Superluminal Travel

in two dimensions

with Sergey Sibiryakov
arXiv:0806.1534

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What is the weirdest QFT?

Whether superluminal signals are possible in a *consistent Lorentz invariant quantum* field theory?

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0-th order motivation
(excuse?):

The answer is *Yes!*

At least in 1+1 dimensions and if the spatial parity is broken

Some more motivations:

- Many people feel that locality is an approximate notion in gravitational theories (no local observables, information recovery from black holes). Instantaneous signal propagation definitely qualifies as a non-local effect.

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> Theories described in this talk provide an example of non-local Lorentz invariant microscopic QFT's.

- Theorist’s curiosity

▸ Theorist's curiosity

a bit more concretely:

Early GUT/string model builder's dream

Unique theory with the unique vacuum

‣ Theorist's curiosity

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Real life

Landscape of 10^{500} vacua

What's next? Is there a natural border of the Landscape?

Theorist's curiosity

a bit more concretely:

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Landscape + Swampland?

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Real life

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What's next? Is there a natural border of the Landscape?

Landscape + Swampland?

or

Landscape + Wonderland??

▸ Interesting candidate habitants of the swampland

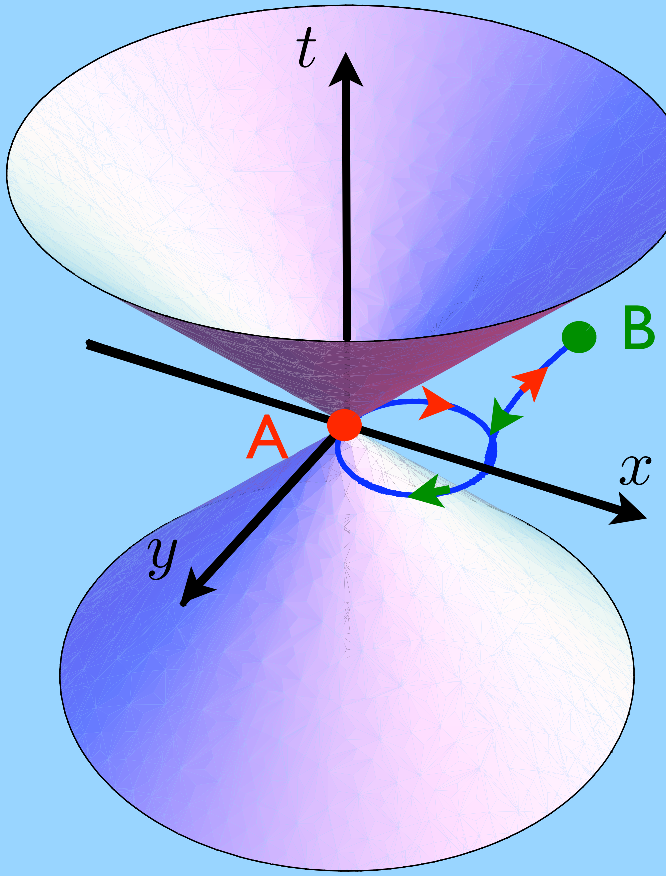
- DGP model
- 4d Higgs phases of gravity (ghost condensate and more general models of massive gravity)

Rich unexpected phenomenology: anomalous precession of the Moon perihelion, massive gravity waves, bumpy black holes...

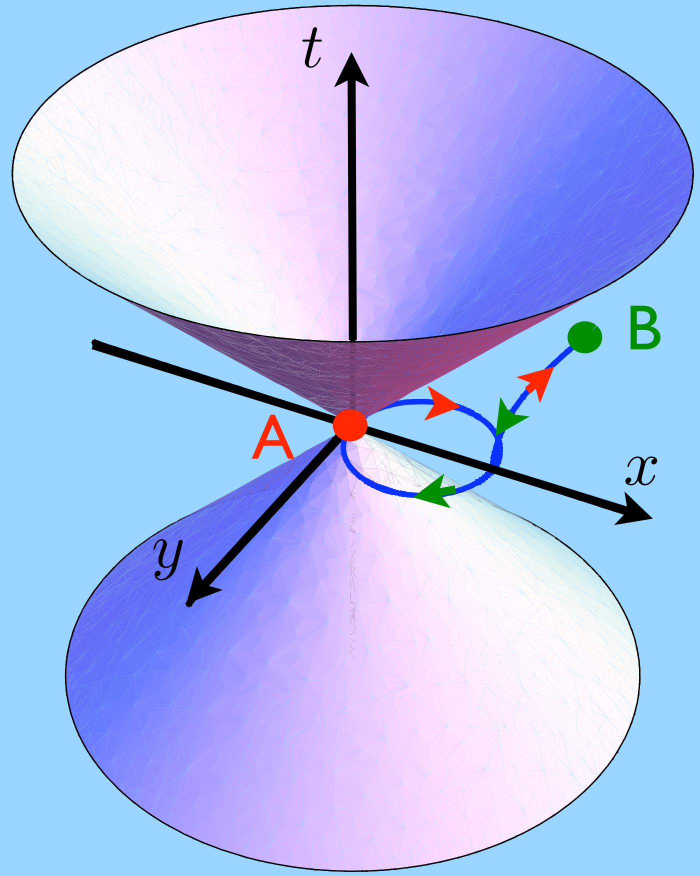
New theoretical opportunities: bouncing cosmologies,...

Tensions with causality and physics of horizons

High school argument why superluminal signals are impossible



High school argument why superluminal signals are impossible

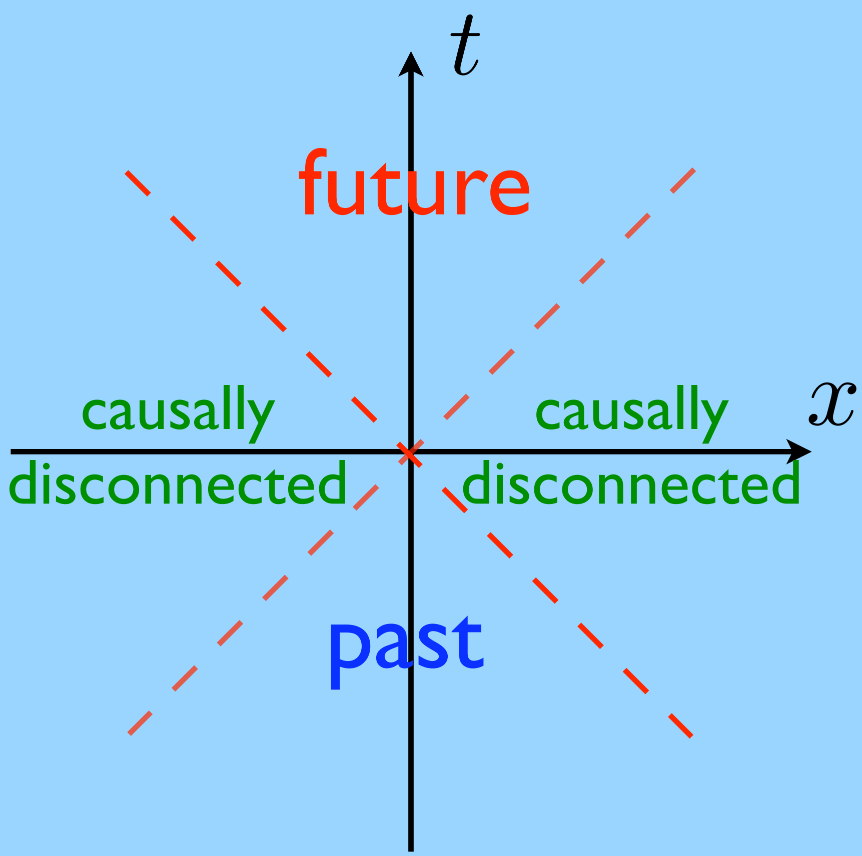


doesn't work in (1+1)d if

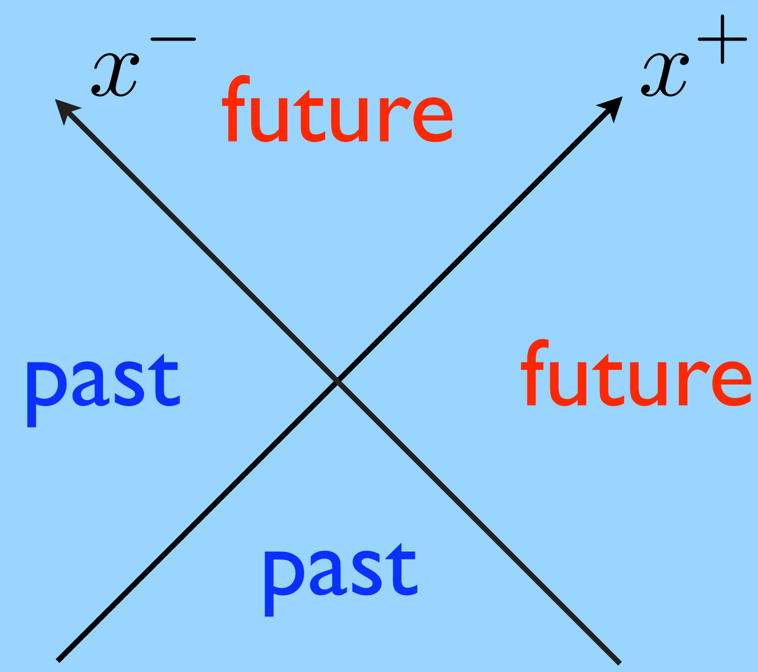
$$x \longrightarrow -x$$

is broken

Equivalently, in (1+1)d one has *two* causal structures compatible with the Poincare group



standard



"instantaneous"

Are there QFT's with instantaneous causal structure?

Straightforward way...

$$S = -\int d^2x (\partial_- A_+)^2$$

No propagating degrees of freedom, c.f. grav. potential in Newton's theory of gravity

A bit less trivial model:

$$S = \int d^2x \left\{ -(\partial_- A_+)^2 + (\partial_+ A_-)^2 - m^2 (A_+ A_-)^2 \right\}$$

NB: "Wick rotation" $x^+ \to -ix^+$ *gives positive Euclidean action*

We ended up studying a bit different class of models.

One reason:

It seems hard to keep instantaneous effects small...

conventional massive field:

$$\partial_+\partial_-\phi = m^2\phi$$

constant x^+- surfaces are not good Cauchy slices

The trick is to get a vector with a vev $\langle V^+\rangle \neq 0$

$$S = \int d^2x(\partial\phi)^2 - m^2\phi^2 + (V^+\partial_+\phi)^2$$

$SO(1,1)$ *nonlinear sigma-model*

aka Einstein-aether theory
aka (Lorentzian) nematic liquid crystal

$$\int d^2x \left(-\alpha_1 \partial_\mu V^\nu \partial^\mu V_\nu - \alpha_2 \partial_\mu V^\mu \partial_\nu V^\nu - \alpha_3 \partial_\mu V^\mu \epsilon^{\nu\lambda} \partial_\nu V_\lambda + \lambda(V^\mu V_\mu - 1)\right)$$

$$V_\pm = \frac{1}{\sqrt{2}} \mathrm{e}^{\mp\psi}$$

$$\int dx_+ dx_- \left\{ \frac{1}{g^2} \partial_+\psi \partial_-\psi + \frac{\beta_+}{2g^2} (\partial_+\psi)^2 \mathrm{e}^{2\psi} + \frac{\beta_-}{2g^2} (\partial_-\psi)^2 \mathrm{e}^{-2\psi} \right\}$$

Three cases:

$$\beta_+ = \beta_- \ , \quad \beta_+ = -\beta_- \ , \quad \text{or } \beta_- = 0$$

$$\beta_+ = -\beta_-$$

$$S = \int dx_+ dx_- \left\{ \frac{1}{g^2} \partial_+ \psi \partial_- \psi + \frac{\beta}{2g^2} \left((\partial_+ \psi)^2 \mathrm{e}^{2\psi} - (\partial_- \psi)^2 \mathrm{e}^{-2\psi} \right) \right\}$$

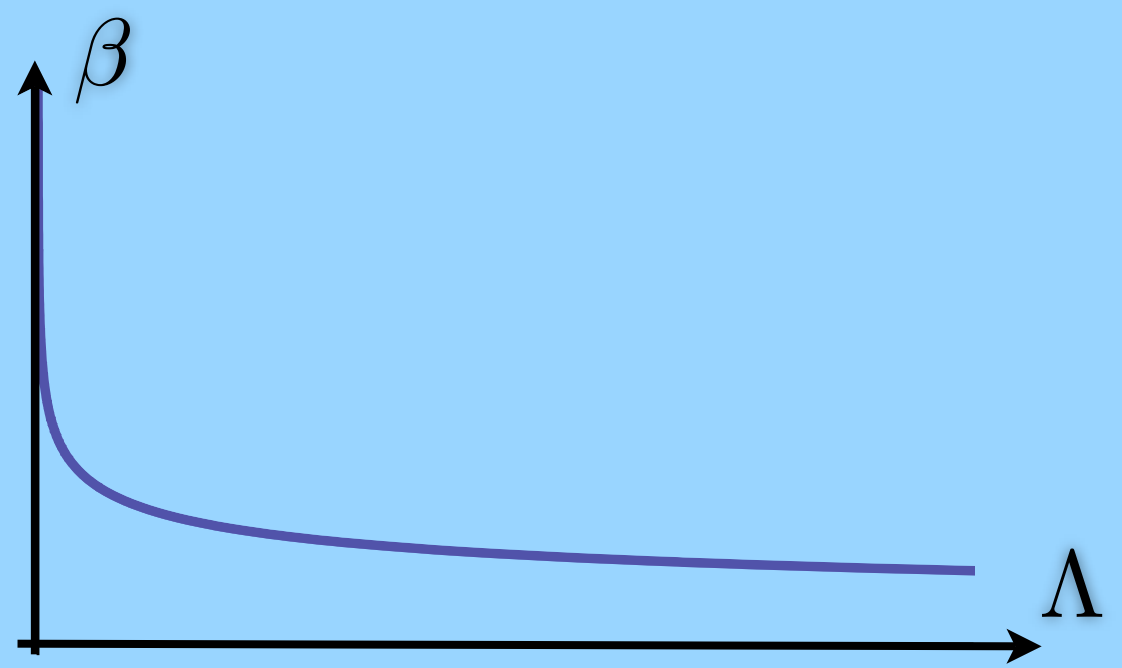
- $\psi(x^+, x^-) \to \psi(\mathrm{e}^\gamma x^+, \mathrm{e}^{-\gamma} x^-) + \gamma$
- Renormalizable
- $x^+ \;\to\; -ix^+$ gives $\mathrm{Re}(S_E) > 0$
- Positive definite Hamiltonian
- Asymptotically free

$$S = \int dx_+ dx_- \left\{ \frac{1}{g^2}\partial_+\psi\partial_-\psi + \frac{\beta}{2g^2}\left((\partial_+\psi)^2 e^{2\psi} - (\partial_-\psi)^2 e^{-2\psi}\right) \right\}$$

1-loop RGE (all orders in β):

$$g = const$$

$$\frac{d\beta}{d\log\Lambda} = -\frac{g^2\beta}{\pi\sqrt{1+\beta^2}}$$



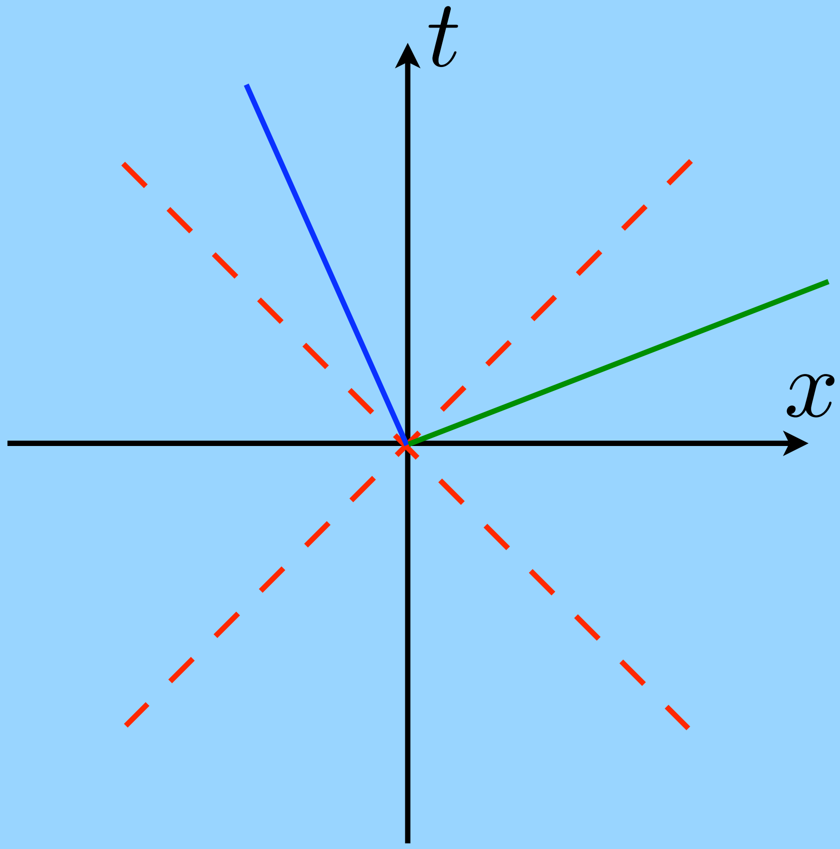
UV: $\frac{1}{g^2}\partial_+\psi\partial_-\psi$

IR: $\frac{1}{2\varkappa^2}\left\{(\partial_+\psi)^2 e^{2\psi} - (\partial_-\psi)^2 e^{-2\psi}\right\}$

$$\varkappa^2 = \frac{g^2}{\beta} \to 0$$

Dispersion relation around $\psi = 0$

$$c_R = \beta + \sqrt{1+\beta^2}\ ,\quad c_L = -\beta + \sqrt{1+\beta^2}$$



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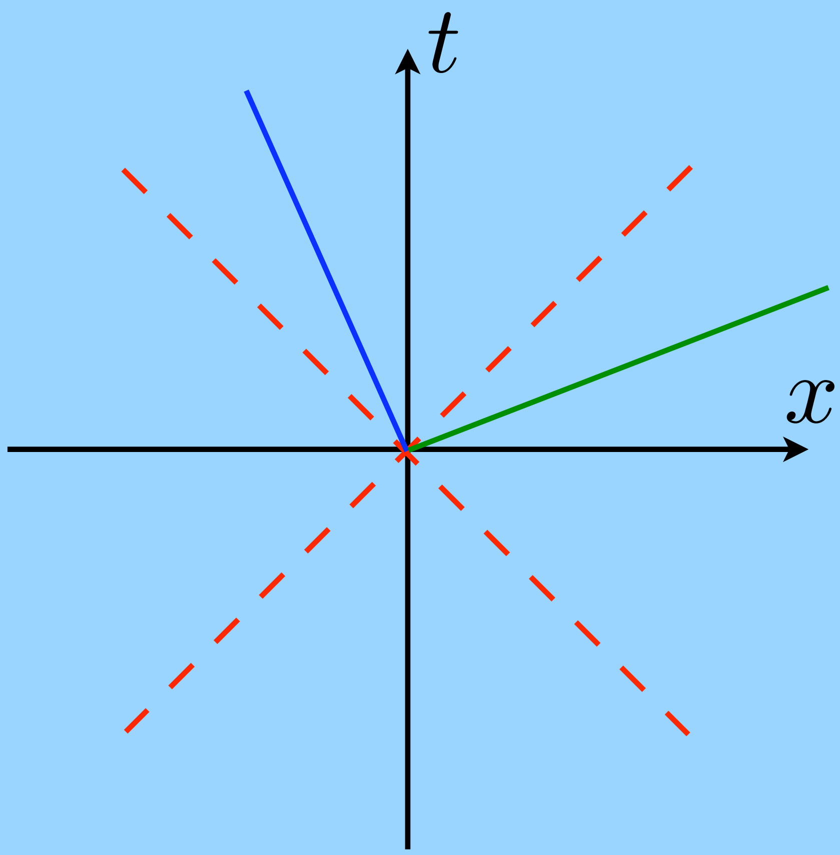
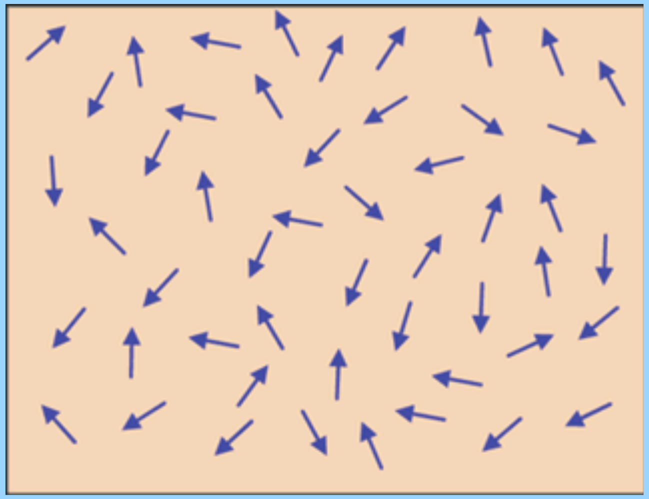


How does this agree with Coleman-Mermin-Wagner?



At long distances directions of the spin average out, but perturbation theory is a good guide for other properties

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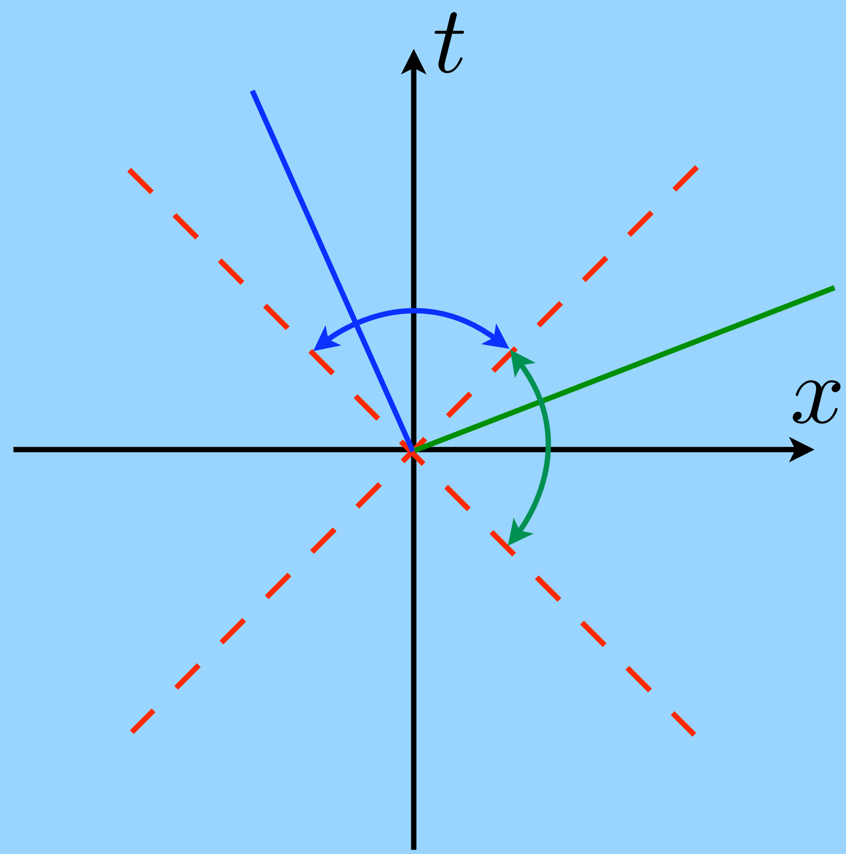

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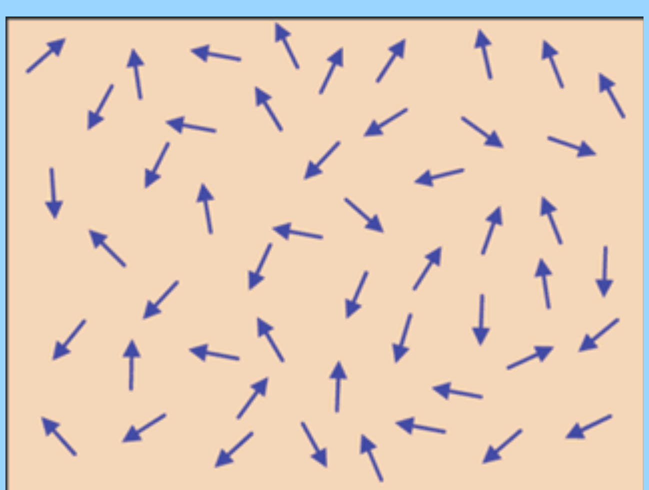

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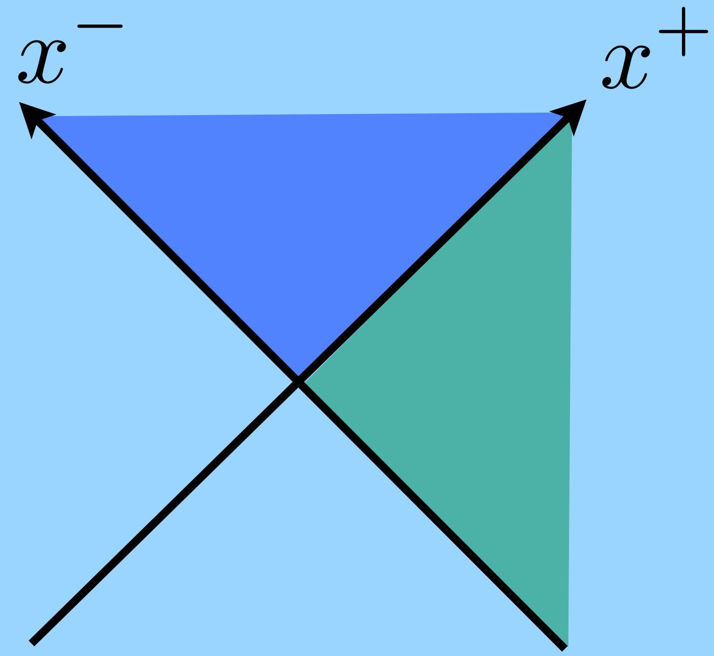

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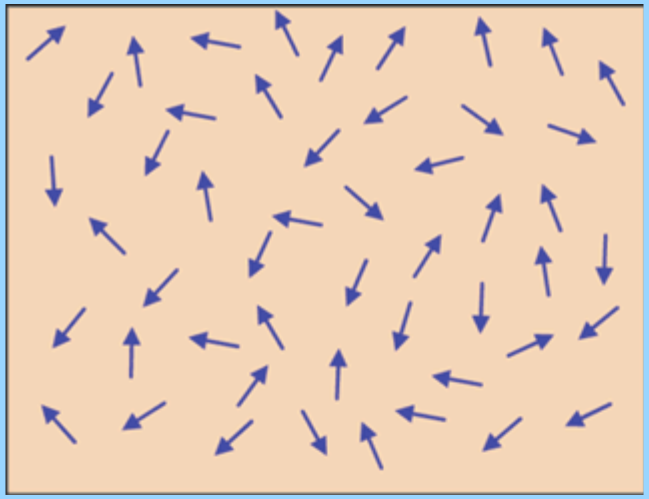


At long distances directions of the spin average out, but perturbation theory is a good guide for other properties

$$\beta_- = 0$$

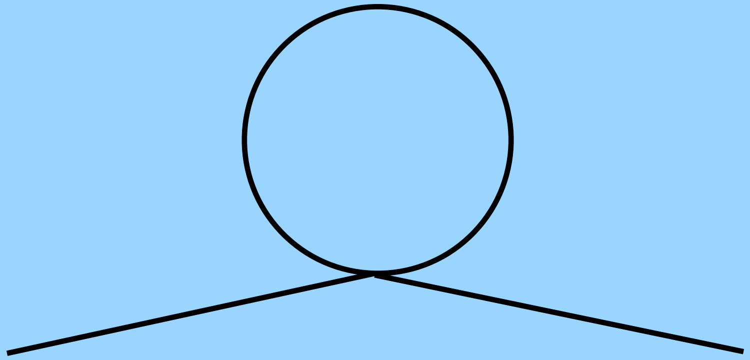
$$S = \int dx_+ dx_- \left\{ \pm \frac{1}{g^2} \partial_+ \psi \partial_- \psi + \frac{\beta}{2g^2} (\partial_+ \psi)^2 \mathrm{e}^{2\psi} \right\}$$

- β can be changed by the shift of ψ
- half+ of the conformal group

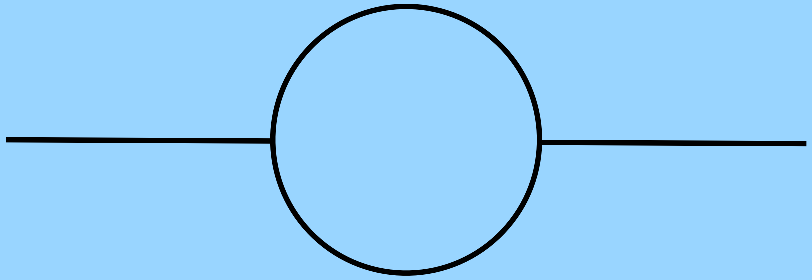
$$\psi(x^+, x^-) \to \psi(g(x^+), f(x^-)) + \frac{1}{2} \log f'(x^-) - \frac{1}{2} \log g'(x^+)$$

where f is arbitrary, $g = \dfrac{ax^+ + b}{cx^+ + d}$, $ad - bc = 1$

- all UV divergences can be removed by normal ordering



normal ordering removes ∞'s



normal ordering doesn't remove ∞'s

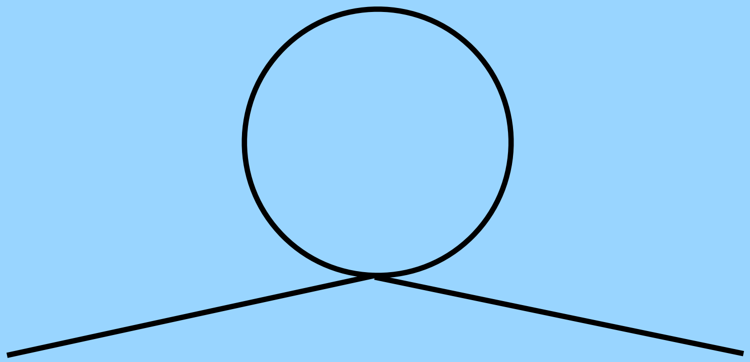
In (1+1)d:

$$\partial_+\psi\partial_-\psi - :U(\psi):$$

finite

$$\partial_+\psi\partial_-\psi - :U(\partial,\psi):$$

not finite



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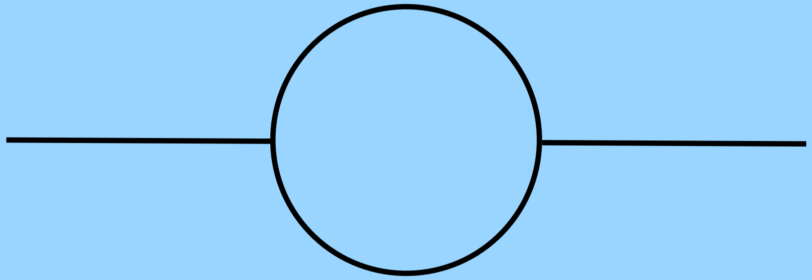

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In (1+1)d:

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finite

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not finite

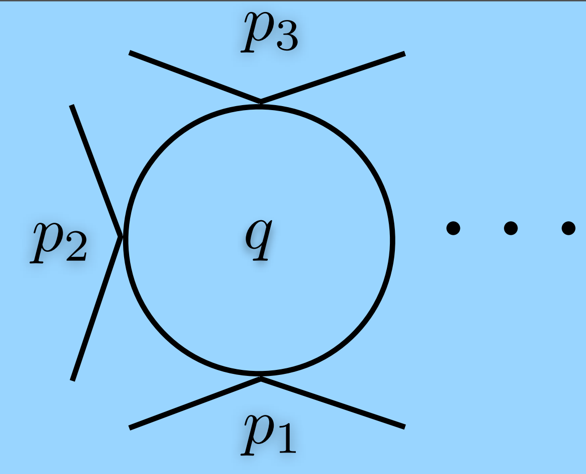
$$\partial_+\psi\partial_-\psi - :U(\partial_+,\psi):$$

finite again!

e.g., a family of Lorentz invariant models:

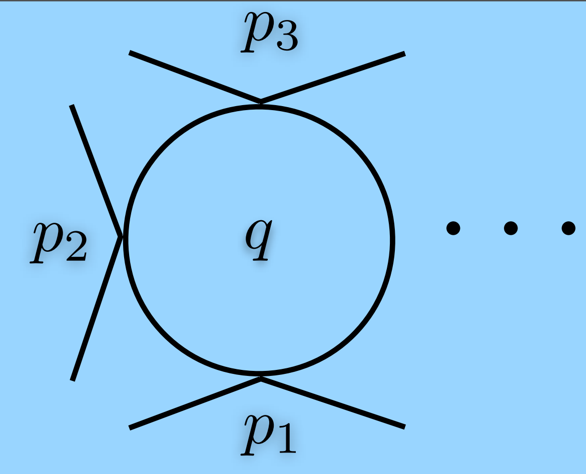
$$\partial_+\psi\partial_-\psi - :U(\partial_+ e^{\psi}):$$

one-loop example



$$\int \frac{d^2q\; q_+^n}{\prod_i \left((q+p_i)^2+\mu^2\right)} = \int_0^\Lambda dq\; q^n \int_0^{2\pi} \frac{d\phi\; \mathrm{e}^{in\phi}}{\prod_i \left(q^2+p_i^2+\mu^2+2qp\cos\left(\phi-\phi_i\right)\right)}$$

one-loop example



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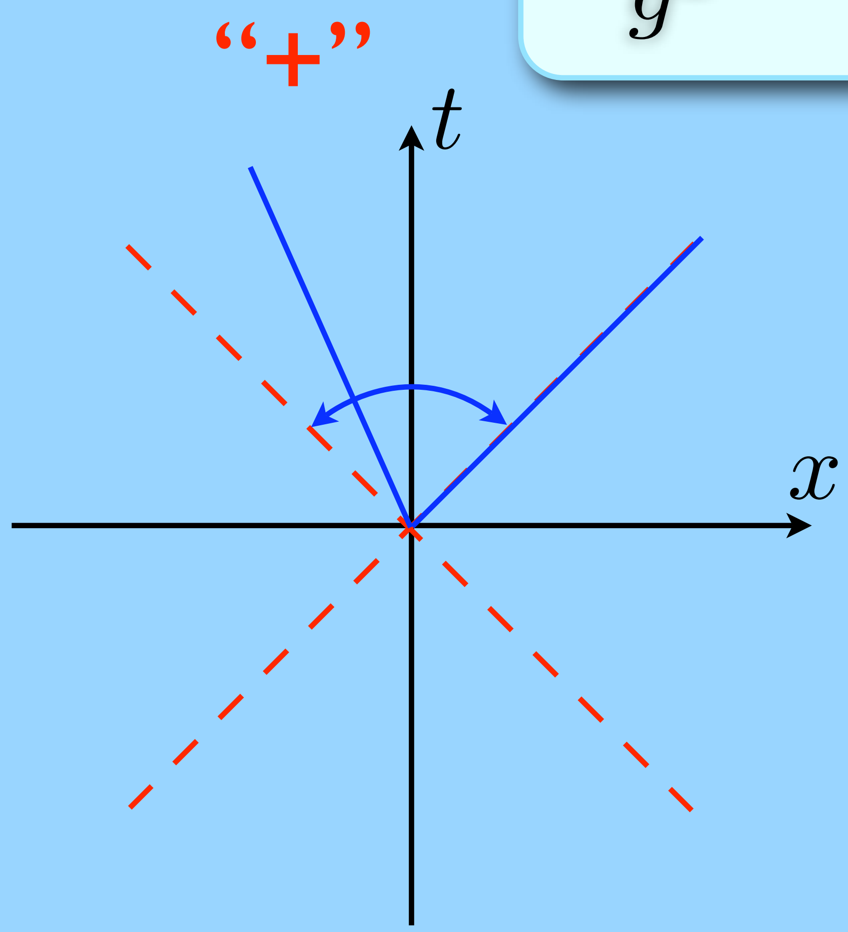
one may hope to obtain non-perturbative information/solve the theory. work in progress...

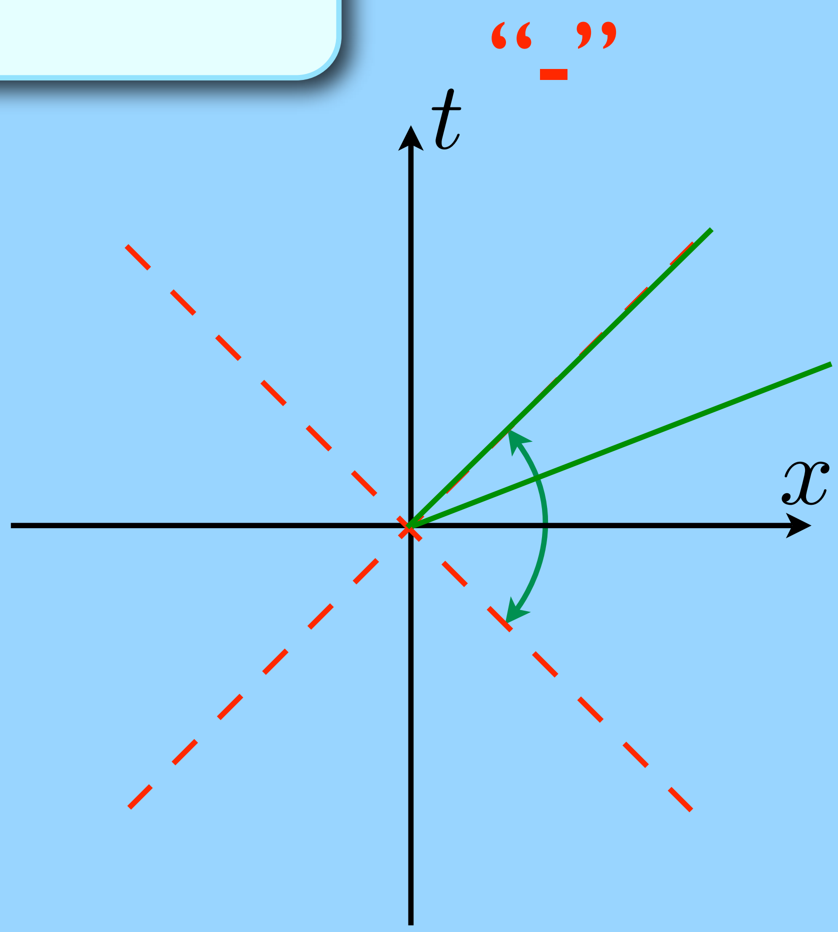
one consequence: $\pm\frac{1}{g^2}\partial_+\psi\partial_-\psi + \frac{\beta}{2g^2}(\partial_+\psi)^2\mathrm{e}^{2\psi}$

$g = const \qquad \beta = Z_\beta\tilde{\beta}$ --- to all orders

scale invariance is preserved at the quantum level

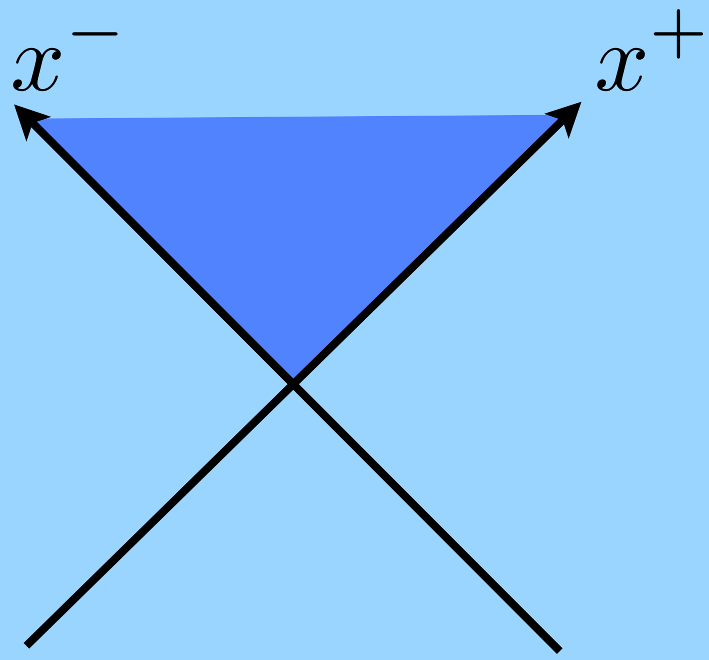
$$\pm \frac{1}{g^2}\partial_+\psi\partial_-\psi + \frac{\beta}{2g^2}(\partial_+\psi)^2 e^{2\psi}$$



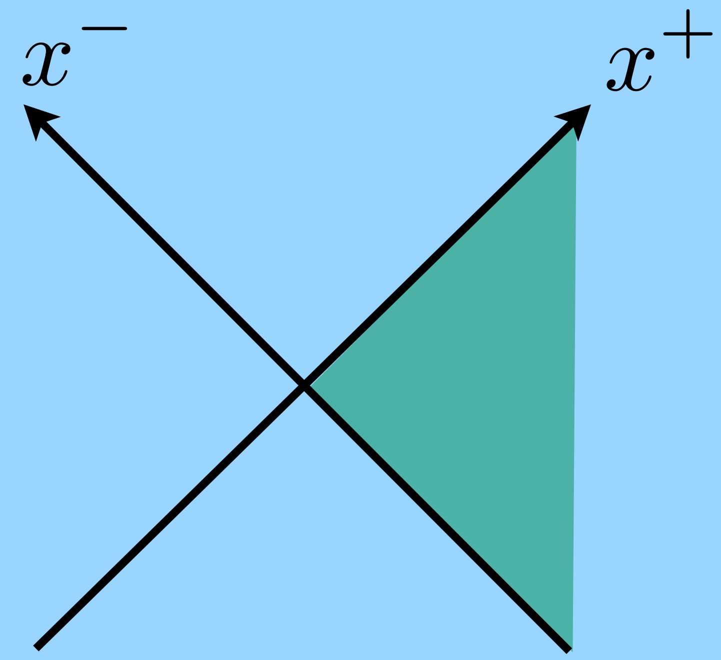


$$\pm\frac{1}{g^2}\partial_+\psi\partial_-\psi+\frac{\beta}{2g^2}(\partial_+\psi)^2\mathrm{e}^{2\psi}$$

"+"

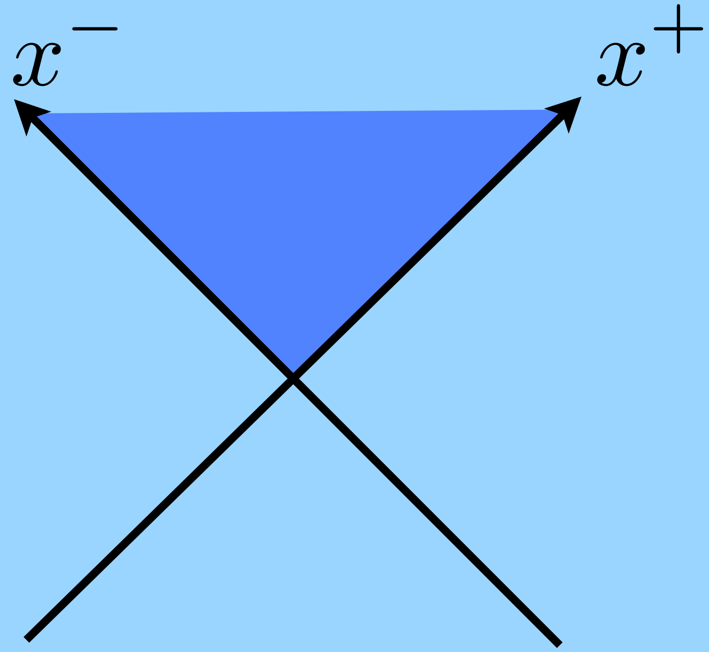


"-"

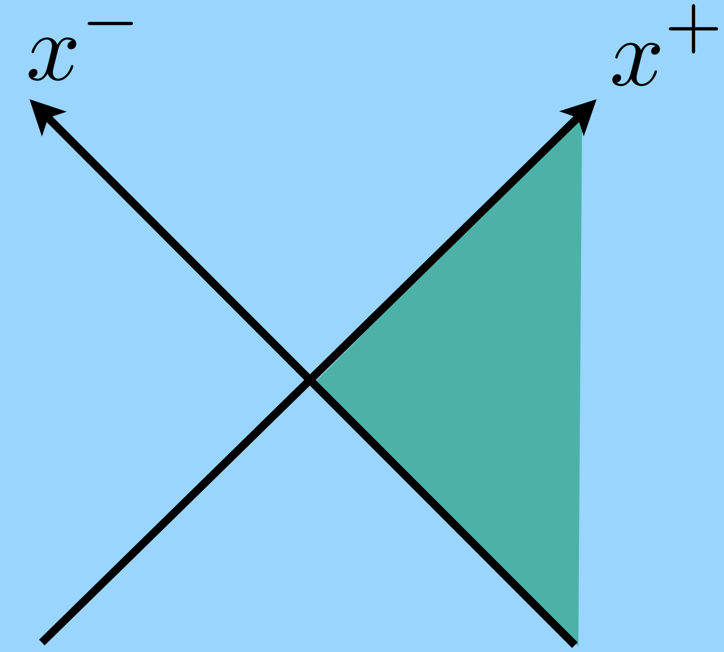


$$\pm\frac{1}{g^2}\partial_+\psi\partial_-\psi + \frac{\beta}{2g^2}(\partial_+\psi)^2\mathrm{e}^{2\psi}$$

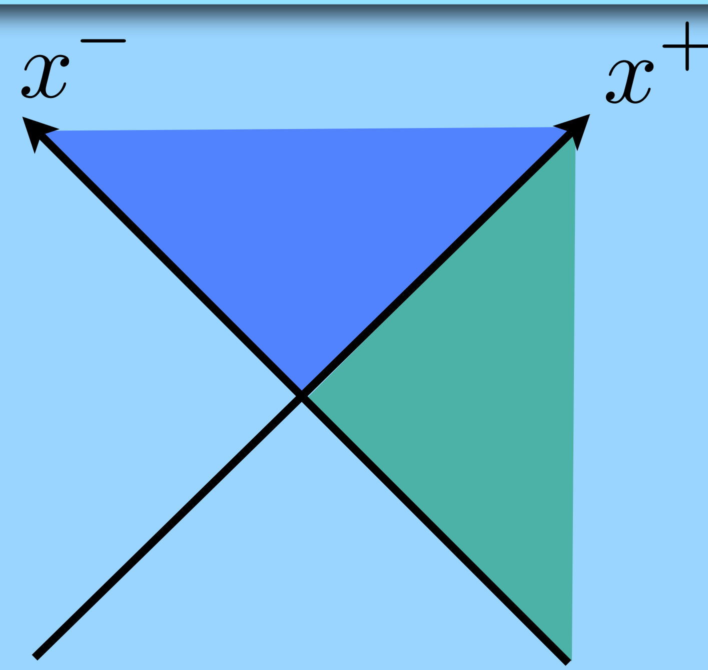
“+”



“-”



$$\frac{1}{g^2}(\partial_+\psi\partial_-\psi - \partial_+X\partial_-X) + \frac{\beta}{2g^2}(\partial_+\psi)^2\mathrm{e}^{2\psi} + \frac{\beta_X}{2g^2}(\partial_+X)^2\mathrm{e}^{2\psi}$$



$$\beta_+ = \beta_-$$

$$S = \int dx_+ dx_- \left\{ \frac{1}{g^2} \partial_+ \psi \partial_- \psi + \frac{\beta}{2g^2} \left((\partial_+ \psi)^2 \mathrm{e}^{2\psi} + (\partial_- \psi)^2 \mathrm{e}^{-2\psi} \right) \right\}$$

$$c_\psi^2 = \frac{1-\beta}{1+\beta}$$

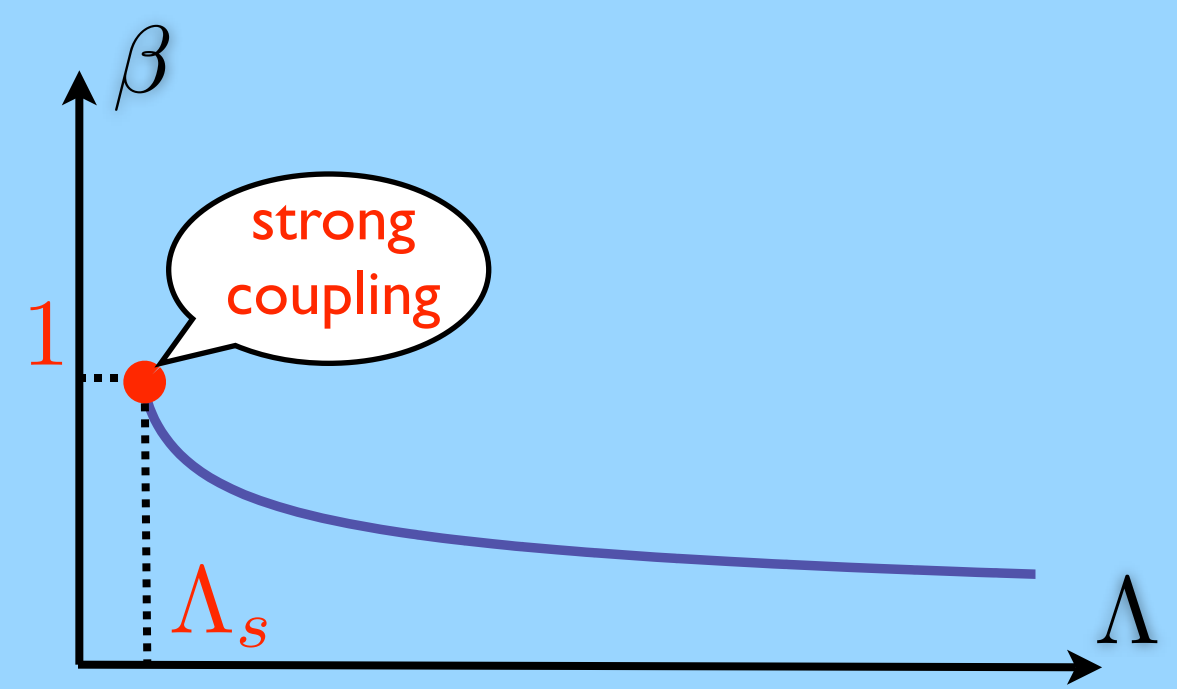
$\Rightarrow$

$|\beta| > 1$ unstable

$0 < \beta < 1$ stable subluminal

$-1 < \beta < 0$ stable superluminal

$$\frac{d\beta}{d \log \Lambda} = - \frac{g^2 \beta}{\pi \sqrt{1 - \beta^2}}$$

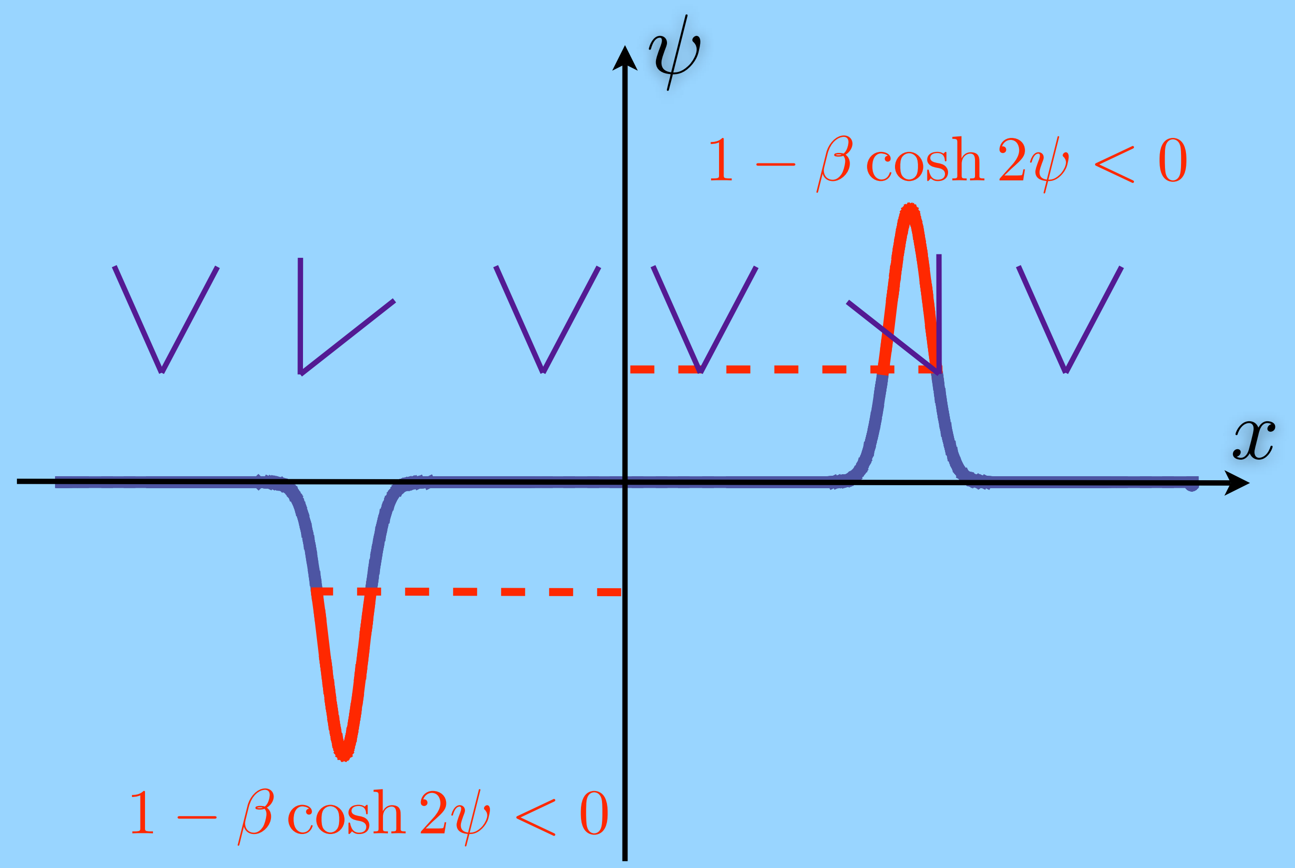


Non-perturbative (in)stability

$$H = \frac{1}{2g^2}\int dx\left\{(1+\beta\cosh 2\psi)\,(\partial_t\psi)^2 + (1-\beta\cosh 2\psi)\,(\partial_x\psi)^2\right\}$$

$$P = \frac{1}{g^2}\int dx\left\{(1+\beta\cosh 2\psi)\,\partial_t\psi\partial_x\psi + \beta\sinh 2\psi\,(\partial_x\psi)^2\right\}$$

subluminal case: $\partial_t\psi(0,x) = 0 \qquad \psi(0,-x) = -\psi(0,x)$



superluminal case: energy on “spacelike” slices is positive

Two options:

- Model doesn't make sense in this case
- Parity is broken spontaneously

In both cases seems interesting to understand what exactly is going on...

Coupling to gravity: preliminary

$$S_{gr} = \frac{1}{2\pi\kappa}\int d^2x\sqrt{-g}R + S_{EA}(g_{\mu\nu}, V_\mu)$$

conformal gauge

$$g_{\mu\nu} = \mathrm{e}^{\phi}\eta_{\mu\nu}$$

$$V_- = \mathrm{e}^{\psi} \qquad V_+ = \mathrm{e}^{\phi-\psi}$$

$$\frac{1}{g^2}\partial_+\psi\partial_-(\psi-\phi) + \frac{\beta_+}{g^2}\mathrm{e}^{2\psi-\phi}(\partial_+\psi)^2 + \frac{\beta_-}{g^2}\mathrm{e}^{\phi-2\psi}\left(\partial_-(\psi-\phi)\right)^2 + S_{Liouv}(\phi)$$

What’s next?

Two ways to look at these models:

(Sophisticated) toys for how non-local physics could look like. Coupling to gravity is clearly the next thing to study. Dilaton gravity in (1+1)d has black holes. An example of a *very radical* resolution of how information gets recovered from the black hole?

2d quantum gravity can be thought of as a world-sheet theory for a (non-critical) string. Is it possible to have a string theory (a theory of extended objects!) with instantaneous excitations on the world-sheet? If yes, will be a way to generate non-local theories in higher dimensions (UV complete Higgs phases of gravity?).

A sample action to start with:

$$S(\phi,\psi,X^i)=\int d^2x\left\{\frac{1}{g^2}\left[\partial_+\psi\partial_-(\psi-\phi)-\partial_+X^i\partial_-X^i\right]+\frac{\beta}{2g^2}e^{2\psi-\phi}\left[(\partial_+\psi)^2+(\partial_+X^i)^2\right]\right\}+S_L(\phi)$$

Both ways, the next question to ask is

Are there 2d CFT's with instantaneous causal structure?

If unitary instantaneous CFT's are there, what happens on the AdS side??

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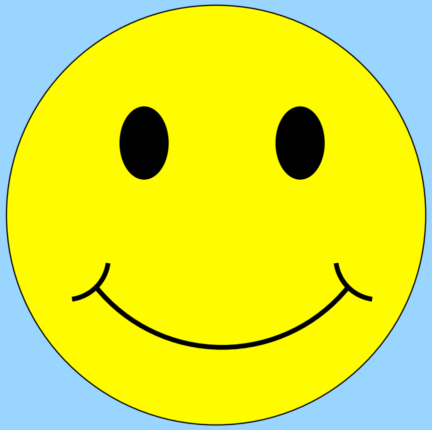
Are there 2d CFT's with instantaneous causal structure?

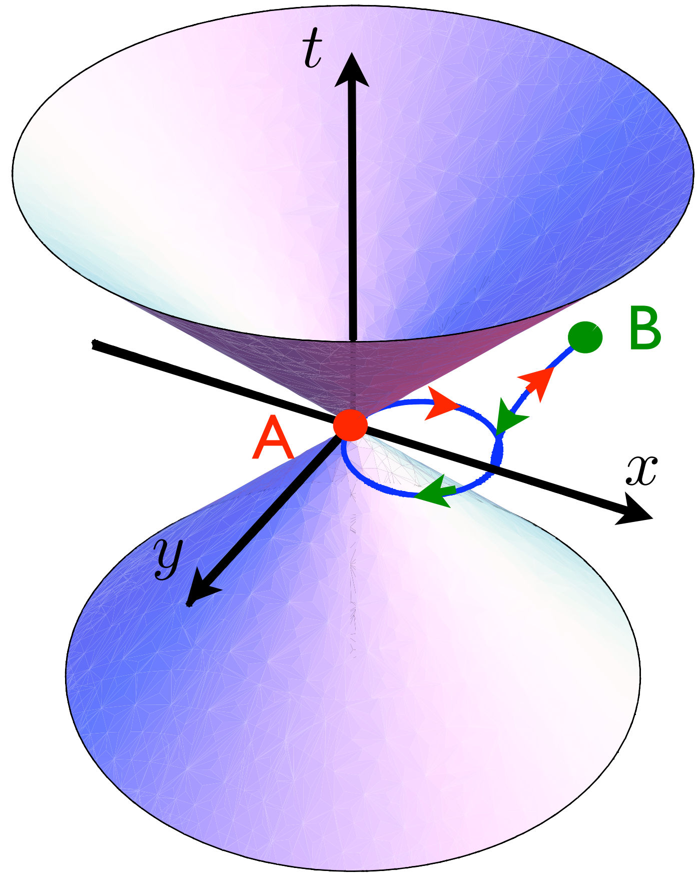
If unitary instantaneous CFT's are there, what happens on the AdS side??

Overall conclusion: instantaneous 2d theories are quite a bit of fun.

Being conservative, one can be quite confident that even if some inconsistency is to be found, it won't be a stupid one and we will learn more both about QFT and gravity.

THANK YOU!





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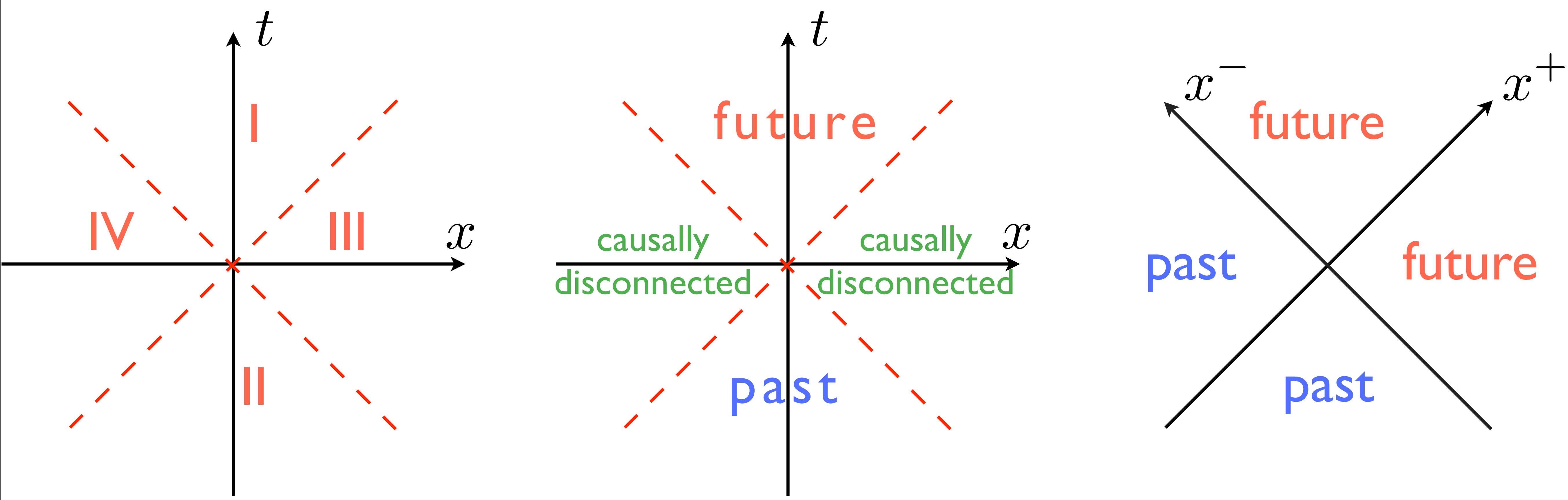

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