Viscosity, quark gluon plasma, and string theory

D. T. Son

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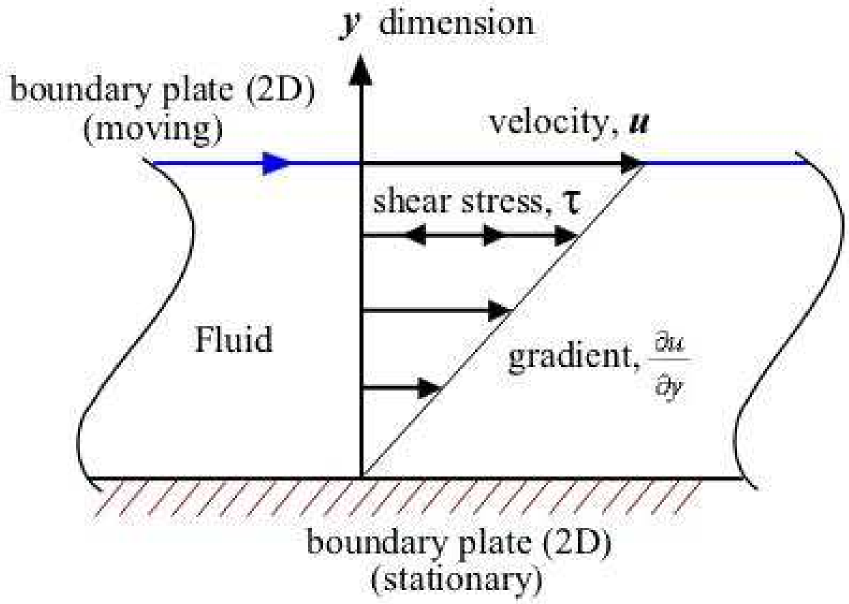
Plan

- Viscosity
- Heavy ion collisions and the quark gluon plasma
- Gauge/gravity duality: a surpising by-product of string theory
- Applications of gauge/gravity duality

Viscosity

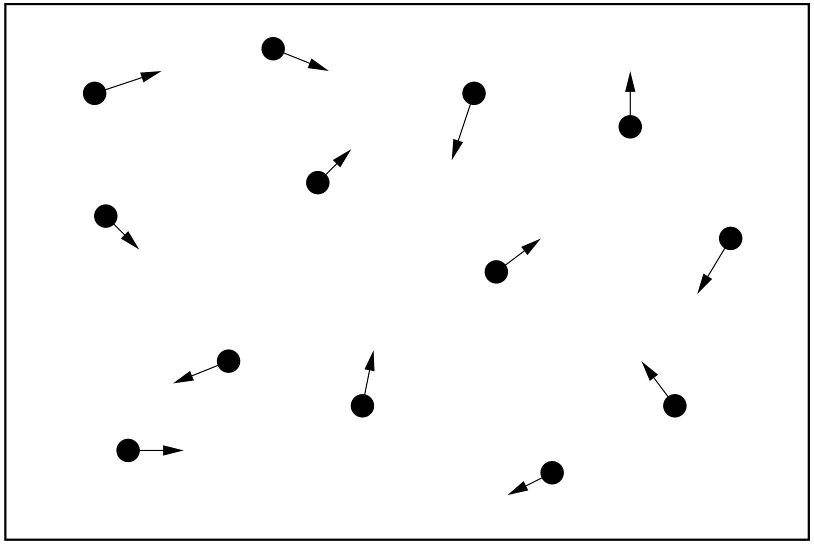
Viscosity: introduced by Claude Navier in 1822 into what would be later called the Navier-Stokes equation.



Friction force between two plates:

$$F = \eta A \frac{\partial u_x}{\partial y}$$

Viscosity and kinetic theory of gas



- Maxwell: in the kinetic theory of gases, viscosity is due to the collisions between gas molecules.

Maxwell's estimate of the viscosity:

$$\eta \sim \rho v \ell = \text{mass density} \times \text{velocity} \times \text{mean free path}$$

Consequence: at fixed temperature viscosity is independent of pressure (density).
Contradicts expectation at the time: the denser the gas, the larger the viscosity

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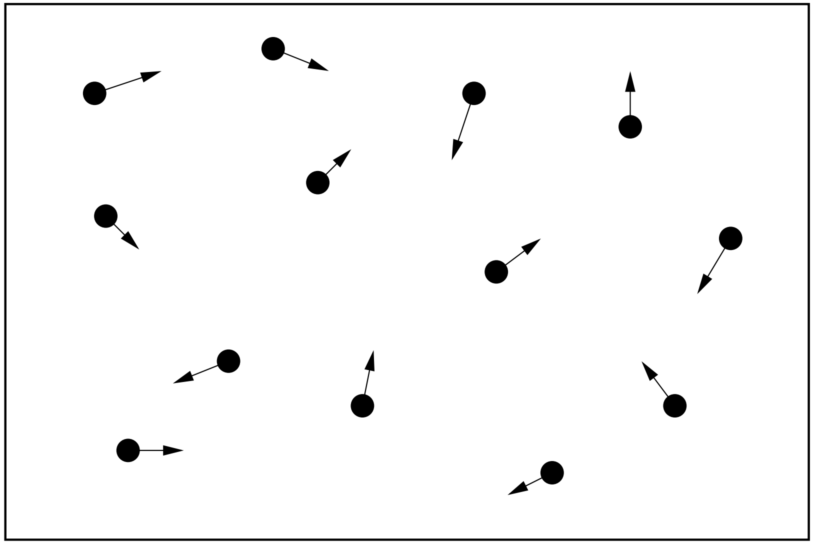

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"Such a consequence of a mathematical theory is very startling, and the only experiment I have met with on the subject does not seem to confirm it" (1860)

Maxwell’s own experiment

During the next few years Maxwell, with the help of his wife, designed and carried out his own expriment.

Reported result in 1865: viscosity of air is independent of pressure when the latter varies from about 1/60 to one atmosphere.

A counter-intuitive behavior

- Imagine one can turn off the interaction between molecules of a gas
- According to Maxwell's formula: viscosity $\to \infty$ (large mean free path)
- Shouldn't one expect no disspation, $\eta \to 0$?

Reason: two limits do not commute:

- The limit of infinitely weak interaction
- The hydrodynamic limit (lengths $\gg$ mean free path)

Viscosity is well defined only when size of experiment $\gg$ mean free path

Viscosity of liquids: a huge range

Viscosity of fluids is much poorer undertsood

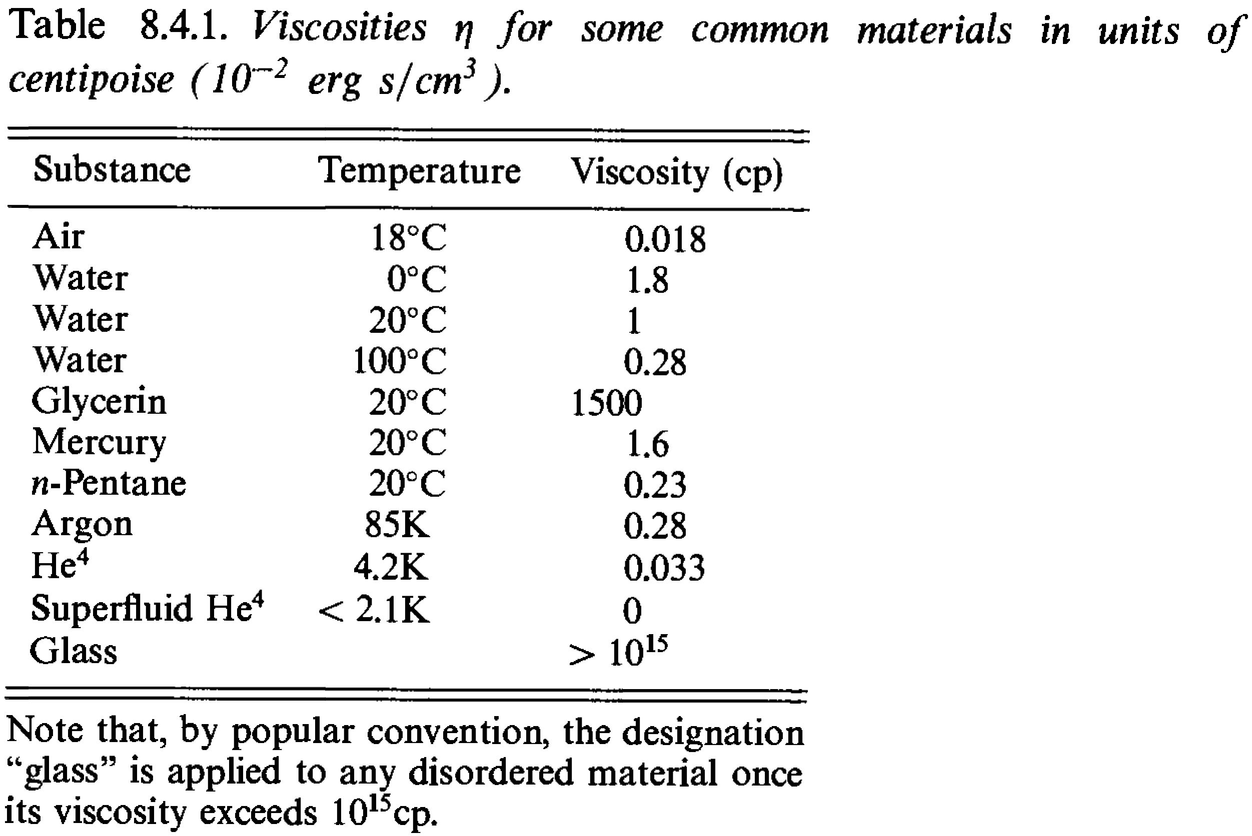

Table 8.4.1. *Viscosities η for some common materials in units of centipoise (10^{-2} erg s/cm^3).*

Substance	Temperature	Viscosity (cp)
Air	18°C	0.018
Water	0°C	1.8
Water	20°C	1
Water	100°C	0.28
Glycerin	20°C	1500
Mercury	20°C	1.6
n-Pentane	20°C	0.23
Argon	85K	0.28
He^4	4.2K	0.033
Superfluid He^4	< 2.1K	0
Glass		$> 10^{15}$

Note that, by popular convention, the designation "glass" is applied to any disordered material once its viscosity exceeds 10^{15}cp.

(from *Chaikin & Lubensky, Principles of Condensed Matter Physics*)

Viscosity spans many orders of magnitudes, despite the fact that the density does not change much.

Viscosity of liquid glycerin

T (C)	η (mPa s)
0	12070
10	3900
20	1410
30	612
40	284
50	142
60	81.3
70	50.6
80	31.9
100	14.8
120	7.8
140	4.7
167	2.8

Liquid viscosity can be very large

The pitch-drop experiment, University of Queensland

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- 2005 Ig Nobel Prize in Physics

Purcell and Weisskopf

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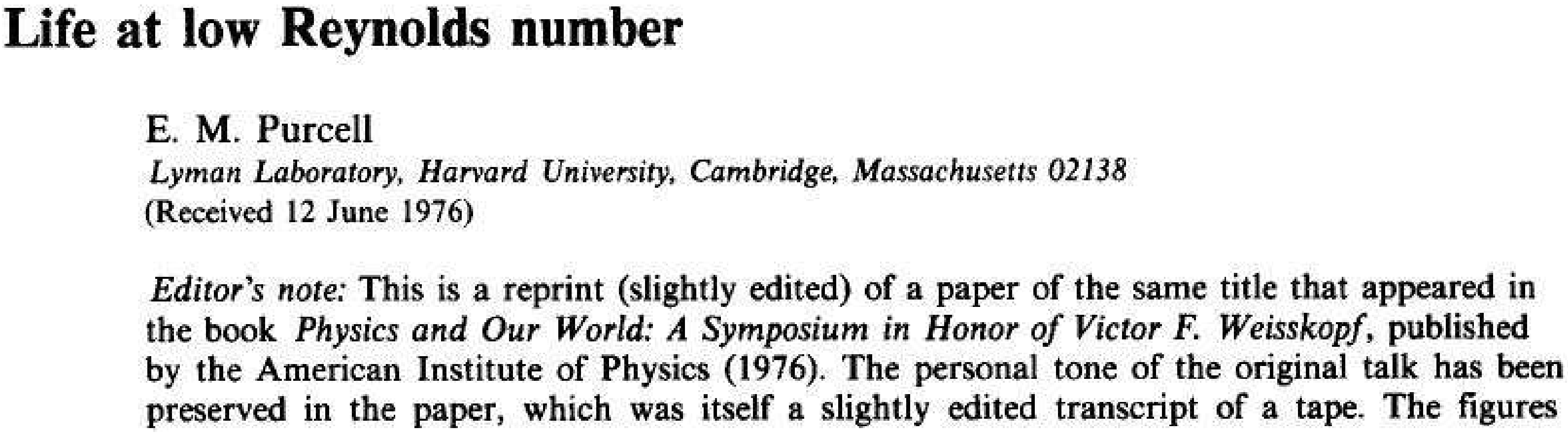

Life at low Reynolds number

E. M. Purcell
Lyman Laboratory, Harvard University, Cambridge, Massachusetts 02138
(Received 12 June 1976)

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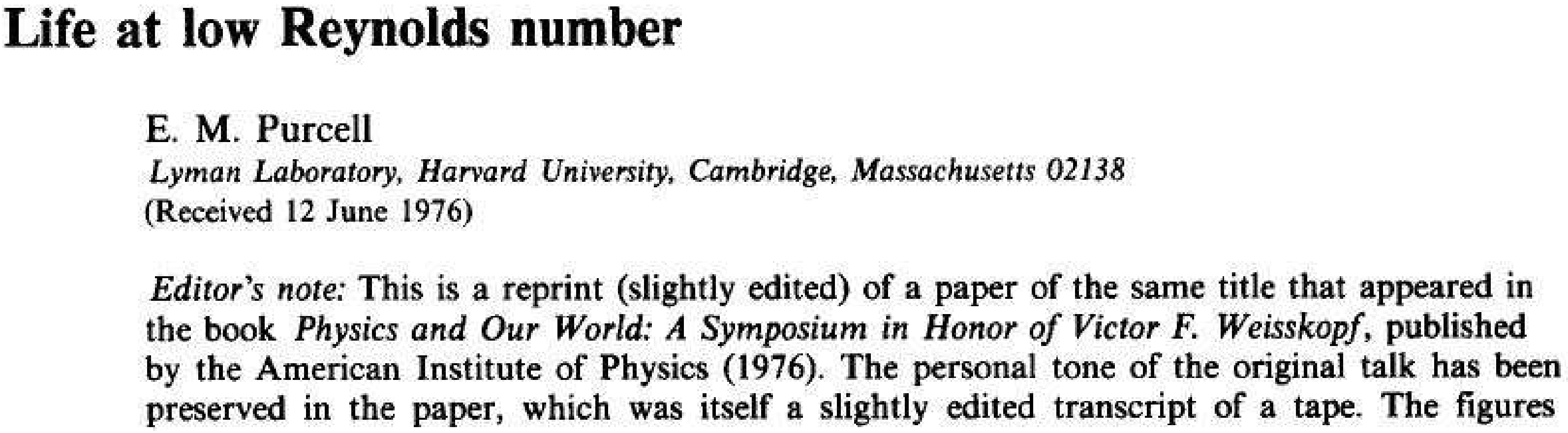

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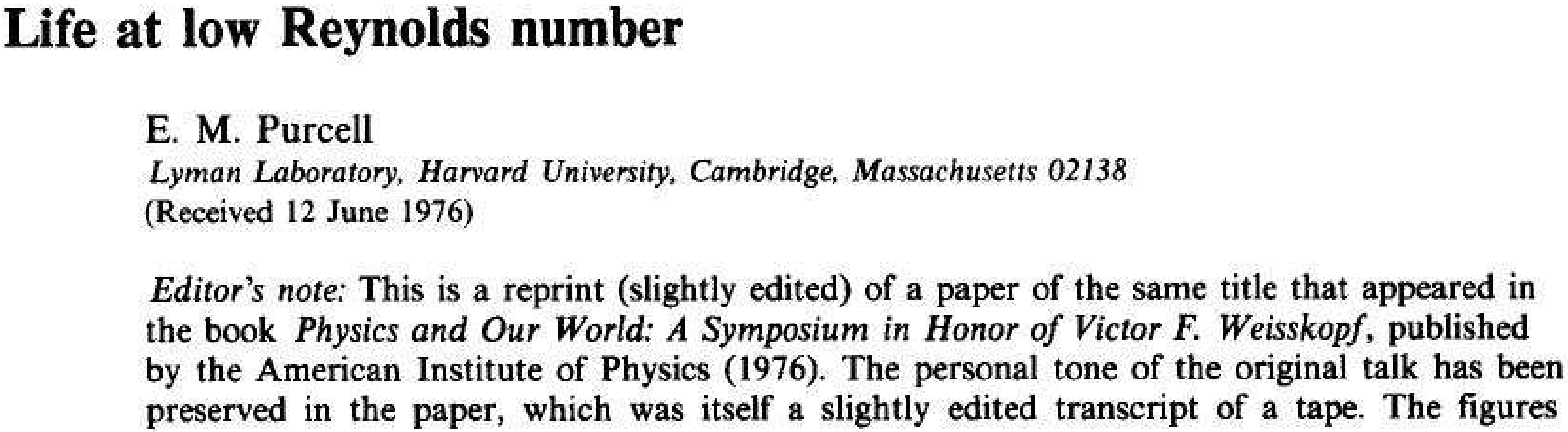

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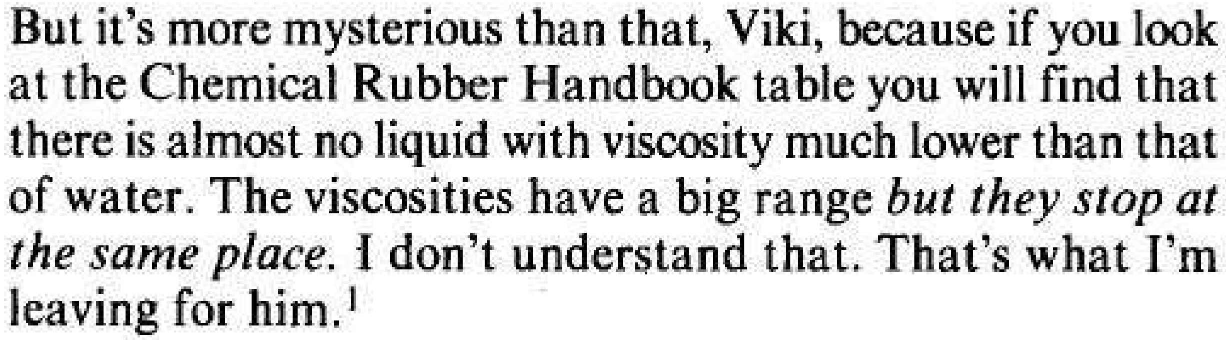
$$\eta \sim \exp(\#)$$

- So he added

Purcell's question



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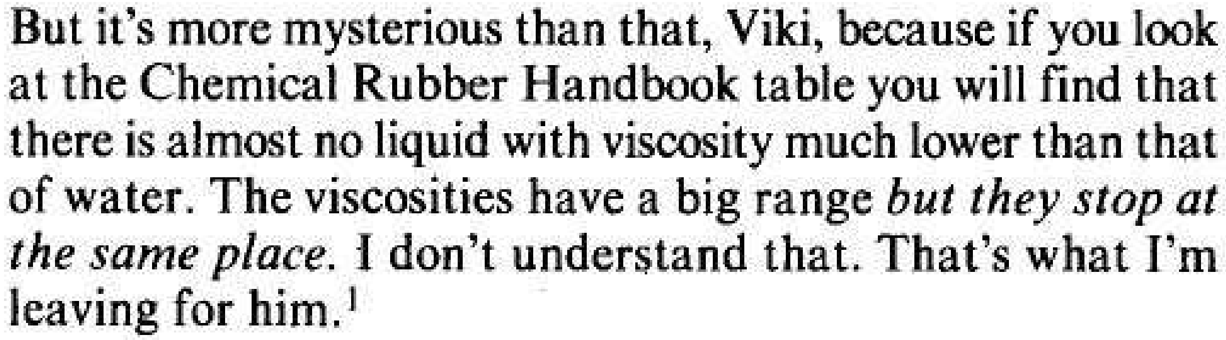
Purcell’s question



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Viscosity of liquids is unbounded from above, but seems to be bounded from below

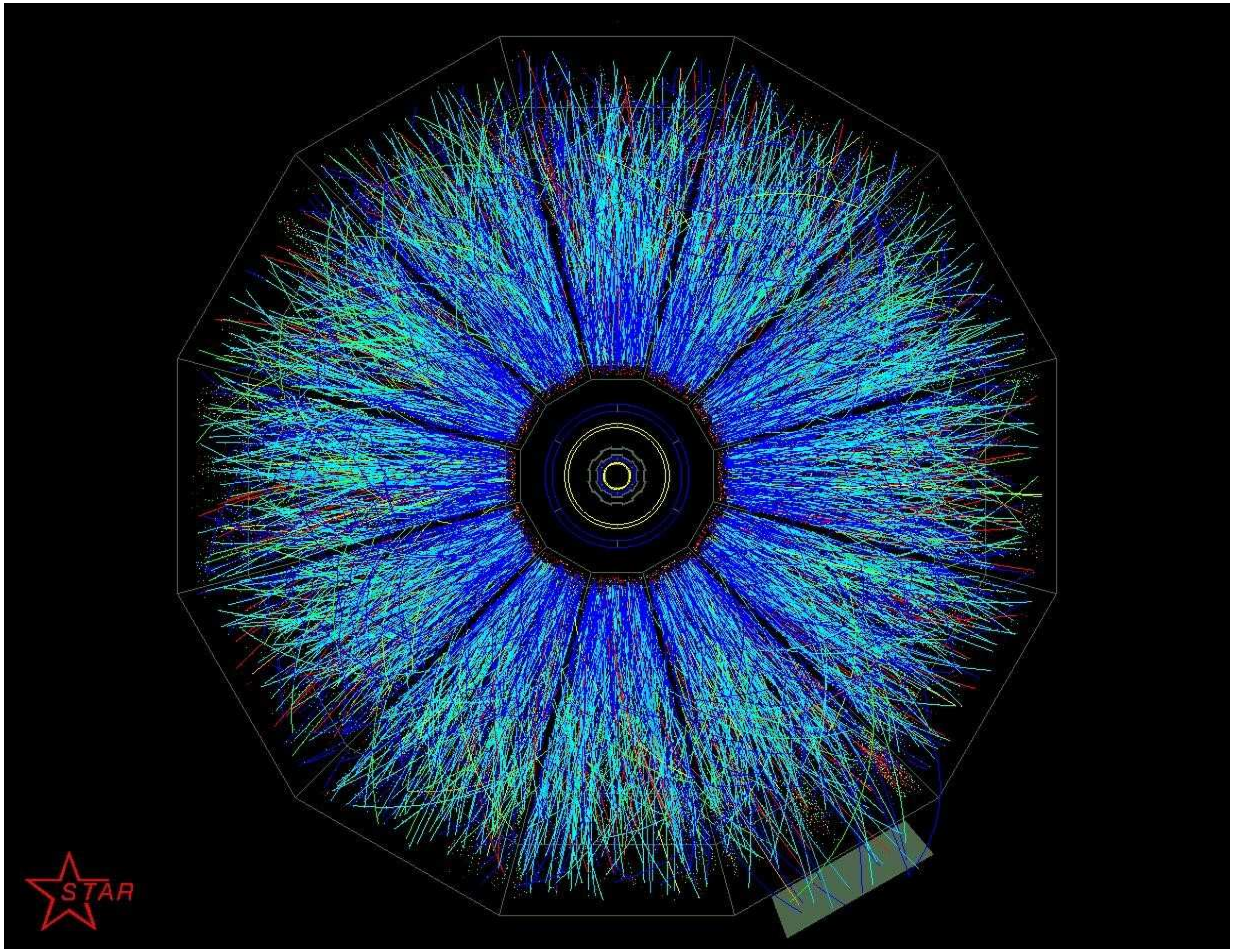
A version of Purcell’s question: By playing with the interaction strength, can one make $\eta \rightarrow 0$?

Modern context: the quark gluon plasma

Heavy ion collisions

- Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Lab
- Au+Au, 200 GeV/pair of nucleons

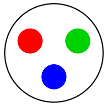
A typical heavy ion collision



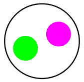
Goal: to create and study the quark gluon plasma

The quark gluon plasma

Hadrons consists of quarks (3 quarks, or 1 quark and 1 antiquark)



protons, neutrons



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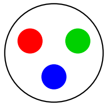

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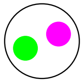

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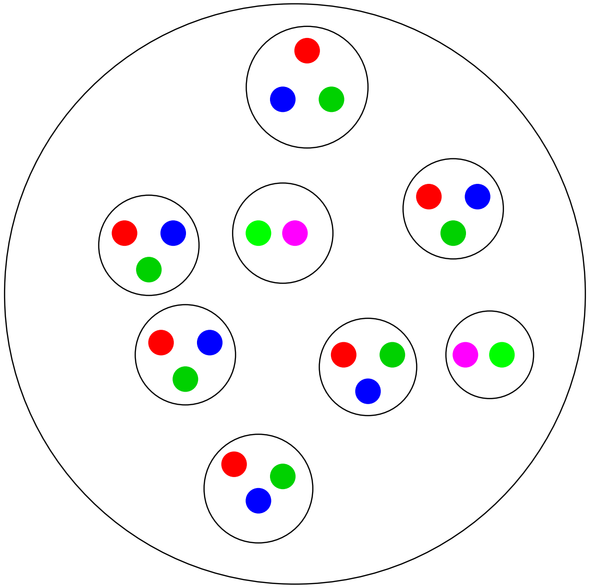
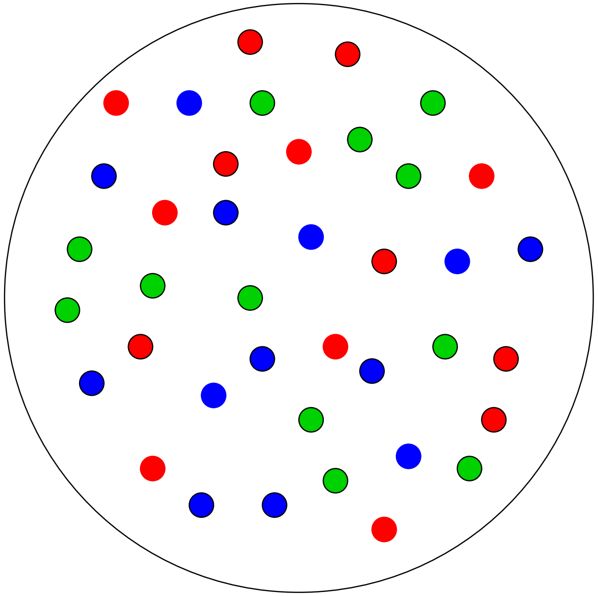


hadron gas

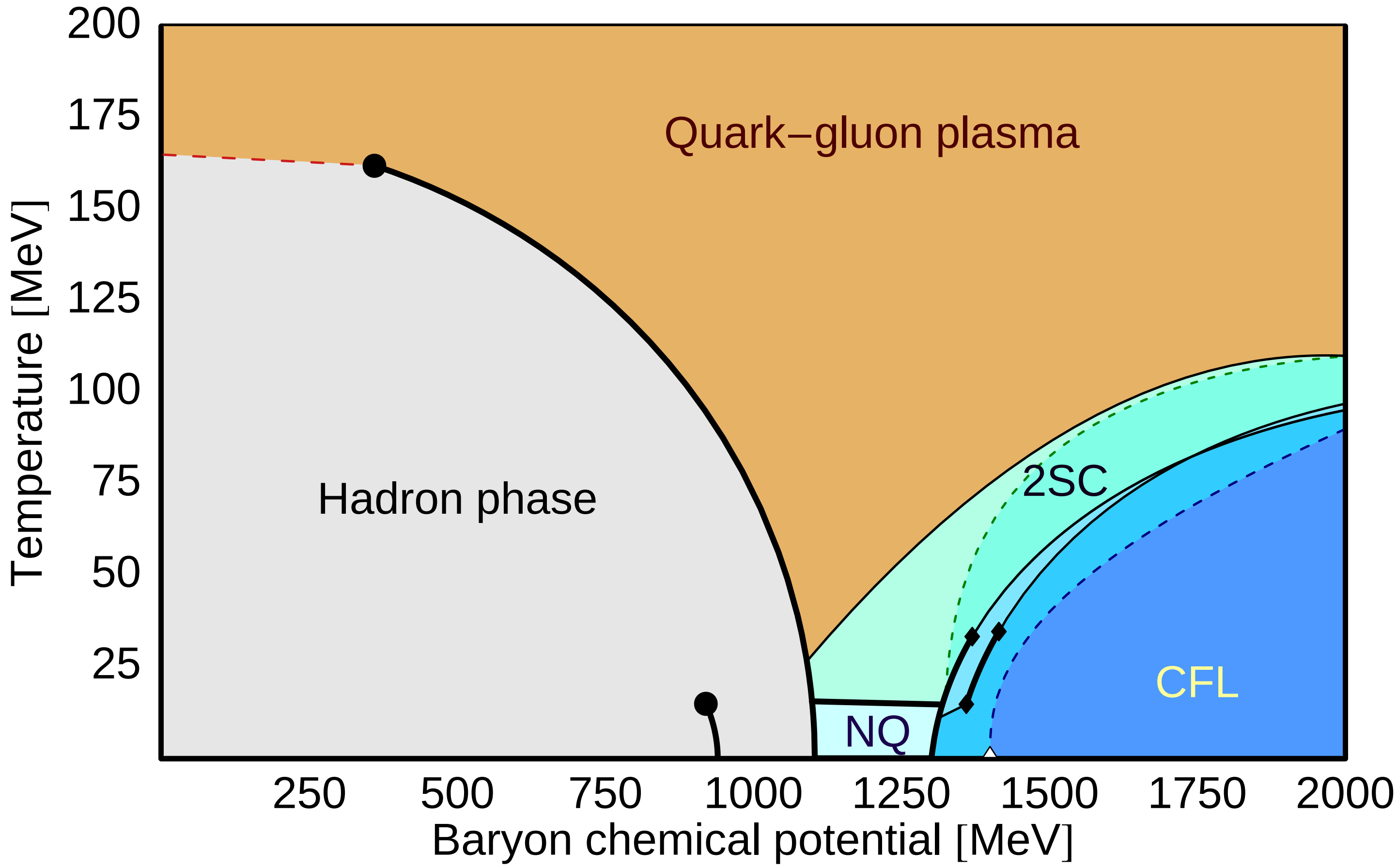


quark−gluon plasma

Quark-gluon plasma: quarks and gluons move relatively freely

Phases of QCD

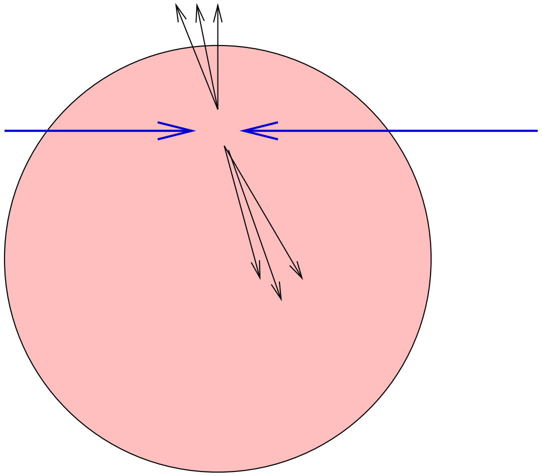
One version of the QCD phase diagram:



(Ruester et al., hep-ph/0503184)

Strongly interacting QGP

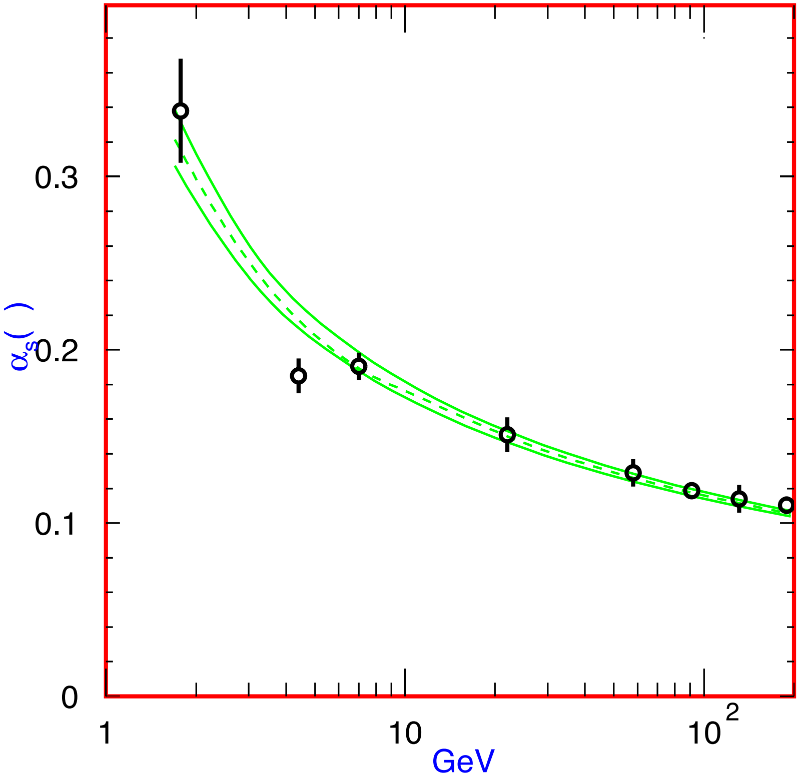
- Unambiguous identification of QGP difficult
- However, RHIC has created some strongly interacting medium
  - Supression of back-to-back jet correlations:



  - “Elliptic flow”: signals that particles push each other (more later)

Can the viscosity be calculated?

- Good: fundamental theory known: Quantum Chromodynamics (QCD)
- Bad: perturbation theory does not work

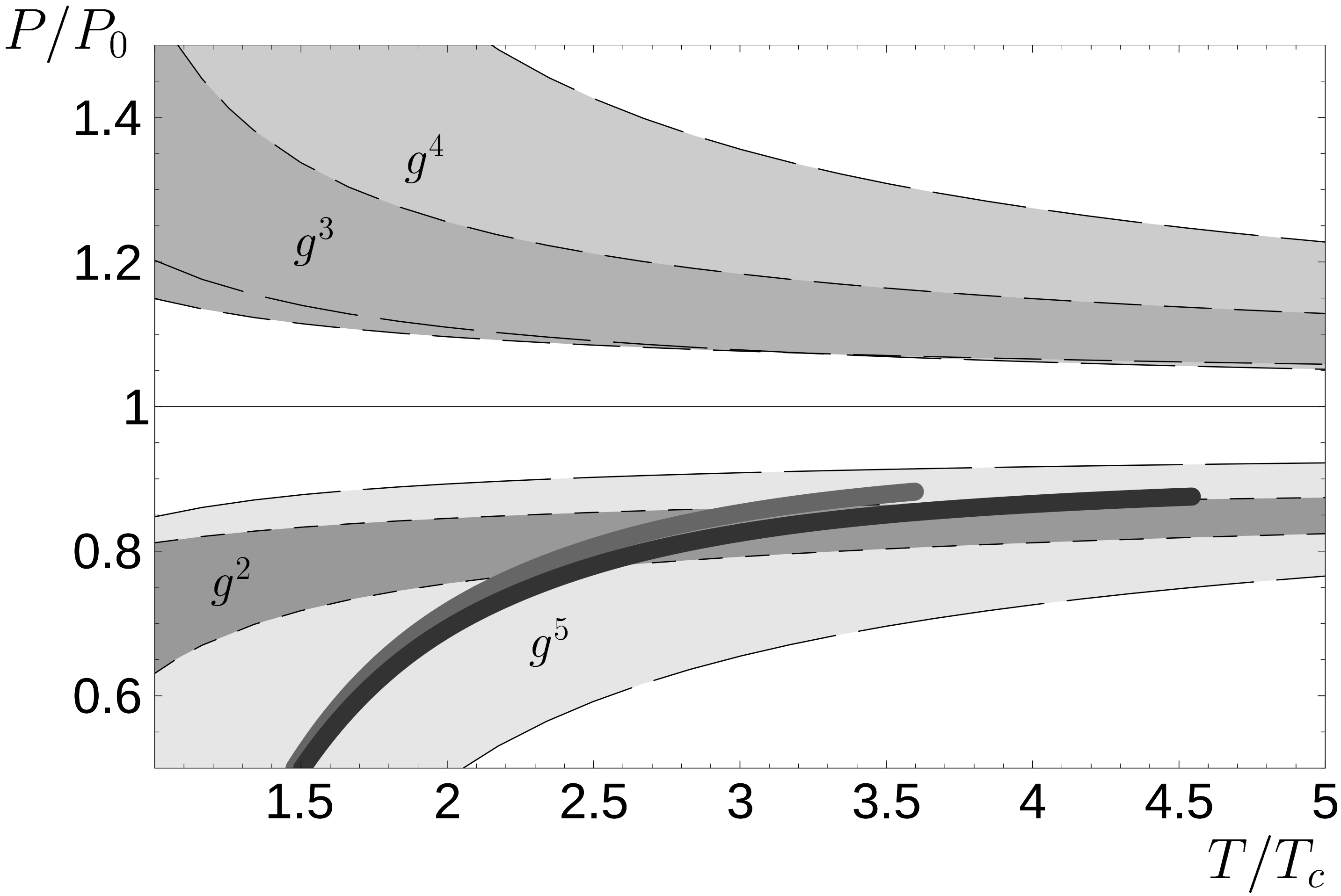


0.2

↑

We are here!

Convergence of perturbation series



$$P/P_0|_{T=2T_c} = 1 - 0.2 + 0.4 + 0.2 - 0.8 + \cdots$$

Kinetic coefficents: worse convergence expected

So what to do?

- Measure the viscosity experimentally, or
- Try a different (simpler theory)

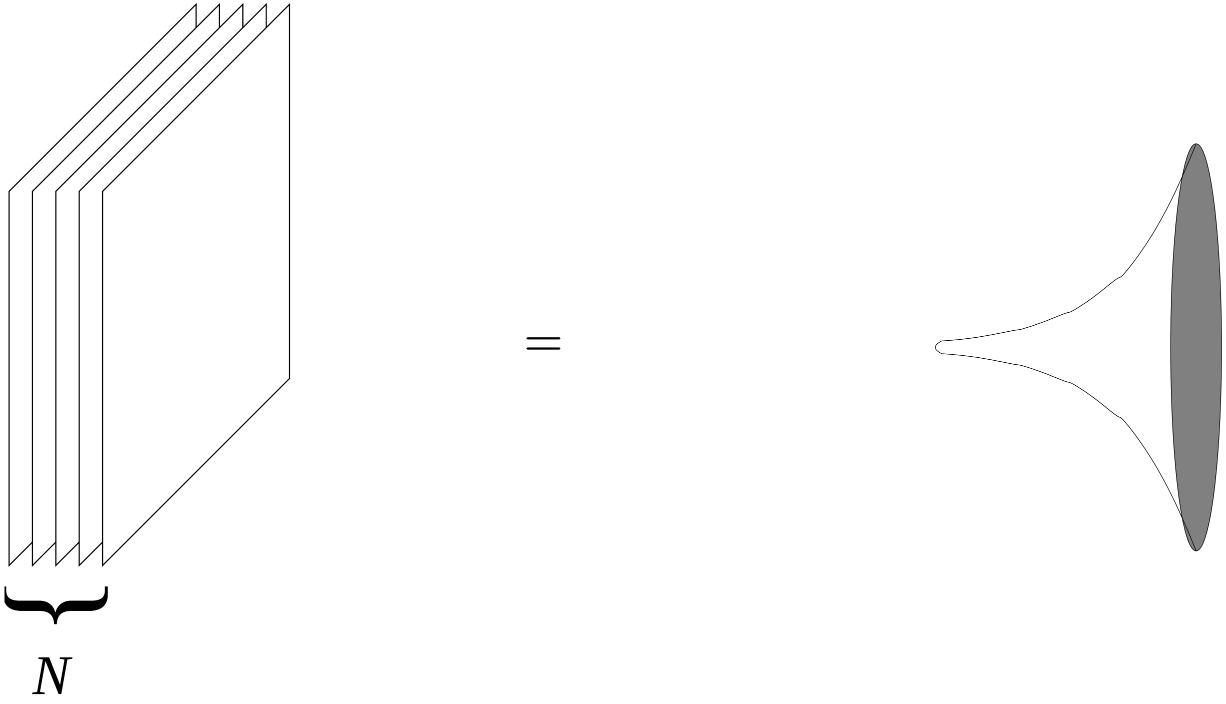
The Gauge/Gravity Duality

Maldacena : stack of N D3-branes in type IIB string theory can be described in two different pictures:

As a quantum field theory describing fluctuations of the branes: $\mathcal{N} = 4$ super-Yang-Mills theory

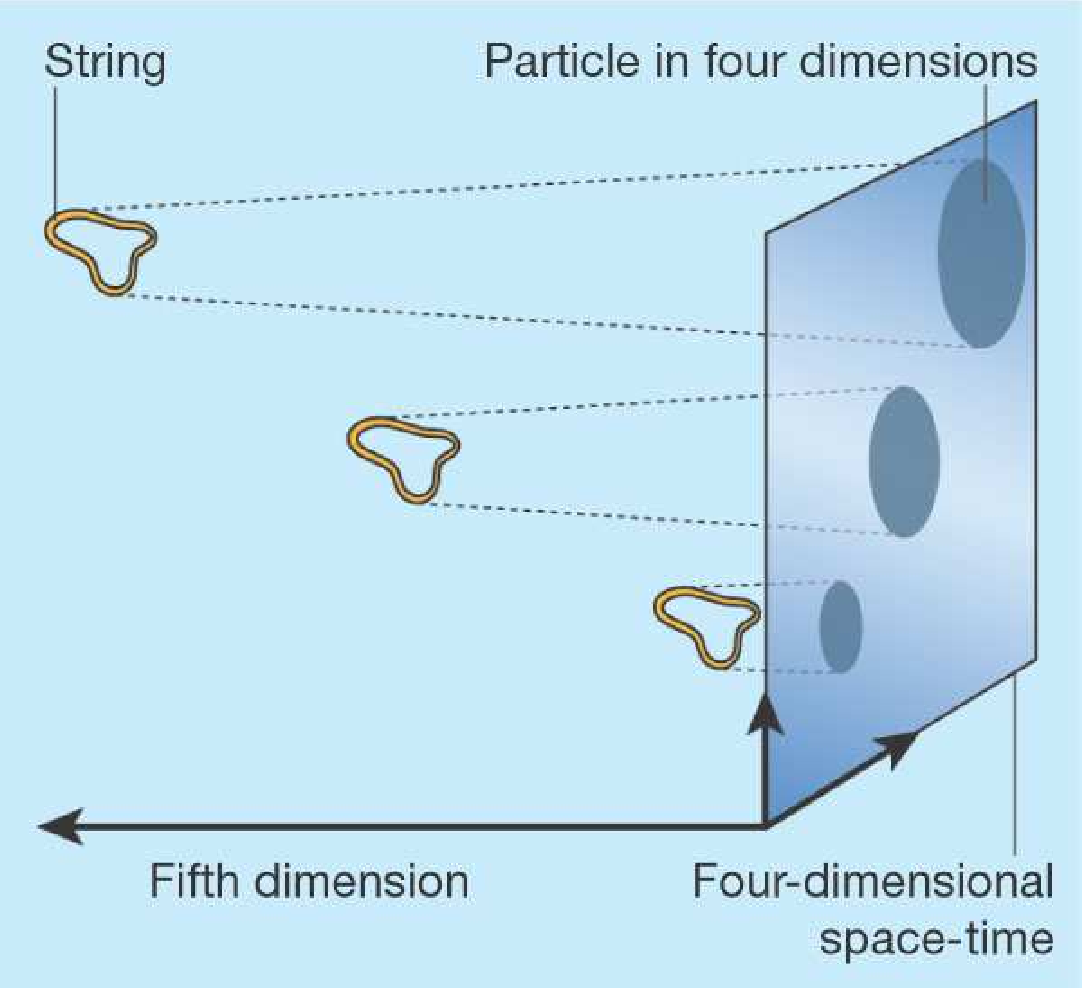
As string theory on a a curved spacetime called AdS$_5 \times$S$_5$

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + d\mathbf{x}^2) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2$$



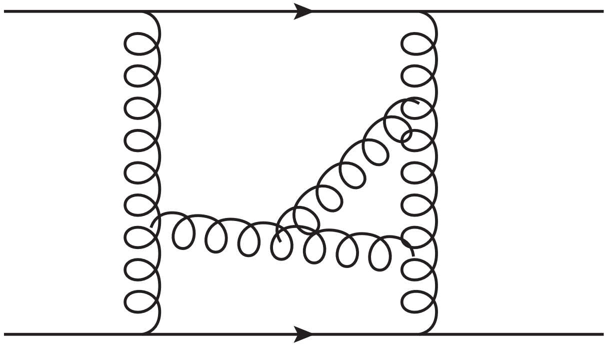
Holography

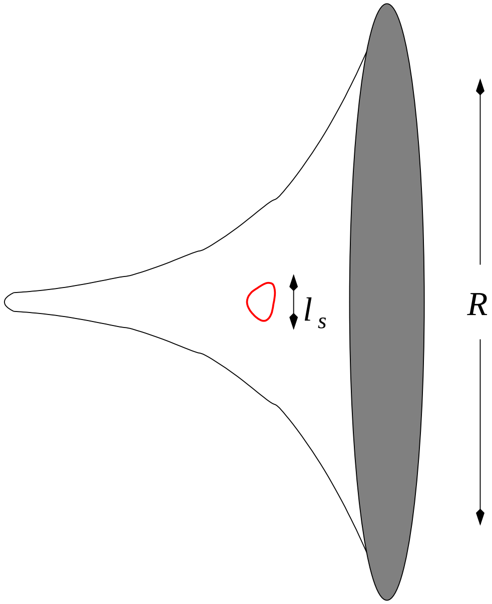
The gauge/gravity duality is a relalization of the idea of holography ('t Hooft, Susskind): a theory without gravity in (3+1)D is equivalent a theory with gravity in higher dimensions



(from Maldacena, Nature 2003)

Mapping of parameters



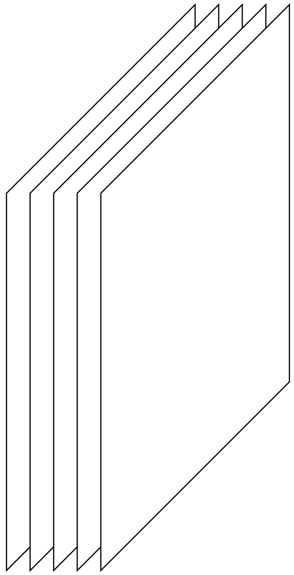


$$g^2 N_c = \frac{R^4}{\ell_s^4}$$

$g^2 N_c \gg 1$: string theory → Einstein's gravity

Difficult regime in field theory = easy regime in string theory

Gauge/gravity duality at finite temperature



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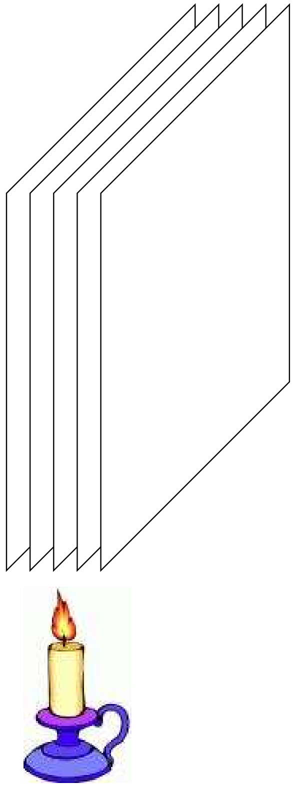

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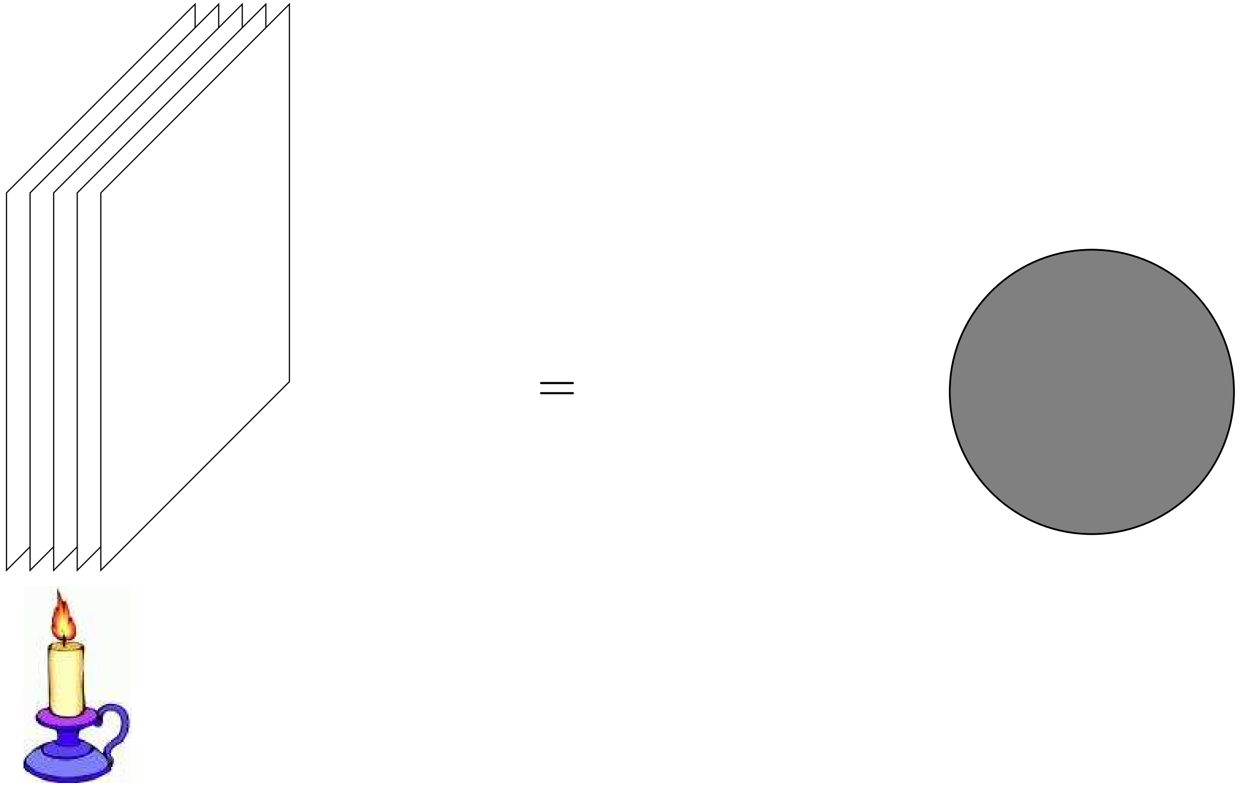

“Quark gluon plasma” = black hole (in anti de-Sitter space)

$$ds^2 = \frac{r^2}{R^2}[-f(r)dt^2 + d\mathbf{x}^2] + \frac{R^2}{r^2 f}dr^2 + R^2 d\Omega_5^2, \quad f(r) = 1 - \frac{r_0^4}{r^4}$$

A theorist’s way of measuring viscosity

How do we measure the viscosity of a system?

- Viscosity $=$ response of the fluid under shear
- Theorist: send gravitational wave through the system

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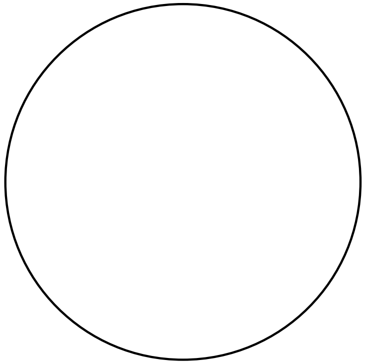

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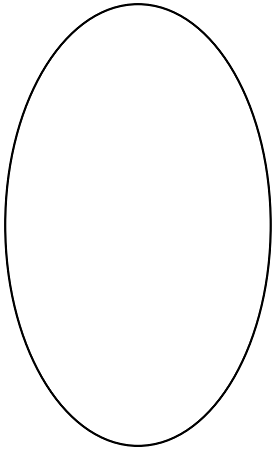

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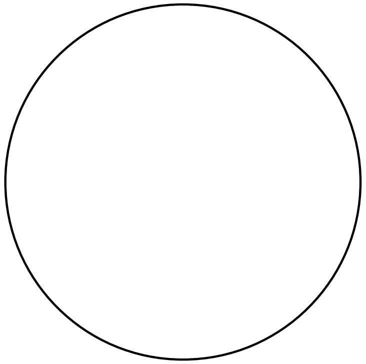

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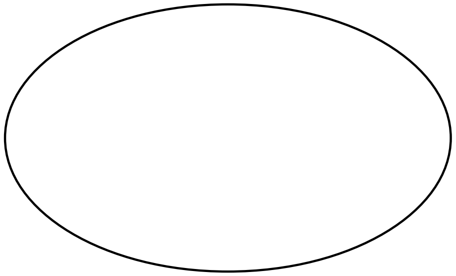

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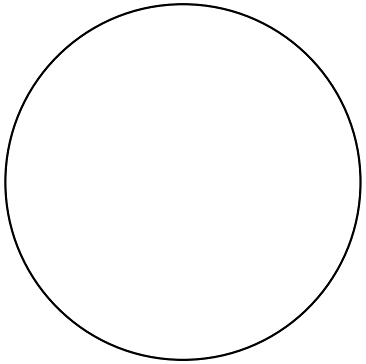
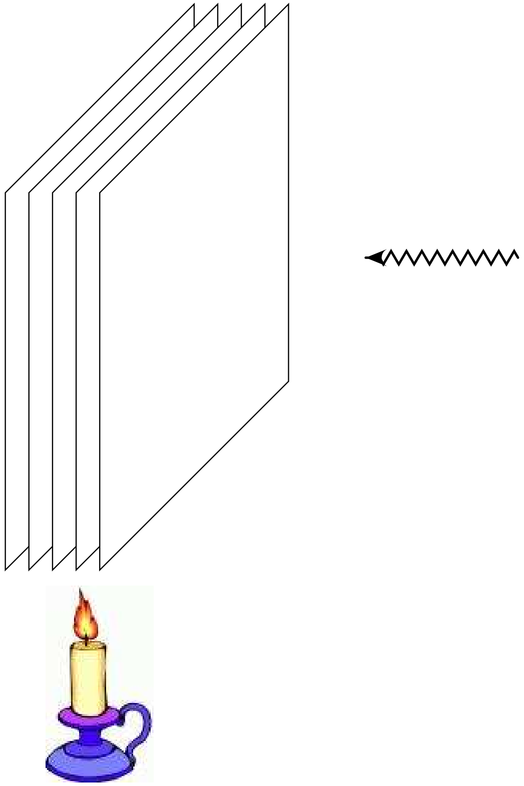


This is the essence of Kubo’s formula

$$\eta = \lim_{\omega\to 0} \frac{1}{2\omega} \int dt\, d\mathbf{x} e^{i\omega t} \langle [T^{xy}(t,\mathbf{x}),\, T^{xy}(0,\mathbf{0})] \rangle$$

Viscosity on the light of duality

Consider a graviton that falls on this stack of N D3-branes
Will be absorbed by the D3 branes.
The process of absorption can be looked at from two different perspectives:



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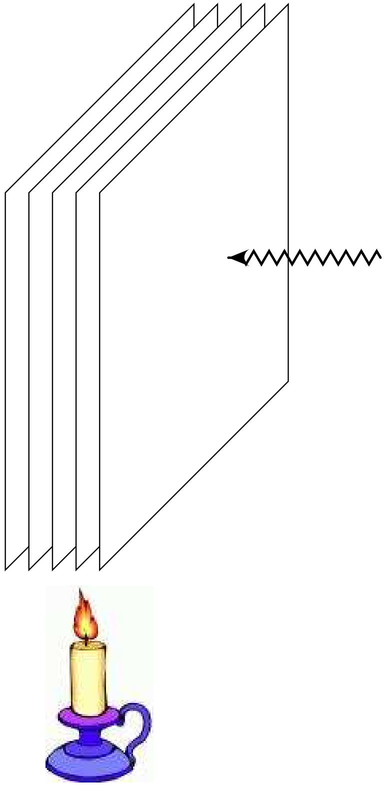

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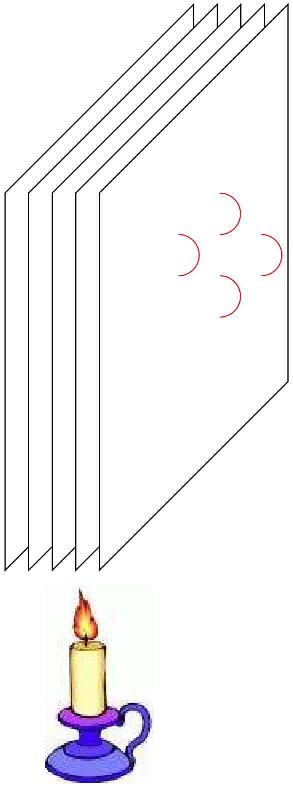

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Absorption by D3 branes ($\sim$ viscosity) = absorption by black hole

Viscosity/entropy density ratio

Collecting facts so far:

- Viscosity $\sim$ absorption cross section for low energy gravitons
- Absorption cross section $=$ area of horizon (a consequence of Einstein equation)
- Entropy: also $\sim$ area of horizon (Bekenstein-Hawking formula)

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

where η is the shear viscosity, s is the entropy per unit volume.

Universality of η/s

Restoring $\hbar$ and c in the formula: a surprise

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}$$

- No velocity of light c
- Finite viscosity at infinitely strong coupling!
- *The same value in all theories with gravity duals* Kovtun, DTS, Starinets 2003

Universality of η/s is related to properties of black hole horizons

Why does η/s have the dimensionality of $\hbar$

In kinetic theory

$$\eta \sim \rho v \ell, \qquad s \sim n = \frac{\rho}{m}$$

$$\frac{\eta}{s} \sim m v \ell \sim \hbar \frac{\text{mean free path}}{\text{de Broglie wavelength}}$$

- In kinetic theory, mean free path cannot be $\ll$ de Broglie wavelength.
- Uncertainty principle: η/s bounded from below by $\#\hbar$, unknown coefficient
- Theories with gravity dual reach $\hbar/4\pi$

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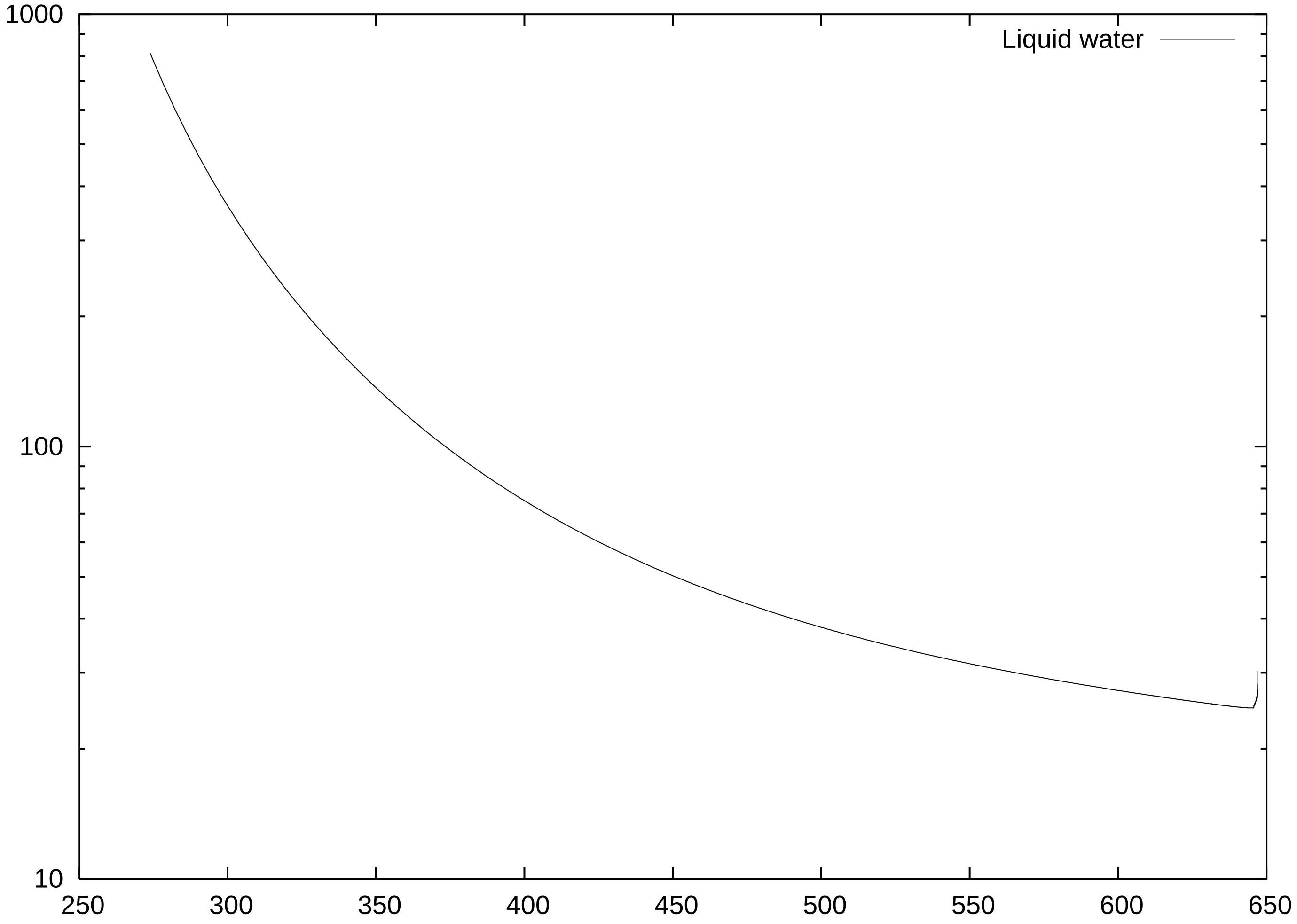
The viscosity bound (conjectured) Kovtun, DTS, Starinets 2003

$$\frac{\eta}{s} \gtrsim \frac{\hbar}{4\pi}$$

Reminds Purcell's question to Weisskopf
No c: $\hbar/(4\pi)$ can be compared with ordinary, nonrelativistic liquids

Ordinary liquids

η/s of liquid water in unit of $\hbar/(4\pi)$, as function of temperature (K), along the saturation curve



General observation on liquids

- η/s reaches minimum near the critical point (liquid$=$gas) Kapusta, McLerran
- but not exactly at the critical point: η diverges there according to theory of dynamic critical phenomena
- The minimal value of η/s varies from substance to substance

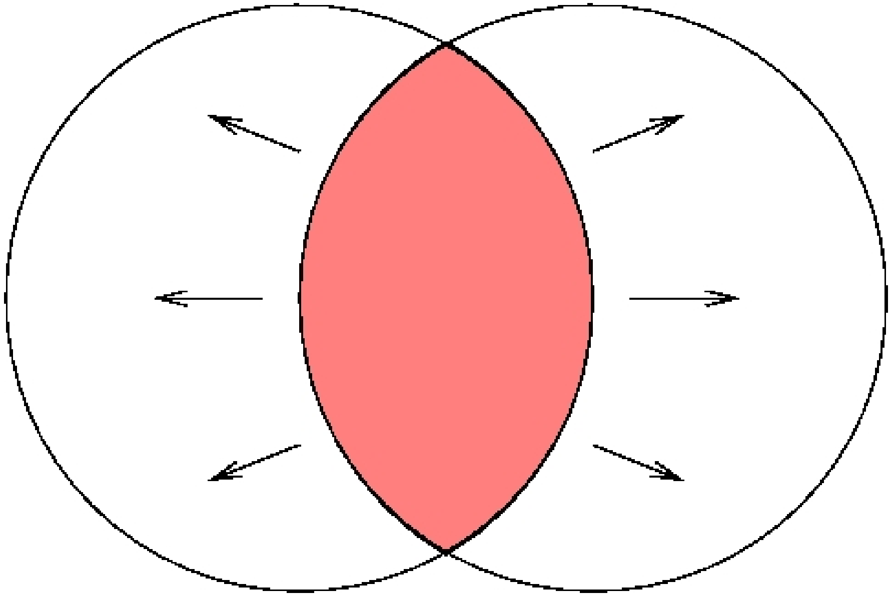
Substance	$\left(\frac{\eta}{s}\right)_{\rm min}$	Substance	$\left(\frac{\eta}{s}\right)_{\rm min}$	Substance	$\left(\frac{\eta}{s}\right)_{\rm min}$
H, He	8.8				
Ne	17	H_2O	25	CO	35
Ar	37	H_2S	35	CO_2	32
Kr	57	N_2	23	SO_2	39
Xe	84	O_2	28		

(η/s is measured in unit of $\hbar/(4\pi)$)

Minimum among substances is reached by the most quantum liquids: hydrogen and helium

Superfluids: normal component finite shear viscosity (Andronikashvili experiments)

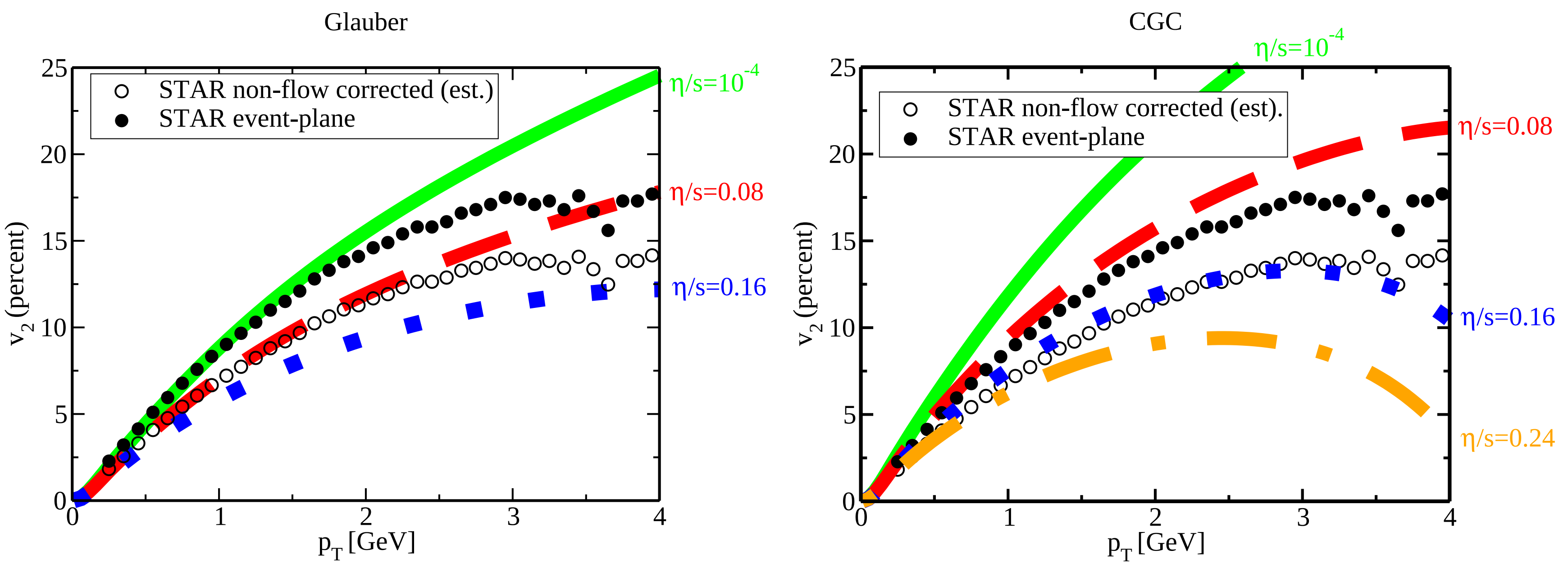
determining viscosity of QGP



- Collisions with nonzero impact parameter
- Distribution of particles over momentum is not axially symmetric: characterized by “elliptic flow” parameter v_2
- explanation: pressure gradient depends on angle

- Viscosity reduces v_2
- Hydrodynamic simulations can give estimates for η

Recent numerical simulations



(from Luzum and Romatschke, arXiv:0804)

$$\frac{\eta}{s} = 0.1 \pm 0.1(\text{th}) \pm 0.08(\text{exp})$$

Not too far way from $1/4\pi$

QGP is strongly coupled (sQGP)

High or low viscosity?

Brookhaven National Lab press release 2005: “the degree of collective interaction, rapid thermalization, and extremely low viscosity of the matter being formed at RHIC make this the most nearly perfect liquid ever observed.

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Estimating the viscosity of the QGP:

$$\eta \sim \frac{\hbar}{4\pi} s \sim \frac{10^{-27}\text{erg} \cdot \text{s}}{(10^{-13}\text{cm})^3} \sim 10^{14}\text{cp}$$

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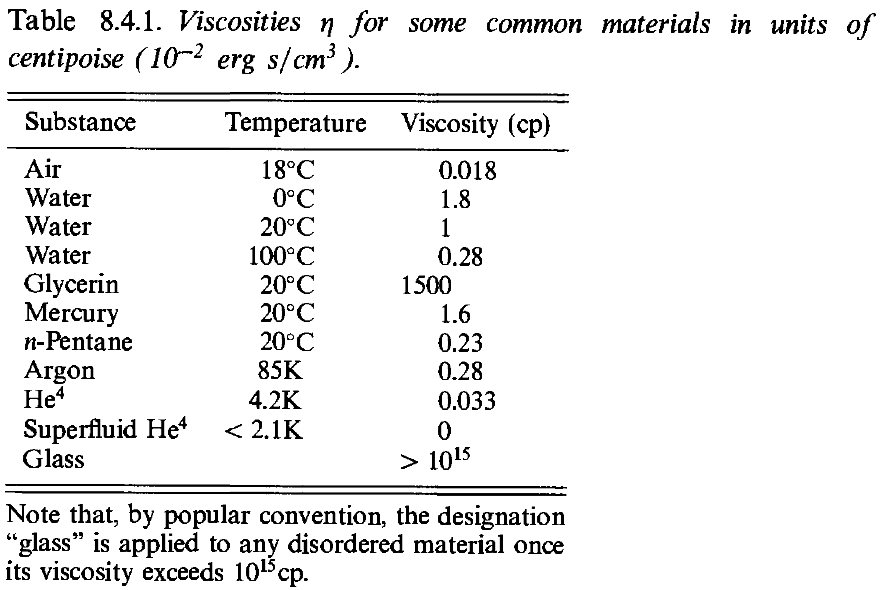

Table 8.4.1. *Viscosities η for some common materials in units of centipoise (10^{-2} erg s/cm^3).*

Substance	Temperature	Viscosity (cp)
Air	18°C	0.018
Water	0°C	1.8
Water	20°C	1
Water	100°C	0.28
Glycerin	20°C	1500
Mercury	20°C	1.6
n-Pentane	20°C	0.23
Argon	85K	0.28
He^4	4.2K	0.033
Superfluid He^4	< 2.1K	0
Glass		$> 10^{15}$

Note that, by popular convention, the designation “glass” is applied to any disordered material once its viscosity exceeds 10^{15}cp.

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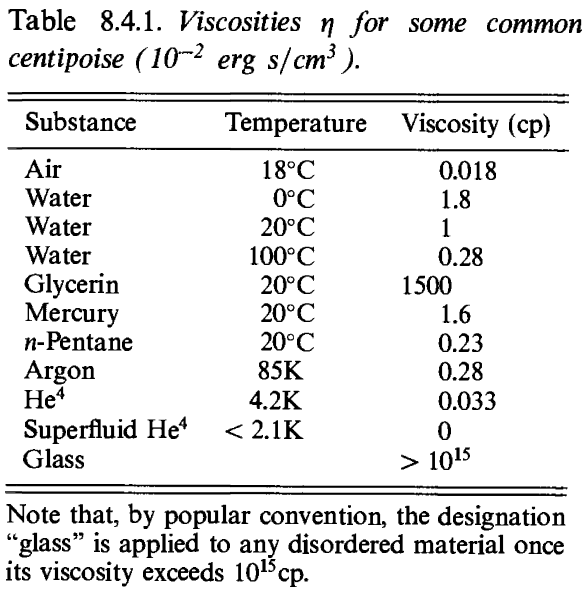

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In absolute terms, QGP is almost as viscous as glass!

Low viscosity: in the sense of small η/s, suggested by gauge/gravity duality

Condensed matter analogy

Mott’s minimal metallic conductivity hypothesis: conductivity is bounded from below by uncertainty principle

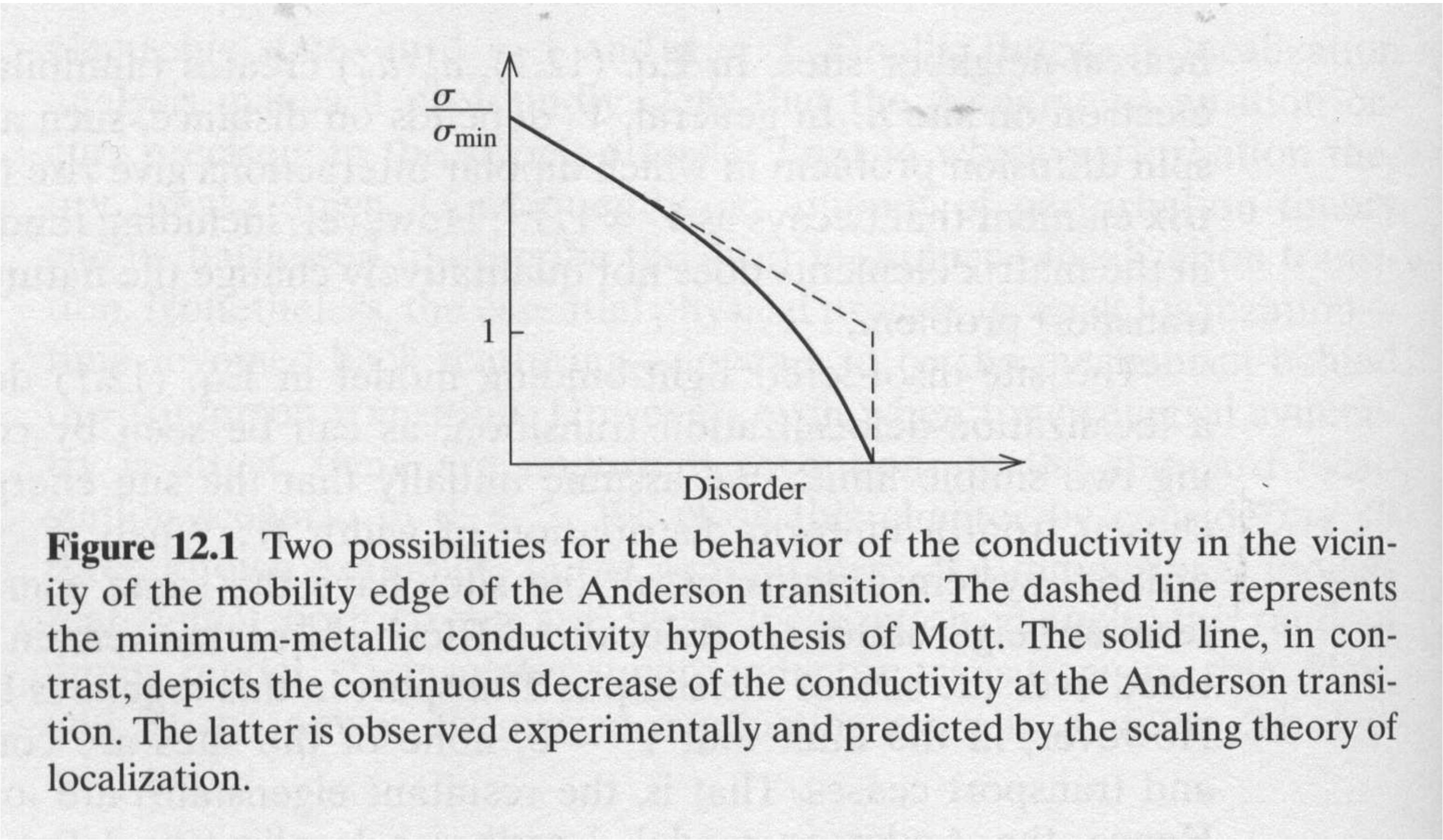

Figure 12.1 Two possibilities for the behavior of the conductivity in the vicinity of the mobility edge of the Anderson transition. The dashed line represents the minimum-metallic conductivity hypothesis of Mott. The solid line, in contrast, depicts the continuous decrease of the conductivity at the Anderson transition. The latter is observed experimentally and predicted by the scaling theory of localization.

(*Phillips, Advanced Solid State Physics*)

Mott’s minimal metallic conductivity is a useful concept, but not a real, absolute minimum.

Is there a viscosity bound?

- $\frac{\eta}{s} \geq \frac{\hbar}{4\pi}$ for all fluids? Kovtun, DTS, Starinets 2005
- There exist theories where

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}\left(1 - \frac{c}{N}\right), \qquad N \gg 1$$

Kats, Petrov; Buchel, Myers, Sinha

- No reliable way to go to small N.
- Model studies: bad things happen before η/s reaches 0 Brigante etc. PRL 2008
  - Within a model: causality is violated when $\eta/s > \frac{16}{25}\frac{\hbar}{4\pi}$
- What is the real minimum of η/s ?

Further applications

Chiral separation in quark matter:

- Rotating volume of quark matter with nonzero chemical potential
- Left and right handed quarks separate along the axis of rotation
- Seen in gauge-gravity duality, and understood to be general feature of chiral relativistic fluids
- Originates from *quantum anomalies*

Conclusion

Surprising applications of string theory to real-world problems

- New perspective on old problems
  - QGP = black hole
  - viscosity = gravity absorption
  - etc
- Suggested η/s as a relevant ratio
- Suggested the value $\hbar/(4\pi)$ as particularly interesting.

Most serious challenge: connect to the correct theory of strong interactions, QCD

The unity of physics

We have also learned a lesson that physics is one single subject:

- Hydrodynamics
- Quantum field theory
- Statistical mechanics
- Gravity
- String theory

The End