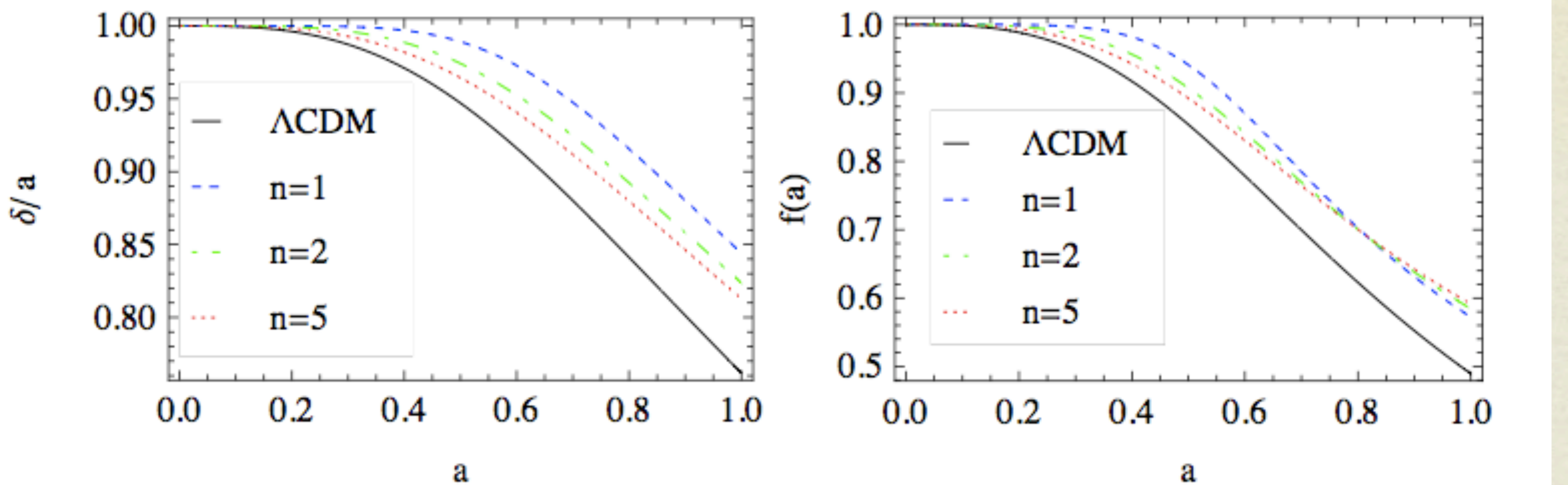
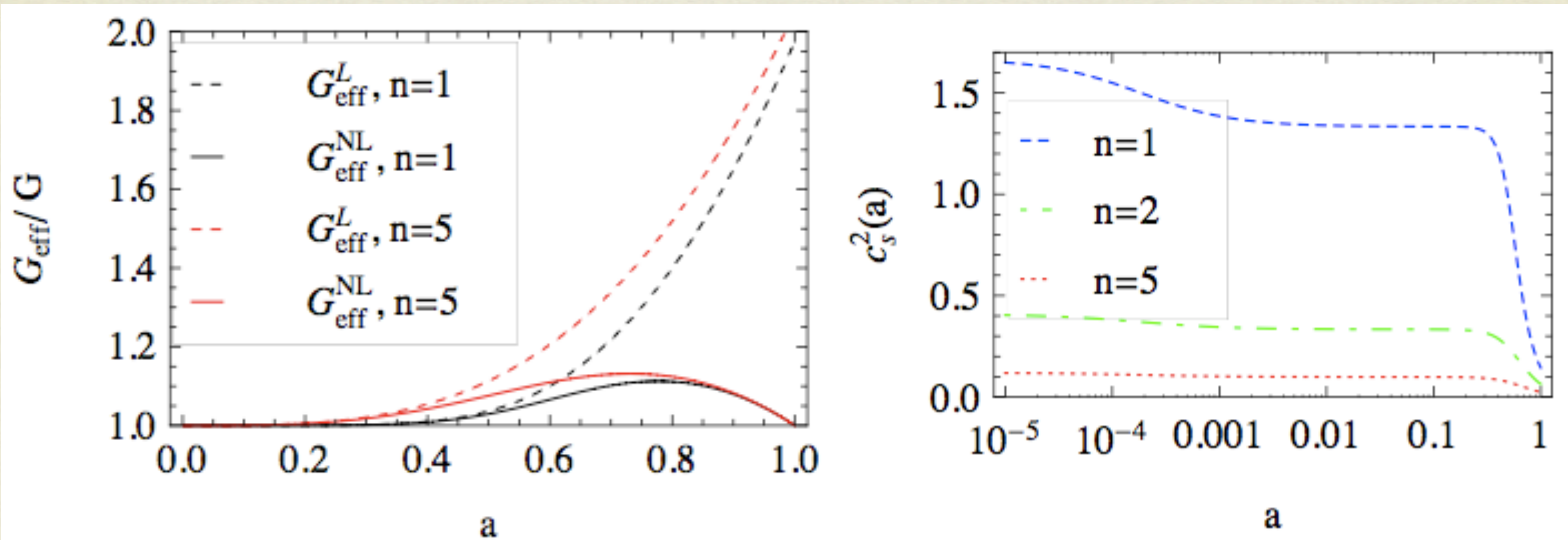


FIG. 4: Left panel: The growth factor divided by scale factor as a function of scale factor for the Λ CDM model (solid curve) and the kinetic braiding model $n = 1$ (dashed curve), $n = 2$ (dash-dotted curve), and $n = 5$ (dotted curve), respectively. Right panel: The linear growth rate as a function of scale factor.

Kinetic Gravity Braiding with $G \propto X^n$

arXiv:1011.2006v2 [astro-ph.CO], *Rampei Kimura, Kazuhiro Yamamoto*

EFFECTIVE NEWTON CONSTANT FOR PERTURBATIONS AND THE SOUND SPEED



Kinetic Gravity Braiding with $G \propto X^n$

arXiv:1011.2006v2 [astro-ph.CO], *Rampei Kimura, Kazuhiro Yamamoto*

THANKS A LOT FOR YOUR ATTENTION!