Phenomenology of Continuum Superpartners

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Introduction

we are considering a supersymmetric theory with an approximate conformal sector. The conformal sector is soft broken at TeV scale, the superpartners of SM particles develop a continuum above a mass gap.

the Ads5 metric

The 5D AdS_5 metric written in the conformal coordinates :

$$ds^2 = \left(\frac{R}{z}\right)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right) .$$

In RS2 model, there is only a UV cut off. The spectrum is Unparticles without Mass Gap.

In RS1 model, there are two branes, one UV brane $z_{UV} = \epsilon$ and one IR brane $z_{IR} = L$. The spectrum is discrete KK modes, the splitting of those KK modes depends on the position of the IR brane.

SUSY fields in Ads5

The $\mathcal{N} = 1$ SUSY in 5D is equivalent to $\mathcal{N} = 2$ SUSY in 4D. An $\mathcal{N} = 2$ hypermultiplet Ψ can be decomposed into two $\mathcal{N} = 1$ chiral superfields $\Phi = \{\phi, \chi, F\}$ and $\Phi_c = \{\phi_c, \psi, F_c\}$, where the two Weyl fermions χ and ψ form a Dirac fermion. The 5D action for matter fields can be written as:

$$S = \int d^4x\, dz \left\{ \int d^4\theta \left(\frac{R}{z}\right)^3 [\Phi^* \Phi + \Phi_c \Phi_c^*] + \right.$$

$$\left. + \int d^2\theta \left(\frac{R}{z}\right)^3 \left[\frac{1}{2} \Phi_c \partial_z \Phi - \frac{1}{2} \partial_z \Phi_c \Phi + m(z) \frac{R}{z} \Phi_c \Phi\right] + h.c. \right\} ,$$

for $m(z)R = c$, we get a supersymmetric Randall-Sundrum Model. Wavefunctions for the bulk fields are Bessel Functions.

soft wall model

Here we want to realize the scenario of one zero mode plus continuum spectrum with a mass gap. Taking $m(z)R = c + \mu\ z$, Soft breaking CFT in the large IR will generate a mass gap between the zero mode and the continuum spectrum.

Figure 1: zero mode plus continuum spectrum in the soft wall model

matter fields in the bulk

Decompose the 5D field into the product of 4D field and a profile:

$$\begin{aligned}\chi(p,z) &= \chi_4(p)\left(\frac{z}{z_{UV}}\right)^2 f_L(p,z), \quad \phi(p,z)=\phi_4(p)\left(\frac{z}{z_{UV}}\right)^{3/2} f_L(p,z),\\ \psi(p,z) &= \psi_4(p)\left(\frac{z}{z_{UV}}\right)^2 f_R(p,z), \quad \phi_c(p,z)=\phi_{c4}(p)\left(\frac{z}{z_{UV}}\right)^{3/2} f_R(p,z),\end{aligned}$$

- Susy relates the profiles of scalars and fermions, χ and ϕ has the same 5D profiles.
- f_L and f_R are related by the first order 5D differential equations.

the 5D equations of motion

the Equations of Motion for the profiles are:

$$\frac{\partial^2}{\partial z^2} f_R + \left(p^2 - \mu^2 - 2\frac{\mu c}{z} - \frac{c(c-1)}{z^2} \right) f_R = 0$$

$$\frac{\partial^2}{\partial z^2} f_L + \left(p^2 - \mu^2 - 2\frac{\mu c}{z} - \frac{c(c+1)}{z^2} \right) f_L = 0$$

- solutions are first kind and second kind of Whittaker Functions.
- zero mode profiles are $f_L^0(z) \sim e^{-\mu z} z^{-c}$ and $f_R^0(z) \sim e^{\mu z} z^c$.
- when $z \to \infty$, the coefficients of the second term goes to $(p^2 - \mu^2)$, $\Rightarrow$ a continuum with mass gap.

the 5D profile functions

their solutions can be expressed as:

$$
\begin{aligned}
f_L(p,z) \quad &= a \cdot M(-\frac{c\mu}{\sqrt{\mu^2-p^2}}, \frac{1}{2}+c, 2\sqrt{\mu^2-p^2}z) \\
&+ b \cdot W(-\frac{c\mu}{\sqrt{\mu^2-p^2}}, \frac{1}{2}+c, 2\sqrt{\mu^2-p^2}z) \\
f_R(p,z) \quad &= a \cdot \frac{2(1+2c)\sqrt{\mu^2-p^2}}{p} M(-\frac{c\mu}{\sqrt{\mu^2-p^2}}, -\frac{1}{2}+c, 2\sqrt{\mu^2-p^2}z) \\
&+ b \cdot \frac{p}{(\mu+\sqrt{\mu^2-p^2})} W(-\frac{c\mu}{\sqrt{\mu^2-p^2}}, -\frac{1}{2}+c, 2\sqrt{\mu^2-p^2}z) \\
&\kappa \equiv -\frac{c\,\mu}{\sqrt{\mu^2-p^2}} \ ,
\end{aligned}
$$

M is the first kind Whittaker Function and W is the second kind Whittaker Function. a and b are determined by boundary conditions.

gauge fields in the bulk

A 5D $\mathcal{N} = 1$ vector supermultiplet can be decompose into a 4D $\mathcal{N} = 1$ vector supermultiplet $V = (A_\mu, \lambda_1, D)$ and a 4D $\mathcal{N} = 1$ chiral supermultiplet $\chi = ((\Sigma + iA_5)/\sqrt{2}, \lambda_2, F_\chi)$.

$$
\begin{aligned}
S_V &= \int d^4x dz \cdot \frac{R}{z}\frac{1}{4}\int d^2\theta W_\alpha W^\alpha \Phi + h.c. \\
&+ \int d^4x dz \cdot \frac{R}{z}\frac{1}{2}\int d^4\theta(\partial_z V - \frac{R}{z}\frac{(\chi + \chi^\dagger)}{\sqrt{2}})^2(\Phi + \Phi^\dagger)
\end{aligned}
$$

dilaton field can gain a VEV $\langle\Phi\rangle = e^{-2uz}/g_5^2$, which will generate a mass gap for the continuum mode.

adding gauge fixing term (unitary gauge):

$$S_{GF} = -\int d^5x \frac{R}{z} \cdot \frac{e^{-2uz}}{g_5^2} \frac{1}{2} \left(\partial_\mu A^\mu + \frac{z}{R} \partial_z (\frac{R}{z} A_5) + A_5 \partial_z (\ln \Phi) \right)^2$$

by analogy with the matter fields, for the gauge fields, we have:

$$\begin{aligned} \lambda_1(p,z) &= \chi_4(p) e^{uz} \left(\frac{z}{z_{UV}} \right)^2 h_L \quad A_\mu(p,z) = A_{\mu 4}(p) e^{uz} \left(\frac{z}{z_{UV}} \right)^{1/2} h_L \\ \lambda_2(p,z) &= \psi_4(p) e^{uz} \left(\frac{z}{z_{UV}} \right)^2 h_R \quad \Sigma = \phi_4(p) e^{uz} \left(\frac{z}{z_{UV}} \right)^{3/2} h_R \end{aligned}$$

- h_L and h_R are f_L and f_R evaluated at $c = 1/2$.
- zero mode profile for the gauge boson is constant $\Rightarrow$ zero mode gauge boson coupling to matter fields are universal.

5D scalar propagator

The propagator for the left handed scalar field satisfies the following homogeneous equations in momentum-position space:

$$\left(\partial_z^2 - \frac{3}{z}\partial_z + (c^2 + c - \frac{15}{4})\frac{1}{z^2} + \frac{2c\mu}{z} + (\mu^2 - p^2)\right) P(p, z, z')$$

$$= \left(\frac{z_{UV}}{z}\right)^{-3} i\delta(z - z')$$

- in the region of $z < z'$ and $z > z'$, the solutions are Whittaker functions.
- propagator in the two region will be matched at $z = z'$.
- we prefer to choose another two independent solutions $K(p, z)$ and $S(p, z)$, which are linear combinations of Whittaker functions. This definition can be generalized to warped space with generic metric.

two independent solutions

$$
K(p,z) = \left(\frac{z}{z_{UV}}\right)^{3/2} \frac{W(\kappa,\frac{1}{2}+c,2\sqrt{\mu^2-p^2}\,z)}{W(\kappa,\frac{1}{2}+c,2\sqrt{\mu^2-p^2}\,z_{UV})}
$$

$$
\begin{aligned}
S(p,z) = {} & \left(\frac{z}{z_{UV}}\right)^{3/2} \frac{1}{2\sqrt{\mu^2-p^2}} \frac{\Gamma(1+c-\kappa)}{\Gamma(2+2c)} \\
& \Big(M(\kappa,\tfrac{1}{2}+c,2\sqrt{\mu^2-p^2}\,z)\; W(\kappa,\tfrac{1}{2}+c,2\sqrt{\mu^2-p^2}\,z_{UV}) \\
& -W(\kappa,\tfrac{1}{2}+c,2\sqrt{\mu^2-p^2}\,z)\; M(\kappa,\tfrac{1}{2}+c,2\sqrt{\mu^2-p^2}\,z_{UV})\Big)
\end{aligned}
$$

$$
\kappa \equiv -\frac{c\,\mu}{\sqrt{\mu^2-p^2}}\ ,
$$

these two functions satisfying the following boundary conditions ($K(p,z)$ damping in the z region):

$$
K(p,z_{UV}) = 1; \qquad S(p,z_{UV}) = 0 \;\text{ and }\; S'(p,z_{UV}) = 1
$$

boundary condition for propagator

- for the solution$P_<(p, z, z')$ in the $z < z'$ region, we can impose UV boundary condition, at the $z = z_{UV}$ brane:

$$\left(\partial_z + \frac{1}{z}(-\frac{3}{2} + c + \mu z)\right) P_<(p, z, z')\Big|_{z=z_{UV}} = 0$$

- for the solution $P_>(p, z, z')$ in the $z > z'$ region, we require the propagator exponentially damping for large Euclidean momenta and large z , so that it can only contain the function $K(p, z)$.

- at the point of $z = z'$, $P_<(p, z, z')$ and $P_>(p, z, z')$ can be matched by the following two connection conditions.

$$P_<(p, z, z') - P_>(p, z, z')|_{z=z'} = 0$$
$$\partial_z P_<(p, z, z') - \partial_z P_>(p, z, z')|_{z=z'} = i\left(\frac{z_{UV}}{z}\right)^{-3}$$

expression for propagator

In the basis of $K(p,z)$ and $S(p,z)$, our scalar propagator can be expressed as, for $z < z'$:

$$P(p,z,z') = \frac{K(p,z)K(p,z')}{\Pi(p)} - S(p,z)K(p,z')$$

for $z > z'$, exchange the position of z and z'. The second term in the propagator will vanish on the UV brane.

expression for $\Pi(p)$ is also concise in this basis:

$$\Pi(p) = \frac{p^2}{(\mu + \sqrt{\mu^2 - p^2})} \cdot \frac{W\left(-\frac{c\mu}{\sqrt{-p^2+\mu^2}}, \frac{1}{2} - c, 2\sqrt{-p^2+\mu^2}\; z_{UV}\right)}{W\left(-\frac{c\mu}{\sqrt{-p^2+\mu^2}}, \frac{1}{2} + c, 2\sqrt{-p^2+\mu^2}\; z_{UV}\right)}$$

interesting phenomenology

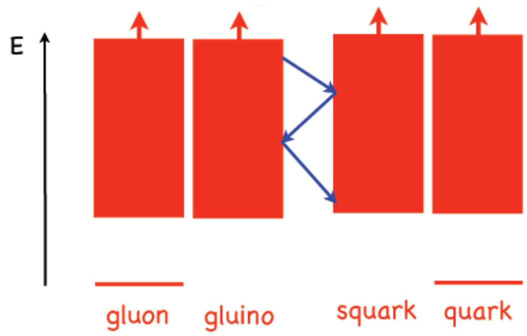

Figure 1: possible extended decay chain with continuum spectrum

neutrilino decay in soft wall model

we are considering one decay chain similar to Figure 2, neutrilino decays into selectron then decays back into neutrilino.

p_1 is the four momentum for χ_1, k_1 is the four momentum for χ_2, k_2 is the four momentum for e^-, and k_3 is the four momentum for e^+. $q = p_1 - k_2$ and m is the mass for the selectron.

when the intermediate selectron is an single particle, we can use the narrow width approximation:

$$\frac{1}{q^2 - m^2 + im\ \Gamma_{total}} = P(\frac{1}{q^2 - m^2}) + i\pi\delta(q^2 - m^2)$$

if $\Gamma_{total} \ll m$, the imaginary part of the propagator gives the main contribution and on shell decaying dominates.

when the intermediate mode is a continuum spectrum, situation may become different:

- Putting an IR brane in the soft wall model, sothat the continuum mode will become quasi continuum. The splitting between the KK modes should be less than the mass gap.

- we need to integrate the overlapping of neutrilino wavefunctions and the selectron propagator to get the vertex.

$$
\begin{aligned}
v(p_1, q, k_1) &= N_e^2 N_{\chi_1} N_{\chi_2} \int_{z_{UV}}^{z_{IR}} dz \int_{z_{UV}}^{z_{IR}} dz' e^{(u-\mu)z} z^{1/2-c} h_L(p_1, z) \\
&\cdot e^{(u-\mu)z'} z'^{1/2-c} h_L(k_1, z') \cdot P(q, z, z')
\end{aligned}
$$

- we can calculate the differential decay rate with respect to the first electron energy for the three body neutrilino decay:

$$
\frac{d\Gamma}{dE_2} = e^4 \ |v(p_1, q, k_1)|^2 \frac{E_2^2(2E_2\sqrt{p_1^2} - p_1^2 + k_1^2)}{32(2E_2 - \sqrt{p_1^2})\sqrt{p_1^2}\pi^3}
$$

- fixing the mass of initial neutrilino, and summing over all the finial neutrilino states, from the mass gap μ to the initial neutrilino mass $\sqrt{p_1^2}$. here is an example:

  selectron: $c = 0.5$, and $\mu = 0.4$ TeV.

  neutrilino: $u = 0.2$ TeV.

  Initial neutrilino mass: 1.59 TeV

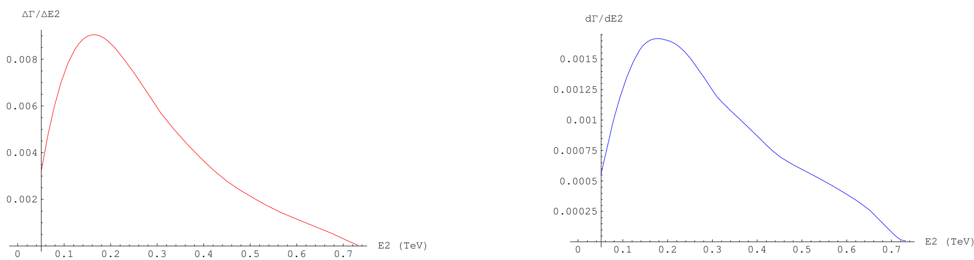

Figure 1: $z_{UV} = 10^{-3}$ TeV^{-1}, $z_{IR} = 30$ TeV^{-1}. Left one (red one) is for two body decay and right one (blue one) is for three body decay

conclusion

- the two body decay rate is peaked at small electron energy. neutrilino prefers to decay into selectron whose mass is close to it. Reducing the mass of selectron increases the phase space but decreases the profile overlaping.

- for the three body decay, after summing over all the final states, the decay rate is also peaked at small energy.

- Extended decay chain is possible in continuum spectrum situation