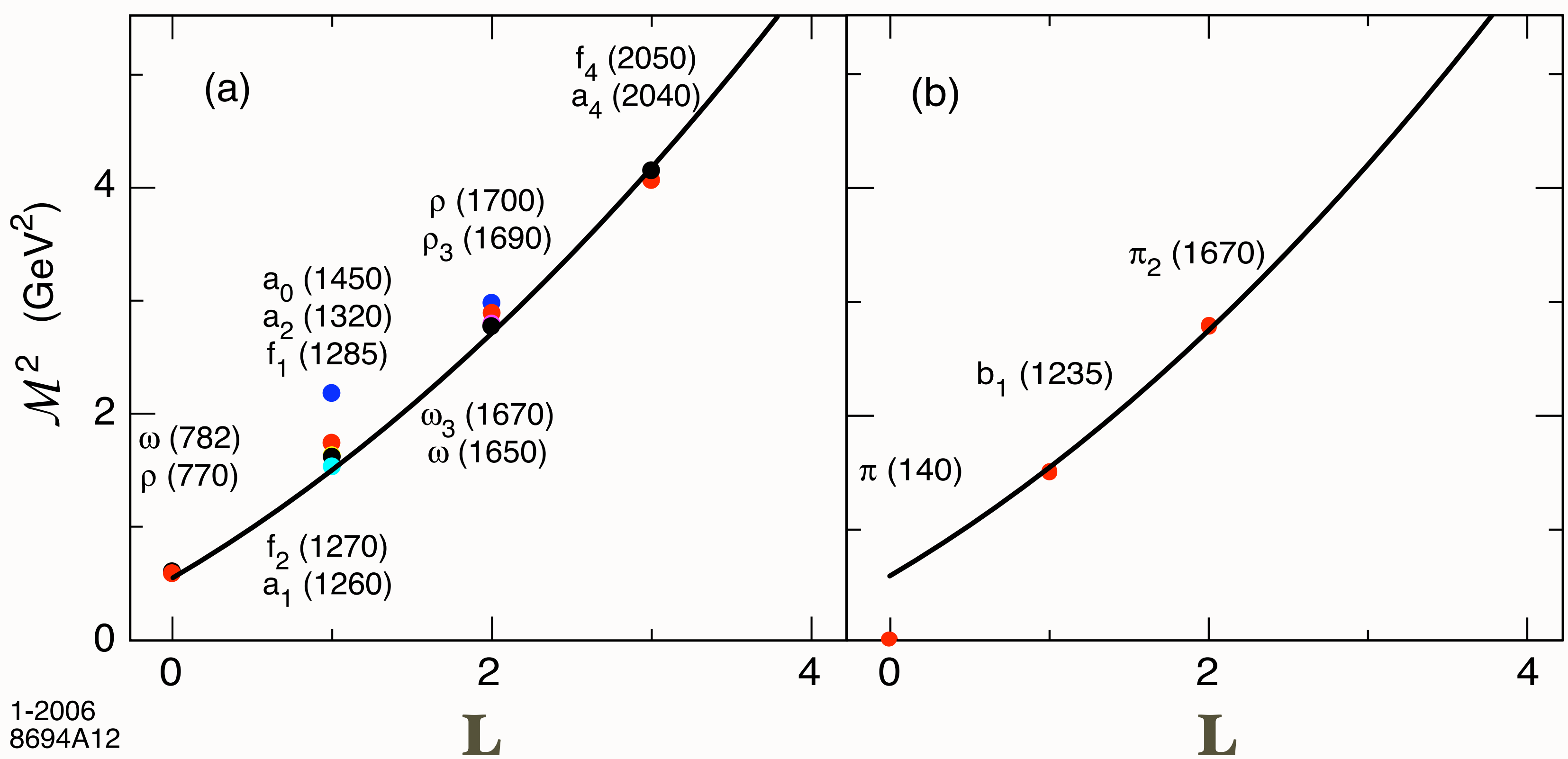


Light meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

Guy de Teramond
SJB

Baryon Spectrum

- Baryon: twist-three, dimension $\frac{9}{2} + L$

$$\mathcal{O}_{\frac{9}{2}+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \quad L = \sum_{i=1}^{m} \ell_i .$$

Wave Equation: $$\boxed{\left[z^2\, \partial_z^2 - 3z\, \partial_z + z^2 \mathcal{M}^2 - \mathcal{L}_\pm^2 + 4\right] f_\pm(z) = 0}$$

with $\mathcal{L}_+ = L + 1$, $\mathcal{L}_- = L + 2$, and solution

$$\Psi(x, z) = C e^{-iP\cdot x} z^2 \Big[J_{1+L}(z\mathcal{M})\, u_+(P) + J_{2+L}(z\mathcal{M})\, u_-(P) \Big].$$

- 4-d mass spectrum $\Psi(x, z_o)^{\pm} = 0 \quad \Longrightarrow$ <u>parallel Regge trajectories for baryons !</u>

$$\mathcal{M}_{\alpha,k}^+ = \beta_{\alpha,k} \Lambda_{QCD}, \quad \mathcal{M}_{\alpha,k}^- = \beta_{\alpha+1,k} \Lambda_{QCD}.$$

- Ratio of eigenvalues determined by the ratio of zeros of Bessel functions !

Predictions of AdS/CFT

Only one parameter!

Entire light quark baryon spectrum

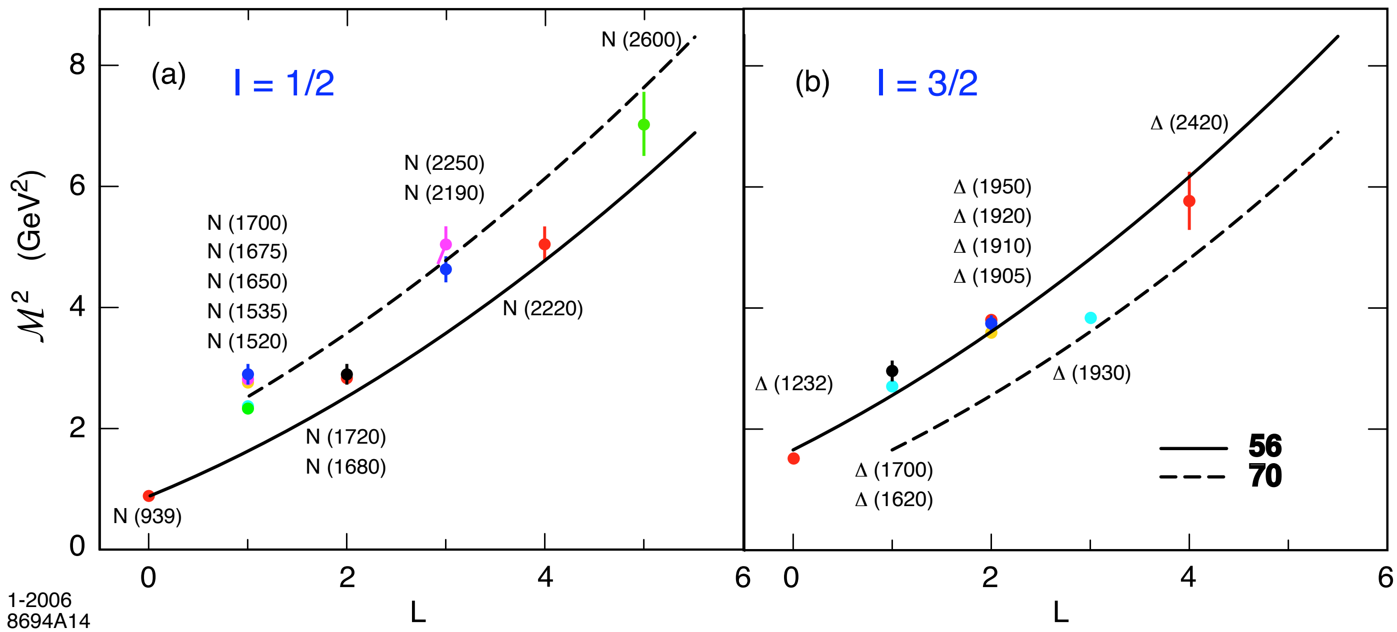


Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV. The **56** trajectory corresponds to L even $P = +$ states, and the **70** to L odd $P = -$ states.

Guy de Teramond
SJB

Glueball Spectrum

- AdS wave function with effective mass μ:

$$\left[z^2\,\partial_z^2 - (d-1)z\,\partial_z + z^2\,\mathcal{M}^2 - (\mu R)^2\right] f(z) = 0,$$

where $\Phi(x,z) = e^{-iP\cdot x}\, f(z)$ and $P_\mu P^\mu = \mathcal{M}^2$.

- Glueball interpolating operator with twist -dimension minus spin- two, and conformal dimension $4+L$

$$\mathcal{O}_{4+L} = F D_{\{\ell_1} \ldots D_{\ell_m\}} F,$$

where $L = \sum_{i=1}^{m} \ell_i$ is the total internal space-time orbital momentum.

- Normalizable scalar AdS mode (d = 4):

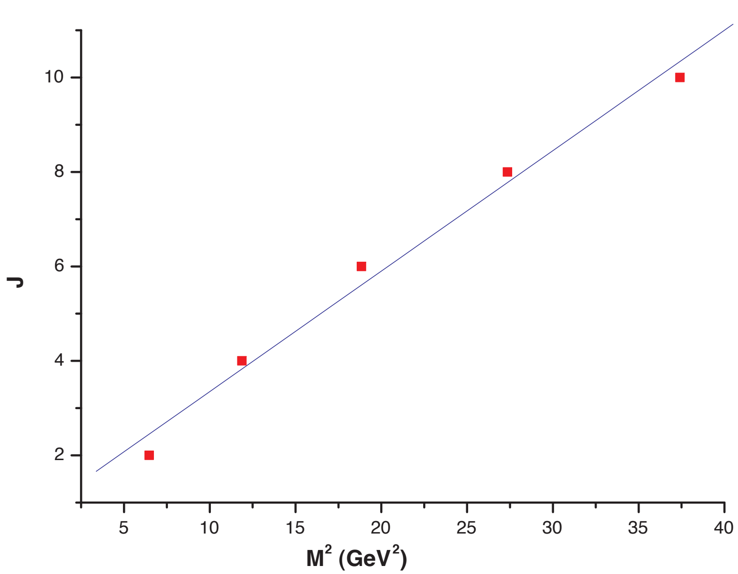
$$\Phi_{\alpha,k}(x,z) = C_{\alpha,k} e^{-iP\cdot x} z^2 J_\alpha\left(z\,\beta_{\alpha,a}\Lambda_{QCD}\right)$$

with $\alpha = 2+L$ and scaling dimension $4+L$.

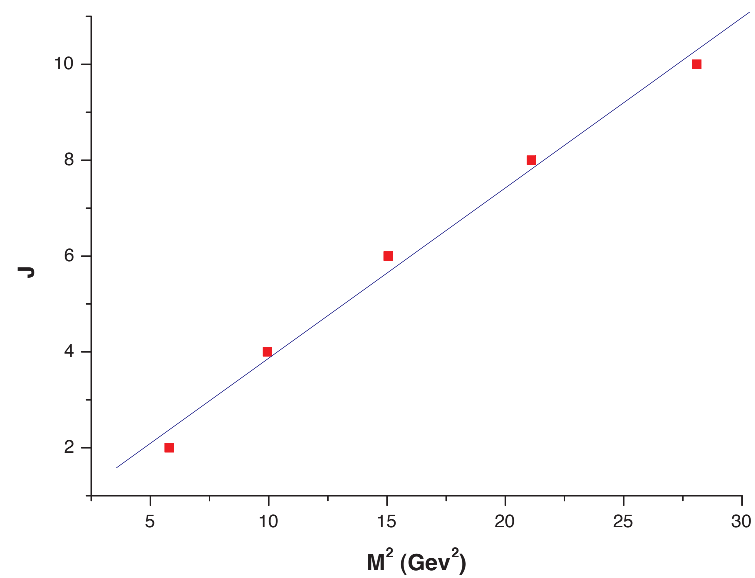
Glueball Regge trajectories from gauge/string duality and the Pomeron

Henrique Boschi-Filho,* Nelson R. F. Braga,† and Hector L. Carrion‡

Instituto de Física, Universidade Federal do Rio de Janeiro,



Neumann Boundary Conditions



Dirichlet Boundary Conditions

Hadronic Form Factor in Space and Time-Like Regions

- The form factor in AdS/QCD is the overlap of the normalizable modes dual to the incoming and outgoing hadron Φ_I and Φ_F and the non-normalizable mode J, dual to the external source (hadron spin σ):

$$F(Q^2)_{I\to F} = R^{3+2\sigma} \int_0^\infty \frac{dz}{z^{3+2\sigma}} \, e^{(3+2\sigma)A(z)} \Phi_F(z) \, J(Q,z) \, \Phi_I(z)$$

$$\simeq R^{3+2\sigma} \int_0^{z_o} \frac{dz}{z^{3+2\sigma}} \, \Phi_F(z) \, J(Q,z) \, \Phi_I(z),$$

- $J(Q,z)$ has the limiting value 1 at zero momentum transfer, $F(0)=1$, and has as boundary limit the external current, $A^\mu = \epsilon^\mu e^{iQ\cdot x} J(Q,z)$. Thus:

$$\lim_{Q\to 0} J(Q,z) = \lim_{z\to 0} J(Q,z) = 1.$$

- Solution to the AdS Wave equation with boundary conditions at $Q=0$ and $z\to 0$:

$$J(Q,z) = zQK_1(zQ).$$

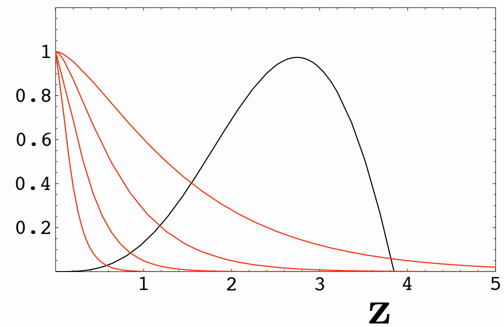
Polchinski and Strassler, hep-th/0209211; Hong, Yong and Strassler, hep-th/0409118.

Hadron Form Factors from AdS/CFT

- Propagation of external perturbation suppressed inside AdS.
- At large Q^2 the important integration region is $z \sim 1/Q$.

$$F(Q^2)_{I \to F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q,z) \Phi_I(z)$$

$\mathbf{J}(\mathbf{Q}, \mathbf{z}), \quad \mathbf{\Phi}(\mathbf{z})$



Polchinski, Strassler
de Teramond, sjb

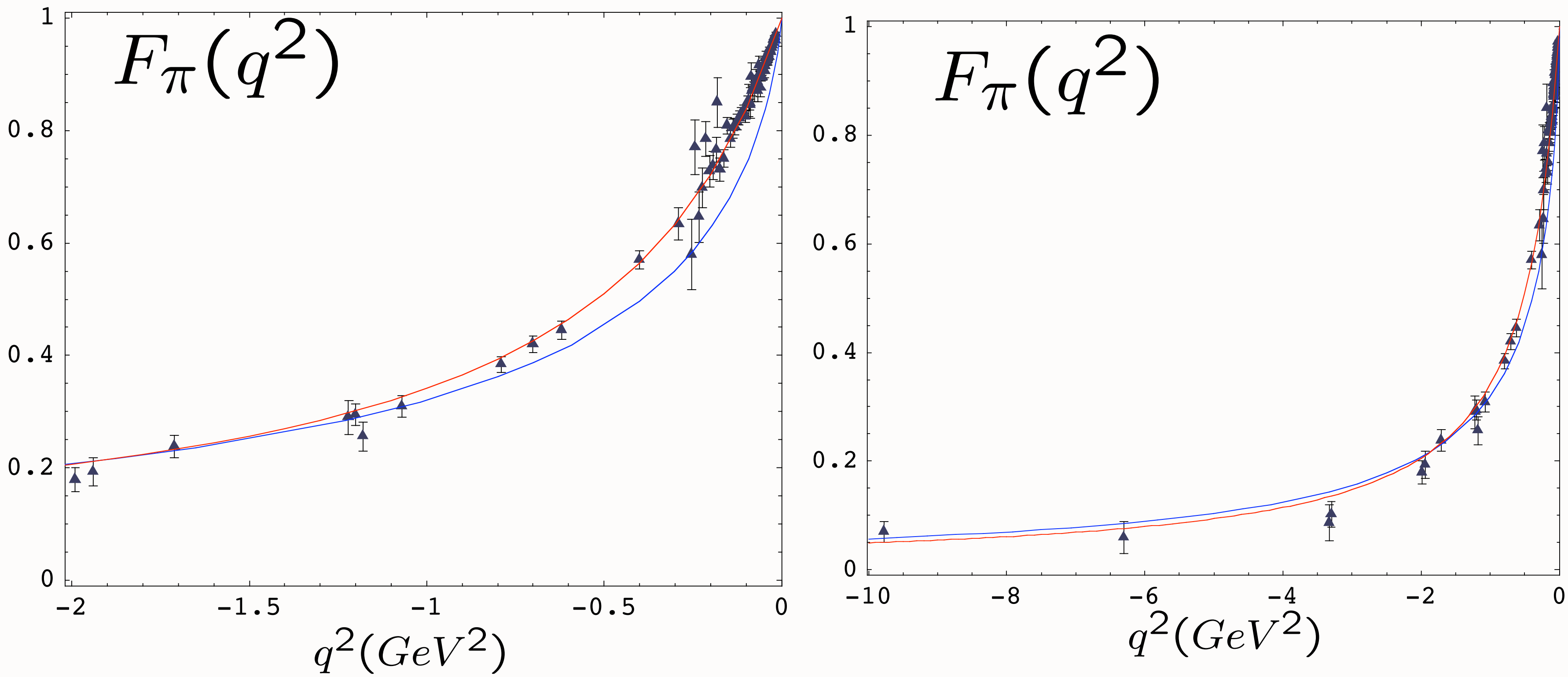
- Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau - 1},$$

Dimensional Quark Counting Rules: General result from AdS/CFT

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT



Data Compilation from Baldini, Kloe and Volmer

Harmonic Oscillator Confinement

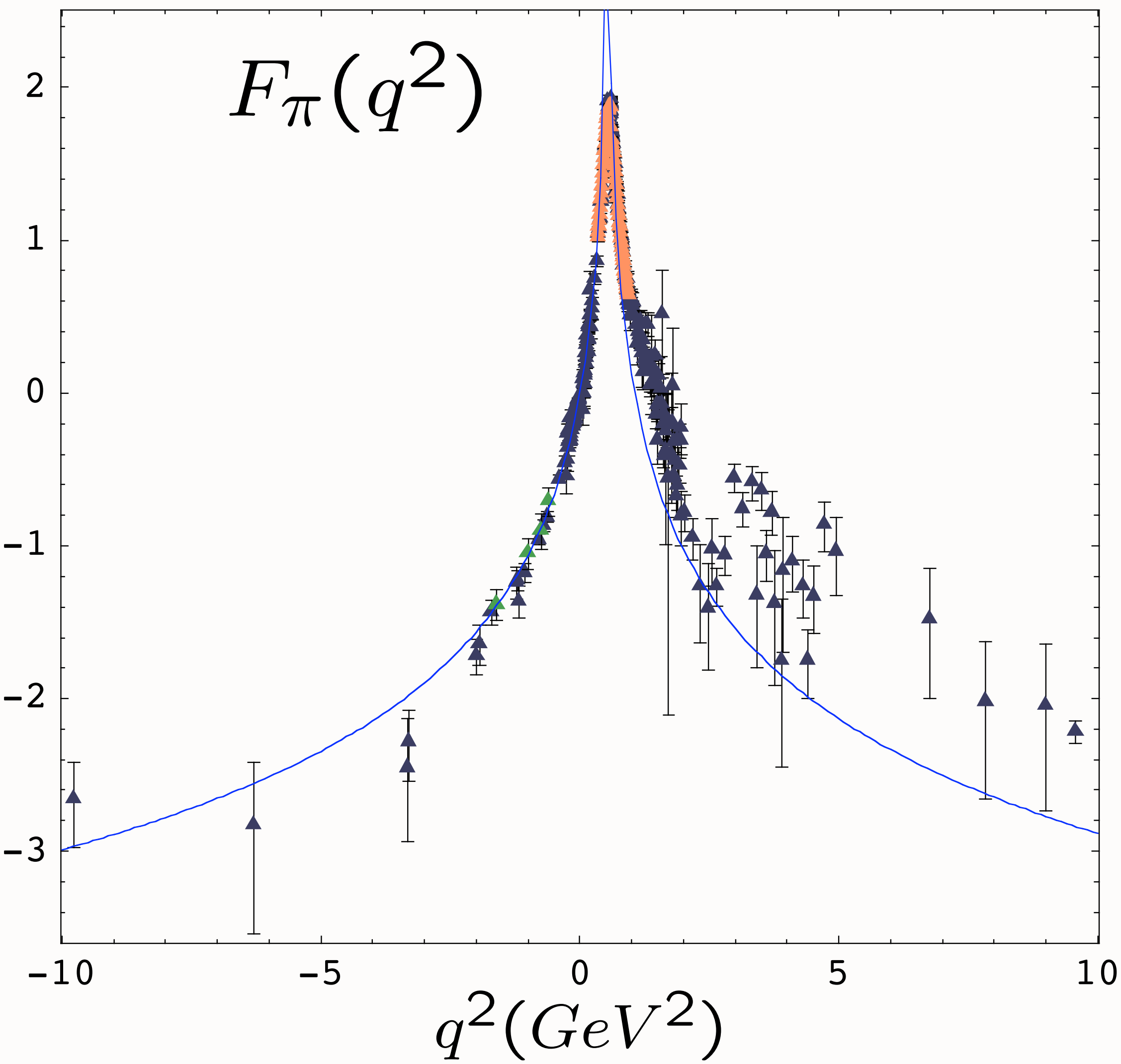
Truncated Space Confinement

One parameter - set by pion decay constant

G. de Teramond, sjb

Spacelike and Timelike Pion form factor from AdS/CFT

G. de Teramond, sjb



One parameter - set by pion decay constant

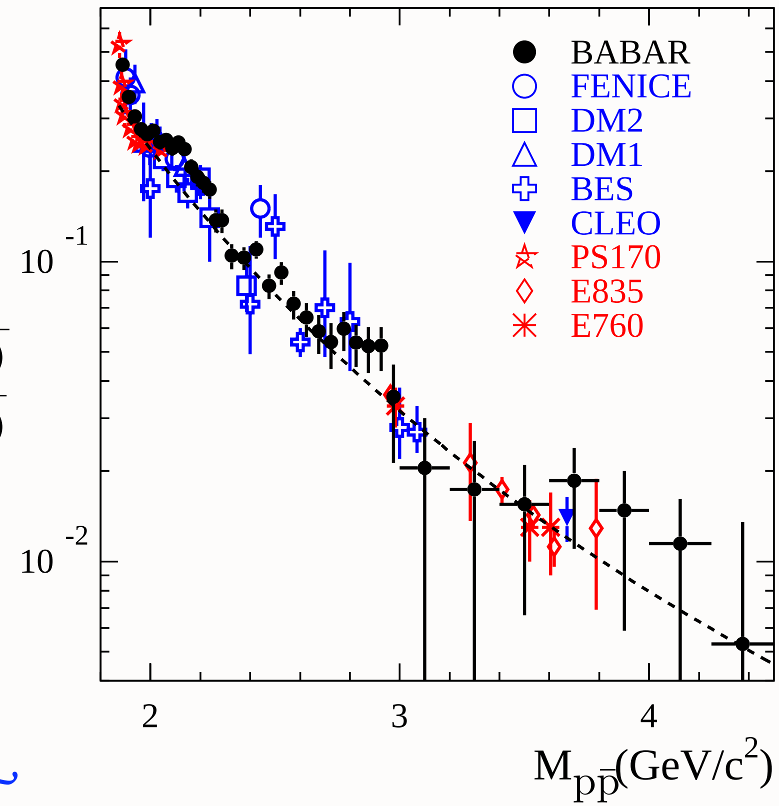
Harmonic Oscillator Confinement
Current modified by metric

Effective timelike proton form factor

$$|F_{\bar{p}p}(M)| \equiv \sqrt{\frac{\sigma_{e^+e^-\to p\bar{p}}(M)}{\sigma^{pt}_{e^+e^-\to p\bar{p}}(M)}}$$

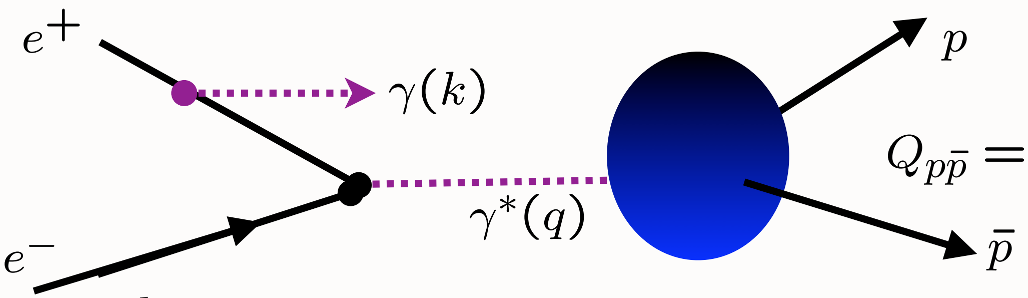
$$G_E = G_M = 1$$

Babar PHYSICAL REVIEW D **73,** 012005 (2006)



- - - - - Lepage, sjb
Chernyak, Zhitnitsky

Babar: radiative return



$$Q_{p\bar{p}} = \sqrt{s}$$

$$|F^{PQCD}_{\bar{p}p}(M)| = \frac{C}{M^4 \log^2 M^2/\Lambda^2_{QCD}}$$

$$\frac{d\sigma}{d\Omega_\gamma dE_\gamma}(e^+e^- \to p\bar{p}\gamma) = P(s, E_\gamma, \Omega_\gamma) \cdot \sigma(Q^2_{p\bar{p}})$$

Baryon Form Factors

- Coupling of the extended AdS mode with an external gauge field $A^\mu(x,z)$

$$ig_5 \int d^4x\, dz\, \sqrt{g}\, A_\mu(x,z)\, \overline{\Psi}(x,z)\gamma^\mu\Psi(x,z),$$

where

$$\Psi(x,z) = e^{-iP\cdot x}\left[\psi_+(z)u_+(P) + \psi_-(z)u_-(P)\right],$$

$$\psi_+(z) = Cz^2 J_1(zM), \qquad \psi_-(z) = Cz^2 J_2(zM),$$

and

$$u(P)_\pm = \frac{1 \pm \gamma_5}{2}\, u(P).$$

$$\psi_+(z) \equiv \psi^\uparrow(z), \quad \psi_-(z) \equiv \psi^\downarrow(z),$$

the LC $\pm$ spin projection along $\hat{z}$.

- Constant C determined by charge normalization:

$$C = \frac{\sqrt{2}\Lambda_{\rm QCD}}{R^{3/2}\left[-J_0(\beta_{1,1})J_2(\beta_{1,1})\right]^{1/2}}.$$

AdS/QCD

Nucleon Form Factors

- Consider the spin non-flip form factors in the infinite wall approximation

$$F_+(Q^2) = g_+ R^3 \int \frac{dz}{z^3} J(Q,z)\, |\psi_+(z)|^2,$$
$$F_-(Q^2) = g_- R^3 \int \frac{dz}{z^3} J(Q,z)\, |\psi_-(z)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(z)$ and $\psi_-(z)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = R^3 \int \frac{dz}{z^3} J(Q,z) |\psi_+(z)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} R^3 \int \frac{dz}{z^3} J(Q,z) \left[|\psi_+(z)|^2 - |\psi_-(z)|^2\right],$$

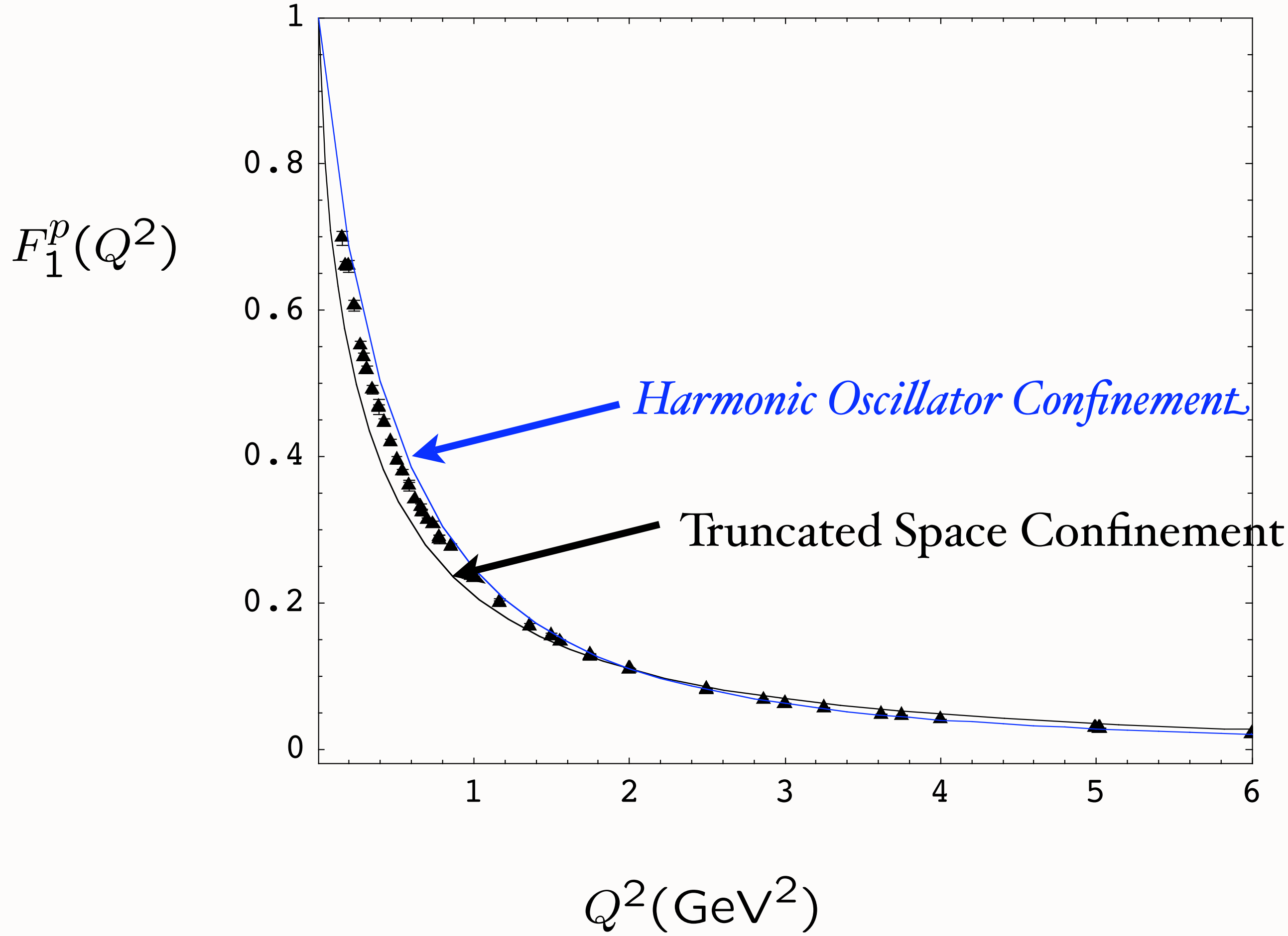
where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

- Large Q power scaling: $F_1(Q^2) \to \left[1/Q^2\right]^2$.

G. de Teramond, sjb

G. de Teramond, sjb

Preliminary



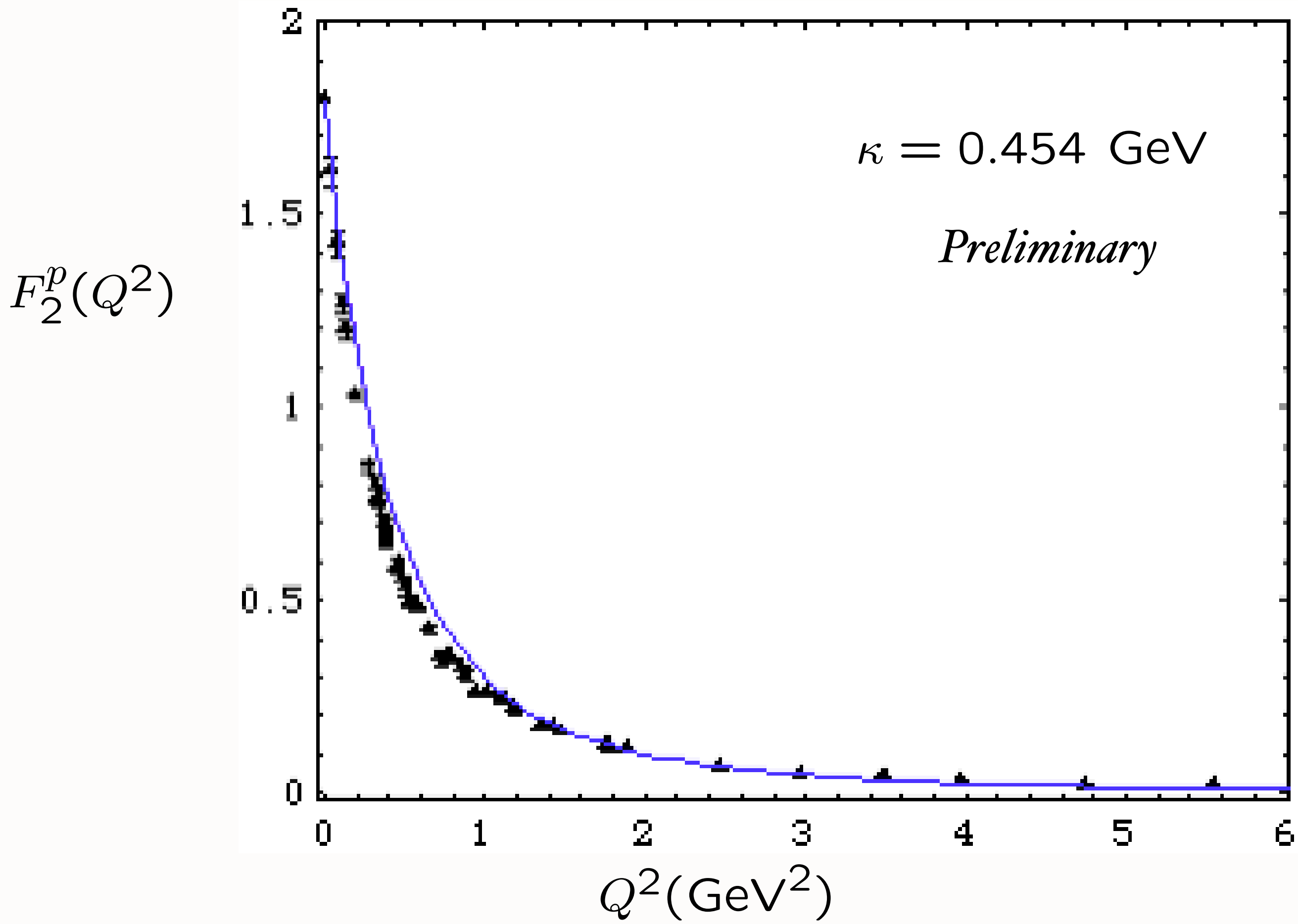
$\kappa = 0.454$ GeV

$\Lambda = 0.2$ GeV

Current modified by metric

$$F_1(Q^2)_{I\to F} = \int \frac{dz}{z^3} \Phi_F^{\uparrow}(z) J(Q,z) \Phi_I^{\uparrow}(z)$$

Harmonic Oscillator Confinement



Current modified by metric

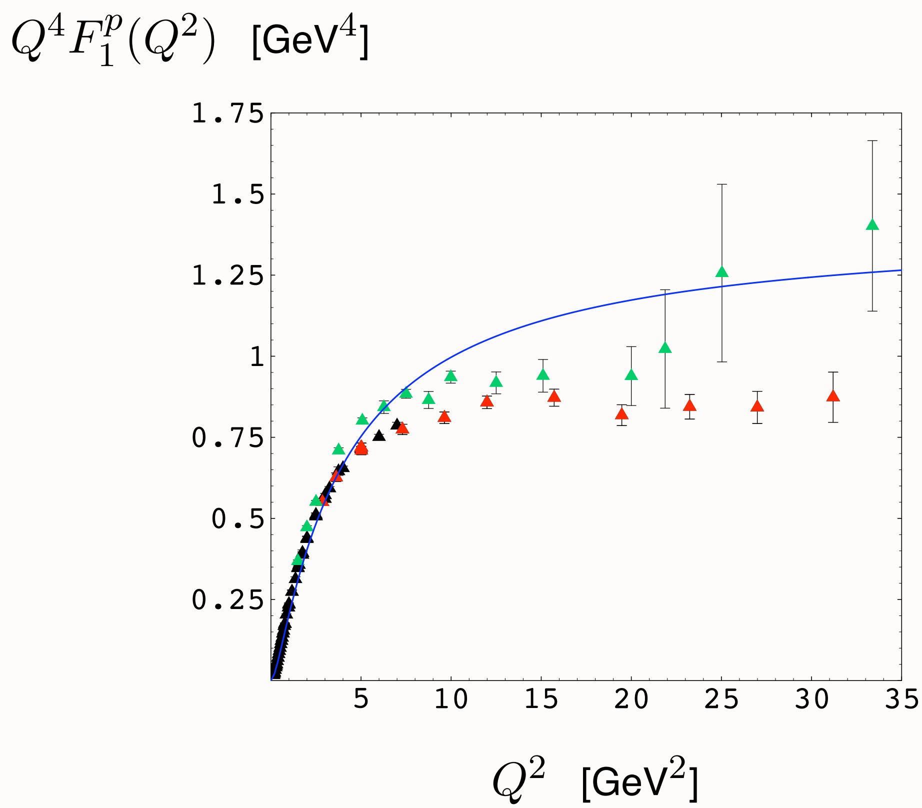
$$F_2(Q^2)_{I\to F} = \int \frac{dz}{z^2} \Phi_F^{\uparrow}(z) J(Q,z) \Phi_I^{\downarrow}(z)$$

AdS/QCD

Dirac Proton Form Factor

(Valence Approximation)

Truncated Space Confinement

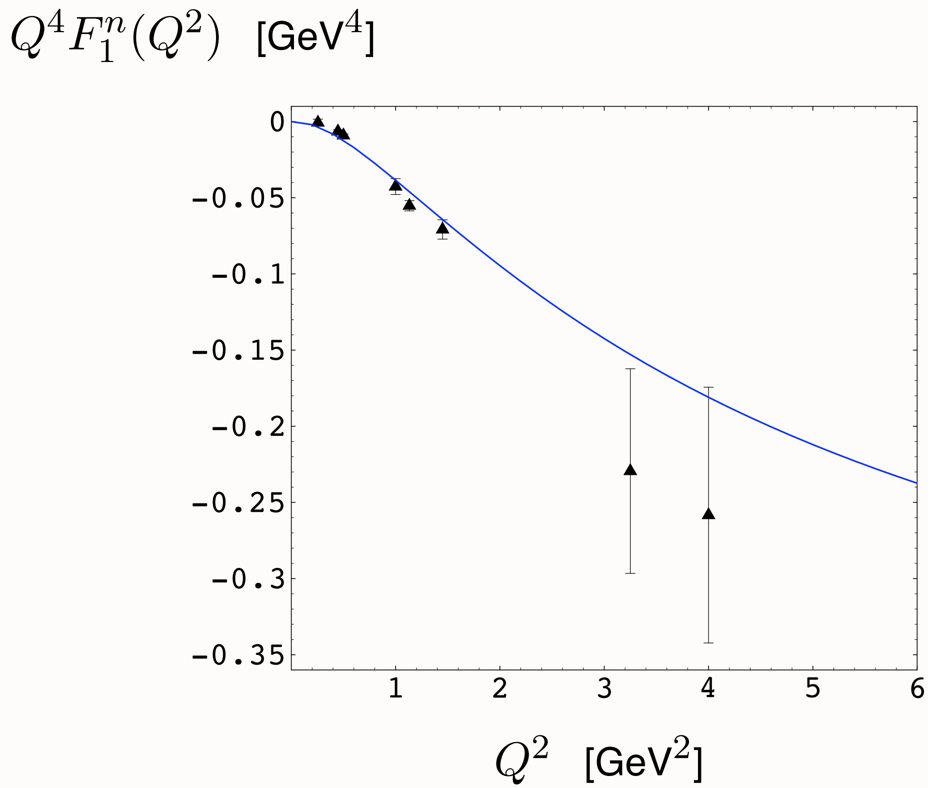


Prediction for $Q^4 F_1^p(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Analysis of the data is from Diehl (2005). Red points are from Sill (1993). Superimposed Green points are from Kirk (1973).

Dirac Neutron Form Factor

(Valence Approximation)

Truncated Space Confinement



Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\mathrm{QCD}} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

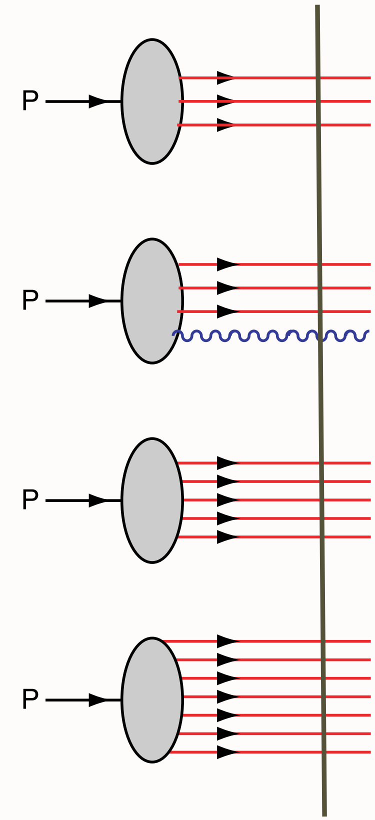
$$\mathrm{H}_{LF}^{QCD} |\psi> = M^2 |\psi>$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi_n(x, k_\perp) \qquad x_i = \frac{k_i^+}{P^+}$$



$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

Invariant under boosts. Independent of P^μ

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2 \; |\Psi_h\rangle$$

$$H_{LC}^{QCD} = P_\mu P^\mu = \quad P^- P^+ - \vec{P}_\perp^2$$

The hadron state $|\Psi_h\rangle$ is expanded in a Fock-state complete basis of non-interacting n-particle states $|n\rangle$ with an infinite number of components

$$\left|\Psi_h(P^+, \vec{P}_\perp)\right\rangle =$$

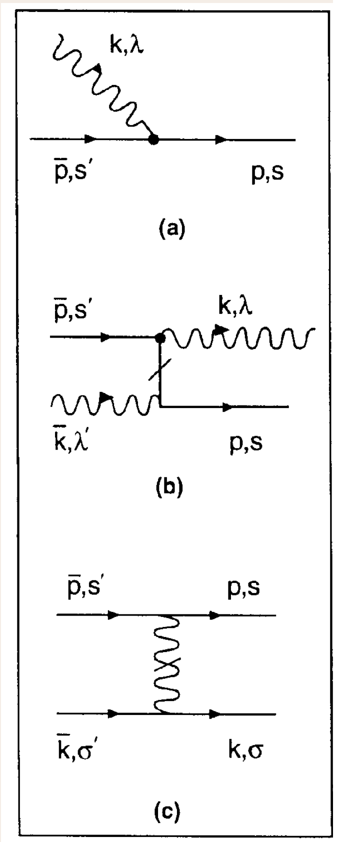
$$\sum_{n,\lambda_i} \int \; [dx_i \; d^2\vec{k}_{\perp i}] \psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i)$$

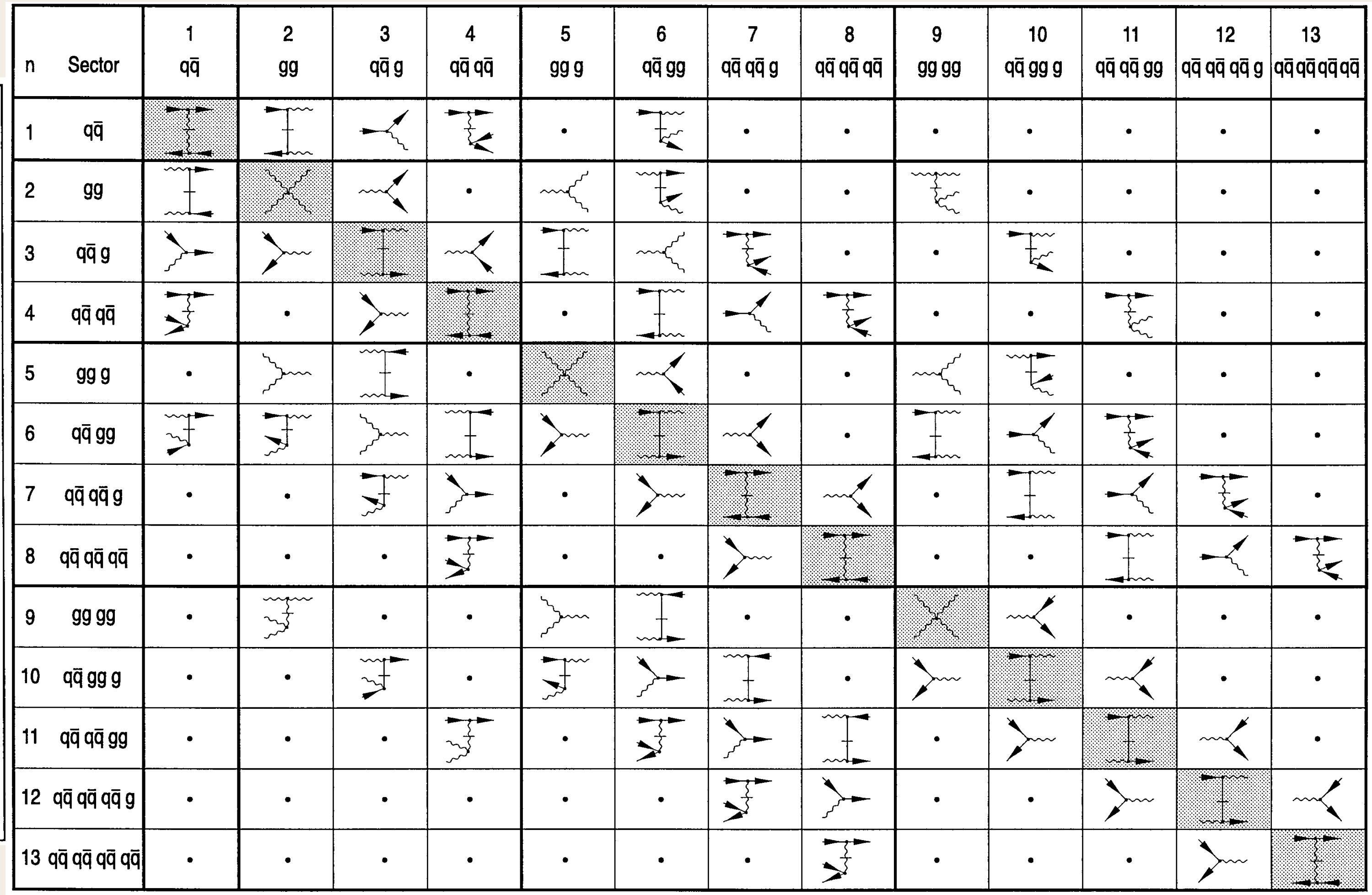
$$\times \; |n : x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle$$

$$\sum_n \int [dx_i \; d^2\vec{k}_{\perp i}] \; |\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i)|^2 = 1$$

Light-Front QCD Heisenberg Equation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2 \; |\Psi_h\rangle$$

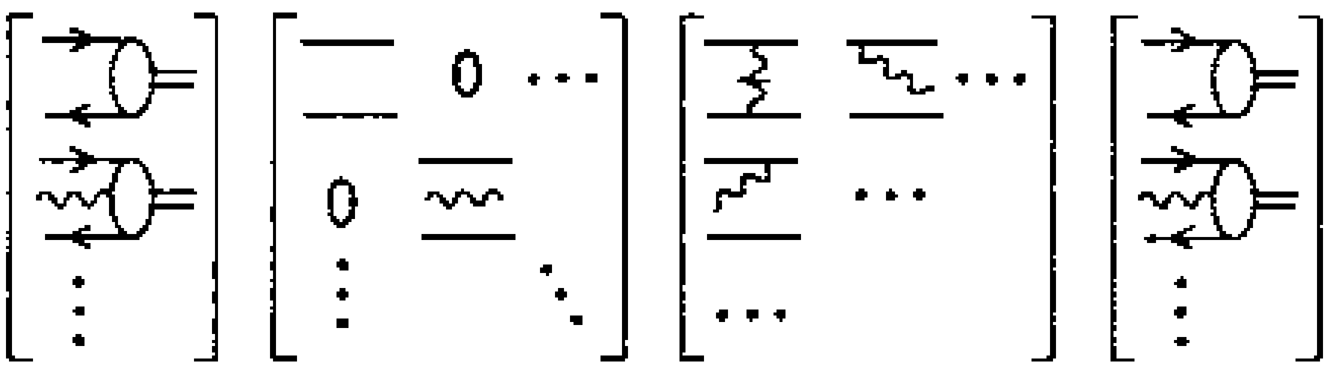




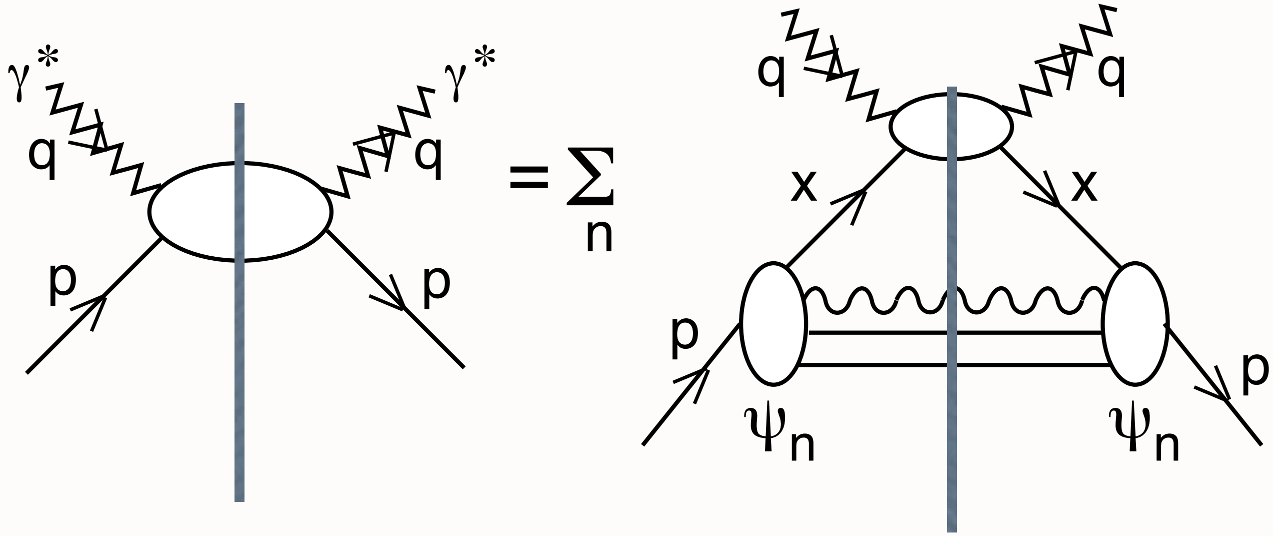
AdS/QCD

LIGHT-FRONT SCHRODINGER EQUATION

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\, V \,|q\bar{q}\rangle & \langle q\bar{q}|\, V \,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\, V \,|q\bar{q}\rangle & \langle q\bar{q}g|\, V \,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$

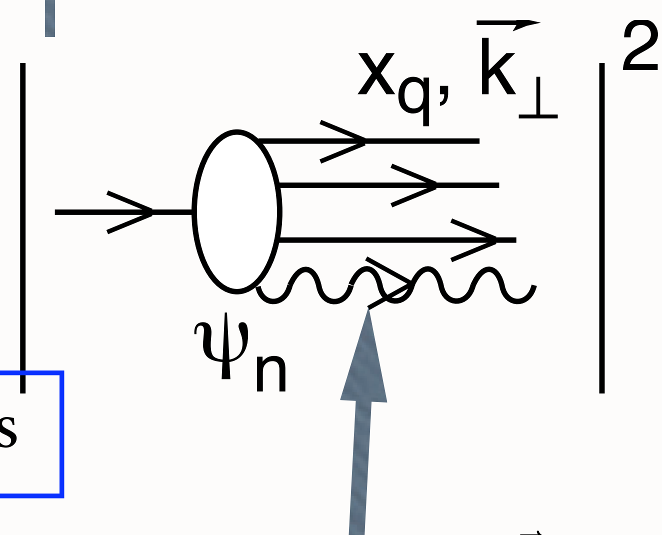


Deep Inelastic Lepton Proton Scattering



Imaginary Part of
Forward Virtual Compton Amplitude

$q(x,Q^2) = \sum_n \int^{k_\perp^2 \le Q^2 \perp} d^2k_\perp \, |\Psi_n(x,k_\perp)|^2$



$x = x_q$

All spin, flavor distributions

Light-Front Wave Functions $\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

Angular Momentum on the Light-Front

A^+=0 gauge: No unphysical degrees of freedom

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

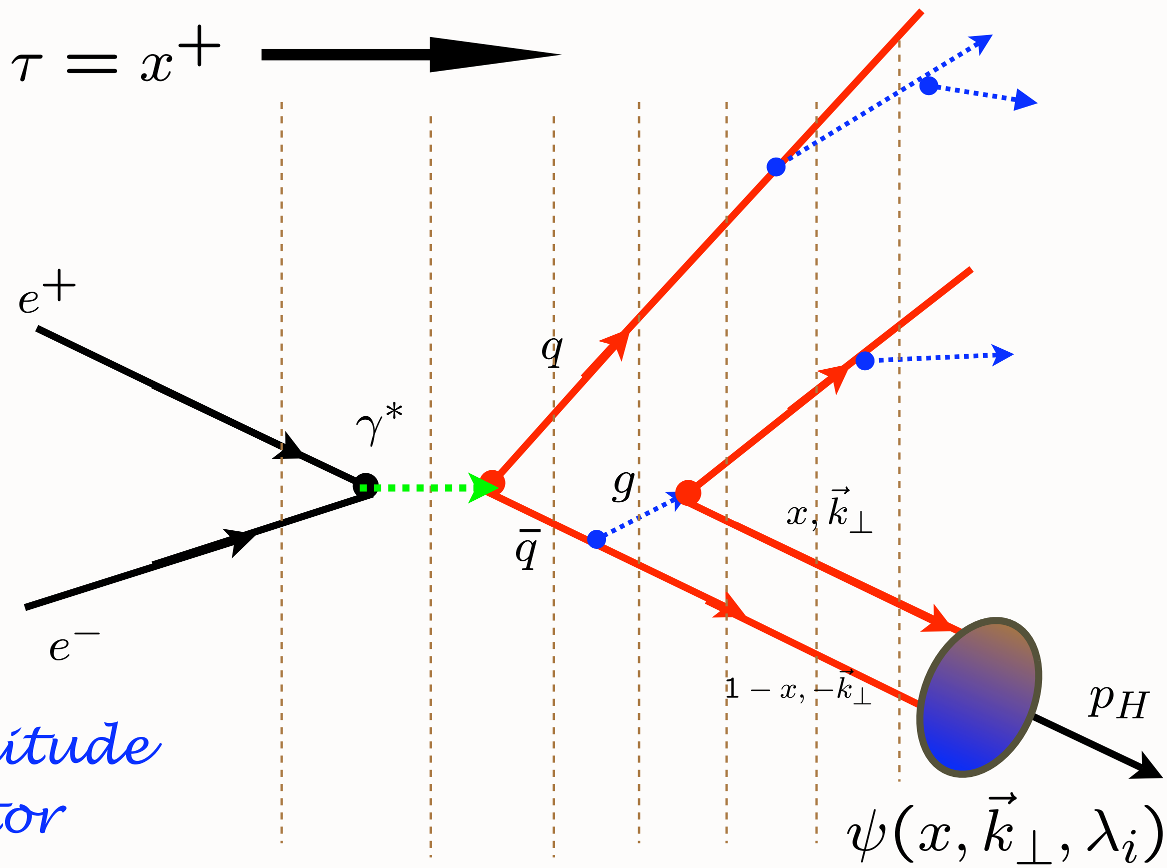
Conserved
LF Fock state by Fock State

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment requires
Nonzero orbital angular momentum

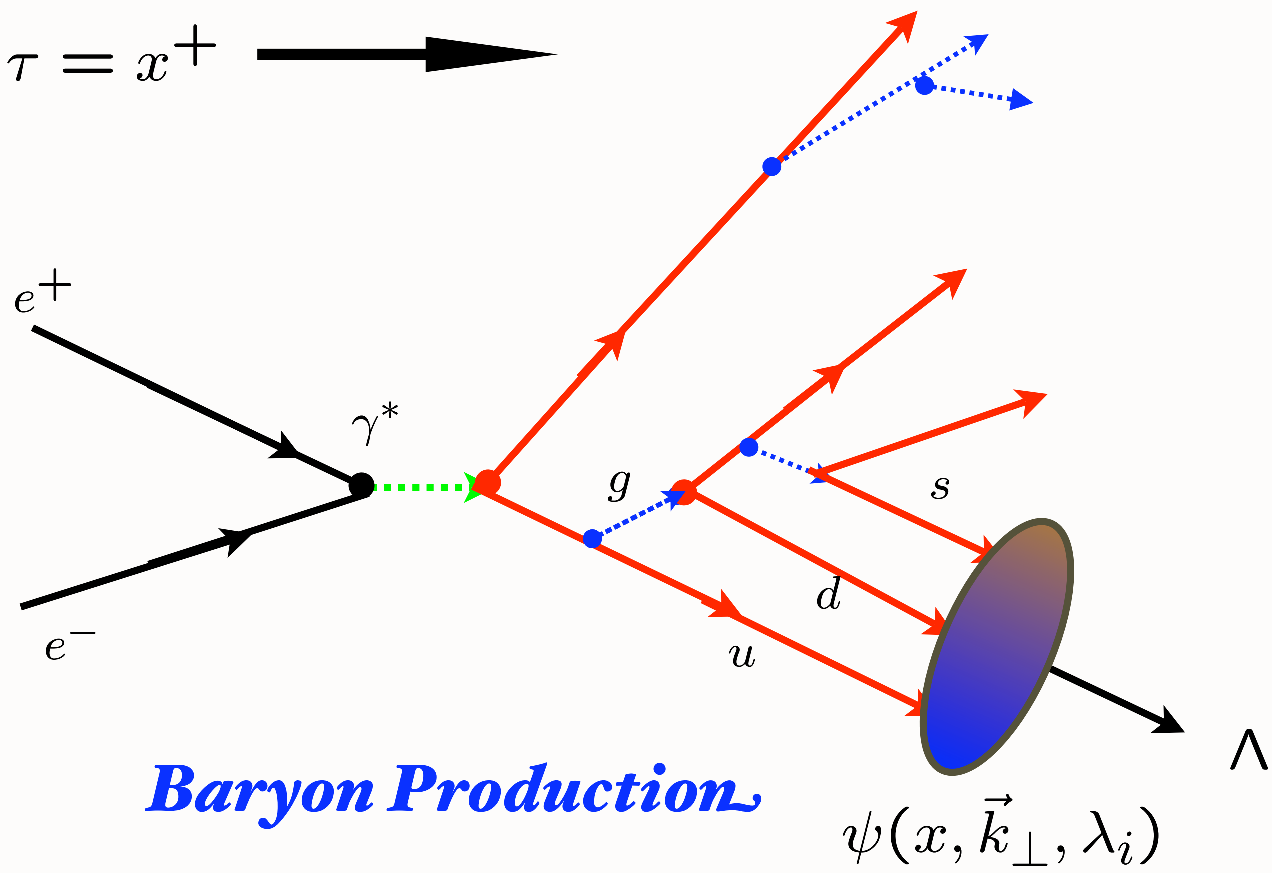
Hadronization at the Amplitude Level



Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level

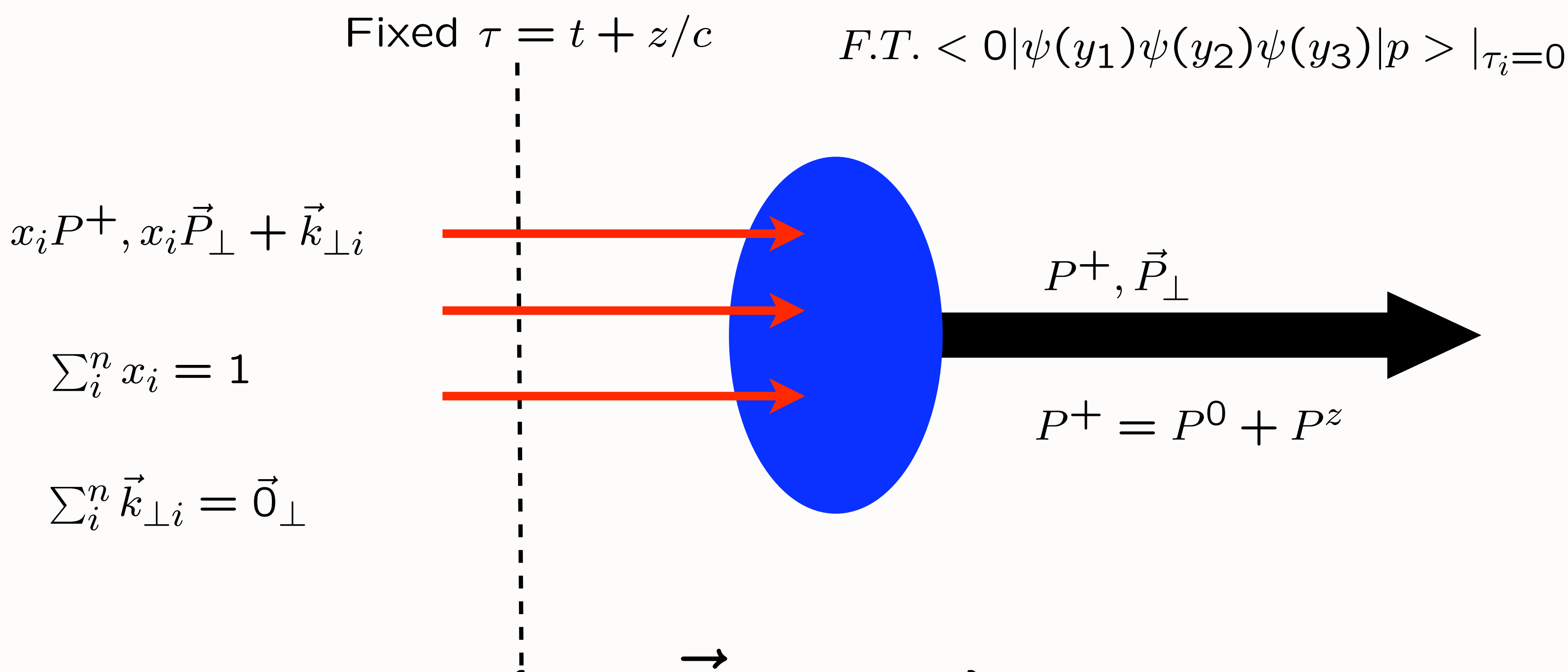


Baryon Production

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

AdS/QCD

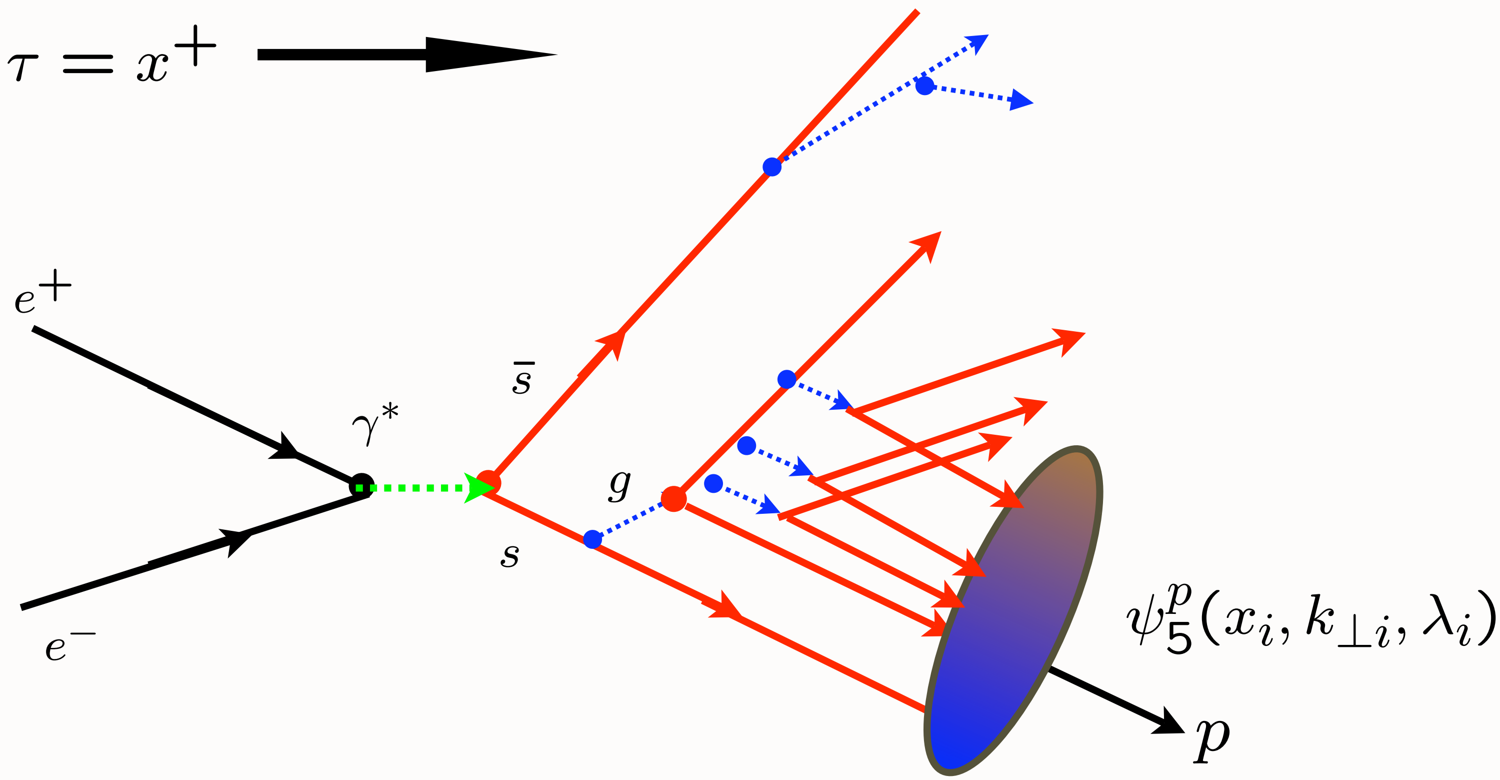
Light-Front Wavefunctions



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Invariant under boosts! Independent of P^μ

Hadronization at the Amplitude Level



Higher Fock State Coalescence $|uuds\bar{s}>$

Asymmetric Hadronization ! $D_{s \to p}(z) \neq D_{s \to \bar{p}}(z)$

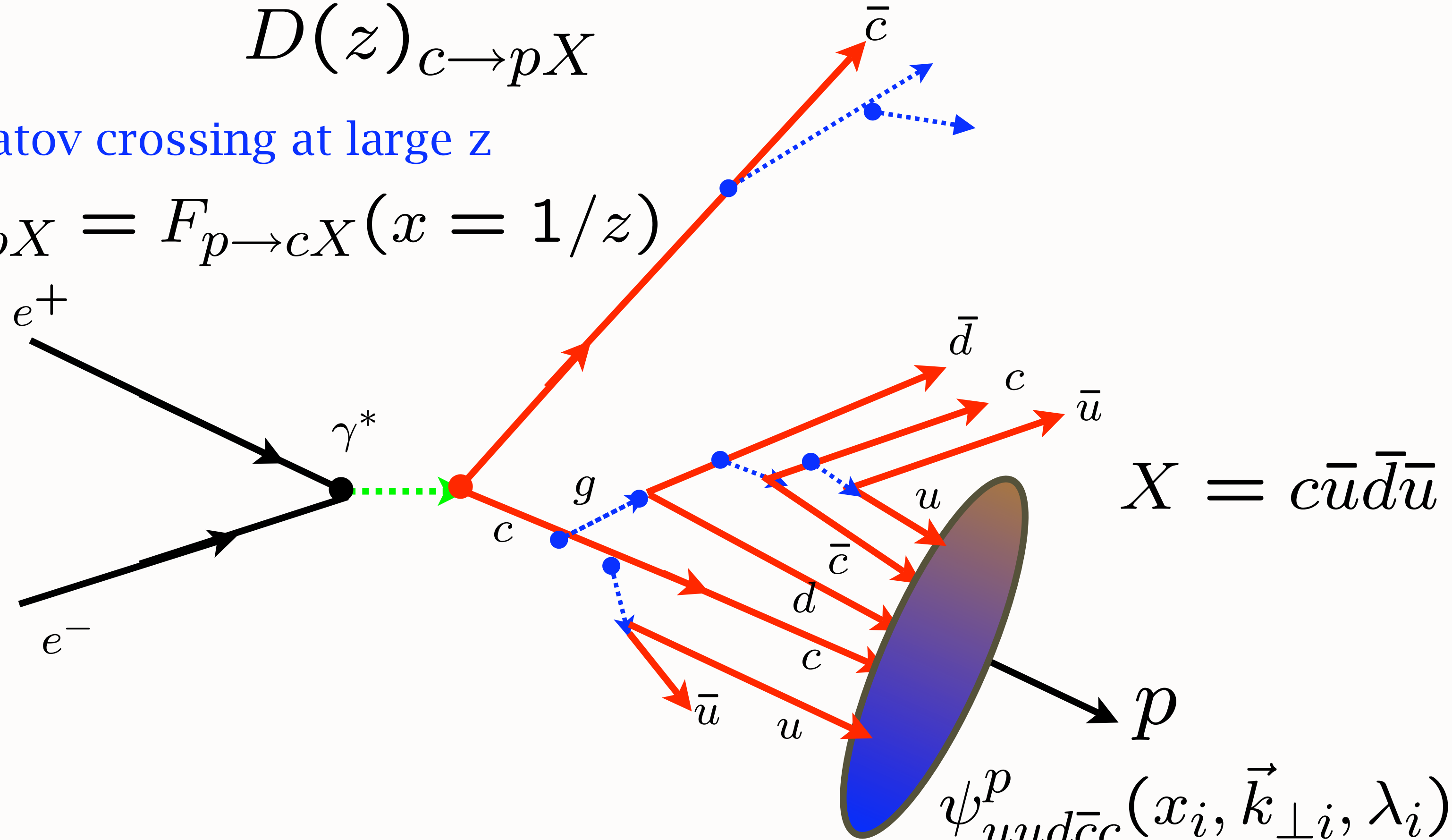
B-Q Ma, sjb

Timelike Test of Charm Distribution in Proton

$$D(z)_{c \to pX}$$

Gribov-Lipatov crossing at large z

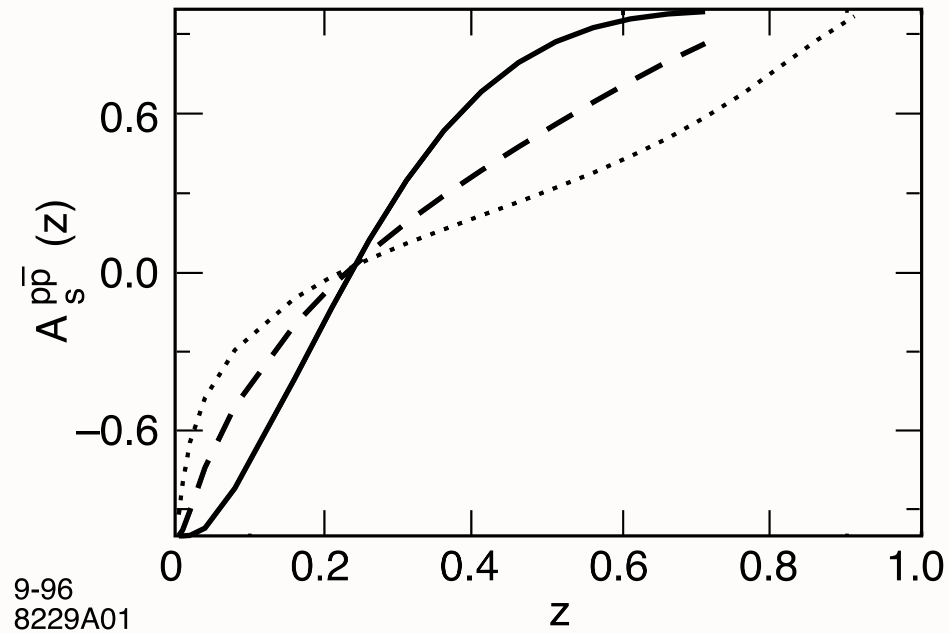
$$zD(z)_{c \to pX} = F_{p \to cX}(x = 1/z)$$



Intrinsic charm model:
predict proton at same rapidity as charm quark: high z

$$z_i \propto m_{\perp i} = \sqrt{m_i^2 + k_\perp^2}$$

$$D_{s\to p}(z) \neq D_{s\to\bar{p}}(z)$$



$$A_s^{p\bar{p}}(z) = \frac{D_{s\to p}(z) - D_{s\to\bar{p}}(z)}{D_{s\to p}(z) + D_{s\to\bar{p}}(z)}$$

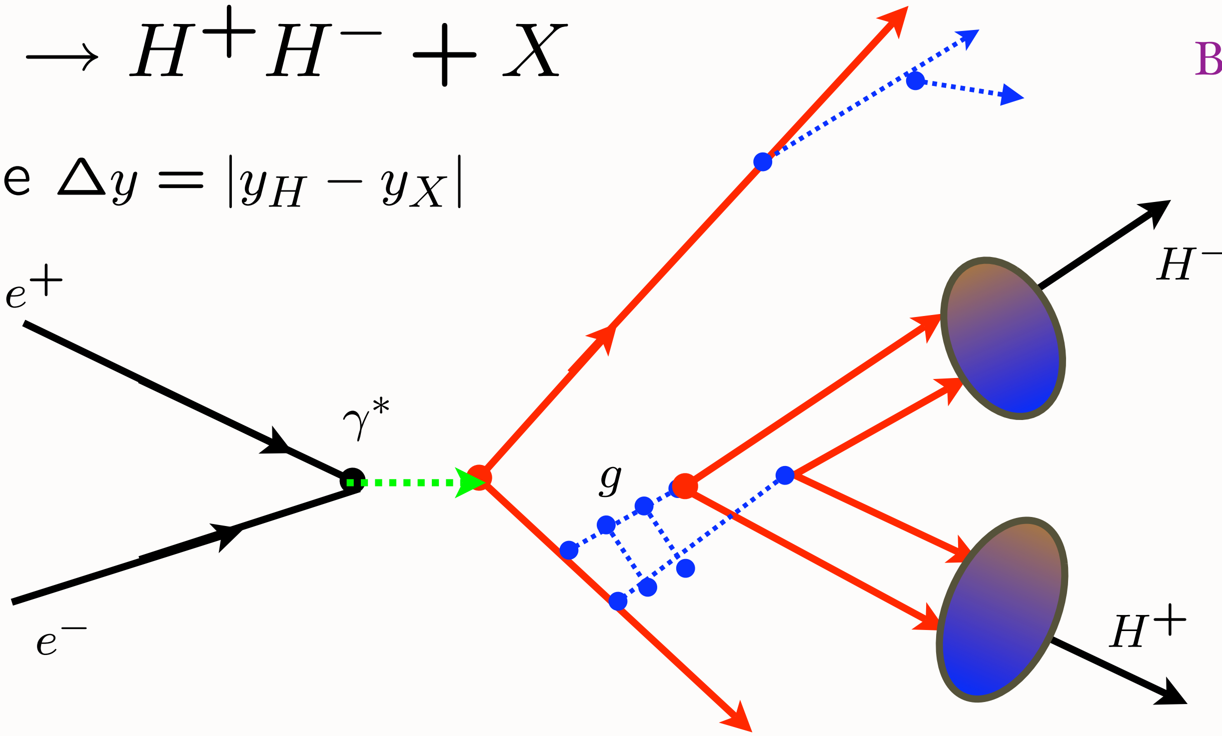
Consequence of $s_p(x) \neq \bar{s}_p(x)$ $\quad |uuds\bar{s} > \simeq |K^+\Lambda >$

Hadronization at the Amplitude Level

$$e^+e^- \to H^+H^- + X$$

Large $\Delta y = |y_H - y_X|$

Bjorken, Lu, sjb
Kopeliovich, Schmidt, sjb



Timelike Pomeron

Large Rapidity Gap Events

C= + Gluonium Trajectory

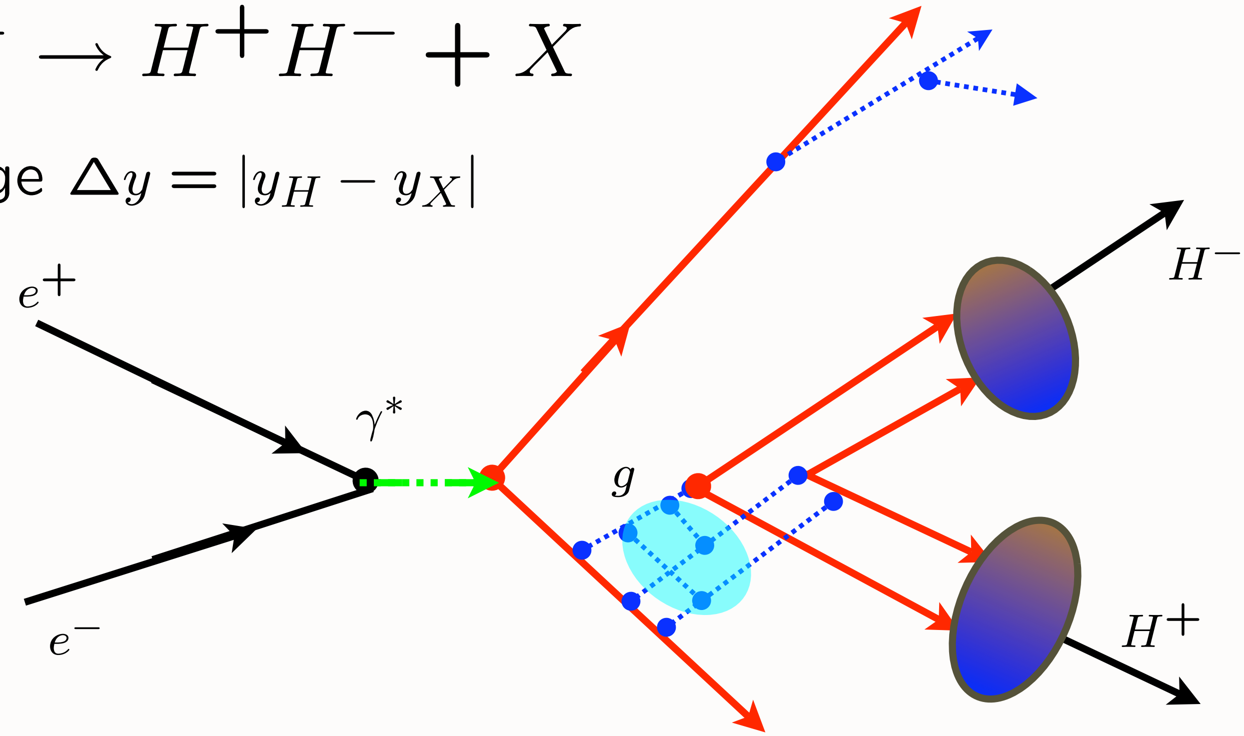
Crossing analog of Diffractive DIS $eH \to eH + X$

Hadronization at the Amplitude Level

Kopeliovich, Schmidt, sjb

$$e^+e^- \to H^+H^- + X$$

Large $\Delta y = |y_H - y_X|$



Timelike Odderon

Large Rapidity Gap Events

C= - Gluonium Trajectory

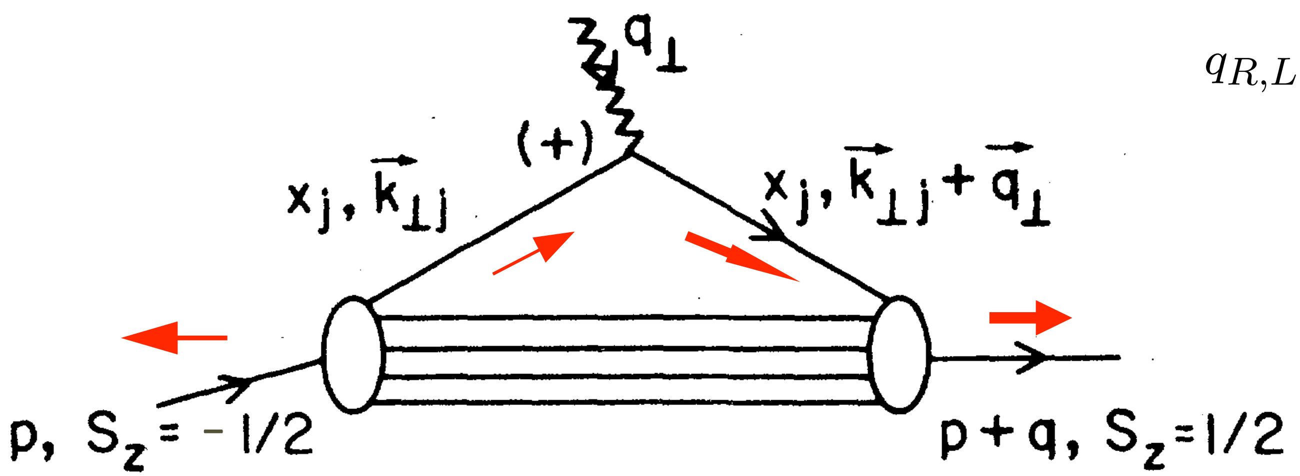
H^+H^- asymmetry from Odderon-Pomeron interference

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$



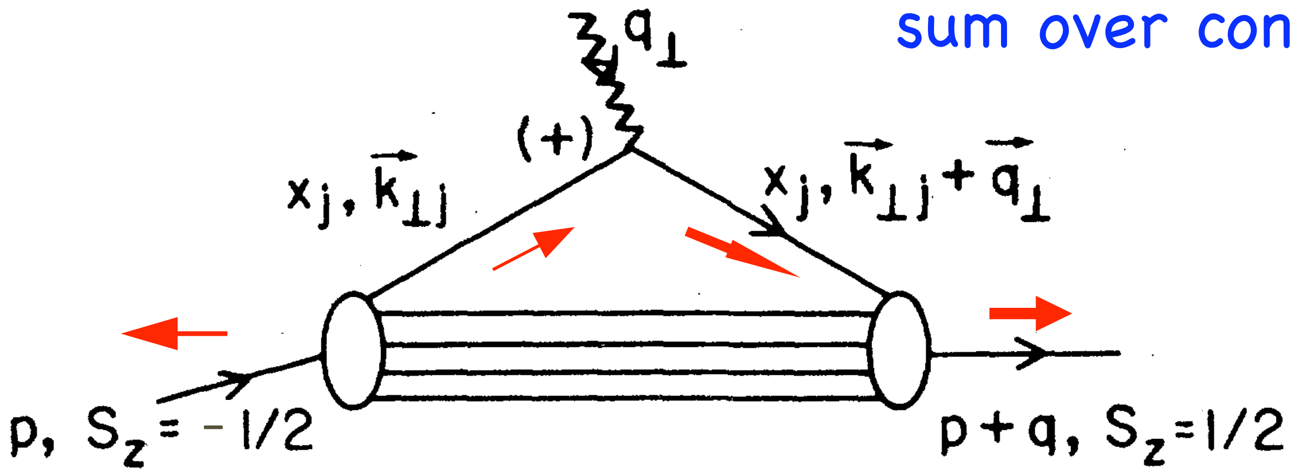
$$q_{R,L} = q^x \pm i q^y$$

Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Anomalous gravitomagnetic moment B(0)

Okun et al: *B(0) Must vanish because of Equivalence Theorem*

graviton



sum over constituents

Hwang, Schmidt, sjb;
Holstein et al

B(0) = 0

Each Fock State

Electric Dipole Form Factor on the Light Front

We consider the electric dipole form factor $F_3(q^2)$ in the light-front formalism of QCD, to complement earlier studies of the Dirac and Pauli form factors. [Drell, Yan, PRL 1970; West, PRL 1970; Brodsky, Drell, PRD 1980]

Recall

$$\langle P', S'_z | J^\mu(0) | P, S_z \rangle =$$

$$\bar{U}(P', \lambda') \left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2M}\sigma^{\mu\alpha}q_\alpha + F_3(q^2)\frac{-1}{2M}\sigma^{\mu\alpha}\gamma_5 q_\alpha \right] U(P, \lambda)$$

$$\kappa = \frac{e}{2M}\left[F_2(0)\right] , \qquad d = \frac{e}{M}\left[F_3(0)\right]$$

We will find a close connection between κ and d, as long anticipated. [Bigi, Uralstev, NPB 1991]

Electromagnetic Form Factors on the Light Front

Interaction picture for $J^+(0)$, $q^+ = 0$ frame,
imply ($q^{R/L} \equiv q^1 \pm iq^2$):

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right],$$

$$\frac{F_3(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{i}{2} \times$$

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) - \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right],$$

$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$ for the struck constituent j and $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp$ for each spectator ($i \neq j$). $q^+ = 0 \Longrightarrow$ only $n' = n$.

Both $F_2(q^2)$ and $F_3(q^2)$ are helicity-flip form factors.

Gardner, Hwang, sjb,

CP-violating phase

$$F_3(q^2) = F_2(q^2) \times \tan\phi$$

Fock state by Fock state

Gardner, Hwang, sjb,

Holographic Model for QCD Light-Front Wavefunctions

- Drell-Yan-West form factor

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp)\, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp\, e^{i\vec{b}_\perp\cdot\vec{k}_\perp}\widetilde{\psi}(x, \vec{b}_\perp)$$

- Find $(b = |\vec{b}_\perp|)$:

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp\, e^{ix\vec{b}_\perp\cdot\vec{q}_\perp}\left|\widetilde{\psi}(x,b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b\, db\, J_0\,(bqx)\, \left|\widetilde{\psi}(x,b)\right|^2, \end{aligned}$$

Soper

Hadronic Form Factor in Space and Time-Like Regions

- The form factor in AdS/QCD is the overlap of the normalizable modes dual to the incoming and outgoing hadron Φ_I and Φ_F and the non-normalizable mode J, dual to the external source (hadron spin σ):

$$F(Q^2)_{I\to F} = R^{3+2\sigma} \int_0^\infty \frac{dz}{z^{3+2\sigma}} \, e^{(3+2\sigma)A(z)} \Phi_F(z) \, J(Q,z) \, \Phi_I(z)$$
$$\simeq R^{3+2\sigma} \int_0^{z_o} \frac{dz}{z^{3+2\sigma}} \, \Phi_F(z) \, J(Q,z) \, \Phi_I(z),$$

- $J(Q,z)$ has the limiting value 1 at zero momentum transfer, $F(0) = 1$, and has as boundary limit the external current, $A^\mu = \epsilon^\mu e^{iQ\cdot x} J(Q,z)$. Thus:

$$\lim_{Q\to 0} J(Q,z) = \lim_{z\to 0} J(Q,z) = 1.$$

- Solution to the AdS Wave equation with boundary conditions at $Q = 0$ and $z \to 0$:

$$J(Q,z) = zQK_1(zQ).$$

Polchinski and Strassler, hep-th/0209211; Hong, Yong and Strassler, hep-th/0409118.

Identical DYW and AdS5 Formulae: Two parton case

- Change the integration variable $\zeta = |\vec{b}_\perp|\sqrt{x(1-x)}$

$$F(Q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int_0^{\zeta_{max}=\Lambda^{-1}_{\rm QCD}} \zeta \, d\zeta \, J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) \left|\widetilde{\psi}(x,\zeta)\right|^2,$$

- Compare with AdS form factor for arbitrary Q. Find:

$$J(Q,\zeta) = \int_0^1 dx J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for the electromagnetic potential in AdS space, and

$$\widetilde{\psi}(x,\vec{b}_\perp) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{0,1}\Lambda_{QCD}\right) \theta\left(\vec{b}_\perp^{\,2} \le \frac{\Lambda^{-2}_{\rm QCD}}{x(1-x)}\right)$$

the holographic LFWF for the valence Fock state of the pion $\psi_{\overline{q}q/\pi}$.

- The variable ζ, $0 \le \zeta \le \Lambda^{-1}_{QCD}$, represents the scale of the invariant separation between quarks and is also the holographic coordinate $\zeta = z$!

- Define effective single particle transverse density by (Soper, Phys. Rev. D **15**, 1141 (1977))

$$F(q^2) = \int_0^1 dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp \cdot \vec{q}_\perp} \tilde{\rho}(x, \vec{\eta}_\perp)$$

- From DYW expression for the FF in transverse position space:

$$\tilde{\rho}(x, \vec{\eta}_\perp) = \sum_n \prod_{j=1}^{n-1} \int dx_j \, d^2\vec{b}_{\perp j} \, \delta(1 - x - \sum_{j=1}^{n-1} x_j) \, \delta^{(2)}(\sum_{j=1}^{n-1} x_j \vec{b}_{\perp j} - \vec{\eta}_\perp) |\psi_n(x_j, \vec{b}_{\perp j})|^2$$

- Compare with the the form factor in AdS space for arbitrary Q:

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} e^{3A(z)} \Phi_{P'}(z) \, J(Q, z) \, \Phi_P(z)$$

- Holographic variable z is expressed in terms of the average transverse separation distance of the spectator constituents $\vec{\eta} = \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j}$

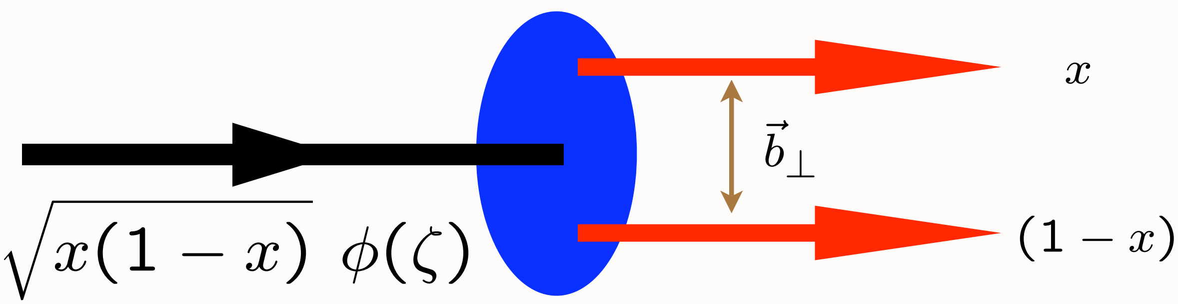
$$z = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j} \right|$$

Mapping between LF(3+1) and AdS5

LF(3+1) | **AdS5**

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x, \vec{b}_\perp) = \sqrt{x(1-x)}\ \phi(\zeta)$$

G. de Teramond and sjb

Map AdS/CFT to 3+1 LF Theory

Effective radial equation:

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

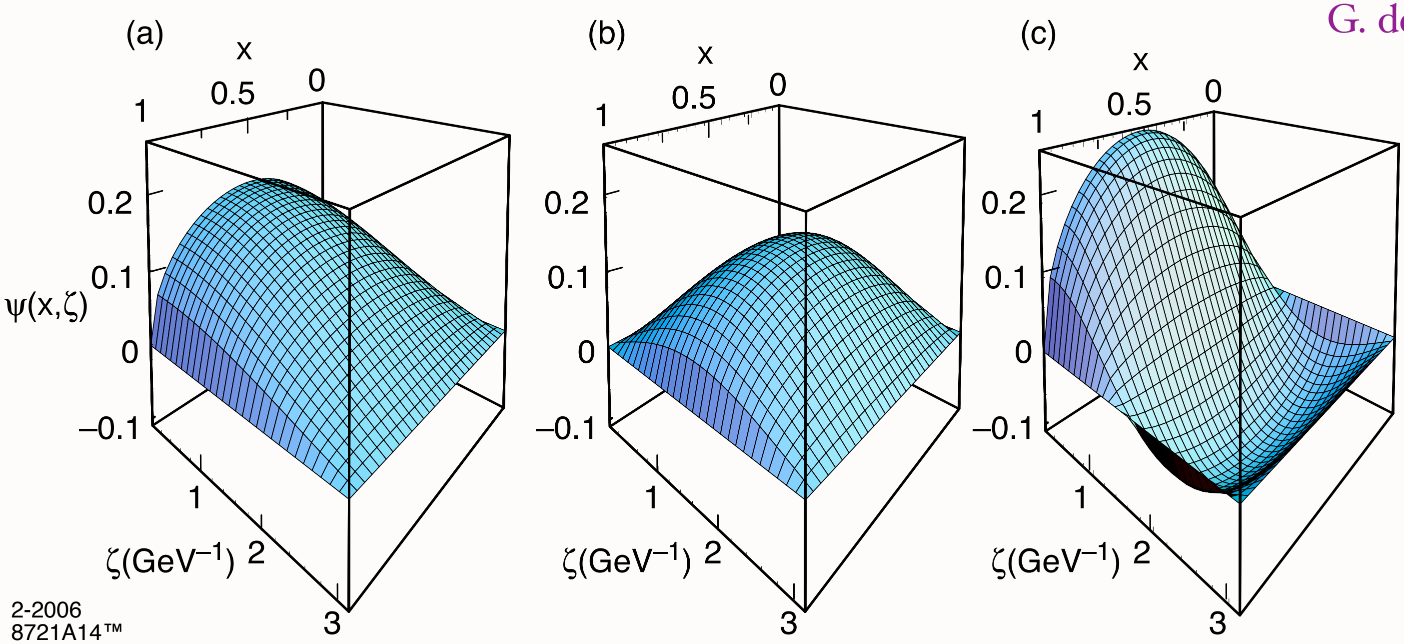
General solution:

$$\widetilde{\psi}_{L,k}(x,\vec{b}_\perp) = B_{L,k}\sqrt{x(1-x)}$$

$$J_L\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\left(\vec{b}_\perp^{\,2} \le \frac{\Lambda^{-2}_{\rm QCD}}{x(1-x)}\right),$$

AdS/CFT Prediction for Meson LFWF

G. de Teramond
SJB



Two-parton holographic LFWF in impact space $\widetilde{\psi}(x,\zeta)$ for $\Lambda_{QCD} = 0.32$ GeV: (a) ground state $L=0,\ k=1$; (b) first orbital exited state $L=1,\ k=1$; (c) first radial exited state $L=0,\ k=2$. The variable ζ is the holographic variable $z = \zeta = |b_\perp|\sqrt{x(1-x)}$.

$$\widetilde{\psi}(x,\zeta) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi} J_1(\beta_{0,1})}\sqrt{x(1-x)}\, J_0\left(\zeta \beta_{0,1} \Lambda_{QCD}\right) \theta\left(z \le \Lambda^{-1}_{\rm QCD}\right)$$

AdS/QCD

AdS/CFT and Integrability

- Conformal Symmetry plus Confinement: Reduce AdS/QCD Equations to Linear Form
- Generate eigenvalues and eigenfunctions using Ladder Operators
- Apply to Covariant Light-Front Radial Dirac and Schrodinger Equations
- L. Infeld, "On a new treatment of some eigenvalue problems", Phys. Rev. 59, 737 (1941).

AdS/CFT LF Equation for Mesons with HO Confinement

Karch, et al.

$$\left(\frac{d^2}{d\zeta^2} + \frac{1-4\nu^2}{4\zeta^2} - \kappa^4\zeta^2 - 2\kappa^2(\nu+1) + \mathcal{M}^2\right)\phi_\nu(\zeta) = 0$$

LF Hamiltonian

$$H^{\nu}_{LF}\phi_\nu = \mathcal{M}^2_\nu \phi_\nu$$

Bilinear $H^{\nu}_{LF} = \Pi^\dagger_\nu \Pi_\nu,$

where

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta} - \kappa^2\zeta\right),$$

and its adjoint

de Teramond, sjb

$$\Pi^\dagger_\nu(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta\right),$$

with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi^\dagger_\nu(\zeta)\right] = \frac{2\nu+1}{\zeta^2} - 2\kappa^2.$$

AdS/QCD

AdS/CFT LF Equation for Mesons with HO Confinement

$$\left(\frac{d^2}{d\zeta^2} + \frac{1-4\nu^2}{4\zeta^2} - \kappa^4\zeta^2 - 2\kappa^2(\nu+1) + \mathcal{M}^2\right)\phi_\nu(\zeta) = 0$$

Define $$b_\nu^\dagger = -i\Pi_\nu = \frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta$$

$$b_\nu = \frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta \qquad b_\nu^\dagger b_\nu = b_{\nu+1} b_{\nu+1}^\dagger$$

Ladder Operator $$b_\nu^\dagger|\nu\rangle = c_\nu|\nu+1\rangle$$

$$\left(-\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta\right)\phi_\nu(\zeta) = c_\nu\phi_{\nu+1}(\zeta)$$

$$\phi_\nu(z) = C z^{1/2+\nu} e^{-\kappa^2\zeta^2/2} G_\nu(\zeta),$$

$$2xG_\nu(x) - G'(x) = xG_{\nu+1}(x)$$

defines the associated Laguerre function $L_n^{\nu+1}(x^2)$

$$\phi_\nu(z) = C_\nu z^{1/2+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2).$$

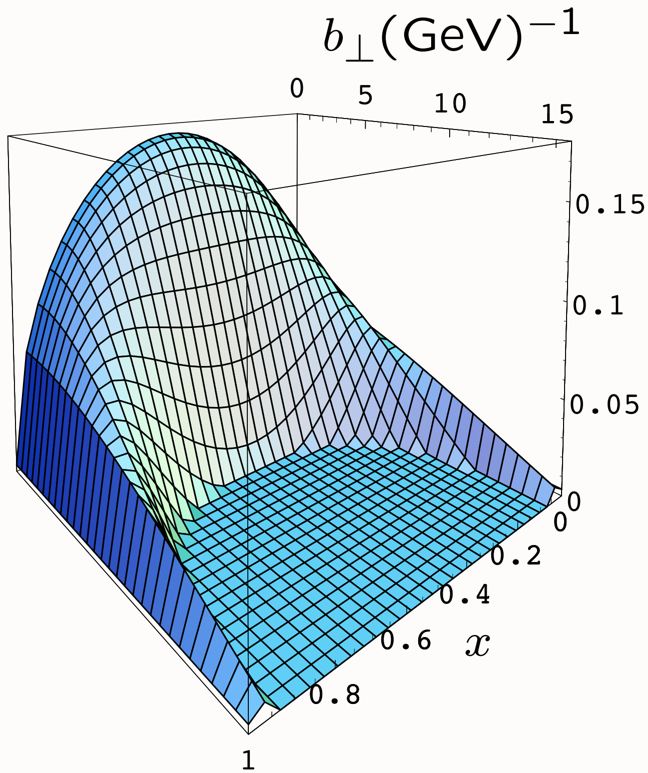
Subtract Vacuum Energy

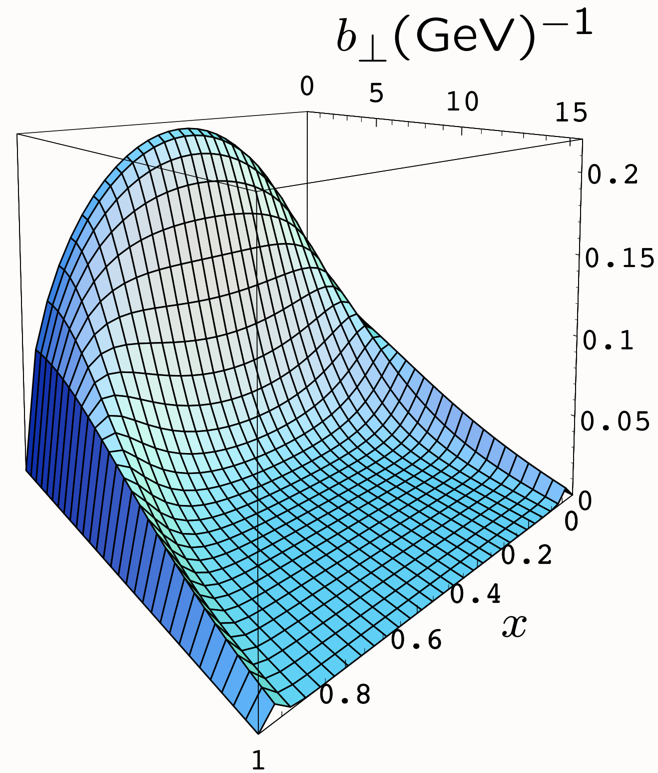
$$\mathcal{M}^2 \rightarrow \mathcal{M}^2 - 2\kappa^2,$$

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + \frac{1}{2}).$$

AdS/QCD

AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$





$\Lambda_{QCD} = 0.32$ GeV $\qquad \kappa = 0.76$ GeV

Truncated Space ***Harmonic Oscillator***