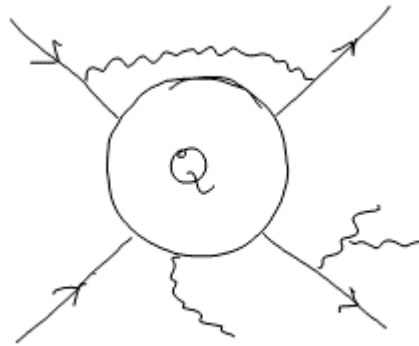


Superleading logarithms in QCD

- Soft gluons in QCD: an introduction.
- Gaps between jets I: the old way (<2001).
- A second example: Higgs plus two jets.
- Gaps between jets II: the new way (<2006).
- Superleading logarithms: the newer way?

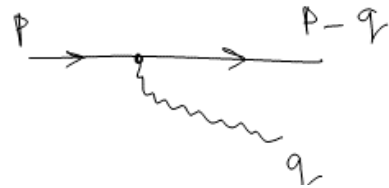
Given a particular hard scattering process we can ask how it will be dressed with additional radiation:



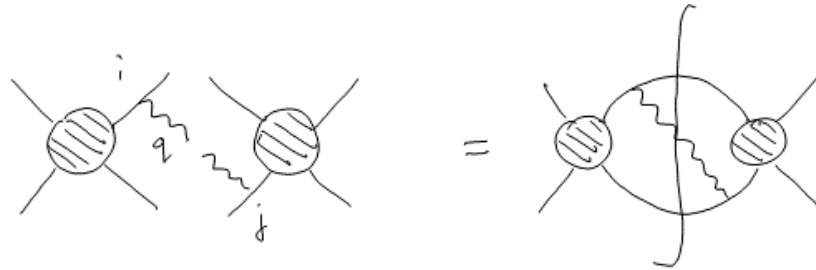
This question may not be interesting a priori because hadronization could wreck any underlying partonic correlations. However experiment reveals that the hadronization process is ‘gentle’.

The most important emissions are those involving either collinear quarks/gluons or soft gluons. By important we mean that the usual suppression in the strong coupling is compensated by a large logarithm.

SOFT GLUONS:



$$= 2gp_\mu \delta_{\lambda\lambda'} T_{ij}^a$$



$$d\sigma_{n+1} = d\sigma_n \frac{\alpha_s}{2\pi} \frac{dE}{E} \frac{d\Omega}{2\pi} \sum_{ij} C_{ij} E^2 \frac{p_i \cdot p_j}{p_i \cdot q p_j \cdot q}$$

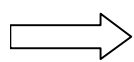
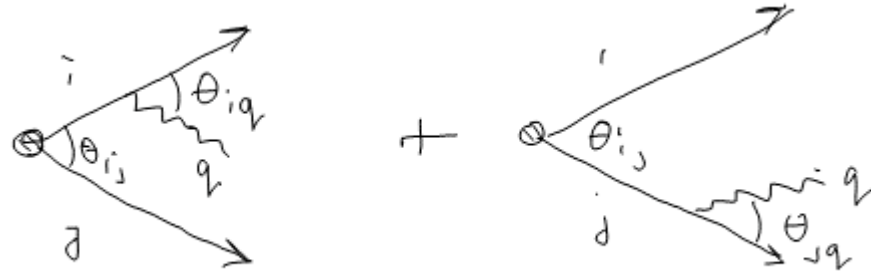
Only have to consider soft gluons off the external legs of a hard subprocess since internal hard propagators cannot be put on shell.

Virtual corrections are included analogously....of which more later....

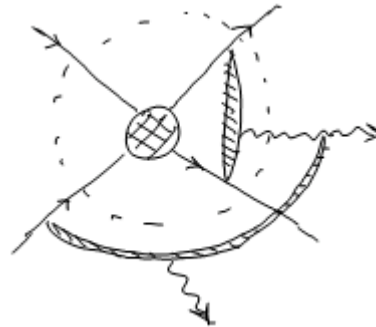
Only need to consider gluons

$$\int \frac{d\phi}{2\pi} E^2 \frac{p_i \cdot p_j}{p_i \cdot q p_j \cdot q} = \frac{\Theta(\theta_{ij} - \theta_{iq})}{1 - \cos \theta_{iq}} + \frac{\Theta(\theta_{ij} - \theta_{jq})}{1 - \cos \theta_{jq}}$$

Note: may not always be safe to average over azimuth.



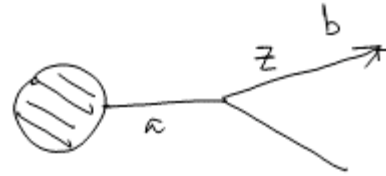
Can build up soft gluon effects starting from a hard scatter and successively adding soft gluons by emitting them off pairs of harder partons into cones of ever decreasing opening angle.



Major difficulty is the colour factor (for hard scattering with >3 external particles).

COLLINEAR EMISSIONS:

Colour structure is easier. It is as if emission is off the parton to which it is collinear ~ “classical branching”.



$$d\sigma_{n+1} = d\sigma_n \frac{\alpha_s}{2\pi} \frac{dq^2}{q^2} dz P_{ba}(z)$$

HERWIG: soft and/or collinear evolution is handled simultaneously using “angular ordered parton evolution”.

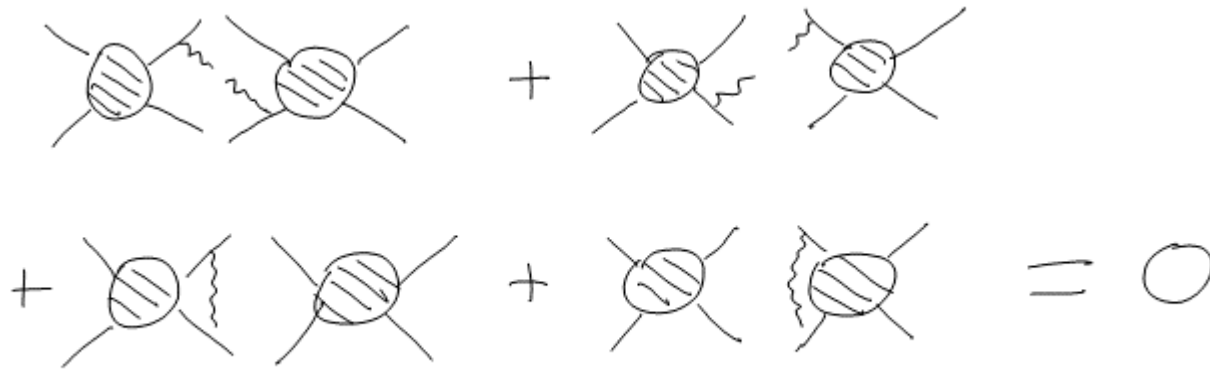
OK in the large N approximation where colour simplifies hugely. Also azimuthal averaging.

Similar approach in PYTHIA.

Not all observables are affected by soft and/or collinear enhancements

Intuitive: imagine the e^+e^- total cross-section. It cannot care that the outgoing quarks may subsequently radiate additional soft and/or collinear particles (causality and unitarity).

Bloch-Nordsieck: *soft gluon corrections cancel in “sufficiently inclusive” observables.*



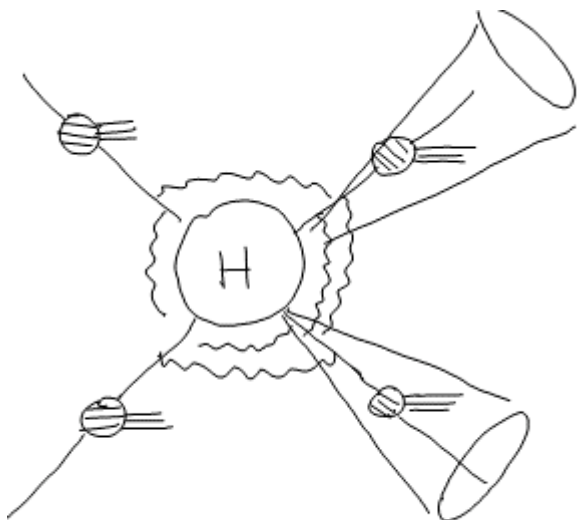
Miscancellation can be induced by restricting the real emissions in some way.

All observables are “sufficiently inclusive” to guarantee that the would-be soft divergence cancels (no detector can detect zero energy particles). But the miscancellation may leave behind a logarithm, e.g. if real emissions are forbidden above μ

$$\alpha_s \int_{\mu}^Q \frac{dE}{E} = \alpha_s \ln \frac{Q}{\mu}$$

A similar cancellation occurs for emissions collinear to the outgoing particles provided they are summed inclusively.

Radiation collinear to the incoming particles is *not* cancelled – need to account for the large collinear logarithms which can always be *factorized* into the incoming parton density functions (DGLAP).



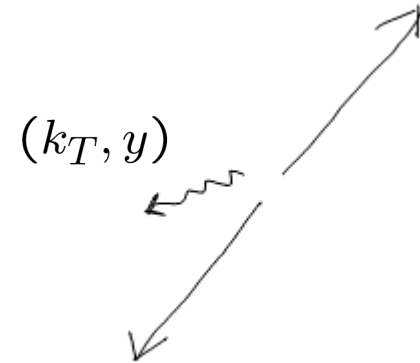
Calculation of a generic observable:
collinear radiation factorizes whilst soft radiation can be accounted for by dressing the hard scattering with soft gluons.

An example: the **thrust** distribution

$$T = \max \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|} \quad T = 1 \text{ for } e^+e^- \rightarrow q\bar{q} \text{ in lowest order}$$

Consider

$$f(\tau) = \int_{1-\tau}^1 dT \frac{1}{\sigma} \frac{d\sigma}{dT}$$



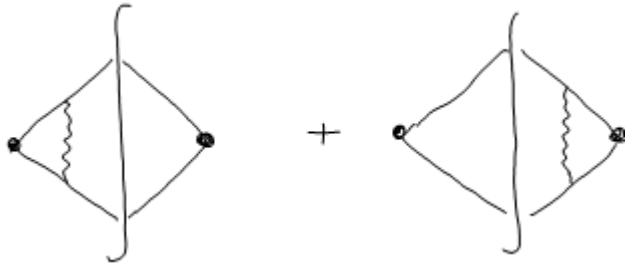
Clearly gluons cannot be emitted at too large an angle if they are to produce a final state which contributes to this integral, i.e. real emissions are forbidden if they satisfy

$$\frac{k_T}{Q} \exp(-|y|) > \tau$$

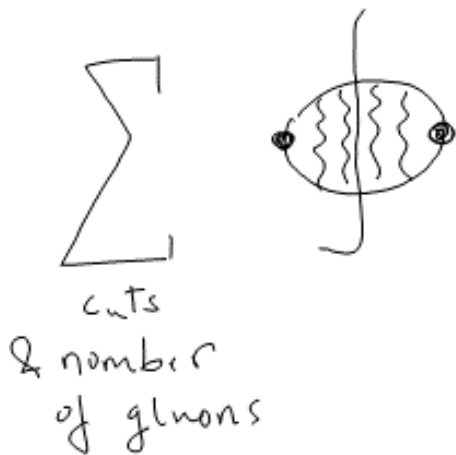
The absence of these real emissions leaves behind uncanceled virtual soft gluon corrections which we must account for....recall that all other emissions cancel between real and virtual graphs due to Bloch-Nordsieck.

The first soft gluon correction, integrated over the disallowed region for real emission, arises after multiplying the lowest order result by $\frac{k_T}{Q} \exp(-|y|) > \tau$

$$\frac{\alpha_s}{\pi} C_F \int_{\tau Q}^Q \frac{dk_T}{k_T} \int_0^{\ln(k_T/(\tau Q))} dy = \frac{\alpha_s}{2\pi} C_F \ln^2 \frac{1}{\tau}$$



$$f(\tau) = 1 - \frac{\alpha_s}{\pi} C_F \ln^2 \frac{1}{\tau}$$



$$f(\tau) = \exp \left(-\frac{\alpha_s}{\pi} C_F \ln^2 \frac{1}{\tau} \right)$$

Double logarithmic suppression....

Double logs because observable restricts gluons which are both soft AND collinear (i.e. energy and transverse momentum are small on the scale of the CM energy Q).

This is a Sudakov suppression. It looks like a poissonian suppression corresponding to the probability not to emit soft-collinear gluons in the forbidden region.

Next step would be to re-compute to single log accuracy....

Banfi, Salam & Zanderighi: automated resummations
Marchesini & Dokshitzer: classical nature of soft gluon radiation

We have seen that soft gluon corrections will be important for observables which insist on only *small deviations from lowest order kinematics*.

In such cases real radiation is constrained to a small corner of phase space and BN miscancellation induces large logarithms.

If V measures 'distance' from the lowest order kinematics:

Event shapes such as thrust ($V = 1 - T$)

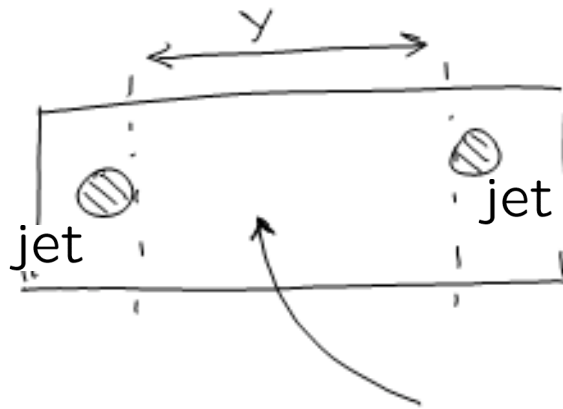
Production near threshold (top, W/Z) ($V = 1 - M^2/\hat{s}$)

Drell-Yan at low p_T (W/Z or Higgs) ($V = p_T^2/\hat{s}$)

Deep-inelastic scattering at large x ($V = 1 - x$)

Gaps between jets....

GAPS BETWEEN JETS:



Jets produced with $p_T = Q \gg Q_0$

No radiation in between jets with $k_T > Q_0$

Observable restricts soft emission in the gap region therefore expect

$$\alpha_s^n \ln^n(Q/Q_0)$$

i.e. do not expect collinear enhancement since we sum inclusively over the collinear regions of the incoming and outgoing partons.

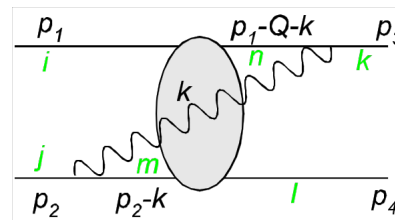
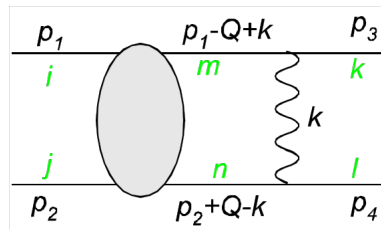
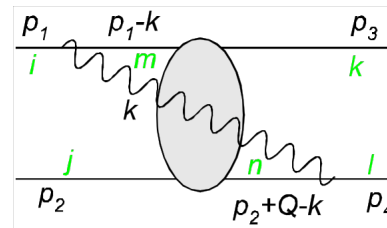
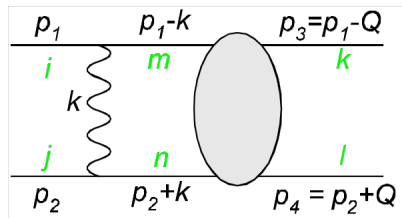
We start with the original calculation of Oderda & Sterman...and work only with quark-quark scattering.

Real emissions are forbidden in the phase-space region

$$-Y/2 < y < Y/2$$

$$k_T > Q_0$$

“By Bloch-Nordsieck, all other real emissions cancel and we therefore only need to compute the virtual soft gluon corrections to the primary hard scattering.”



The soft gluon is integrated over “in gap” momenta.

(plus two others)

The amplitude can be projected onto a colour basis:

$$(M)_{ij}^{kl} = M^{(1)} C_{ijkl}^{(1)} + M^{(8)} C_{ijkl}^{(8)}$$

$$C_{ijkl}^{(8)} = (T^a)_{ik} (T^a)_{jl}$$

$$C_{ijkl}^{(1)} = \delta_{ik} \delta_{jl}$$

i.e.
$$\mathbf{M} = \begin{pmatrix} M^{(1)} \\ M^{(8)} \end{pmatrix} \quad \text{and} \quad \sigma = \mathbf{M}^\dagger \mathbf{S}_V \mathbf{M}$$

$$\mathbf{S}_V = \begin{pmatrix} N^2 & 0 \\ 0 & \frac{N^2-1}{4} \end{pmatrix}$$

Iterating the insertion of soft virtual gluons builds up the Nth order amplitude:

$$\mathbf{M} = \exp \left(-\frac{2\alpha_s}{\pi} \int_{Q_0}^Q \frac{dk_T}{k_T} \mathbf{\Gamma} \right) \mathbf{M}_0$$

The factorial needed for exponentiation arises as a result of ordering the transverse momenta of successive soft gluons, i.e.

$$Q_0 \ll k_{T1} \dots \ll k_{TN} \ll Q$$

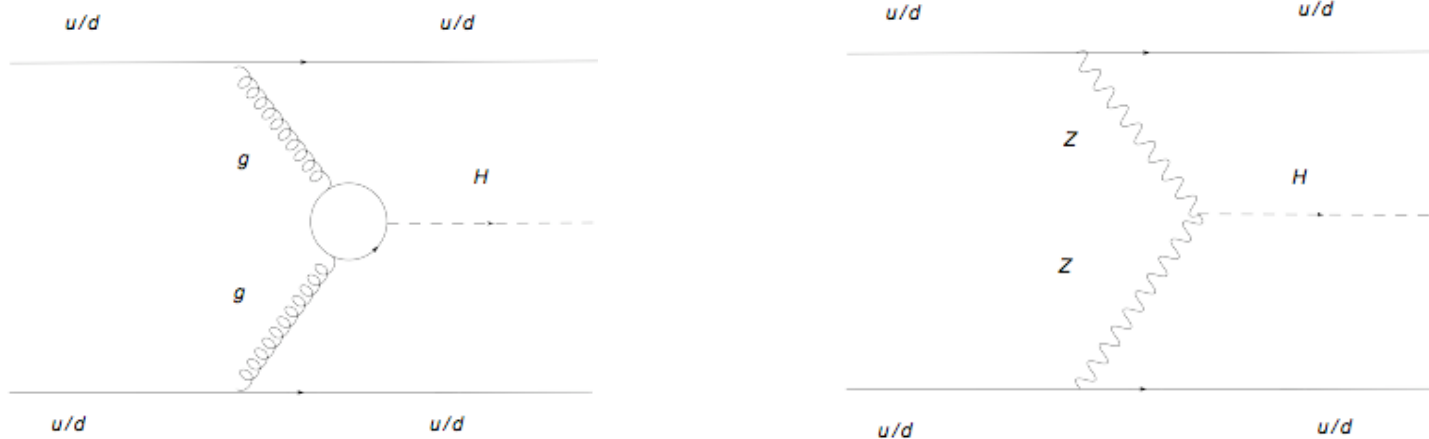
where the evolution matrix is

$$\mathbf{\Gamma} = \begin{pmatrix} \frac{N^2-1}{4N} \rho(Y, \Delta y) & \frac{N^2-1}{4N^2} i\pi \\ i\pi & -\frac{1}{N} i\pi + \frac{N}{2} Y + \frac{N^2-1}{4N} \rho(Y, \Delta y) \end{pmatrix}$$

Δy = distance between jet centres

Y = size of gap

An example: Higgs plus two jets



- To reduce backgrounds and to focus on the VBF channel, experimenters will make a veto on additional radiation between the tag jets, i.e. no additional jets with

$$k_T \geq Q_0$$

- Soft gluon effects will induce logarithms:

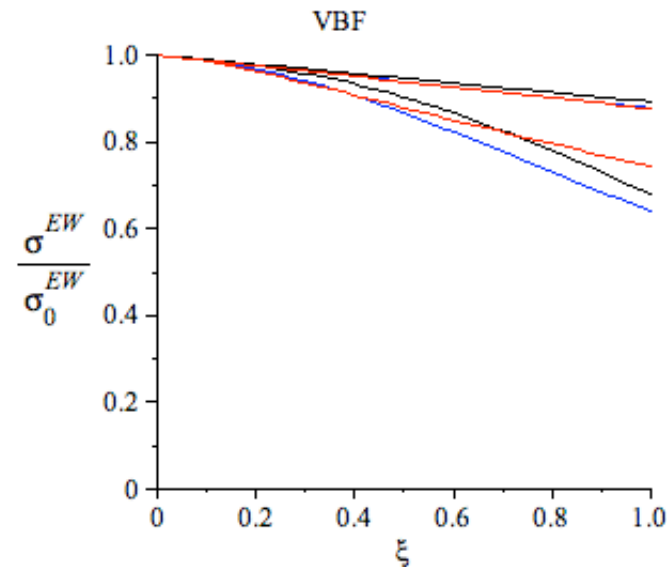
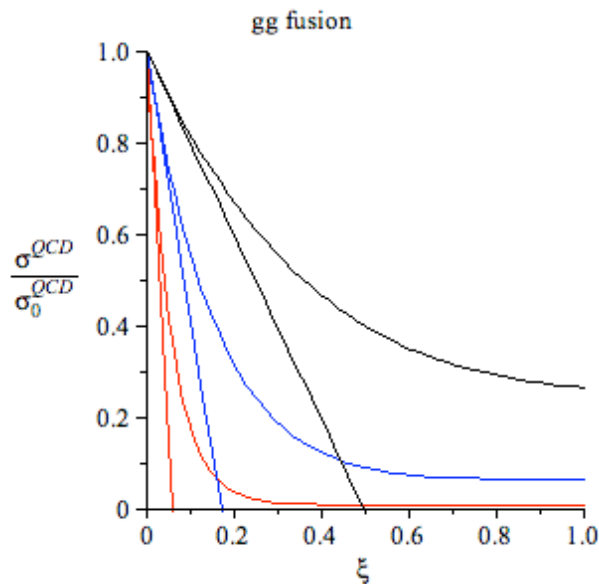
$$\alpha_s^n \log^n (Q/Q_0)$$

$$Q = \text{Transverse momentum of tag jets}$$

Resummation proceeds almost exactly as for “gaps between jets”

$$\mathbf{M} = \exp \left(-\frac{2\alpha_s}{\pi} \int_{Q_0}^Q \frac{dk_T}{k_T} \mathbf{\Gamma} \right) \mathbf{M}_0 \quad \rho(Y, \Delta y) \rightarrow \frac{1}{2}(\rho(Y, 2|y_3|) + \rho(Y, 2|y_4|))$$

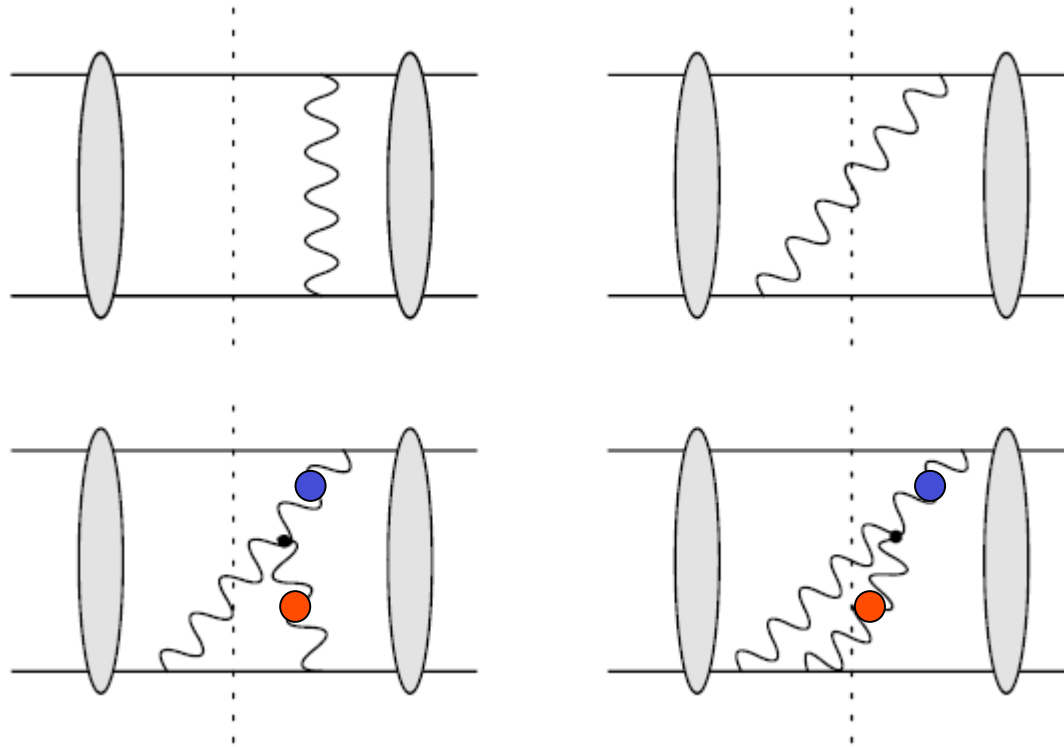
$Y = 1$ (black), 3 (blue), 9 (red)



$$\xi = \int_{Q_0}^Q \frac{dq}{q} \alpha_s(q) \approx 0.2 \quad \text{for 100 GeV jets and a 20 GeV veto, i.e. resummation is important at LHC}$$

- Fixed order calculations cannot account adequately for the effect of a veto.
- How much is this physics already present in parton shower Monte Carlos?
- gg-VBF interference is present but is negligibly small ($<1\%$).

But there is a big fly in the ointment: these observables are *non-global*



Such real & virtual corrections cancel.

But these do not if the gluon marked with a **red** blob is in the forbidden region: the 2nd cut is not allowed.

So the cancellation does not hold.....

real and virtual

It fails only once we start to evolve emissions (such as those denoted by the **blue** blob in the above) which lie *outside of the gap* region and which have $k_T > Q_0$

$$|y| > Y/2$$

If $k_T < Q_0$ then subsequent evolution also has $k_T < Q_0$ and cancellation works.

The miscancellation is telling us that this observable is sensitive to soft gluon emissions outside of the gap, even though the observable sums inclusively over that region.

Perhaps oughtn't to be a surprise once we realise that emissions outside of the gap can subsequently radiate back into the gap.

We must therefore include any number of emissions outside of the gap and their subsequent evolution.

Colour structure makes this impossible using current technology.

We can either aim to compute the all orders non-global corrections in the leading N approximation. Dasgupta, Salam, Appleby, Seymour, Delenda, Banfi

Instead we choose to compute the “one emission out of the gap” contribution without any approximation on the colour.

Two new ingredients still sticking to quark-quark scattering:

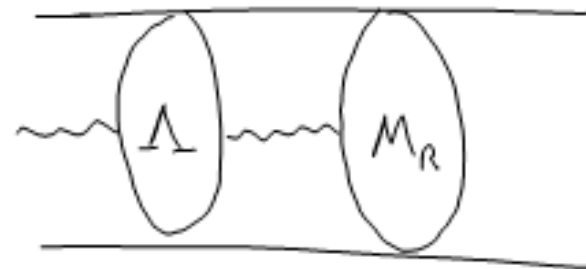
1) How to add a real gluon to the four-quark amplitude

$$\mathbf{M}_R = \mathbf{D} \cdot \mathbf{M}$$



2) How to evolve the five-parton amplitude

$$\mathbf{M}_R(Q_0) = \exp \left(-\frac{2\alpha_s}{\pi} \int_{Q_0}^{k_T} \frac{dk'_T}{k'_T} \mathbf{\Lambda} \right) \mathbf{M}_R(k_T)$$



$$\mathbf{D}^\mu = \begin{pmatrix} \frac{1}{2}(-h_1^\mu - h_2^\mu + h_3^\mu + h_4^\mu) & \frac{1}{4N}(-h_1^\mu - h_2^\mu + h_3^\mu + h_4^\mu) \\ 0 & \frac{1}{2}(-h_1^\mu - h_2^\mu + h_3^\mu + h_4^\mu) \\ \frac{1}{2}(-h_1^\mu + h_2^\mu + h_3^\mu - h_4^\mu) & \frac{1}{4N}(h_1^\mu - h_2^\mu - h_3^\mu + h_4^\mu) \\ 0 & \frac{1}{2}(-h_1^\mu + h_2^\mu - h_3^\mu + h_4^\mu) \end{pmatrix} \quad h_i^\mu = \frac{1}{2}k_T \frac{p_i^\mu}{p_i \cdot k}$$

$$\mathbf{\Lambda} = \begin{pmatrix} \frac{N}{4}(Y - i\pi) + \frac{1}{2N}i\pi & \left(\frac{1}{4} - \frac{1}{N^2}\right)i\pi & -\frac{N}{4}s_y Y & 0 \\ i\pi & \frac{N}{4}(2Y - i\pi) - \frac{3}{2N}i\pi & 0 & 0 \\ -\frac{N}{4}s_y Y & 0 & \frac{N}{4}(Y - i\pi) - \frac{1}{2N}i\pi & -\frac{1}{4}i\pi \\ 0 & 0 & -i\pi & \frac{N}{4}(2Y - i\pi) - \frac{1}{2N}i\pi \end{pmatrix}$$

$$+ \begin{pmatrix} N & 0 & 0 & 0 \\ 0 & N & 0 & 0 \\ 0 & 0 & N & 0 \\ 0 & 0 & 0 & N \end{pmatrix} \frac{1}{4}\rho(Y, 2|y|)$$

$$+ \begin{pmatrix} C_F & 0 & 0 & 0 \\ 0 & C_F & 0 & 0 \\ 0 & 0 & C_F & 0 \\ 0 & 0 & 0 & C_F \end{pmatrix} \frac{1}{2}\rho(Y, \Delta y)$$

$$+ \begin{pmatrix} \frac{N}{4}\left(-\frac{1}{2}\lambda\right) & 0 & \frac{N}{4}\left(-\frac{1}{2}s_y\lambda\right) & \frac{1}{4}\left(\frac{1}{2}s_y\lambda\right) \\ 0 & \frac{N}{4}\left(-\frac{1}{2}\lambda\right) & 0 & \frac{N}{4}\left(\frac{1}{2}s_y\lambda\right) \\ \frac{N}{4}\left(-\frac{1}{2}s_y\lambda\right) & 0 & \frac{N}{4}\left(-\frac{1}{2}\lambda\right) & \frac{1}{4}\left(-\frac{1}{2}\lambda\right) \\ \frac{1}{2}s_y\lambda & \left(\frac{N}{4} - \frac{1}{N}\right)\left(\frac{1}{2}s_y\lambda\right) & -\frac{1}{2}\lambda & \frac{N}{4}\left(-\frac{1}{2}\lambda\right) \end{pmatrix}$$

The complete cross-section for one real emission outside of the gap is thus

$$\begin{aligned}
 \sigma_R = & -\frac{2\alpha_s}{\pi} \int_{Q_0}^Q \frac{dk_T}{k_T} \int_{\text{out}} \frac{dy d\phi}{2\pi} \\
 & \mathbf{M}_0^\dagger \exp \left(-\frac{2\alpha_s}{\pi} \int_{k_T}^Q \frac{dk'_T}{k'_T} \Gamma^\dagger \right) \mathbf{D}_{\mu}^\dagger \exp \left(-\frac{2\alpha_s}{\pi} \int_{Q_0}^{k_T} \frac{dk'_T}{k'_T} \Lambda^\dagger \right) \mathbf{S}_R \\
 & \exp \left(-\frac{2\alpha_s}{\pi} \int_{Q_0}^{k_T} \frac{dk'_T}{k'_T} \Lambda \right) \mathbf{D}^\mu \exp \left(-\frac{2\alpha_s}{\pi} \int_{k_T}^Q \frac{dk'_T}{k'_T} \Gamma \right) \mathbf{M}_0
 \end{aligned}$$

And the corresponding contribution when the out of gap gluon is virtual is

$$\sigma_V = -\frac{2\alpha_s}{\pi} \int_{Q_0}^Q \frac{dk_T}{k_T} \int_{\text{out}} \frac{dy d\phi}{2\pi}$$

$$\left[M_0^\dagger \exp \left(-\frac{2\alpha_s}{\pi} \int_{Q_0}^Q \frac{dk'_T}{k'_T} \Gamma^\dagger \right) S_V \right.$$

$$\left. \exp \left(-\frac{2\alpha_s}{\pi} \int_{Q_0}^{k_T} \frac{dk'_T}{k'_T} \Gamma \right) \gamma \exp \left(-\frac{2\alpha_s}{\pi} \int_{k_T}^Q \frac{dk'_T}{k'_T} \Gamma \right) M_0 + \text{c.c.} \right]$$

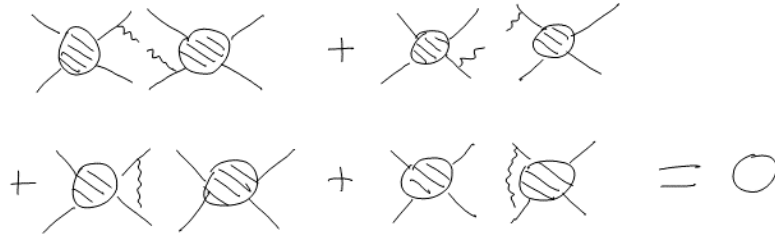
Adds one “out of the gap” virtual gluon

Coulomb gluons

I have skipped over a subtle issue....the real-virtual cancellation of soft gluons occurs point-by-point in (y, k_T) only between the *real parts* of the virtual correction and the real emission.

The imaginary part cancels if the soft gluon is closest to the cut....

But what about subsequent evolution? Might this spoil the claimed cancellation below Q_0 ?

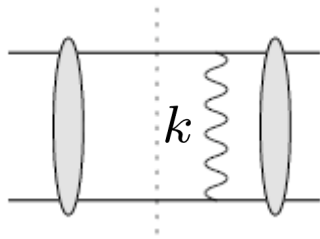


No: the “non-cancelled” $i\pi$ terms exponentiate to produce a pure phase in the amplitude.

$$\Gamma = \left(\begin{array}{c} \frac{N^2-1}{4N} \rho(Y, \Delta y) \\ i\pi \end{array} \quad -\frac{1}{N} i\pi + \frac{N}{2} Y + \frac{N^2-1}{4N} \rho(Y, \Delta y) \right) = \Gamma_{\text{in}} + \Gamma_{\text{coulomb}}$$

$$\gamma = \Gamma_{\text{out}} \quad (\text{purely real})$$

The real parts of the virtual evolution matrix arise as a result of the soft gluon going on-shell. In contrast, the $i\pi$ arise when the fermion propagator goes on-shell. This occurs when $k = (0; \mathbf{k}_T, 0)$.



We shall have more to say on these *Coulomb gluon* contributions in a moment...

Conventional wisdom: when the out of gap gluon becomes collinear with either incoming quark or either outgoing quark the real and virtual contributions should cancel.

This cancellation operates for **final state collinear emission:**

$$\mathbf{D}^{\mu\dagger}(\mathbf{\Lambda}^\dagger)^{n-m}\mathbf{S}_R\mathbf{\Lambda}^m\mathbf{D}_\mu + (\mathbf{\Gamma}^\dagger)^{n-m}\mathbf{S}_V\mathbf{\Gamma}^m\gamma + \gamma^\dagger(\mathbf{\Gamma}^\dagger)^{n-m}\mathbf{S}_V\mathbf{\Gamma}^m = \mathbf{0}$$

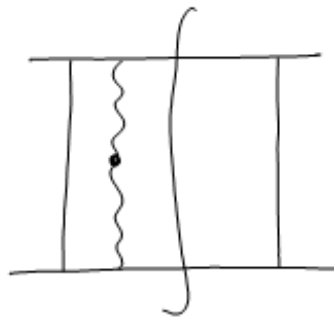
But it fails for **initial state collinear emission:**

The problem is entirely due to the emission of Coulomb gluons.

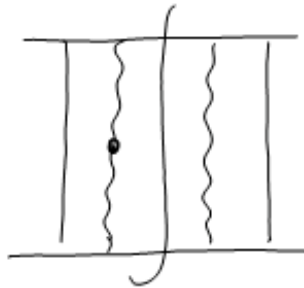
Cancellation does occur for $n = 0, 1$ and 2 gluons but not for larger n .

This is the lowest order where the Coulomb gluons do not trivially cancel.

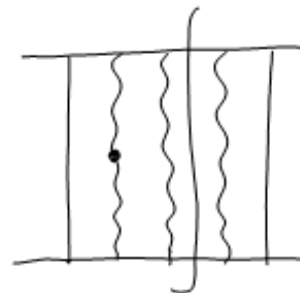
Dot indicates the
out of gap gluon
(purely real)



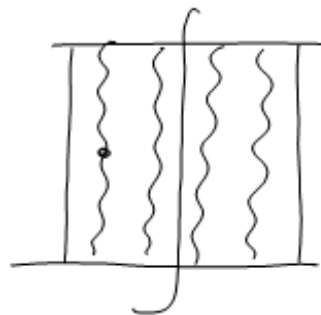
$i\pi$ never appears



only one $i\pi$



$i\pi$'s cancel in sum
over cuts.



$(i\pi)^2$ present for
first time

What are we to make of a non-cancelling collinear divergence?

$$\sigma \sim \sigma_0 \alpha^4 L^4 \pi^2 Y \int_{\text{out}} dy$$

Cannot actually have infinite rapidity with $k_T > Q_0$

Need to go beyond soft gluon approximation in collinear limit:

$$\int d^2 k_T \int_{\text{out}} dy \left. \frac{d\sigma}{dy d^2 k_T} \right|_{\text{soft}} \rightarrow \int d^2 k_T \left[\int^{y_{\text{max}}} dy \left. \frac{d\sigma}{dy d^2 k_T} \right|_{\text{soft}} + \int_{y_{\text{max}}}^{\infty} dy \left. \frac{d\sigma}{dy d^2 k_T} \right|_{\text{collinear}} \right]$$

$$\int_{y_{\text{max}}}^{\infty} dy \left. \frac{d\sigma}{dy d^2 k_T} \right|_{\text{collinear}} = \int_{y_{\text{max}}}^{\infty} dy \left(\left. \frac{d\sigma_{\text{R}}}{dy d^2 k_T} \right|_{\text{collinear}} + \left. \frac{d\sigma_{\text{V}}}{dy d^2 k_T} \right|_{\text{collinear}} \right)$$

Real collinear emission:

$$\begin{aligned}
 \int_{y_{\max}}^{\infty} dy \left. \frac{d\sigma_{\text{R}}}{dy d^2 k_T} \right|_{\text{collinear}} &= \int_0^{1-\delta} dz \frac{1}{2} \left(\frac{1+z^2}{1-z} \right) \frac{q(x/z, \mu^2)}{q(x, \mu^2)} A_{\text{R}} \\
 &= \int_0^{1-\delta} dz \frac{1}{2} \left(\frac{1+z^2}{1-z} \right) \left(\frac{q(x/z, \mu^2)}{q(x, \mu^2)} - 1 \right) A_{\text{R}} + \int_0^{1-\delta} dz \frac{1}{2} \frac{1+z^2}{1-z} A_{\text{R}}
 \end{aligned}$$

Virtual collinear emission:

$$\int_{y_{\max}}^{\infty} dy \left. \frac{d\sigma_{\text{V}}}{dy d^2 k_T} \right|_{\text{collinear}} = \int_0^{1-\delta} dz \frac{1}{2} \left(\frac{1+z^2}{1-z} \right) A_{\text{V}}$$

$y > y_{\max}$
 implies
 $\delta \approx \frac{k_T}{Q} \exp \left(y_{\max} - \frac{\Delta y}{2} \right)$

If $A_{\text{R}} + A_{\text{V}} = 0$

then the divergence would cancel leaving behind a regularized splitting which would correspond to the DGLAP evolution of the incoming quark pdf. These purely collinear logs could then be resummed by selecting the scale of the pdf to be the jet scale Q .

But as we have seen, the Coulomb gluons spoil this cancellation. Instead we have

$$\int_0^{1-\delta} dz \frac{1}{2} \left(\frac{1+z^2}{1-z} \right) (A_R + A_V) = \ln \left(\frac{1}{\delta} \right) (A_R + A_V) + \text{subleading}$$

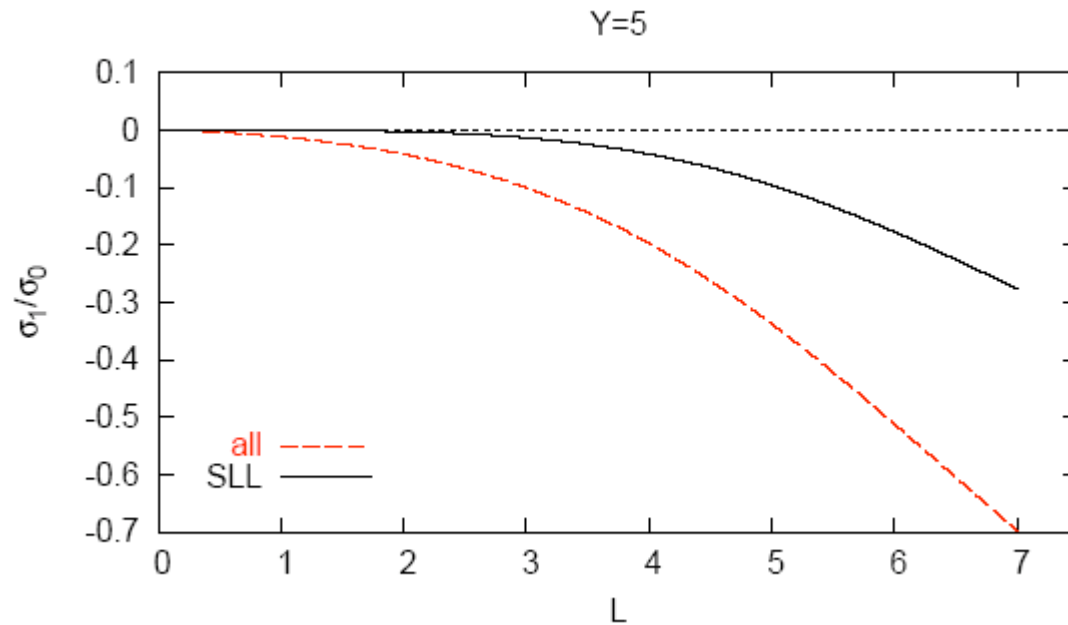
$$\approx \left(-y_{\max} + \frac{\Delta y}{2} + \ln \left(\frac{Q}{k_T} \right) \right) (A_R + A_V)$$

Hence

$$\int_{Q_0}^Q \frac{dk_T}{k_T} \int_{\text{out}} dy \rightarrow \int_{Q_0}^Q \frac{dk_T}{k_T} \left(\int^{y_{\max}} dy + (-y_{\max} + \ln \frac{Q}{k_T}) \right) = \frac{1}{2} \ln^2 \frac{Q}{Q_0}$$

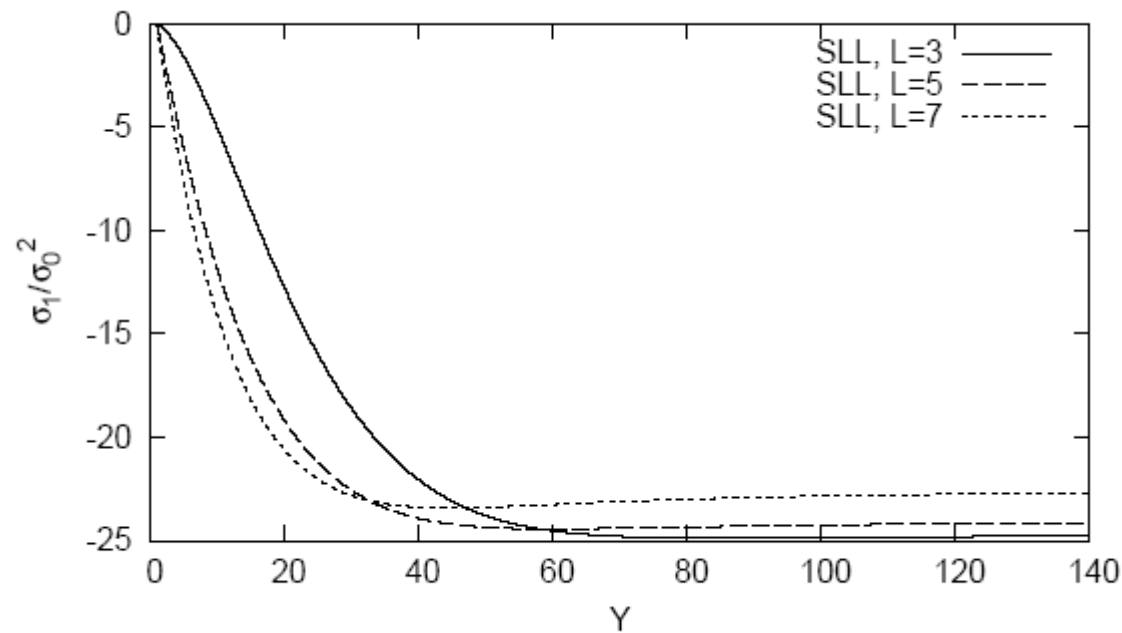
The final result for the “one emission out-of-gap” cross-section is

$$\sigma_{1,\text{SLL}} = -\sigma_0 \left(\frac{2\alpha_s}{\pi} \right)^4 \ln^5 \left(\frac{Q}{Q_0} \right) \pi^2 Y \frac{(3N^2 - 4)}{480}$$



Modest but potentially not a negligible phenomenological impact.

We already knew single non-global logs are potentially important (but can be reduced by taking a small cone radius).
 Appleby & Seymour



Intriguing link to non-linear effects in small-x physics?

Concluding comments on super-leading logs:

- Need to add the contribution from $n > 1$ out-of-gap gluons.
- The $\alpha_s^4 L^5$ term we just computed cannot be cancelled by an $n > 1$ contribution.
- This contribution is formally more important than everything else.
- To get the “leading” logs correct therefore requires a “next-to-leading” calculation of the evolution matrices etc (recent progress: Dixon, Mert Aybat, Sterman)
- Shakes the foundations of soft gluon theory: large collinear enhancements in an observable which sums inclusively over the collinear region.
Conventional wisdom says expect soft enhancement but not soft-collinear.
i.e. constitutes a breakdown of collinear factorization and of “QCD coherence”.
- Implications for other (global?) observables?

Conclusions

- Are the super-leading logarithms really there? Implications?
- Soft resummation is needed for Higgs plus two jet production
- “Standard” non-global effects have not yet been included in Higgs plus two jet production but are expected to be modest.
- Pressing need to establish how reliable existing resummations, based on parton shower Monte Carlos, actually are.
- Ultimate goal: to extract the Higgs coupling to vector bosons.