The QCD Strings of Randall-Sundrum Models

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At Davis, May 24, 2010

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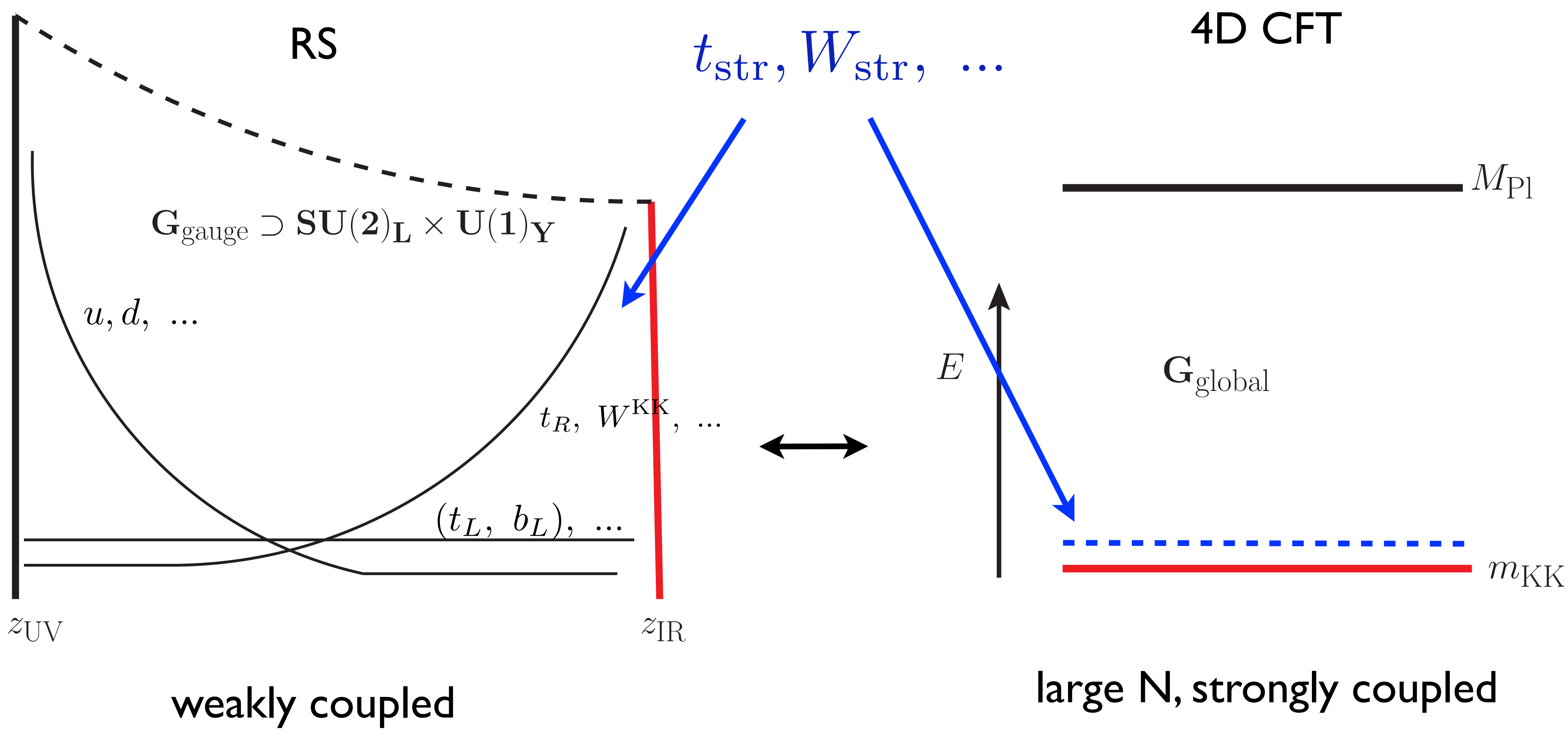
Confining Strings in RS

- I will be discussing Randall-Sundrum constructions with at least the SM electroweak gauge fields in the bulk
- This is dual to gauging global symmetries of some confining, technicolor-like theory
- Such a theory should have confining strings
- How heavy are they? Should they be part of the low-energy effective theory?

Basic setup and result.



- Light string states ~ TeV
  - Higher spin, Regge-like.
  - Studied examples:

spin 2 excitations of SM gauge boson: Perelstein & Spray, arXiv:0907.3496
spin 3/2 excitations the top quark: Hassanain, March-Russell, and Rosa, arXiv:0904.4108

The Short Version

- Two known arguments -- avoiding a Landau pole and completing the confining phase transition -- imply a bound of loosely $N_c < 10$.
- In $AdS_5 \times S^5$, the AdS curvature radius scales as $R^4 = 4\pi g_s N l_s^4$, so the bound on N bounds $m_s/m_{KK} \sim R/l_s .<\sim 10^{1/4}$ (Strassler; Hassanain et al; Perelstein & Spray)
- Our goal: explain these arguments in detail, extend them to various examples, and look for what type of string construction is most promising.

Avoiding Landau Poles

The two-point function of the global symmetry current computes its contribution to the running of $SU(2)_L$ in the SM:

$$\int d^4x\ e^{-iq\cdot x}\,\langle J_\mu(0)J_\nu(x)\rangle_{CFT} = -\frac{b_{CFT}}{16\pi^2}\left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)\log q^2,$$

$$\frac{8\pi^2}{g^2(Q^2)} = \frac{8\pi^2}{g^2(\Lambda_X^2)} + (b_{SM} + b_{CFT})\log\frac{\Lambda_X}{Q}$$

In most examples, $b_{CFT} \sim N$ (from fields in the bifundamental of color and flavor).

Set by $b_{CFT} = 8\pi^2 R/g_5^2$ in 5D theory.

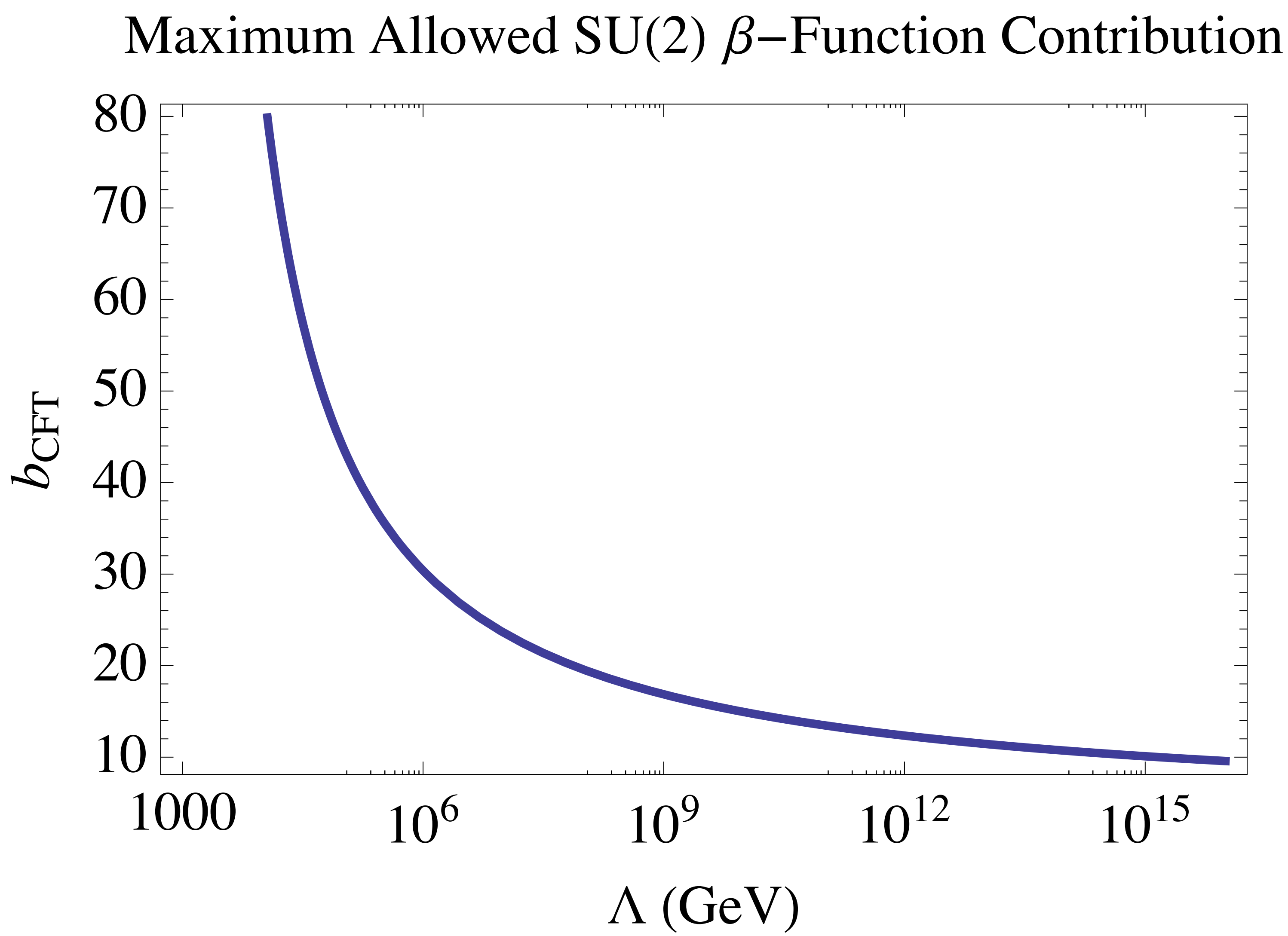


Figure 1: Bound on b_{CFT} as a function of the scale Λ below which we forbid a Landau pole.

GUT-scale hierarchy:

$$b_{CFT} \leq \frac{8\pi^2}{g^2(M_Z)} \frac{1}{\log 10^{12}} + \frac{10}{3} \approx 10.$$

Ways out?

- If the SM gauge bosons are composite -- e.g. emerging from Seiberg duality at the bottom of some cascade -- such bounds do not apply. (Interpret Landau pole as hint of duality.) No longer technicolor-like.
- If b_{CFT} is order-one, as in some M5 brane models (Gaiotto-Maldacena), this bound does not apply.

Cosmology

- If RS is a good description, expect the confinement/deconfinement transition to be of Hawking-Page type.
- $T > T_c$: thermal plasma, dual to AdS-Schwarzschild.
- $T < T_c$: hadronization, dual to AdS on thermal circle
- Phase transition is *first-order*.

Cosmology

- The phase transition is slow (Creminelli et al.; Randall & Servant; Kaplan, Schuster, & Toro)
- Critical temperature: $T_c \sim 2^{1/4}/(\pi z_{IR})$. (Herzog) Scale of KK modes, not string modes.
- Entropy density $O(N^2)$ at high temperatures and $O(1)$ at low temperatures.
- Similarly: change in vacuum energy $O(N^2)$

$$\Delta E_{vac} = \frac{16 M_5^3 R_{\rm AdS}^3}{z_{IR}^4} = \frac{8}{\pi^2} c \frac{1}{z_{IR}^4}.$$

Cosmology

- The danger is the "empty universe problem," explained clearly in this context by Kaplan, Schuster, and Toro.
- Rate of bubble nucleation:

$$\Gamma \sim z_{IR}^{-4} e^{-\mathcal{O}(N^2)}$$

- If $\Gamma < H^4$, bubbles never meet, and the transition never completes.

The Bound

- We can't calculate the bounce action that takes us from thermal AdS to AdS-Schwarzschild. (Approximations exist for Goldberger-Wise stabilization.)

- In general, N^2 replaced with central charge c

$$a_0 z_{IR}^{-4} \exp^{-a_1 c} > c^2 z_{IR}^{-8} M_{Pl}^{-4}$$

- (Unknown order-one numbers a_0, a_1)

$$c \lesssim \frac{1}{a_1} \left(4 \log(M_{Pl} z_{IR}) + \log a_0 - 2 \log c\right)$$

$$c \lesssim 140$$

Summary of Bounds

- These are two known bounds, comparably strong: $b_{CFT} \sim N < 10$ and $c \sim N^2 < 140$.
- We will see that the string scale is related to these numbers raised to small fractional powers, so is tightly bounded.
- Both of these numbers turn out to be very geometric
- Bound on c is more generic ($b_{CFT} \sim 1$ in M5 examples), but could avoid if never at temperature > TeV

4d vs 5d Masses

- We're interested in ratios of masses of 4d states (heavy string modes and light Kaluza-Klein modes)
- Our proxy for this is the ratio of length scales R_{AdS}/l_s in the bulk theory.
- KK masses set by z_{IR}^{-1}, location of the IR wall. String masses set by warped-down string scale at IR wall.

4d vs 5d Masses

- Another way to see this: for a bulk mass m_5^2 in units of R_{AdS} (for a scalar with Dirichlet b.c., for convenience), 4d masses are zeroes of $J_\nu(m_{4d} z_{IR})$ with

$$\nu = \sqrt{4 + m_5^2 R_{\mathrm{AdS}}^2}$$

- The first such zero goes as:

$$\left(\nu + 1.856\nu^{1/3} + \mathcal{O}(1)\right)$$

- Thus $m_{4d}\, z_{IR} \sim m_{5d}\, R_{\mathrm{AdS}}$ at large m_{5d}

R_{AdS} vs. l_s in N=4 SYM

- Before looking at more examples, let's remind ourselves of $AdS_5 \times S^5$, where $R_{AdS}{}^4 = 4\pi g_s N l_s{}^4$.
- What's happening here can be thought of as moduli stabilization: need to fix the radius of the S^5 compactification.
- Two terms in potential: curvature ~ $1/R^2$ and flux ~ $g_s{}^2 N^2/\mathrm{Vol}(S^5)^2$, in string units.
- Comparable size at minimum, sets R_{AdS}.

c Bound and Geometry

Assuming we start with 10d string theory, reduce to 5d AdS to obtain a Planck scale:

$$M_5^3 = \frac{1}{(2\pi)^7 g_s^2 l_s^8} \mathrm{Vol}_{M_5} .$$

Read off the central charge from the ⟨TT⟩

correlator as $c = 2\pi^2 M_5^3 R_{\mathrm{AdS}}^3$:

$$c = \left(\frac{R_{\mathrm{AdS}}^4}{8\pi l_s^4 g_s} \right)^2 \left(\frac{\mathrm{v}_{M_5}}{\pi^3} \right)$$

Here v_{M5} is the volume of M_5 in units of R_{AdS}.

c Bound, Numerically

- We see that c is expressed in terms of (R_{AdS}/l_s), the number we wish to bound, along with $g_s < 1$ (by S-duality) and v_{M5}.
- So our goal is to make M_5, the internal manifold, small compared to the AdS space.
- Normalize using $\text{AdS}_5 \times S^5$:

$$\frac{m_{\text{str}}}{m_{\text{KK}}} \lesssim \left(140 \times 64\pi^2 \frac{\pi^3}{\text{v}_{M_5}}\right)^{1/8} \approx 4.2 \left(\frac{\pi^3}{\text{v}_{M_5}}\right)^{1/8}.$$

b_{CFT} Bound and Geometry

For the Landau pole bound on b_{CFT}, we need gauge fields in the bulk. There are different routes to this, but let's focus on D7 branes (Karch-Katz).

These must wrap a 3-manifold $M_3 \subset M_5$. (May have a tachyon above the Breitenlohner-Freedman bound.)

$$S_{\mathrm{DBI}} = -\tau_7 \int d^8\sigma \, \mathrm{tr}\sqrt{-\det(G_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta})}$$

$$\Rightarrow \quad \frac{R_{\mathrm{AdS}}}{g_5^2} = R_{\mathrm{AdS}} \times \frac{\mathrm{Vol}_{M_3}}{g_7^2} = \frac{\mathrm{v}_{M_3}}{2\pi^2} \frac{2\pi^2}{2g_s(2\pi)^5} \left(\frac{R_{\mathrm{AdS}}}{l_s}\right)^4$$

b_{CFT} Bound, Numerically

The bulk gauge coupling determines the coefficient in the $\langle J\,J \rangle$ correlator and hence b_{CFT}:

$$b_{CFT} = 8\pi^2 \frac{R_{\mathrm{AdS}}}{g_5^2} = \frac{\mathrm{v}_{M_3}}{2\pi^2} \left(\frac{R_{\mathrm{AdS}}}{l_s} \right)^4 \frac{1}{4\pi g_s}.$$

Similarly to what we found for *c*, we have expressed *bCFT* in terms (R_{AdS}/l_s), the number we wish to bound, along with $g_s < 1$ and v_{M3}.

$$\frac{m_{\mathrm{str}}}{m_{\mathrm{KK}}} \lesssim \left(4\pi g_s \frac{2\pi^2}{\mathrm{v}_{M_3}} \left(\frac{8\pi^2}{g^2(m_Z^2)} \frac{1}{\log(\Lambda_{UV}/\Lambda_{TC})} + \frac{10}{3} \right) \right)^{1/4} \lesssim 3.3 \left(g_s \frac{2\pi^2}{\mathrm{v}_{M_3}} \right)^{1/4}$$

Generic Einstein Manifolds

- The previous formulas express the b_{CFT} and c bounds in terms of geometry
- For N D3 branes on a cone over an Einstein manifold, there is a further relation that clarifies how these relate to number of colors:

$$R_{\mathrm{AdS}}^4 = 4\pi N g_s l_s^4 \frac{\pi^3}{\mathrm{v}_{M_5}}$$

$$\Rightarrow \qquad b_{CFT} = \frac{\mathrm{v}_{M_3}}{2\pi^2} \frac{\pi^3}{\mathrm{v}_{M_5}} N. \qquad \text{cf. hypermultiplet}$$

Orbifolds

- One way to reduce the volume of the internal geometry is to orbifold it.
- S^5 can be thought of as a circle fibered over CP^2; mod out by Z_k subgroup
- Doesn't change AdS_5 part of geometry: same R/l_s, but b_{CFT}, c lower by factor of k.
- Heavier strings at no cost!

Orbifolds

- However, run into a limit: size of the fiber shrinks from R to R/k, becomes l_s eventually
- Bound: $k < N^{1/4}$.
- Our bound on N was strict enough that this gives us only a small improvement.
- HOP instability: space decays if $k > N^{1/4}$.

$Y_{p,q}$

- More complicated examples have qualitatively similar features: for instance, an infinite class of spaces $Y_{p,q}$. (Gauntlett et al.)
- The small volume limit is $p >> q$; in that case volume ~$1/p$.
- Very similar to orbifold: circle direction has length ~ R/p, require $p < N^{1/4}$

Cascading Geometries

- We have been discussing "hard-wall" backgrounds: good description for confinement induced by relevant ops (Polchinski, Strassler)
- Solve hierarchy problem $\Rightarrow$ gentler RG flow, marginal (or nearly so) operators
- Example: Klebanov-Strassler throat

Klebanov-Strassler

- Finite temperature transition is similar, and depends on total number of degrees of freedom near the tip of the throat (Hassanain et al.)
- Beta function now runs as $\log^2$: increasing number of d.o.f. in UV

$$R^4(r) = \frac{81}{8}(g_s M)^2 \alpha'^2 \log(r/r_s).$$

Small internal dimensions by tuning

- Recently Polchinski and Silverstein constructed an F-theory background with small internal geometry
- Their trick: cancel a $1/R^2$ curvature term with a term from D7-branes, leaving an ε/R^2
- Scalings:

$$R_f \sim \epsilon R_{\mathrm{AdS}}, \;\; R \sim \epsilon^{1/2} R_{\mathrm{AdS}}, \;\; R_{\mathrm{AdS}}^4 \sim \frac{N}{\epsilon^3} l_s^4$$

Polchinski/Silverstein

- The interval volume is of size $\varepsilon^3 R_{AdS}^5$.
- Smallest allowed $\varepsilon \sim 1/N$ (string-scale fiber)
- Thus $c \sim N^5$ with $R \sim N$ (instead of $c \sim N^2$)
- So $m_{\rm str}/m_{\rm KK} < c_{max}^{1/5}$ (rather than $c_{max}^{1/8}$)
- Similarly weaker b_{CFT} bound:
  $m_{\rm str}/m_{\rm KK} < b_{max}^{1/2 \text{ or } 2/5}$ rather than $b_{max}^{1/4}$
- Helps! But not dramatically.

D4/D8 Constructions

- D4 branes compactified on a circle with SUSY-breaking boundary conditions give an interesting confining theory (Witten ’98)
- Can add D8 branes intersecting the D4’s in 3+1 dimensional subspace to get flavors (Sakai/Sugimoto ’04)
- Can use this as a variation on RS; loosely “RS×UED” (but not universal)

D4/D8 Constructions

3+1D matter, but 4+1D gluons. Power-law, rather than logarithmic, running: stronger Landau-pole bound.

$$\frac{1}{g_4^2} \sim \frac{1}{9\pi^2}\tau_8' R_{\rm AdS}^3 \left(\frac{z_{\rm UV}}{U_{\rm KK}}\right)^{1/3}$$

Thermal phase transition is essentially the same as in AdS_5, so similar bound on N^2.

Theories on M5 Branes

- M5 branes give different physics than D-branes. Have $R^3 = N\, l_{Pl}^3$.
- Gaiotto/Maldacena: M5 flavor branes wrapped on $AdS_5 \times S^1$ give $b_{CFT} = O(1)$. Evade Landau pole bound!
- However, $c \sim N^3$ and similar deconfinement transition $\Rightarrow$ still strong bounds.

The Weak Gravity Conjecture

- Interesting argument from weak gravity: add UV brane, go on branch with one D3 brane a distance R_{AdS} in the bulk, apply bound $m_W < g\, M_{Pl}$ (Arkani-Hamed, Motl, Nicolis, Vafa '06)
- Find that this means a bound on size of internal space, $Vol_d > g_s\, R_{AdS}\, l_s^{d-1}$
- Examples with fluxes generically have a stronger $Vol_d > g_s\, N_{flux}\, R_{AdS}\, l_s^{d-1}$ without tuning.

Weak-Gravity Saturation

- Suppose we knew a construction that saturates the weak-gravity bound $\mathrm{Vol}_d > g_s R_{\mathrm{AdS}} l_s^{d-1}$ (we don't)
- It would have $c_{sat} \sim (R_{\mathrm{AdS}}/l_s)^4$. (Contrast $(R_{\mathrm{AdS}}/l_s)^8$ in $\mathrm{AdS}_5 \times S^5$) Similarly for b_{CFT}
- Would be intrinsically interesting, plus the best route to decoupling strings. Does it exist?

Noncritical Strings?

- Noncritical string theory: central charge defect sources string-scale curvature
- $R_{\mathrm{AdS}} \sim l_s$, so saturate weak gravity?
- Actually, $g_s \sim 1/N$ (not large 't Hooft coupling), so still satisfy the stronger bound.

Resummation & Concavity: Stringless Argument

- Resumming one-gluon exchanges and extrapolating to large λ gives $-\sqrt{\lambda}/r$ Coulomb potential (Erickson, Semenoff, Zarembo)
- Bachas: static potential is concave
- Long distances: $V(r) \sim \sigma r$ (confinement)
- Assume Coulomb until $r \sim z_{IR}$
- Learn: $m_{str}\, z_{IR} \sim \sqrt{\sigma}\, z_{IR} < \lambda^{1/4}$

Precision Electroweak

- One advantage of an RS description of a strongly-coupled sector is that quantities are calculable, e.g. the S, T, U parameters.
- Light strings could give $O(1)$ corrections, but probably don't change conclusions about viability.
- E.g., custodial symmetry still protects T.

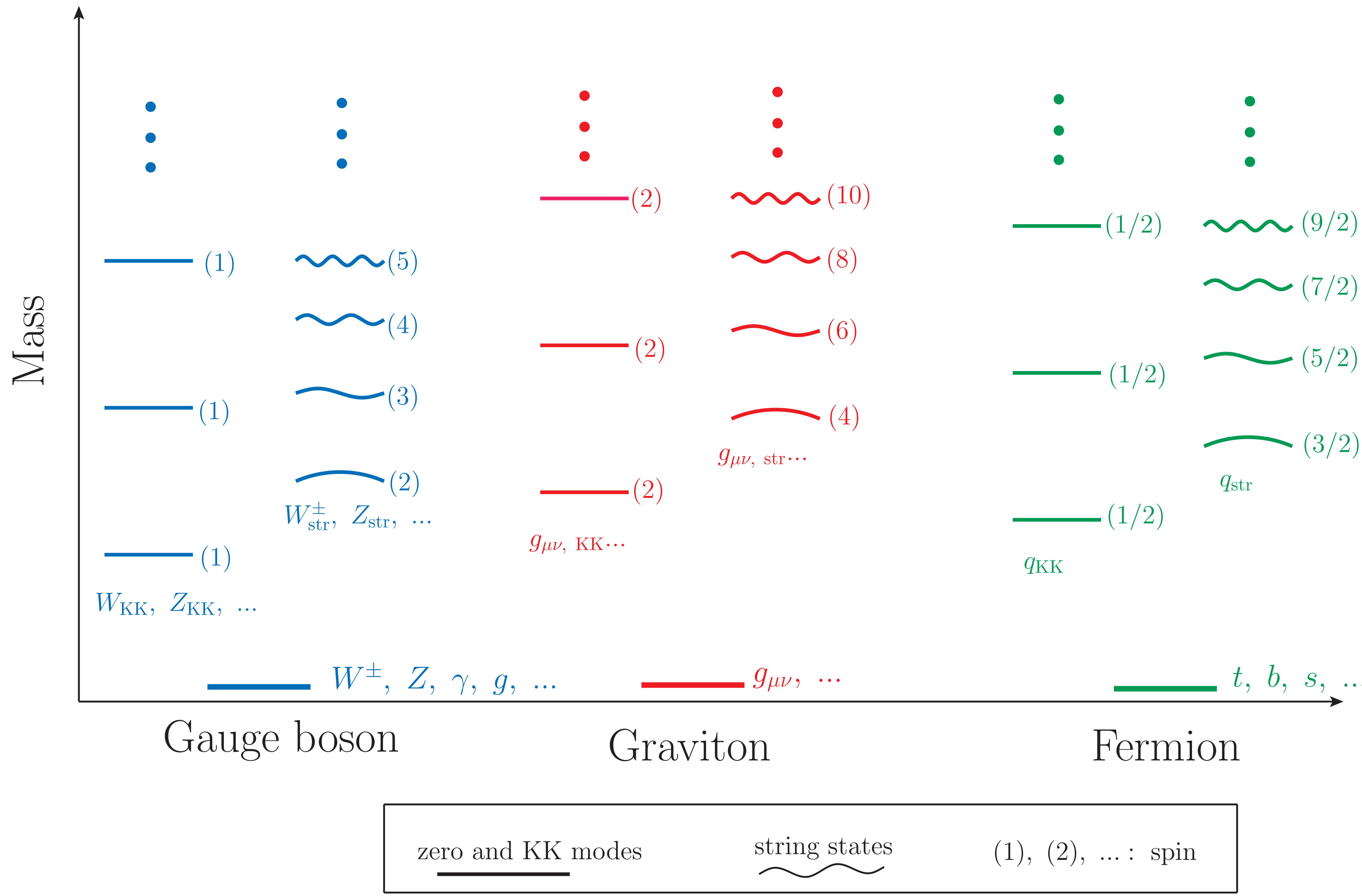
S-Parameter

- One example of a challenge for RS model-building is the S-parameter. Strings will change it by an unknown order-one amount.
- Approaches: either use composite Higgs, (v/M_{KK}) small (still viable)
- Or: Higgsless limit, tune fermion profiles ("delocalization") to cancel S: still viable, just different tuning.

Stringy States

- What sort of states do we expect?
- Higher-spin *W* and *Z* bosons.
- Fermions model-dependent; possibly spin-3/2 top, bottom, etc.
- KK modes on internal directions.
- Higher-spin "KK gravitons" (closed strings)
- A whole zoo; challenging spectroscopy.

Spectrum


Mass
(1)
(1)
(1)
$W_{\rm KK}$, $Z_{\rm KK}$, ...
(5)
(4)
(3)
(2)
$W^{\pm}_{\rm str}$, $Z_{\rm str}$, ...
$W^{\pm}$, Z, γ, g, ...
(2)
(2)
(2)
$g_{\mu\nu}$, KK...
(10)
(8)
(6)
(4)
$g_{\mu\nu}$, str...
$g_{\mu\nu}$, ...
(1/2)
(1/2)
(1/2)
$q_{\rm KK}$
(9/2)
(7/2)
(5/2)
(3/2)
$q_{\rm str}$
t, b, s, ...
Gauge boson
Graviton
Fermion
zero and KK modes
string states
(1), (2), ... : spin

Event Shapes?

Recently Strassler conjectured that large ‘t Hooft coupling theories will lead to *spherical* events: rather than jets, see particles moving in all directions.

This was confirmed by Hofman and Maldacena for conformal theories (also: Hatta, Iancu, Mueller)

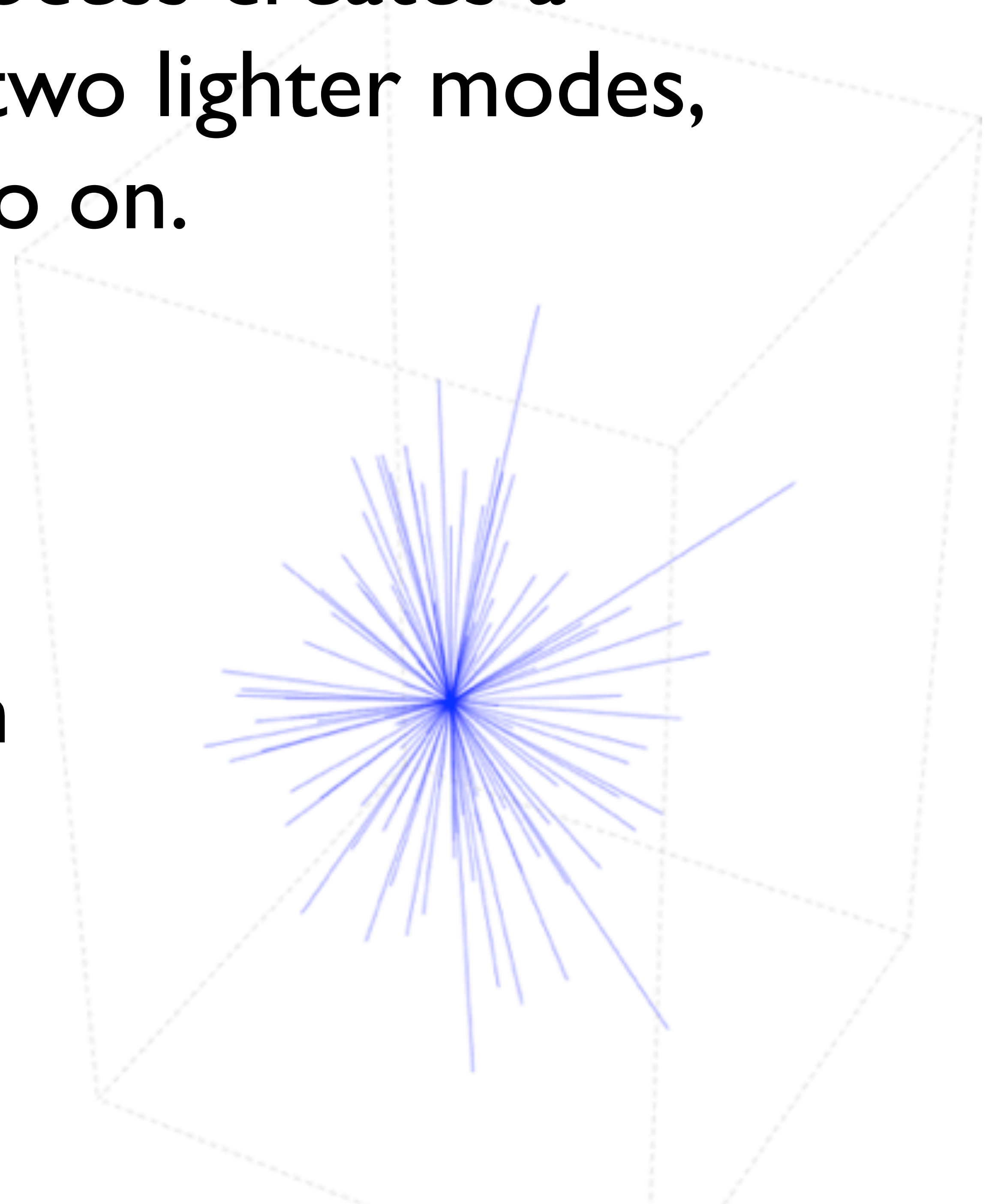
Cute, lowbrow version in Randall-Sundrum: approximate conservation of KK number.

Sphericity in RS

(C. Csáki, M.R., J. Terning, 0811.3001)

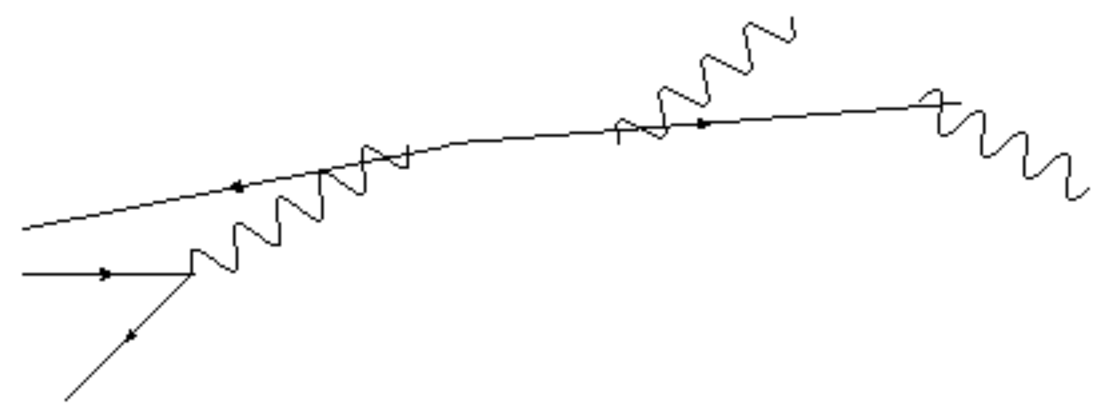
Suppose a high-energy process creates a heavy mode. It decays to two lighter modes, which in turn decay, and so on.

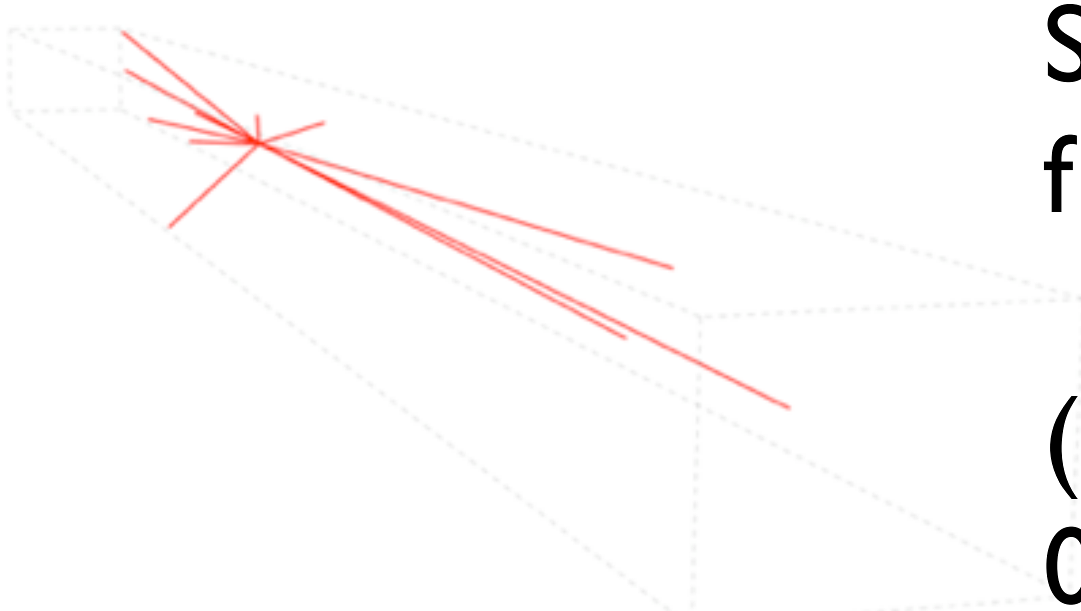
Approximately conserved momentum in the extra dimension: #10 prefers to go to #8 + #1 rather than #2 + #1. Small phase space, no preferred directions.



QCD is Jetty; So are Strings?

At high energies, QCD is weakly coupled; produce a few particles, radiate mostly collinear or soft emissions. Result: “jets” (Sterman, Weinberg)





Similar shapes from snapping flux tubes at large N.

(C. Csáki, M.R., J. Terning, 0811.3001)

Event Shapes

- RS with a very high string scale would give spherical events.
- The bounds we discuss mean the theory is at best at moderately large 't Hooft coupling.
- Don't obviously have QCD-like jets, because $\lambda \approx 10$ or 100, not small.
- What would continuum events look like?
- Not clear there's any reliable calculational scheme (need strings on RR backgrounds...)

Conclusions

- Surveying a variety of string constructions, we find that phenomenological bounds on number of d.o.f. imply light strings.
- Maybe not in reach of LHC, given precision constraints. But still interesting.
- Approaches like the Polchinski/Silverstein construction might be of interest.
- Are there spaces saturating weak gravity?