TeV Physics and Conformality

UC Davis
September 27, 2010
1. Collaboration with George Fleming and Ethan Neil
   1) arXiv:0712.0609, PRL 100, 171607, 2008
   2) arXiv:0901.3766 PR D79, 076010, 2009

   2) arXiv: this week (David Schaich)

USQCD clusters, LLNL BG/L, NSF Teragrid
New Strong Dynamics at TeV Energies?

New, SM-singlet Sector
Conformal behavior

Electroweak breaking
Near-conformal infrared behavior: walking technicolor, conformal technicolor (M. Luty et al)

Other possible uses of (near) conformal-symmetry
Flavor hierarchies (Nelson&Strassler), SUSY flavor problem
Conformal Symmetry / Scale Invariance / Dilatation Symmetry

No fixed scales (masses)

Might expect at high energies or temperatures

Examples \((h/2\pi = c = 1)\):

\[\sigma (e^+e^- \rightarrow \text{Hadrons}) \sim \frac{1}{E_{\text{cm}}^2}\]

\[F(T) \sim T^4\]
Quantum field theories *typically* depend on a renormalization scale $\Lambda$. 

$$\alpha \equiv g^2/4\pi = \alpha(q^2 / \Lambda^2)$$

For a Yang-Mills theory

$$q^2 \to \infty \quad \sim 1/ \log(q^2 / \Lambda^2)$$

Asymptotic freedom
QCD

\begin{align*}
\alpha_s(Q) &= \frac{1}{s^2} \\
\text{Data} &\quad \text{Theory} \\
\text{Deep Inelastic Scattering} &\quad \text{NLO} \\
\text{e^+e^- Annihilation} &\quad \text{NNLO} \\
\text{Hadron Collisions} &\quad \text{Lattice} \\
\text{Heavy Quarkonia} &\quad \text{Lattice}
\end{align*}

\begin{align*}
\Lambda_{\overline{MS}}^{(5)} &\quad \alpha_s(M_Z) \\
\text{QCD} &\quad 251 \text{ MeV} \quad 0.1215 \\
O(\alpha_s^4) &\quad 213 \text{ MeV} \quad 0.1184 \\
&\quad 178 \text{ MeV} \quad 0.1153
\end{align*}
QCD Infrared Features

1. Quark and gluon confinement
   \[ \Lambda \sim 200 \text{ MeV} \]

2. Spontaneous chiral symmetry breaking
   \[ \text{SU}(2)_L \times \text{SU}(2)_R \rightarrow \text{SU}(2) \]
   3 Pions = PNGB’s
   \[ F = 93 \text{ MeV} \]
   \[ \langle \bar{\psi} \psi \rangle \sim 4 \pi F^3 \]
TeV-Scale Theories

• SU(N) Gauge Theories with $N_f$ Massless Fermions (fundamental and other representations)

• Asymptotically-free (can take lattice spacing to zero) $N_f < N_{af}$

• Vary $N_f$ and study how the infrared behavior changes (Chiral symmetry breaking and confinement versus infrared conformal behavior.)

SUSY SU(N) Theories: Strange and interesting possibilities
Conformal Window

$N_{af} > N_f > N_{fc}$ (Fermion screening)

Perturbative for $N_f \rightarrow N_{af}$

Gross and Wilczek, antiquity

William Caswell, 1974  

Banks and Zaks, 1982

$\alpha^*$ increases as $N_f$ decreases.

$N_{fc}$: $\alpha^*$ becomes large enough to trigger chiral symmetry breaking and confinement.

(Probably not accessible in perturbation theory)
NEAR-CONFORMAL BEHAVIOR
For \( N_f \) just below \( N_{f_c} \) (Walking)

\[ \beta(\mu) \sim \text{IRFP} \]

\[ \alpha^*(\mu) \]

\[ \alpha_c \]

\[ \Lambda_{\text{conf}} \approx 250 \text{ GeV} \]

in TC
Questions

1. Value of $N_{fc}$?

2. Order of the phase transition?

2. Correlation functions and anomalous dimensions inside the conformal window ($N_f > N_{fc}$)

3. Below and near the transition ($N_f < N_{fc}$)

   Approximate IRFP (Walking)? Condensate enhancement? Parity Doubling? EW precision studies? Dilaton? Implications for the LHC?
Topics

(1) $N_{fc}$ in SU(N) QCD

→ Walking

(2) Dilaton (1 Slide)

(3) Chiral Symmetry Breaking and Condensate Enhancement

(4) Parity Doubling and EW Precision Studies
(1) $N_{fc}$ in SU(N) QCD

- **Degree-of-Freedom Inequality** (Cohen, Schmaltz, TA 1999). Fundamental rep:
  \[ N_{fc} < 4N[1 - 1/18N^2 + \ldots] \]
  \[ \leftarrow \text{Conjecture} \]

- **Gap-Equation Studies, Instantons** (1996 ….):
  \[ N_{fc} \approx 4N \]
  \[ \text{Pioneers:} \]
  Maskawa & Nakajima 1974
  Kugo & Fukuda 1975
  \[ \leftarrow \text{Suspect methods} \]
Lattice-Simulation Study of the Extent of the Conformal window in an SU(3) Gauge Theory with $N_f$ Fermions in the Fundamental Representation

Conformality violated by $a, L$ !!

Earlier, inconclusive lattice work

Fleming, Neil, TA
2007 …….
Focus: Gauge Invariant and Non-Perturbative Definition of the Running Coupling from the Schroedinger Functional of the Gauge Theory

ALPHA Collaboration: Luscher, Sommer, Weisz, Wolff, Bode, Heitger, Simma, …

Transition amplitude from a prescribed state at $t=0$ to one at $t=T = L \pm a$ (Dirichlet BC). ($m = 0$)
Using Staggered Fermions as in

Miyazaki & Kikukawa

Focus on $N_f = $ multiples of 4:

16: Perturbative IRFP
12: IRFP “expected”, Simulate
  8 : IRFP uncertain , Simulate
  4 : Confinement, ChSB
SF Running Coupling

\[ \frac{1}{\bar{g}^2(L)} = \frac{1}{2} \left[ \frac{1}{\bar{g}^2(L, L-a)} + \frac{1}{\bar{g}^2(L, L+a)} \right] \]

One Large Scale!

In perturbation theory:

- \( N_f = 16 \) IRFP at \( g_{SF}^* = 0.47 \) \( (g_{SF}^*/4\pi \approx 0.04) \)
- \( N_f = 12 \) IRFP at \( g_{SF}^* = 5.18 \) \( (g_{SF}^*/4\pi \approx 0.4) \)
- \( N_f \leq 8 \) No perturbative IRFP
Lattice Simulations

MILC Code (Urs Heller)
Staggered Fermions

$N_f = 8, 12$

Range of Lattice Couplings $g_0^2 (= 6/\beta)$ and Lattice Sizes $L/a \rightarrow 20$

O(a) Lattice Artifacts due to Dirichlet Boundary Conditions
$N_f = 8$ Data with Fits
$N_f = 8$ Continuum Running
$N_f = 12$ Data with Fits
$N_f = 12$ Continuum Running
Conclusions

1. Lattice evidence that for an SU(3) gauge theory with $N_f$ Dirac fermions in the fundamental representation $8 < N_{fc} < 12$

2. $N_f=12$: Relatively weak IRFP


Employing the Schroedinger-functional running coupling defined at the box boundary $L$
Summary of lattice results at $N_f = 12$ (3-color fundamental)

<table>
<thead>
<tr>
<th>Group</th>
<th>Fermion action</th>
<th>Gauge action</th>
<th>Method</th>
<th>Conformal?</th>
<th>Refs (arXiv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fleming, Neil, TA</td>
<td>staggered</td>
<td>unimproved (plaquette)</td>
<td>SF coupling</td>
<td>Yes</td>
<td>0712.0609 0901.3766</td>
</tr>
<tr>
<td>Jin &amp; Mawhinney</td>
<td>staggered</td>
<td>improved (DBW2)</td>
<td>Spectrum</td>
<td>No</td>
<td>0910.3216</td>
</tr>
<tr>
<td>Hasenfratz</td>
<td>improved staggered</td>
<td>unimproved (plaquette)</td>
<td>MCRG</td>
<td>Yes</td>
<td>0911.0646 priv. comm.</td>
</tr>
<tr>
<td>Kuti, Holland, Fodor, et al.</td>
<td>improved staggered</td>
<td>improved (Symanzik)</td>
<td>Spectrum</td>
<td>No</td>
<td>0911.2463</td>
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<tr>
<td>Deuzeman, Lombardo, Pallante</td>
<td>improved staggered</td>
<td>improved (Symanzik)</td>
<td>Thermal trans.</td>
<td>Yes</td>
<td>0904.4662</td>
</tr>
</tbody>
</table>
Lots of Related Activity

1. Other $N_f$ values such as $N_f=10$.
   - Yamada et al

2. Other gauge groups and representation assignments for the fermions
   - Sannino, del Debbio, DeGrand, Shamir, Svetitsky; Hietenan et al,
     Sinclair, Kogut, Catterall. .......

3. Examine physical quantities such as the static potential (Wilson loop) and correlation functions.
   - Bilgici et al [arXiv:0902.3768]
Lattice is now a tool for studying strongly coupled gauge theories.

So far, consistent with a single $N_{fc}(N)$
(2) Dilaton

An (approximate) NGB (a PNGB) associated with the spontaneous breaking of (approximate) scale symmetry

\[ M_d^2 \sim (N_{f_c} - N_f)\Lambda_{conf}^2 \]

Yang Bai and TA

arXiv:1006.4375

Dilaton Phenomenology:
Goldberger, Grinstein, Skiba
PRL 2008
(3) Chiral Symmetry Breaking and Condensate Enhancement (LSD)

Walking:

As the conformal window is approached \((N_f \rightarrow N_{fc})\),
\(\langle \bar{\psi} \psi \rangle\) is enhanced relative to its nominal value \(4\pi F^3\).

LSD Program:

Search for enhancement of \(\langle \bar{\psi} \psi \rangle / F^3\) by starting at \(N_f = 2\),
and then \(N_f = 6\). (Creeping Toward the Conformal Window)
\((\Lambda = a^{-1})\)

arXiv:0910.2224
PRL 104, 071601 (2010)
WHY?

\[ \Lambda_i \]

\[ \langle \overline{U}_L U_R \rangle \]

\[ M^{(u)}_{ii} \approx 4\pi \frac{F^3}{\Lambda_i^{2}} \]

\[ \exp \left\{ \int \frac{d\mu}{\mu} \gamma[\alpha_{TC}(\mu)] \right\} \]

\[ d=3-\gamma \]
\[ \gamma \rightarrow 1 \, ? \]
\[ \gamma \rightarrow 2 \, ? \]
M. Luty
Some Details

- Domain-wall fermions, Iwasaki improved action
- USQCD: Chroma, CPS

- $32^3 \times 64$ lattice ($L_s = 16$)
- $m_f = 0.005, 0.01, 0.015, 0.02, 0.025$, $m = m_f + m_{\text{res}}$

- $N_f^2 - 1$ PNGB’s

- Simulate: $M_P$, $F$, $<\bar{\psi} \psi>$, $M_V$, $M_P L > 4$
Extrapolate to $m=0$ with Chiral Perturbation Theory

- $M^2_{Pm} = 2m \langle \psi \psi \rangle / F^2 \{1 + zm [\alpha_{M1} + (1/N_f) \log(zm)] + \ldots\} \quad z \equiv 2 \langle \bar{\psi} \psi \rangle / (4\pi)^2 F^4$
- $F_m = F \{1 + zm [\alpha_{F1} - (N_f/2) \log(zm)] + \ldots\}$
- $\langle \bar{\psi} \psi \rangle_m = \langle \bar{\psi} \psi \rangle \{1 + zm [\alpha_{C1} - ((N_f^2 - 1)/N_f) \log(zm)] + \ldots\}$

$M_{Vm} = M_V \{1 + \alpha_{R1} zm + \alpha_{R3/2} (zm)^{3/2} + \ldots\}$
$M_{Am} = M_A \{1 + \alpha_{A1} zm + \alpha_{A3/2} (zm)^{3/2} + \ldots\}$
$N_f = 2: \beta = 2.7 \quad N_f = 6: \beta = 2.1$
$R_m = \left[\frac{M_{p_m}^2}{2m_F m}\right]_{6f} / \left[\frac{M_{p_m}^2}{2m_F m}\right]_{2f}$
\( N_f = 2 \)

- Chiral perturbation theory extrapolation:

\[
\frac{\langle \bar{\psi} \psi \rangle}{F^3} = 47.1 \ (17.6)
\]

**QCD Experimental Value:** (renormalized to our lattice scheme - Aoki et al hep-lat/0206013)

\[
\frac{\langle \bar{\psi} \psi \rangle}{F^3} = 36.2 \ (6.5)
\]
$N_f = 6$

Linear Extrapolation $\rightarrow$

Very Conservative Lower Bound on $\frac{\langle \bar{\psi} \psi \rangle}{F^2}$

Very Conservative Upper Bound on $F$

Thus

$$\frac{\langle \bar{\psi} \psi \rangle}{F^3} \geq 60.0 \ (8.0)$$
(4) Resonance Spectrum and the S parameter (LSD)

Is $S$ naturally small as $N_f \rightarrow N_{fc}$ due to approximate parity doubling?

$$S = 4\pi \left(\frac{N_f}{2}\right) [\Pi'_{VV}(0) - \Pi'_{AA}(0)] - \Delta S_{SM}$$

$$= \int_0^{\infty} \frac{ds}{s} \left\{ \left(\frac{N_f}{2}\right) [\text{Im} \Pi_{VV}(s) - \text{Im} \Pi_{AA}(s)] - \frac{1}{12\pi} \left[ 1 - \left(1 - \frac{m_{H,\text{ref}}}{s} \right)^3 \right] \theta(s - m_{H,\text{ref}}^2) \right\}$$

Peskin and Takeuchi
~Same Details

- Domain-wall fermions, Iwasaki improved action
- USQCD: Chroma, CPS

- $32^3 \times 64$ lattice ($L_s = 16$)
- $m_f = .005, .01, .015, .02, .025$, $m = m_f + m_{\text{res}}$

\[ M_p L > 4 \]
Vector and Axial-Vector Masses

![Graph showing Vector and Axial-Vector Masses](image)
$S_{\text{free}} = \frac{1}{2\pi}(N/3)(N_f/2)$  \hspace{1cm}  $S_{\text{QCD}} = 0.3 (N/3)(N_f/2)$
S Parameter

Nf = 2: S is smooth

Extrapolation:

Nf = 6: \( S \sim \frac{1}{12\pi} [N_f^2/4 - 1] \log \left(\frac{1}{m}\right) \)

Cut off by PNGB masses

DelDebbio et al
arXiv:0909.4931
Shintani et al
arXiv:0806.4222

3 EW doublets
Features

When $N_f$ is increased from 2 to 6:

1. The lightest vector and axial states become more parity doubled.

2. The $S$ parameter per electroweak doublet decreases
   (In the chiral limit $m \rightarrow 0$, the full answer will depend logarithmically on PNGB masses.)

3. Single pole dominance ($S = 4\pi [ \frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2} ]$) works to within 20% at $N_f = 2$ and at least as well at $N_f = 6$, showing the relative decrease of $S$ per electroweak doublet.

4. Not true of the WSR’s ($F_V^2 - F_A^2 = F_P^2$, …… )
Summary

• Lattice simulations are beginning to teach us about novel features of strongly coupled gauge theories other than QCD.

• So far, lots of focus on the transition toward infrared conformal behavior. Relevant to real world?
  Condensate enhancement, tendency to parity doubling, decreased S parameter, ........

Next
1. Push on toward $N_{fc}$
2. Jump into the conformal window ($N_f > N_{fc}$) (mass-deformed CFT)
3. → SUSY Theories
   Check methods
   Other applications