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YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Tensor perturbation constraint on inflation models with non-negligible spatial curvature in landscape scenario (Too long)

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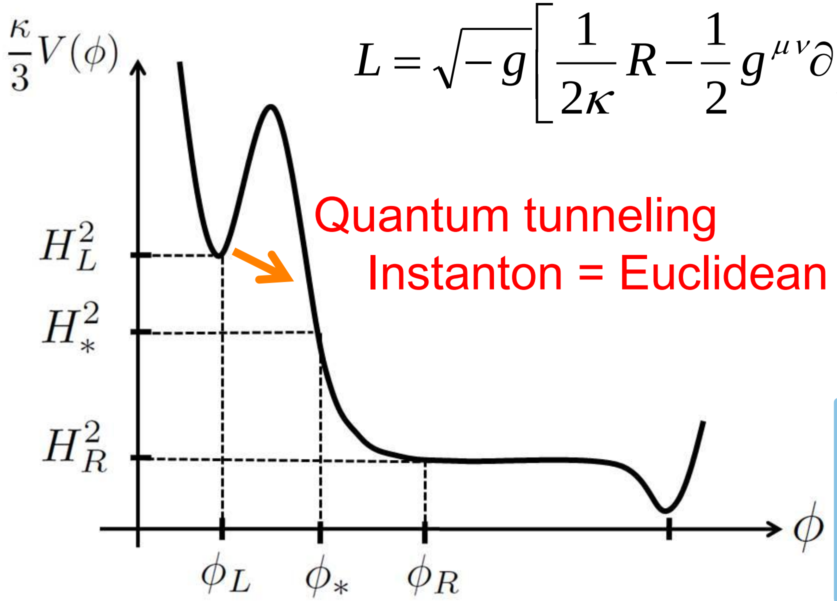
§ One bubble open inflation

- False vacuum decay with gravity

$$L=\sqrt{-g}\left[\frac{1}{2\kappa}R-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi-V(\phi)\right]$$



O(3,1)-symmetric bubble

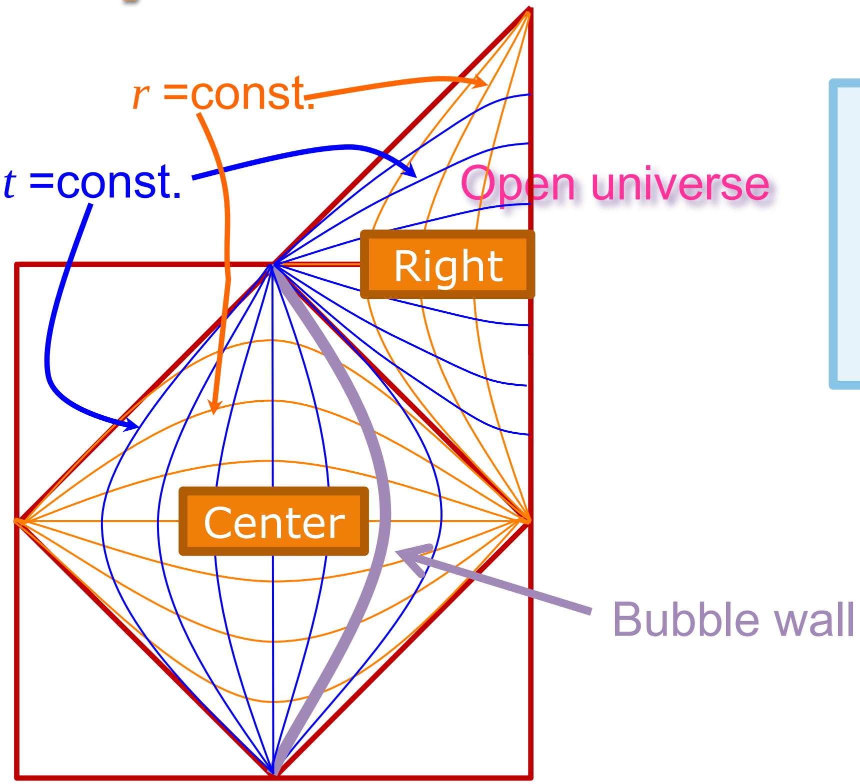
$$ds^2=dt^2+a^2(t)\left(-dr^2+\cosh^2 r\,d\Omega^2\right)$$

$$\phi=\phi(t)$$

$$\left(\frac{\dot a}{a}\right)^2-\frac{1}{a^2}=\frac{\kappa}{3}\left(\frac{1}{2}\dot\phi^2-V(\phi)\right)$$

$$\ddot\phi+3\frac{\dot a}{a}\dot\phi=\frac{dV(\phi)}{d\phi}$$

Analytic continuation to open universe



$$r_C = r_R - \frac{\pi i}{2}$$
$$dt_C = -i dt_R$$
$$a_C = -i a_R$$

Not t=const. surface but r =const. surface is a Cauchy surface in the Center region

◆ Spatially open but homogeneous universe is formed inside the bubble.

◆ Quantum fluctuation is determined by the vacuum state whose mode functions are chosen by the analyticity when they are continued to the Euclidean region.

Tensor perturbation (Garriga, Montes, Sasaki, Tanaka ('98, '99))

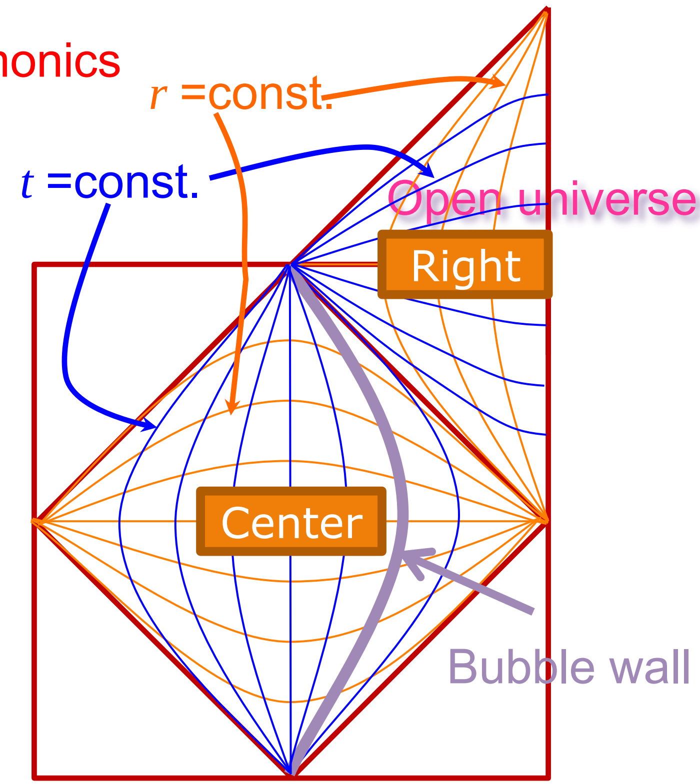
Tensor perturbation in the open universe can be decomposed:

$$\delta h_{\mu\nu} = a^2(t_R)\sum U_{plm}(t_R)\,Y_{\mu\nu}^{(\pm)p\ell m}$$

3-dim tensor harmonics

$$\left[\frac{1}{a(t_R)^3}\partial_{t_R} a(t_R)^3 \partial_{t_R} + \frac{p^2+1}{a(t_R)^2}\right]U_{plm}(t_R) = 0$$

But t=const. surface in the Right region is not a Cauchy surface, but r =const. surface in the Center region is a Cauchy surface.



For the decomposition in the Center region:

- 2+1+1 decomposition is necessary
- Reduction of the quadratic action by means of gauge fixing and constraints:

$$\int dr \int dt\, L = \sum_{\ell,m} \frac{2}{\kappa(\ell-1)\ell(\ell+1)(\ell+2)} \int dr_C \int \frac{dt_C}{2a^3}\left[\cosh^2 r_C \frac{\partial w^{\ell m}}{\partial r_C}\hat{K}\frac{\partial w^{\ell m}}{\partial r_C} - w^{\ell m}\hat{K}\left\{\ell(\ell+1) + (\hat{K}+1)\cosh^2 r_C\right\}w^{\ell m}\right]$$

$$\hat{K} = -\frac{\partial^2}{\partial \eta_C^2} + \frac{\kappa}{2}\phi'^2$$

Reduced action for master variable for each l,m mode

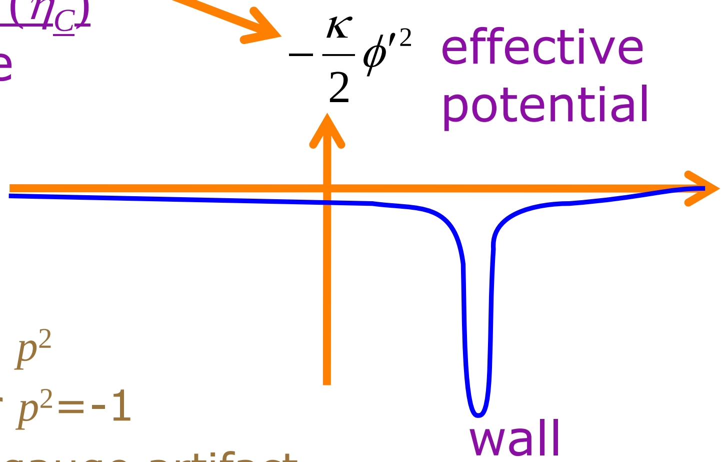
$$\int dr \int dt\, L = \sum_{\ell,m} \frac{2}{\kappa(\ell-1)\ell(\ell+1)(\ell+2)} \int dr_C \int \frac{dt_C}{2a^3} \left[\cosh^2 r_C \frac{\partial w^{\ell m}}{\partial r_C} \hat{K} \frac{\partial w^{\ell m}}{\partial r_C} - w^{\ell m} \hat{K} \left\{ \ell(\ell+1) + \left(\hat{K}+1\right)\cosh^2 r_C \right\} w^{\ell m} \right]$$

$$d\eta_C = \frac{dt_C}{a_C(t_C)} \qquad \hat{K} = -\frac{\partial^2}{\partial \eta_C^2} + \frac{\kappa}{2}\phi'^2$$

Expand the function $w^{lm} = \Sigma f^{pl}(r_C)\, w^p(\eta_C)$ in terms of the eigen function of the operator $\hat{K}$.

$$\hat{K}\, w^p = p^2 w^p$$

η_C is a spatial coordinate.



$p^2 > 0$: continuum, two modes for each p^2

$p^2 < 0$: no discrete spectrum except for $p^2 = -1$

This unique discrete mode is gauge artifact.

$f^{pl}(r_C)$ is fixed to satisfy KG normalization and regularity for Euclidean extension.

$$f^{p\ell} = \sqrt{\frac{(\ell-1)\ell(\ell+1)(\ell+2)}{8\kappa p^3 \sinh \pi p}}\, P^{-\ell-1/2}_{ip-1/2}(i \sinh r_C)$$

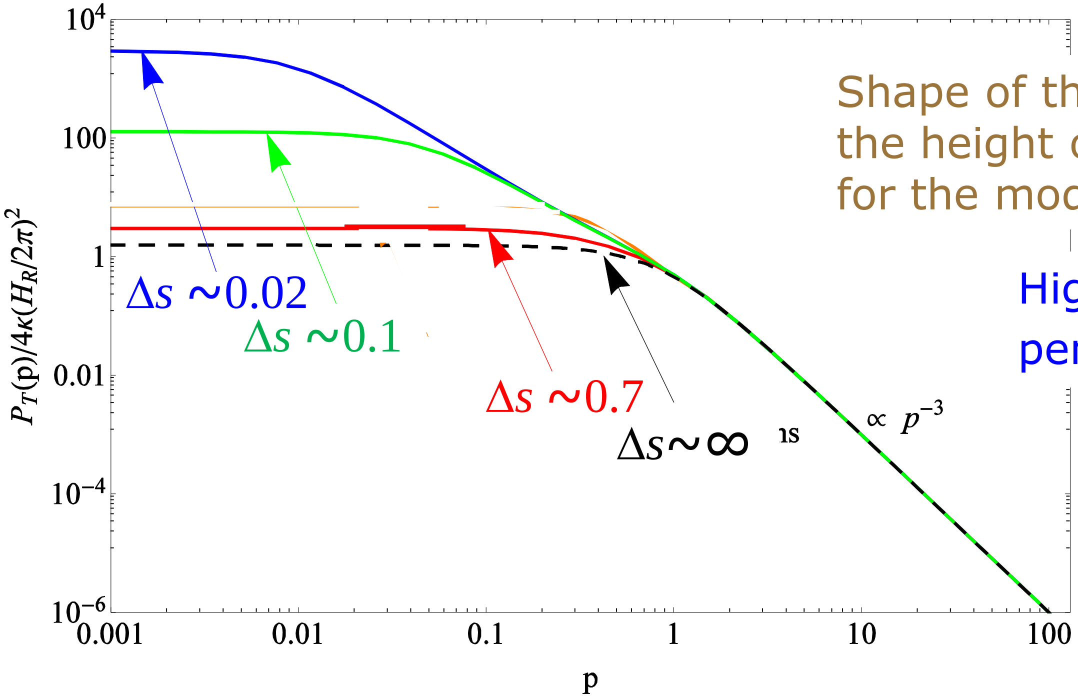
After analytic continuation to the Right open universe, w^p is related to the amplitude of tensor perturbation as

$$U_{p\ell m}(t_R) = -\sqrt{\frac{\kappa}{p(1+p^2)}}\, \frac{1}{a_R} \frac{d}{dt_R}\left(a_R w^p\right) \qquad \delta h_{\mu\nu} = a^2(t_R) \sum U_{plm}(t_R)\, Y^{(\pm)p\ell m}_{\mu\nu}$$

Typical shape of the spectrum

- Thin wall approximation
- Pure de Sitter inside the bubble

$$\Delta s \equiv \frac{\kappa}{2} \int d\eta \, \phi'^2(\eta_C)$$



Shape of the spectrum depends on the height of the effective potential for the mode function

Higher barrier allows less penetration for small p modes

Total power of fluctuation $\int dp \, P_T(p)$ is IR divergent if $\Delta s = 0$ (no barrier)

$$\int dp \, P_T(p) \approx \frac{2\kappa H_*^2}{\Delta s} f(\Delta s \, \eta_w)$$

η_w is the parameter to specify the location of the wall, but it is not so important

$$f(x) = 2e^{-(x+|x|)/2} - 1 + x \approx x \pm 1$$

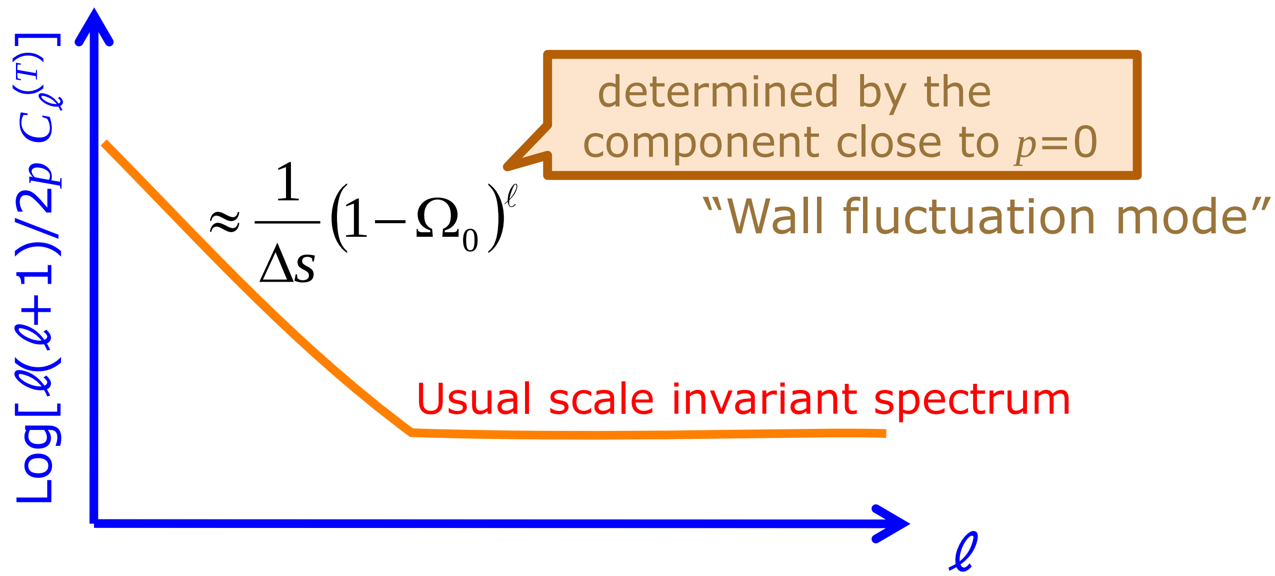
Typical shape of CMB spectrum

We simply evaluate Sachs-Wolfe effect

Unit vector pointing the line-of-sight direction

$$\frac{\Delta T}{T}(\hat{n}) = -\frac{1}{2}\int_{\eta_{LSS}}^{\eta_0} d\eta\, \delta g'_{ij}\left(\eta, x^i(\eta)\right)\hat{n}^i \hat{n}^j$$

For $1-\Omega_0 << 1$, shape of CMB spectrum is quite simple



§ Renewed interest

The original open inflation with $1-\Omega_0 \approx 0.3$ is observationally ruled out but….

moderately small $1-\Omega_0$ might be preferred.

(Freivogel and Susskind (2004))

Primordial probability distribution:

Smaller e-folds during inflation might be preferred

Anthropic pressure:

Smaller $1-\Omega_0$ is preferred for structure formation

⇒ $1-\Omega_0=10^{-2}\sim10^{-3}$

§ A little more hypothetical constraints

1) A typical false vacuum has Planck scale energy.

It will be so, if the energies of false vacua distribute uniformly in linear scale.

large hierarchy

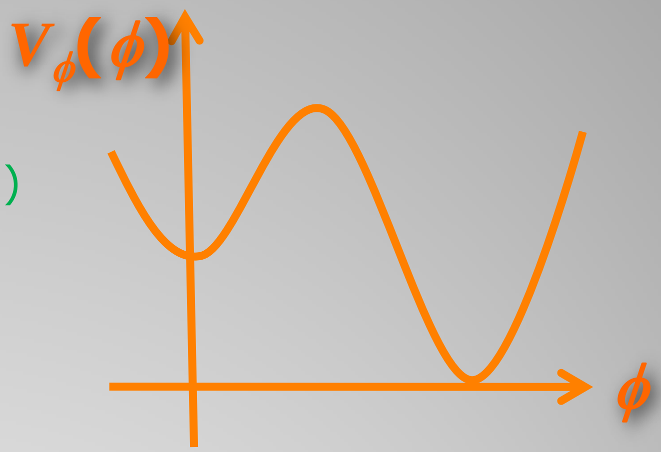
2) The last inflation is slow roll type at relatively low energies.

It will be so, if KKLT-like scenario is typical.

§ Models

1) Decoupled two-field model:

(Linde, Mezhlumian ('95))



$$V(\phi,\sigma)=V_\phi(\phi)+\frac{m_\sigma^2}{2}\sigma^2$$

quasi-open inflation model.

On large scales the state after tunneling is quite inhomogeneous.

If m_σ is sufficiently small, there exists a supercurvature (discrete) mode, which gives CMB anisotropy:

$$\frac{\ell(\ell+1)}{2\pi}C_\ell \approx \frac{\kappa}{\varepsilon_{\inf}}\left(\frac{H_L}{2\pi}\right)^2 (1-\Omega_0)^\ell$$

(Sasaki, Tanaka ('96))

This supercurvature fluctuation has the amplitude determined by the Hubble rate in the false vacuum.

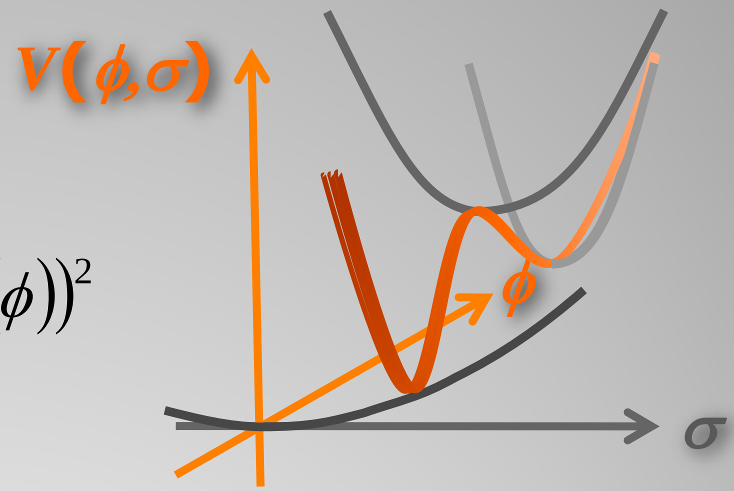
⇒ Conflict with observation!

§ Models

2) Coupled two-field model:

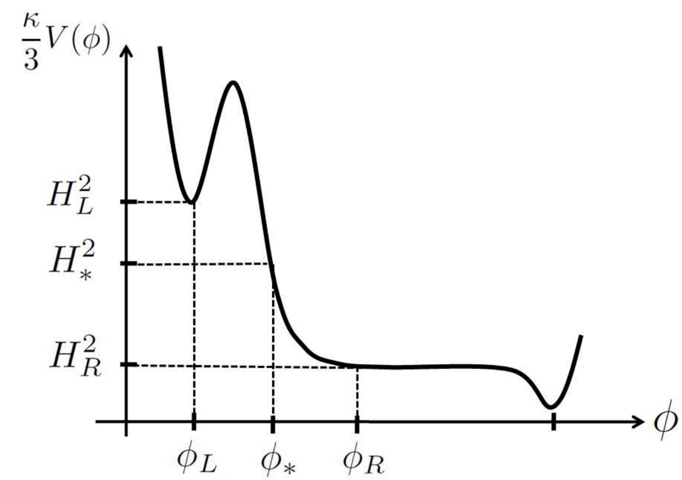
$$V(\phi,\sigma)=V_\phi(\phi)+\frac{m_\sigma^2(\phi)}{2}(\sigma-\sigma_0(\phi))^2$$

Large initial mass eliminates the supercurvature mode.



3) Single field model after integrating out massive degrees of freedom:

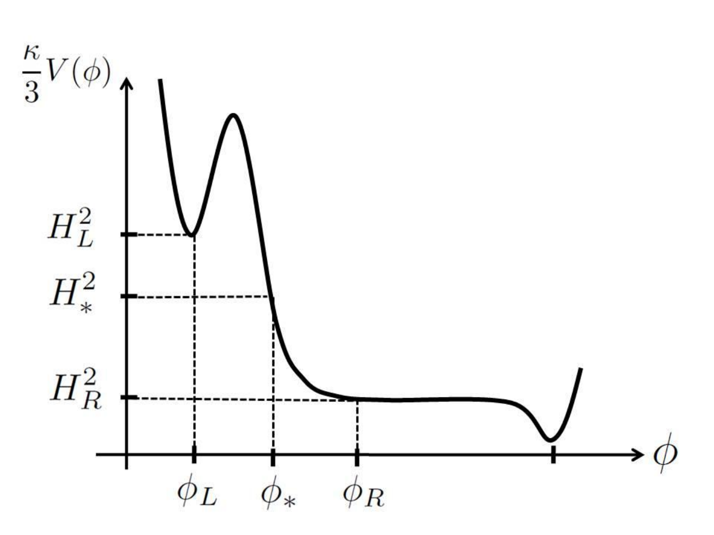
The above model will be equivalent after integrating out the direction perpendicular to the background trajectory.



§ Tensor perturbation

If the tunneling energy scale is Planckian, it may produce robust signature in tensor perturbation.

$$P_T(p) \approx 4\kappa\left(\frac{H}{2\pi}\right)^2 \; \textbf{?}$$



Memory of H_L (Hubble rate in the false vacuum) may remain in the curvature scale perturbation.

Thin wall approximation: for simplicity

$$\Rightarrow \quad H_L^2 \approx H_*^2$$

fast roll down phase

At the beginning, cosmic expansion is dominated by curvature term

$$a_R \approx t \qquad \dot{\phi} \approx -\frac{V'(\phi)}{4} t$$

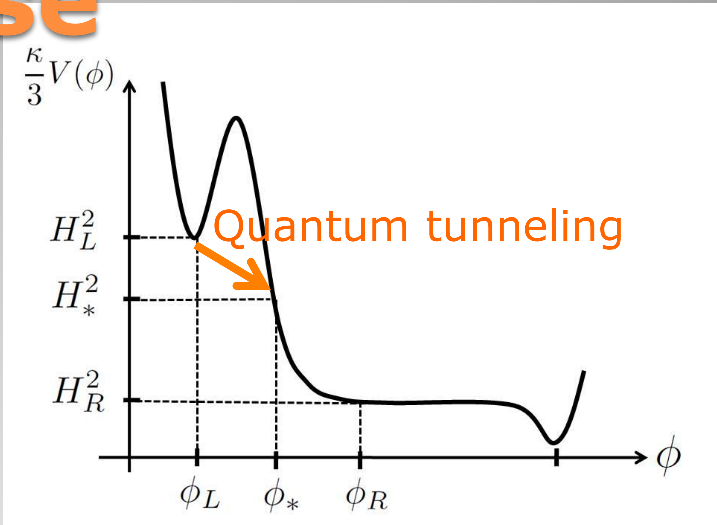
1)Extrapolating this relation, $\dot{\phi}^2 \approx V$ is realized at $t \approx H_*^{-1} / \sqrt{\varepsilon_*}$

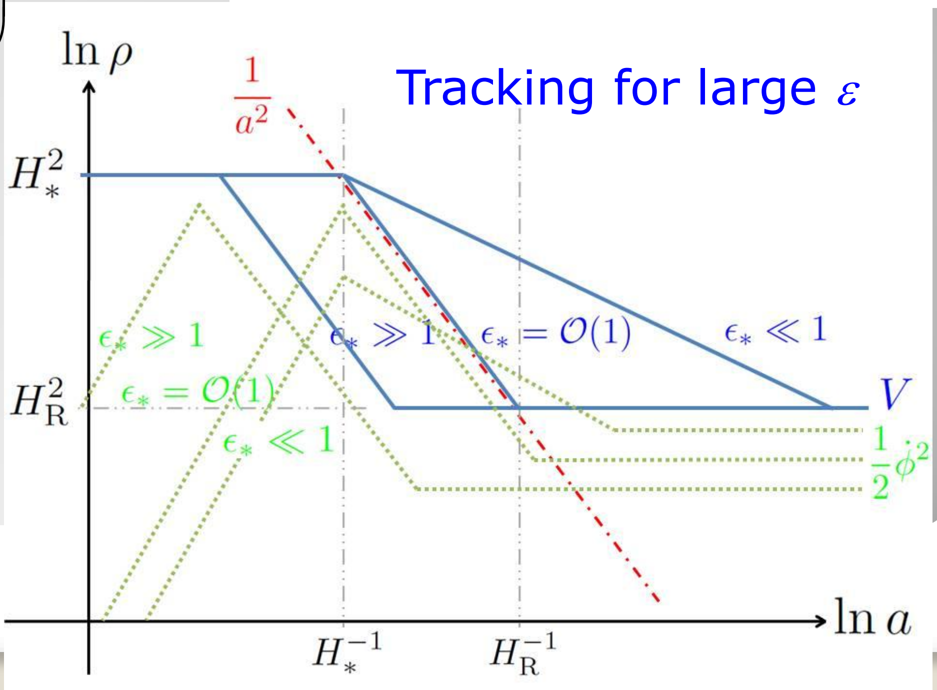
Slow roll parameter: $\varepsilon \equiv \frac{1}{2\kappa}\left(\frac{V'}{V}\right)$

$$\frac{d \ln \rho}{d \ln a_R} = -\frac{6\dot{\phi}^2}{\dot{\phi}^2 + 2V(\phi)}$$

Energy density starts to decay after $t \approx H_*^{-1} / \sqrt{\varepsilon_*}$

2)If the potential does not decay, curvature dominance terminates at $t \approx H_*^{-1}$





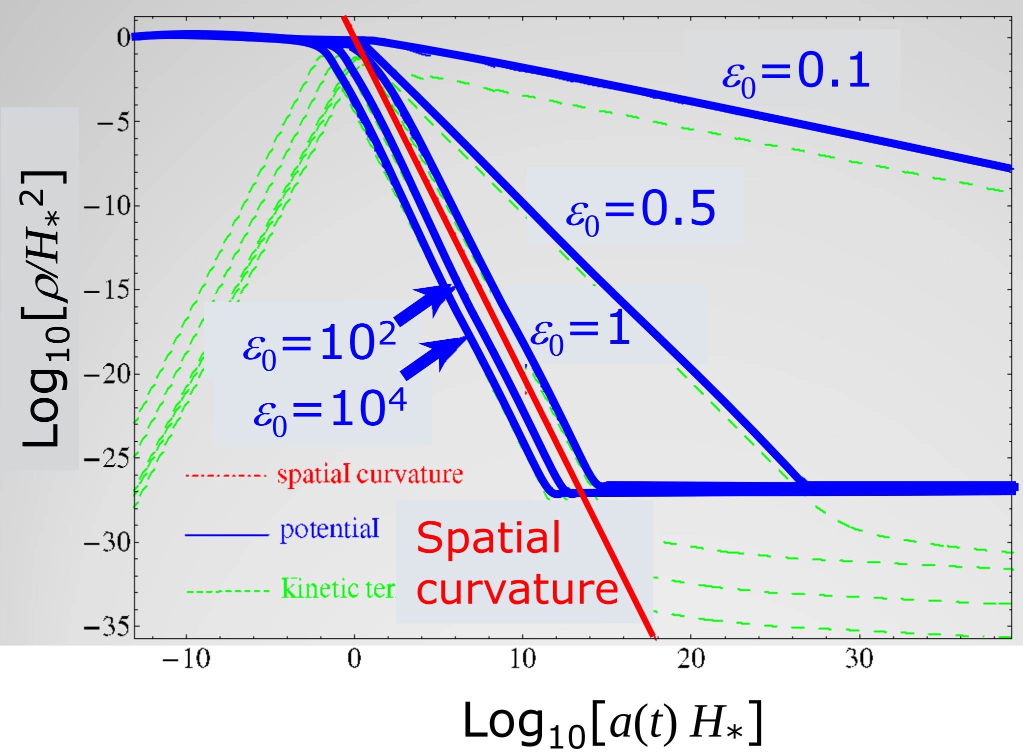
A Simple Model

Exact tracking for exponential potential

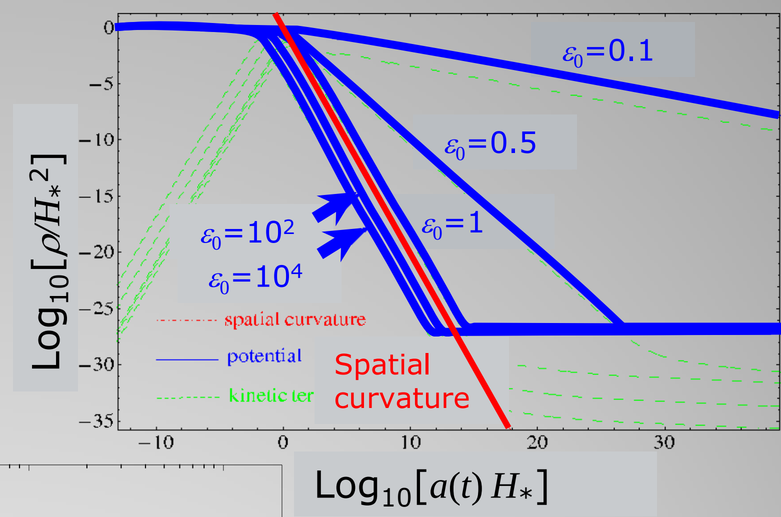
ε =const. ⟹ V'/V=const. ⟹ $V \propto \exp\left(\sqrt{2\kappa\varepsilon}\,\phi\right)$

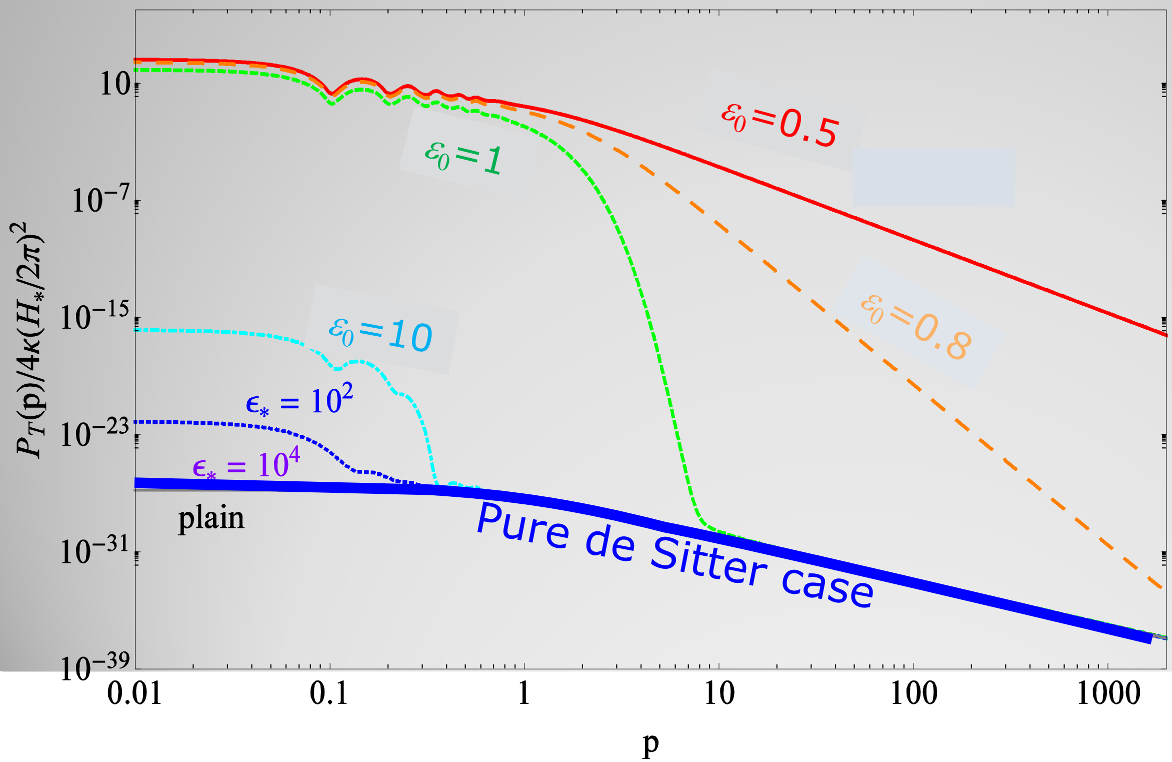
To add late-time slow roll inflation,

$$V = \left(H_*^2 - H_R^2\right)\exp\left(\sqrt{2\kappa\varepsilon_0}\,(\phi-\phi_*)\right) + H_R^2$$



Spectrum



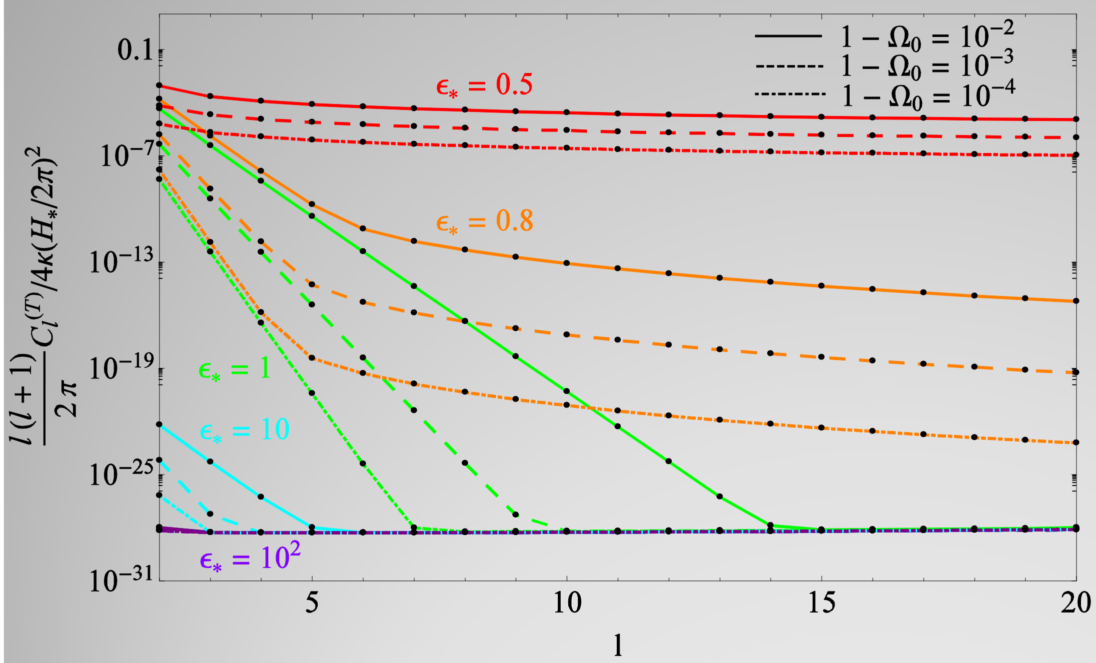


ε<<1: usual slow roll

ε=1: only small p modes remembers the initial large Hubble rate.

ε>>1: No memory of large Hubble rate

CMB Spectrum



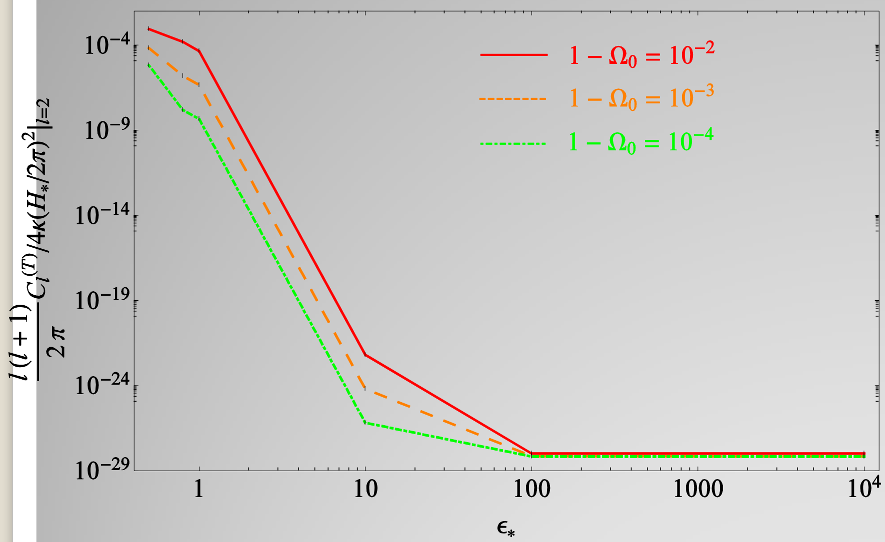
ε<<1: usual slow roll

ε=1: only small p modes remembers the initial large Hubble rate.

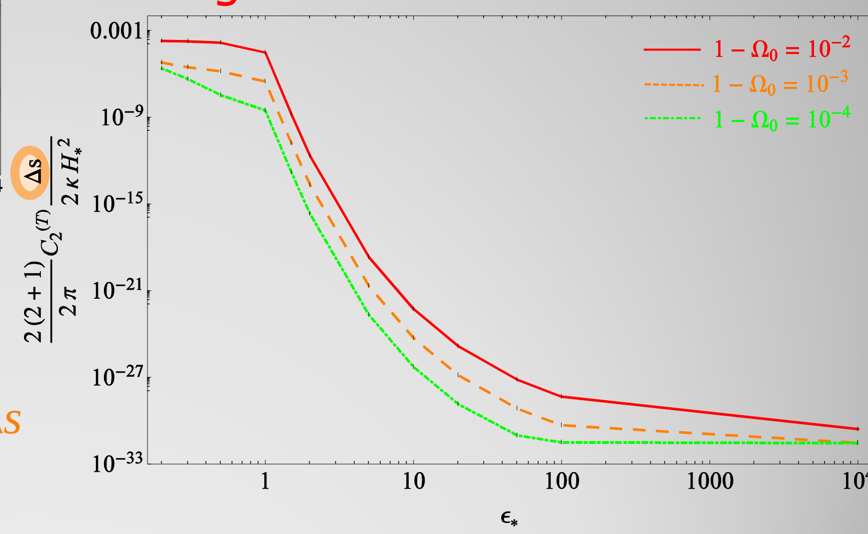
ε>>1: No memory of large Hubble rate

L=2 mode amplitude in CMB

Small wall fluctuation mode



Large wall fluctuation mode



Amplified by the factor $1/\Delta s$

§ Summary

- Open inflation scenario with large hierarchy was considered.
- There is a possibility of rapid roll phase after tunneling if the slow roll parameter ε is not small, and it can alter the CMB spectrum.

  ε<<1: usual slow roll

  ε~1: large angle scale anisotropies have a memory of the initial large Hubble rate.

  ε>>1: No memory of large Hubble rate
- We also have enhancement by the wall fluctuation, which can be large if the tension of the wall is small.

$$C_\ell^{(T)} \approx 2\kappa H^2_{\text{horizoncrossing}} / \Delta s \qquad \Delta s \approx (\text{Wall tension}) / H_* M_{pl}^2$$

  Too small tension also leads to bubble collision.

  (Sasaki, Tanaka, Yakushige ('97))
- Fortunately or unfortunately, depending on the detail of the models, it seems possible to construct models consistent with current observational constraints.