Symmetries, Horizons, and Black Hole Entropy

Steve Carlip
U.C. Davis

UC Davis
June 2007

Black holes behave as thermodynamic objects

$$\boldsymbol{T = \frac{\hbar \kappa}{2\pi c}}$$

$$\boldsymbol{S_{BH} = \frac{A}{4\hbar G}}$$

Quantum ($\hbar$) and gravitational ($\boldsymbol{G}$)

Does this thermodynamic behavior have a microscopic explanation?

The problem of “universality” of black hole entropy

Black hole entropy counts:

- Weakly coupled string and D-brane states
- Horizonless “fuzzball” geometries
- States in a dual conformal field theory “at infinity”
- Spin network states crossing the horizon
- Spin network states inside the horizon
- “Heavy” degrees of freedom in induced gravity
- Points in a causal set in the horizon’s domain of dependence
- Entanglement entropy (maybe holographic)
- No local states—it’s inherently global
- Nothing—it comes from quantum field theory in a fixed background, and doesn’t know about quantum gravity

Answer: apparently, all of the above

> Is there an underlying mechanism that can explain why these approaches all agree?

A small detour:
entropy and the Cardy formula

Any two-dimensional conformal field theory
can be characterized by generators $L[\xi]$ and $\bar{L}[\bar{\xi}]$
of holomorphic and antiholomorphic diffeomorphisms

Virasoro algebra:

$$[L[\xi], L[\eta]] = L[\eta\xi' - \xi\eta'] + \frac{c}{48\pi}\int dz\, (\eta'\xi'' - \xi'\eta'')$$

Central charge c (“conformal anomaly”) depends on theory
Conserved charge $L_0 \sim$ energy

Consider a conformal field theory with
- central charge c
- lowest eigenvalue Δ_0 of $L[\xi_0]$

Cardy:
For $L_0 = \Delta$ large, the density of states is asymptotically

$$\boxed{\ln\rho(L_0) \sim 2\pi\sqrt{\frac{(c - 24\Delta_0)\Delta}{6}}}$$

Entropy is fixed by symmetry, independent of details!

Why this might help:

matter near a horizon looks conformal

Black hole in “tortoise” coordinates:

$$ds^2 = N^2(dt^2 - dr_*{}^2) + ds_\perp{}^2$$

($N \longrightarrow 0$ at horizon)

Scalar field:

$$(\Box - m^2)\varphi = \frac{1}{N^2}(\partial_t^2 - \partial_{r_*}^2)\varphi + O(1)$$

Mass and transverse excitations become negligible
Effective two-dimensional conformal field (at each point)

Wilczek, Robinson, Iso, Morita, Umetsu:
two-dimensional CFT gives Hawking flux, spectrum

Medved, Martin, Visser:
conformal symmetry is generic at Killing horizon

The (2+1)-Dimensional Example

Rotating black hole
in three spacetime dimensions (BTZ black hole):

– standard horizon, causal structure

– asymptotically anti-de Sitter

– entropy $S = \frac{2\pi r_+}{4\hbar G}$

– but no propagating degrees of freedom

Anti-de Sitter “boundary” is a cylinder

– asymptotic symmetries $\Rightarrow$ Virasoro algebra

– classical central charge

– Cardy formula $\Rightarrow$ correct entropy

– source: induced “boundary” conformal field theory
(early case of AdS/CFT correspondence)

Hard to generalize directly, but some lessons. . .

Horizons and constraints
or how to ask about a black hole
in quantum gravity

Standard approach:
Fix black hole background, ask about quantum fields,
gravitational perturbations, etc.

You can't do that in quantum gravity!

Alternative:
Ask a conditional question. . .
impose black hole characteristics as constraints

For example:
– Restrict path integral to metrics with horizons, or
– Add constraints to canonical theory requiring a horizon

A model:

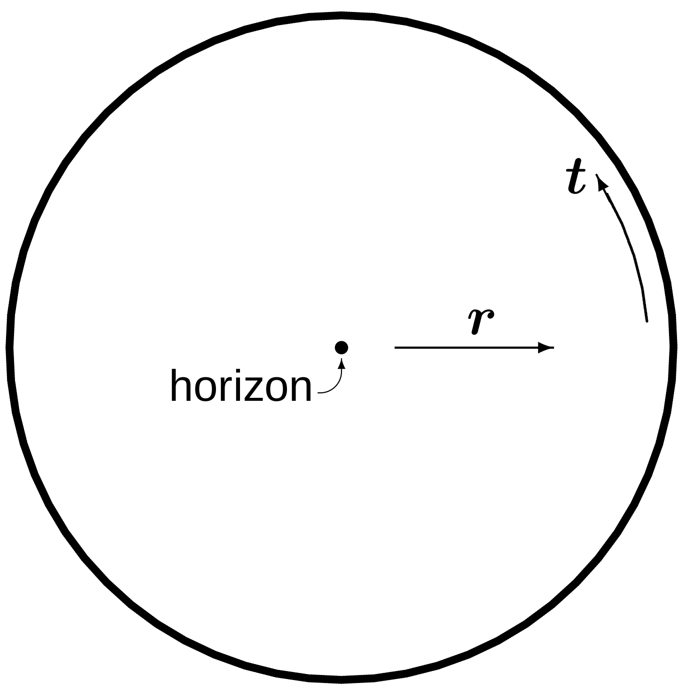
Two-dimensional Euclidean dilaton gravity with horizon constraints

Dimensionally reduce to "r–t plane":

$$I = \frac{1}{2}\int d^2x\sqrt{g}\left[\varphi R + V[\varphi] - \frac{1}{2}W[\varphi]h_{IJ}F^I_{ab}F^{Jab}\right]$$

Continue to "Euclidean" signature and evolve radially:

$$ds^2 = N^2 f^2 dr^2 + f^2(dt + \alpha dr)^2$$



$$t^2 - r^2 = 0 \Rightarrow t^2 + r^2 = 0$$

Find constraints:

$$\mathcal{H}_{\|} = \dot{\varphi}\pi_{\varphi} - f\dot{\pi}_{f} = 0$$
$$\mathcal{H}_{\perp} = f\pi_{f}\pi_{\varphi} + f\left(\frac{\dot{\varphi}}{f}\right)^{\cdot} - \frac{1}{2}f^{2}\hat{V} = 0$$
$$\mathcal{H}_{I} = \dot{\pi}_{I} - c^{J}{}_{IK}A^{K}\pi_{J} = 0$$

Combine to form Virasoro generators:

$$L[\xi] = \frac{1}{2}\int dt\xi(\mathcal{H}_{\|} + i\mathcal{H}_{\perp})$$
$$\bar{L}[\bar{\xi}] = \frac{1}{2}\int dt\xi(\mathcal{H}_{\|} - i\mathcal{H}_{\perp})$$

So far, central charge $c = 0\ldots$

Geometrical quantities:

$$\text{expansion} \qquad s = f\pi_f - i\dot{\varphi}$$

$$\text{surface gravity} \qquad \hat{\kappa} = \pi_\varphi - i\dot{f}/f + f^2\frac{d\omega}{d\varphi}$$

(conformal factor ω to be determined)

Stretched horizon constraints:

$$K = s - a(\hat{\kappa} - \hat{\kappa}_H) = 0$$
$$\bar{K} = \bar{s} - a(\bar{\hat{\kappa}} - \bar{\hat{\kappa}}_H) = 0$$

Dirac-Bergmann-Komar brackets:

Let Δ_{ij} be the inverse of $\{K_i, K_j\}$. Then

$$O^* = O - \sum_{i,j} \int dudv \{O, K_i(u)\} \Delta_{ij}(u,v) K_j(v)$$

will have vanishing Poisson brackets with the K_i.

Horizon algebra:

Fix conformal factor ω, constant a
by demanding that $L^*[\xi]$ and $\bar{L}^*[\bar{\xi}]$ have nice algebra

Find Virasoro algebra with

$$c = \bar{c} = \frac{3\varphi_H}{4G}, \qquad \Delta = \bar{\Delta} = \frac{\varphi_H}{16G}\left(\frac{\kappa_H\beta}{2\pi}\right)^2$$

$$\text{Cardy formula} \Rightarrow S = \frac{2\pi\varphi_H}{4G}\left(\frac{\kappa_H\beta}{2\pi}\right)$$

2π times standard Bekenstein-Hawking entropy
(summed over periodic time?)

Universality again

> If this mechanism is universal,
> the same symmetry-breaking should be present
> in other derivations of black hole entropy.

String theory and AdS/CFT:

Near-extremal black holes have near-horizon structure
$$\approx \mathbf{BTZ\ black\ hole \times trivial}$$
Compute BTZ entropy from AdS/CFT correspondence

This involves a conformal field theory at infinity. . .

	$\underline{BTZ}$	$\underline{horizon}$
$modes$	$\xi_n \sim e^{in(t\pm\ell\phi)/\ell}$	$\xi_n \sim e^{in\kappa_H t}$
c	$\dfrac{3\ell}{2G}$	$2\pi \cdot \dfrac{3\varphi_H}{4\pi G}$
$\Delta_{L,R}$	$\dfrac{(r_+ \pm r_-)^2}{16G\ell}$	$2\pi \cdot \dfrac{\varphi_H}{32\pi G}\left(\dfrac{\kappa_h\beta}{2\pi}\right)^2$

Same entropy, but different central charges, conformal weights

But. . .

- match mode frequencies:

  restriction to modes $\boldsymbol{\xi_{Nn}} \Rightarrow$

  $\boldsymbol{c \rightarrow cN, \;\; \Delta \rightarrow \Delta/N}$

  where here, $\boldsymbol{N = \ell/r_+}$

- switch to corotating coordinates at horizon:

  $\boldsymbol{\phi' = \phi - (r_-/r_+\ell)t} \Rightarrow$

  $\boldsymbol{\beta_\pm = (1 \pm r_-/r_+)\beta}$

Then conformal field theories match perfectly!

Loop quantum gravity:

Loop horizon states $\Rightarrow$ SL(2,**C**) Chern-Simons theory
with $\boldsymbol{k = iA/8\pi\gamma G}$

Induced boundary Liouville theory has $\boldsymbol{c = 6k}$

For $\boldsymbol{\gamma = i}$, this matches central charge here
(relation to Alexandrov's Lorentz-invariant approach?)

"Horizon as boundary" approach:

Central charges match

"$\boldsymbol{\Delta}$ as Komar integral" (Emparan and Mateos):

Conformal weights agree with 2-dimensional Komar integral

Path integral:

Work in progress. . .

What are the states?

Standard treatment of constraints (Dirac):

$$L[\xi]|phys\rangle = \bar{L}[\xi]|phys\rangle = 0$$

Not consistent with Virasoro algebra with $c \neq 0$:

Must weaken constraints—
e.g., only require positive-frequency part annihilate $|phys\rangle$
$\Rightarrow$ formerly nonphysical "gauge" states become physical
e.g., descendant states in CFT (relation to Cardy formula?)

Analogy: Nambu-Goldstone bosons

For scalar: ground state breaks rotational invariance,
states differing by rotation $\Rightarrow$ massless degrees of freedom

For black hole: horizon constraints break diffeo invariance,
states differing by relevant diffeomorphism $\Rightarrow$ horizon degrees of freedom