BREAKDOWN OF THE NARROW WIDTH APPROXIMATION

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- The narrow width approximation
- Calculational tools
- PDF effects
- Non-resonant contributions
- Resonant matrix element effects

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Introduction – the Narrow Width Approximation

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Step 2: phase space is separable (exact)

$$
\begin{aligned}
d\Phi^n &= (2\pi)^4\delta^4\left(k_1+k_2-\sum_{i=1}^{n} p_i\right)\prod_{i=1}^{n}\frac{d^3p_i}{(2\pi)^3 2E_i} \\
&= (2\pi)^4\delta^4\left(k_1+k_2-q-\sum_{i=3}^{n} p_i\right)\prod_{i=3}^{n}\frac{d^3p_i}{(2\pi)^3 2E_i}\times\frac{d^3q}{(2\pi)^3 2E_q^0} \\
&\quad \times\frac{dq^2}{2\pi}\times(2\pi)^4\delta^4(q-p_1-p_2)\,\frac{d^3p_1}{(2\pi)^3 2E_1}\,\frac{d^3p_2}{(2\pi)^3 2E_2}
\end{aligned}
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Step 3: assume the s-channel propagator is separable (iffy)

Step 3 in detail:

$$\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \left| \frac{1}{q^2 - m^2 + im\Gamma} \right|^2 = \int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \frac{1}{(q^2 - m^2)^2 + (m\Gamma)^2}$$

$$[\text{change of variables}] = \int_{q^2_{\rm min}-m^2}^{q^2_{\rm max}-m^2} dx \frac{1}{x^2 + (m\Gamma)^2}$$

$$[q^2_{\rm min} = 0,\, q^2_{\rm max} = s] = \int_{-m^2}^{s-m^2} dx \frac{1}{x^2 + (m\Gamma)^2}$$

$$[s \to \infty,\, 0 \to -\infty] \approx \int_{-\infty}^{\infty} dx \frac{1}{x^2 + (m\Gamma)^2}$$

$$[\text{known maths}] = 2\int_{0}^{\infty} dx \frac{1}{x^2 + (m\Gamma)^2} = 2 \cdot \frac{\pi}{2} \cdot \frac{1}{m\Gamma} = \boxed{\frac{\pi}{m\Gamma}}$$

$\longrightarrow$ just a numerical factor

Conventional wisdom:

NWA is good to $\mathcal{O}(\Gamma/m)$; off-shell rarely needed ($e^+e^- \to W^+W^-$)

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▶ examine 1 , 2 & 3 in this talk

New physics case study

To study, we need new physical states (we expect this anyway):
$\longrightarrow$ preferably heavy & colored (gives larger Γ, Γ/m)

We examine supersymmetry (SUSY):

- lots of new resonances
- some are heavy ($\mathcal{O}(1)$ TeV), some colored;
  have large widths: $\mathcal{O}(10-20\%)$ may easily happen
- LSP (dark matter candidate) is stable, massive
  $\longrightarrow$ end of decay chains not massless
- well-motivated, well-studied SM extension – in the NWA limit!

▶ SUSY simulations always $2 \to 2$, even for multi-TeV fat sparticles

(some Breit-Wigner kludging in a few cases, but still NWA-like)

How we study off-shell effects in SUSY:

SUSY MADGRAPH

Package is standard MADGRAPH [Stelzer & Long, 1994] plus:

1. MSSM model input files (particles, interactions)
2. routine to read SUSY Les Houches Accord spectrum input
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(R-parity-conserving MSSM, no additional CP violation)

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Testing SUSY MADGRAPH:

$\longrightarrow$ all $e^+e^-, pp \rightarrow$ SUSY pairs checked with literature
$\longrightarrow$ all possible $VV, VH \rightarrow$ SUSY pairs checked for unitarity
$\longrightarrow$ EM gauge invariance checked for EW & WBF processes
$\longrightarrow$ 435 ($2 \rightarrow 2$) processes compared with Whizard & Sherpa

(SHERPA or WHIZARD could equally well be used.)

PDF Effects

Logical steps:

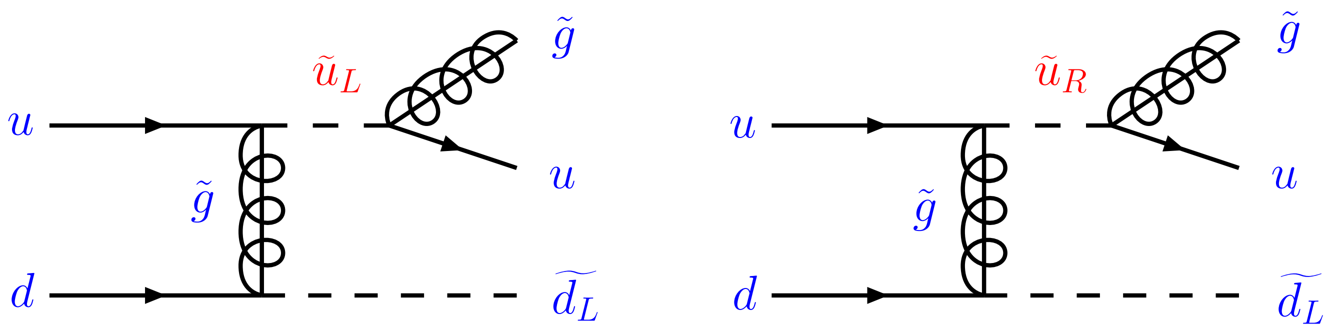
· TeV squarks and gluinos can have multi-hundred GeV widths.

· PDFs in the TeV regime fall steeply.

· Thus, a broad Breit-Wigner may be distorted/suppressed.

E.g.: Focus Point scenario + variations using $ud \to \widetilde{u}_L \to u\widetilde{g}\widetilde{d}_L$:

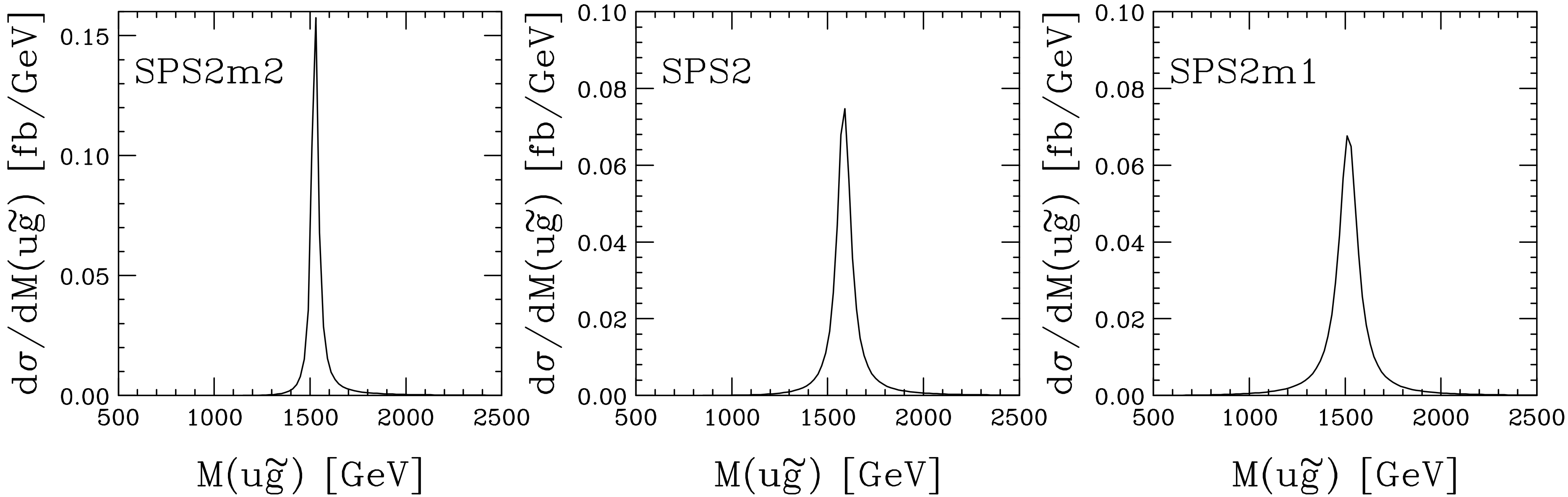
	SPS2m2		SPS2		SPS2m1	
	m [GeV]	Γ [GeV]	m [GeV]	Γ [GeV]	m [GeV]	Γ [GeV]
$\widetilde{u}_L$	1525	43.9	1590	90.0	1525	127
$\widetilde{u}_R$	1525	28.8	1580	73.7	1514	111
$\widetilde{d}_L$	1527	44.0	1592	90.1	1526	127
$\widetilde{d}_R$	1526	26.2	1580	70.9	1515	108
$\widetilde{g}$	1125	0.118	803	3.84×10^{-3}	414	7.36×10^{-5}



PDF effects results for LHC ($\sqrt{s} = 14$ TeV, CTEQ6L1, σ in [fb])

decays	SPS2m1 $\tilde{u}_L$ only	SPS2m1 $\tilde{u}_L, \tilde{d}_L$	SPS2 $\tilde{u}_L$ only	SPS2 $\tilde{u}_L, \tilde{d}_L$	SPS2m2 $\tilde{u}_L$ only	SPS2m2 $\tilde{u}_L, \tilde{d}_L$
ONS	3.11	1.28	4.83	1.88	5.85	1.67
OFS res	2.76	0.96	4.36	1.48	5.60	1.50
shift	-11%	-25%	-9.7%	-22%	-4.3%	-10%

► shifts can easily be larger than the NLO QCD uncertainties!



► WARNING: This is only to make a point about PDFs.
Integrating off-shell without proper interference can be dodgy.

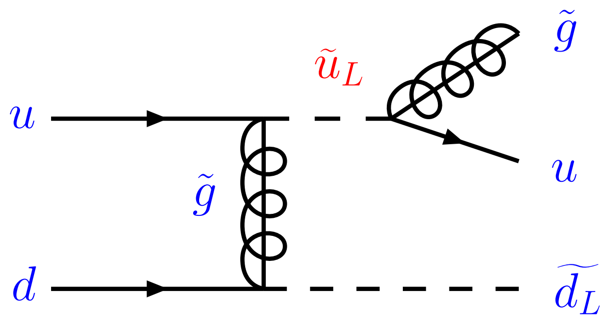
NON-RESONANT INTERFERENCE

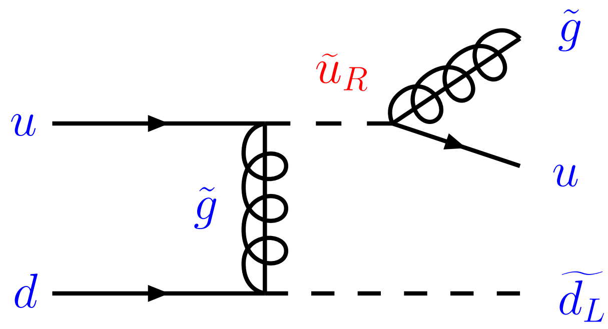
Example 1: heavy squarks, lighter gluino

· TeV squarks and gluinos can have multi-hundred GeV widths.

$\longrightarrow$ plenty of phase space for QCD interference

Study same FP scenarios as for PDFs: $ud \to \tilde{u}_L \widetilde{d}_L \to u\tilde{g}\widetilde{d}_L$:





+ non-resonant $u\tilde{g}\widetilde{d}_L$ production

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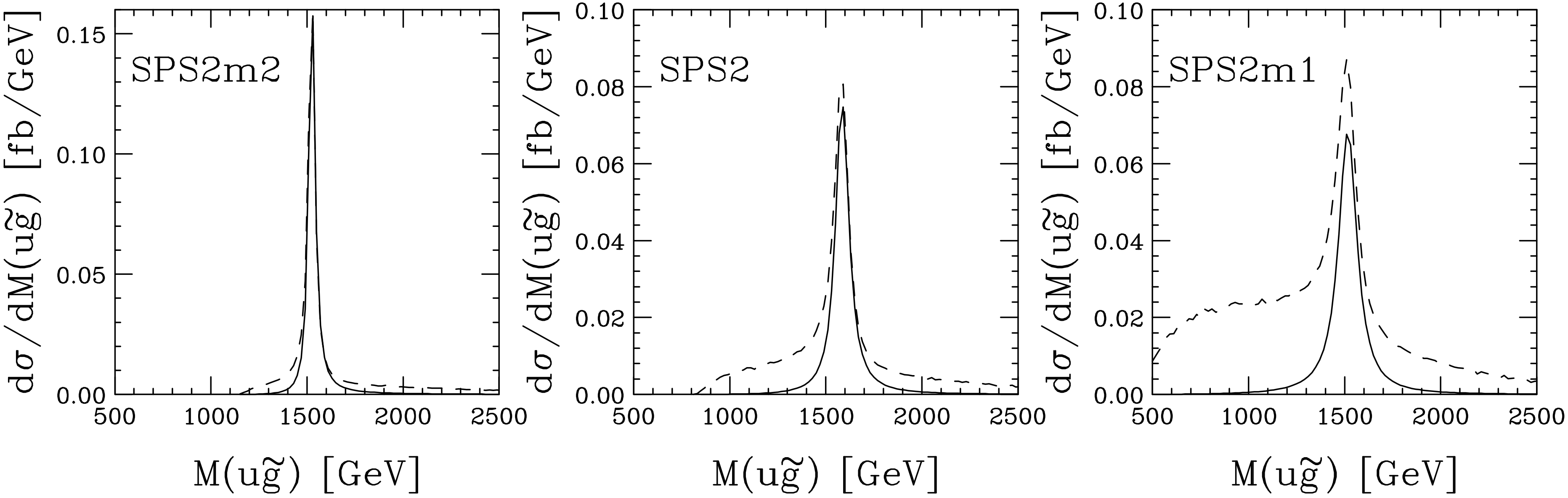
Single heavy squark decay $pp \to u\tilde{g}\tilde{d}_L$

(solid = resonant, dashed = all diagrams)



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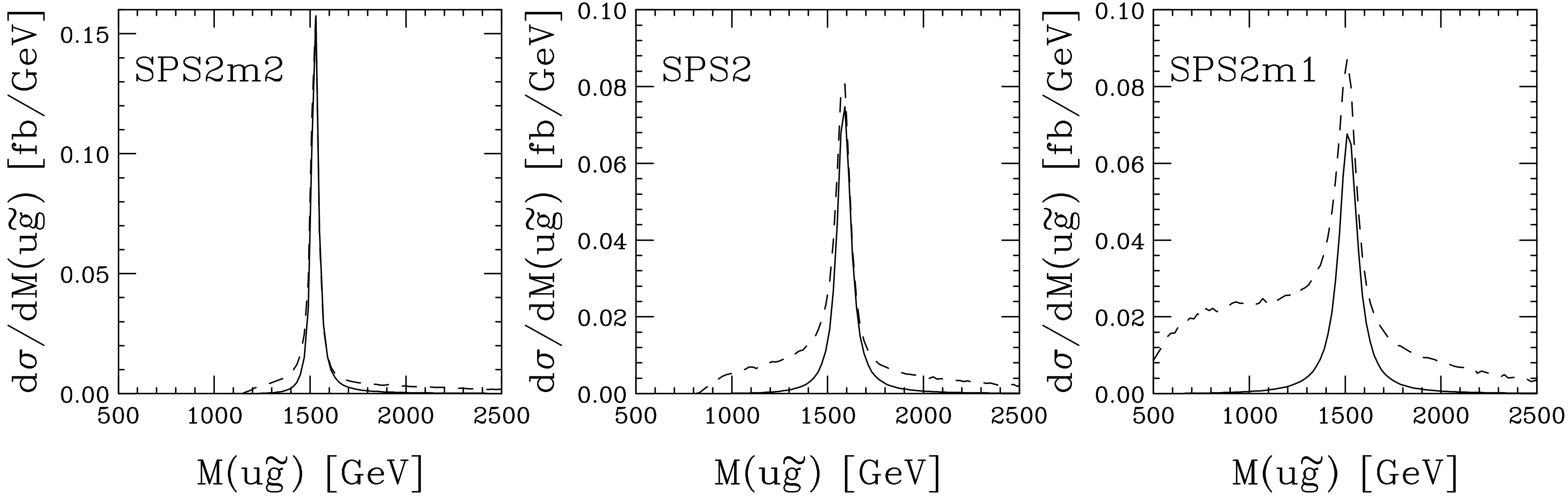
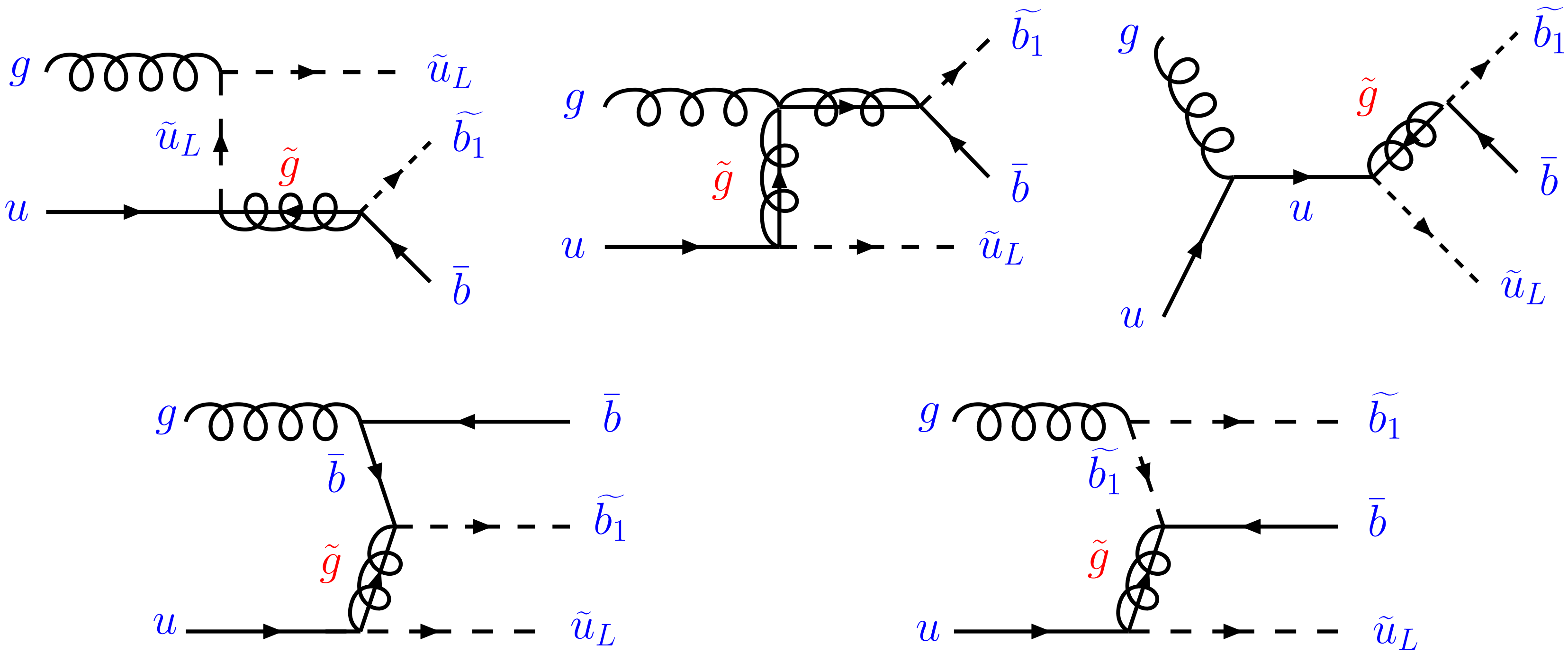


all/res.	SPS2m2	SPS2	SPS2m1
total	+52%	+107%	+280%
2σ	+9.3%	+31%	+85%
1σ	+5.5%	+18%	+50%

- ► effects are many times Γ/m, even in 1σ region
- ► NWA rate wouldn't jive with mass & spin measurements

Example 2: $pp \to \tilde{u}_L \bar{b} \tilde{b}_1$ (squark-gluino pairs, $\tilde{g}$ decay)



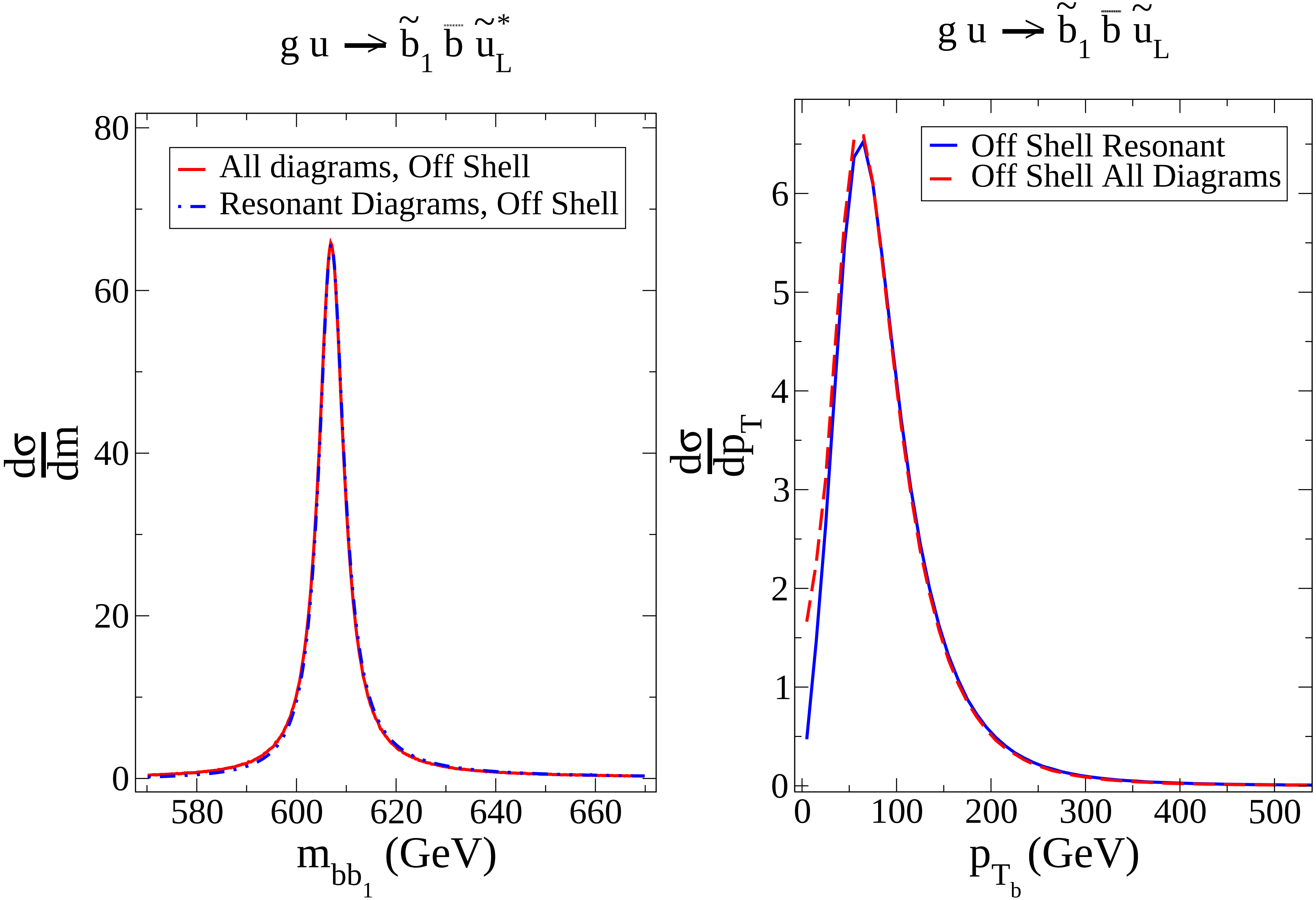
For SPS1a $(m_{\tilde{g}} = 607$ GeV, $\Gamma/m \sim 1\%$, $m_{\tilde{b}_1} = 517$ GeV$)$

$$\begin{aligned} \sigma_{\rm ONS} &= 663 \text{ fb} \\ \sigma_{\rm OFS} &= 633 \text{ fb} \\ R(\rm OFS/ONS) &= 1.05 \end{aligned}$$

► marginal correction to total rate (for one decay only)

→ but does not change kinematics

Kinematics of $gu \to \tilde{u}_L \tilde{g} \to \tilde{u}_L \bar{b} \tilde{b}_1$ off-shell



► little change, but beware of low-p_T enhancement from logs

Example 3: gluino pairs, $pp \to \tilde{g}\tilde{g} \to \bar{b}\tilde{b}_1 b\tilde{b}_1^*$

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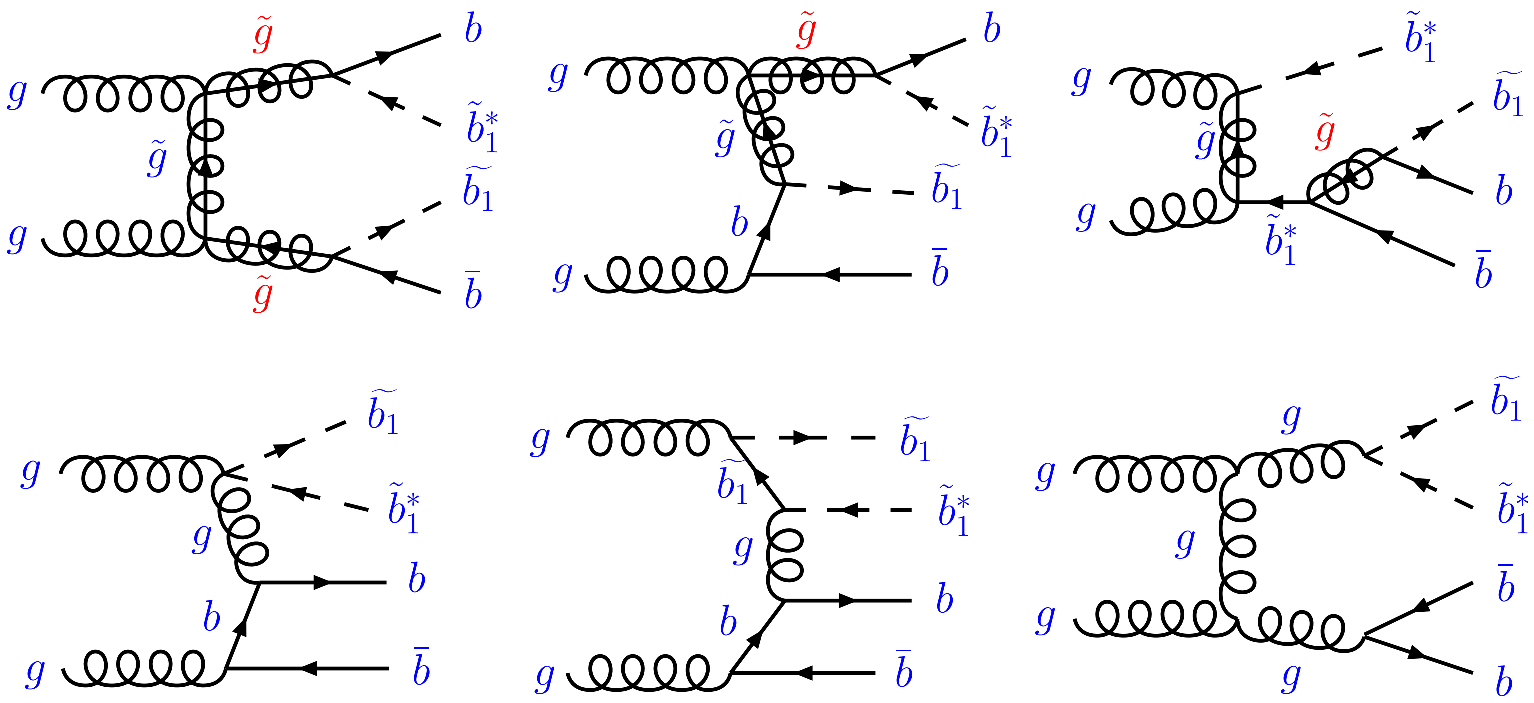
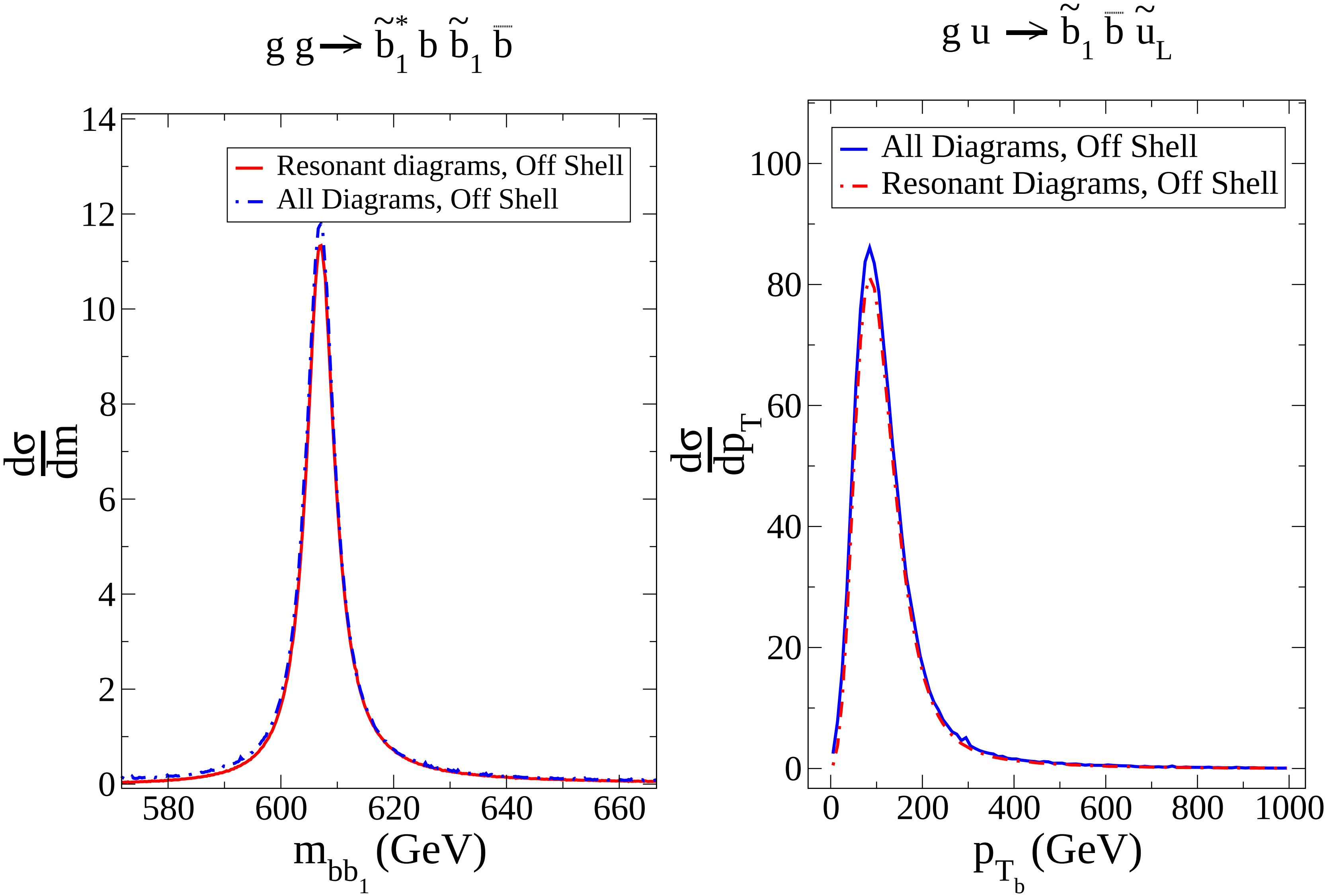


$$\sigma_{\text{ONS}} = 106 \text{ fb}$$
$$\sigma_{\text{OFS}} = 125 \text{ fb}$$
$$R(\text{OFS}/\text{ONS}) = 1.18$$

► large rate correction; NLO $\triangle\sigma \sim 18\%$

$\longrightarrow$ but does not change kinematics!

Kinematics of $pp \to \tilde{g}\tilde{g} \to \bar{b}\tilde{b}_1 b\tilde{b}_1^*$ off-shell



▶ no change! reason: interference is all at $\tilde{g}$ pole

→ expect larger effect for larger $m_{\tilde{g}}$ or smaller $m_{\tilde{b}_1}$ (wider res.)

Same-sign v. opposite-sign gluino pair decays

Opp.-sign: $\tilde{g}\tilde{g} \to b\bar{b}\tilde{b}_1\tilde{b}_1^*$

Same-sign: $\tilde{g}\tilde{g} \to bb\tilde{b}_1^*\tilde{b}_1^*$

$\longrightarrow$ have different non-resonant structures

	OS [fb]	SS [fb]
ONS	106	106
OFS	125	117
shift	$+18\%$	$+10\%$

► OS correction is $\sim$ twice that for SS

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We ask ourselves:

How would such an observation be interpreted without prior knowledge of off-shell effects?

Interference effects summary

We've seen cases where:

- there's no interference effect
- there's an $\mathcal{O}(1)$ rate correction,
  plus significant lineshape distortions
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The LHC is upon us.

Serious studies should use more advanced tools, perform full calculations. No more $2 \longrightarrow 2$ with on-shell cascades.

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Be concerned about jet/lepton edge studies, reconstruction, and data fed into FITTINO/SFITTER.

(Next step: reproduce all the SPS edge studies.)

MATRIX ELEMENT EFFECTS

(altered Breit-Wigner integration)

Let's categorize the decay matrix element types:

Possible renormalizeable 3-point vertices for 2-body decays are:

FFS, FFV, VVV, VVS, VSS, SSS

(ignore 4-point vertices: 3-body decays heavily P.S.–suppressed)

FFV Only in MSSM weak sector: observable V:FF decays ruled out, but F:FV decays may occur, e.g. $\widetilde{\chi}^{\pm} \to \widetilde{\chi}^{0} W^{\pm}$.

FFS Relevant for $\widetilde{g} \to \widetilde{q}\overline{q}$ (F:FS) and $\widetilde{q} \to \widetilde{g}q$ (S:FF).

VVV Nothing new in MSSM.

VVS MSSM Higgs sector only.

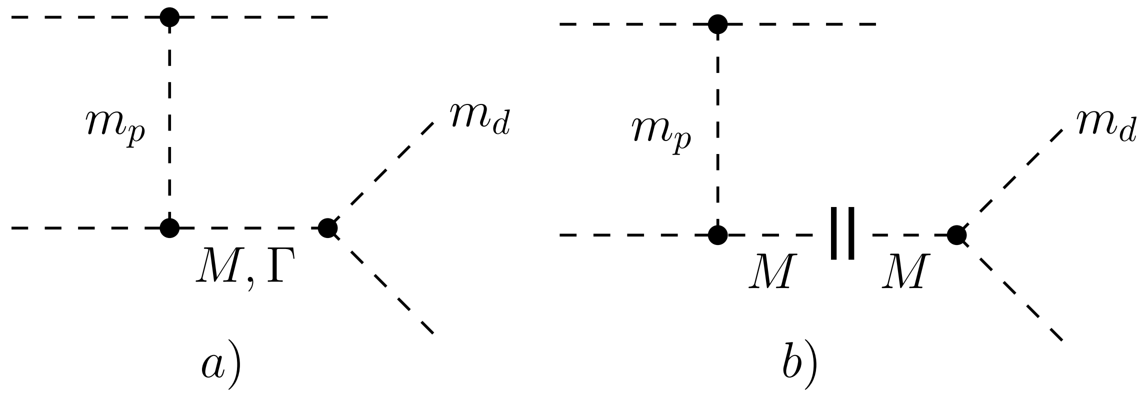
VSS S:SV relevant: $\widetilde{t} \to \widetilde{b}W$ or $\widetilde{b} \to \widetilde{t}W$

SSS Trivial structure – no *decay matrix element* effect.

► we examine S:SS, F:FS, S:FF and S:SV

S:SS type decays (simplest decay type to start with)

→ *consider scalar theory process outside SUSY*
(assign e.g. flavor to limit to this one diagram)



· first, study it analytically; massless scalars except as labeled

$$\frac{\sigma_{OFS}}{\sigma_{NWA}} \sim 1 + \frac{1}{\pi}\frac{\Gamma}{M}\left(\frac{1}{\beta_d^2} - \frac{1}{\beta_M^2}\right) + \ldots$$

where

$$\beta_d = \sqrt{1 - \frac{m_d^2}{M^2}} \quad \text{and} \quad \beta_M = \sqrt{1 - \frac{M^2}{s}}$$

(actually a bigger mess, with $\log(s/m^2)$ terms, but reduces nicely)

S:SS all-scalar case

$$\sqrt{s} \gg M, m_d \to M \qquad\qquad m_d \to 0, \sqrt{s} \to M$$

$$\frac{\sigma_{OFS}}{\sigma_{NWA}} \sim 1 + \frac{1}{\pi}\frac{\Gamma}{M}\frac{1}{\beta_d^2} \qquad\qquad \frac{\sigma_{OFS}}{\sigma_{NWA}} \sim 1 - \frac{1}{\pi}\frac{\Gamma}{M}\frac{1}{\beta_M^2}$$

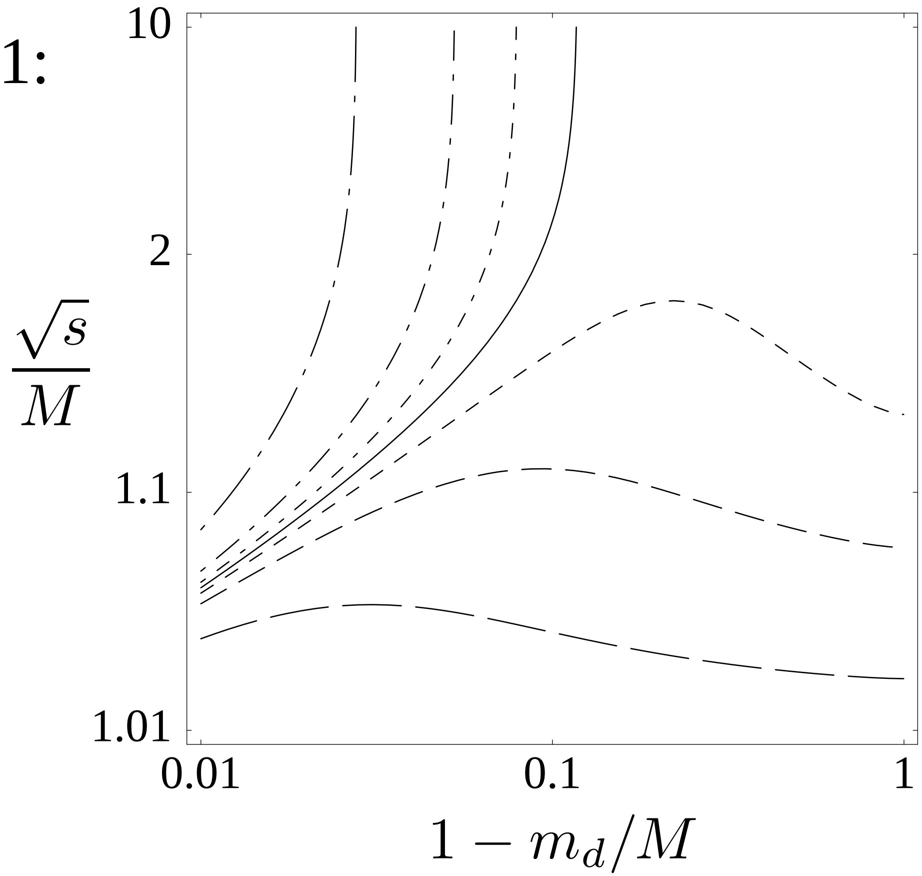
countour plot for $R \equiv \frac{\sigma_{OFS}}{\sigma_{NWA}} - 1$:

solid is $R = 0$

others $R = \{1, 3, 10\}$

dot-dashed $R > 0$

dashed $R < 0$



Not a surprise: $\sigma_{NWA} \to 0$ at any threshold

▶ this is *partly* a phase space effect

Non-SSS Vertex modifications

SSS has no matrix element, but others do. For instance:

S:FF

Decay matrix element is separable:

$$\overline{\sum}|\mathcal{M}_d|^2 = 2\left(q^2 - (m_1 + m_2)^2\right)$$

(F:FS more complicated)

V:SS

Decay matrix element is proportional to:

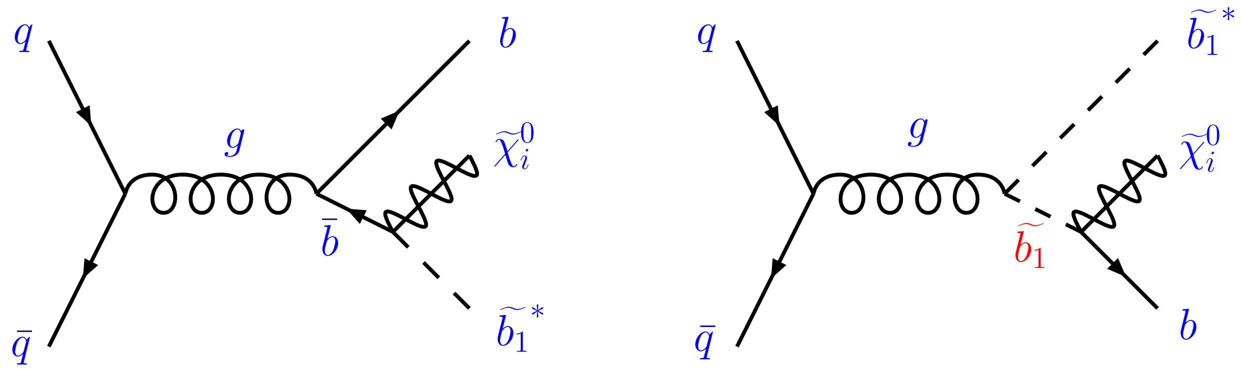
$$\mathcal{M}_d \propto \left(p_1^\mu - p_2^\mu\right) \sim q\,(\text{magnitude})$$

(S:SV more complicated)

S:FF type decays

· simplest realistic decay type in full model

$\longrightarrow$ mostly relevant for squarks, for example:



· other diagrams exist, but may be removed by taking $m_{t-ch.} \longrightarrow \infty$

▶ first study analytically, resonant diagram only, $m_b = m_{\widetilde{b}_1^*} = 0$

(not correct, but good for limits and qualitative behavior)

· use $\widetilde{q} \longrightarrow \widetilde{g}q$ as $\alpha_s \gg \alpha_w$ (partial width is larger)

S:FF

Full result is several pages of Mathematica output.
Leading $1/s$ terms (to match NWA $1/s$ behavior) are:

$$\frac{\sigma_{OFS}}{\sigma_{NWA}} \sim \frac{m_S\Gamma_S}{2\pi\left(m_S^2-m_F^2\right)^2\left(m_S^2+\Gamma_S^2\right)} \times$$

$$\left(\frac{m_S}{\Gamma_S}\left(\left(m_S^2-m_F^2\right)^2+\left(m_S^2-2m_F^2\right)\Gamma_S^2\right)\left(\pi+2\cot^{-1}\left(\frac{m_S\Gamma_S}{m_S^2-m_F^2}\right)\right)\right.$$

$$-\frac{11}{3}m_S^2\left(m_S^2+\Gamma_S^2\right)+m_S^2\left(m_S^2+\Gamma_S^2\right)\log\left(\frac{s^2}{\left(m_S^2-m_F^2\right)^2+m_S^2\Gamma_S^2}\right)$$

$$\left.+m_F^4\log\left(\frac{\left(m_S^2-m_F^2\right)^2+m_S^2\Gamma_S^2}{m_F^4}\right)\right)$$

ostensibly $\mathcal{O}\left(\Gamma/m\right)$, but:

$\longrightarrow$ lots of m_F dependence

$\longrightarrow$ unexpected $\log(s)$ term

S:FF

Let's take the 2 SM limits:

① $m_F \to 0$

$$\frac{\sigma_{OFS}}{\sigma_{NWA}} \xrightarrow{m_F \to 0} \frac{1}{2} + \frac{1}{\pi} \cot^{-1}\left(\frac{\Gamma_S}{m_S}\right) - \frac{11}{6\pi}\frac{\Gamma}{m} + \frac{1}{2\pi}\frac{\Gamma}{m} \log\left(\frac{s^2}{m_S^2\left(m_S^2 + \Gamma_S^2\right)}\right)$$

$\cot^{-1}$ term remains (is known to people)

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$\cot^{-1}$ term remains (is known to people)

② $\Gamma \ll m$

$$\frac{\sigma_{OFS}}{\sigma_{NWA}} \xrightarrow{\Gamma_S \ll m_S} 1 - \frac{17}{6\pi}\frac{\Gamma}{m} + \frac{1}{\pi}\frac{\Gamma}{m}\log\left(\frac{s}{m_S^2}\right)$$

Basically what we expect: $1 + \mathcal{O}(\Gamma/m)$,

but still has $\log(s)$ dependence

S:FF

Let's rewrite the full result in a clearer form:

$$\frac{\sigma_{OFS}}{\sigma_{NWA}} \sim \frac{1}{2\pi}\left(\frac{\beta_F^4 + \left(1 - 2\frac{m_F^2}{m_S^2}\right)\frac{\Gamma_S^2}{m_S^2}}{\beta_F^4\left(1+\frac{\Gamma_S^2}{m_S^2}\right)} \left(\pi + 2\cot^{-1}\left(\beta_F^{-2}\frac{\Gamma_S}{m_S}\right)\right)\right.$$

$$\left. + \beta_F^{-4}\frac{\Gamma_S}{m_S}\left(-\frac{11}{3} + \log\left(\frac{s^2}{m_S^4\left(\beta_F^4 + \frac{\Gamma_S^2}{m_S^2}\right)}\right) + \frac{m_F^4}{m_S^4}\left(\frac{1}{1+\frac{\Gamma_S^2}{m_S^2}}\right)\log\left(\frac{m_S^4\left(\beta_F^4 + \frac{\Gamma_S^2}{m_S^2}\right)}{m_F^4}\right)\right)\right)$$

where $\beta_F = \sqrt{1 - \frac{m_F^2}{m_S^2}}$

S:FF

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where $\beta_F=\sqrt{1-\frac{m_F^2}{m_S^2}}$

Realize that β_F^{-x} blows up for $m_F\to m_S$ (easy in SUSY):

$$\frac{\sigma_{OFS}}{\sigma_{NWA}} \xrightarrow{m_F\to m_S} \frac{1}{2\pi}\left(\left(1-(1+\beta_F^{-4})\frac{\Gamma_S^2}{m_S^2}\right)\left(\pi+2\tan^{-1}\left(\beta_F^2\frac{m_S}{\Gamma_S}\right)\right)\right.$$

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S:FF

But be careful! Partial widths scale as β_F^4:

$$\Gamma_{S:FF} = \frac{g^2}{6\pi} m_S \left(1 - \frac{m_F^2}{m_S^2}\right)^2 = \frac{g^2}{6\pi} m_S \beta_F^4$$

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2 cases to examine: (use $m_{\tilde{g}} = 600$ GeV)

1. One decay mode open: $\Gamma_{\rm tot} = \Gamma_{\tilde{g}q}$

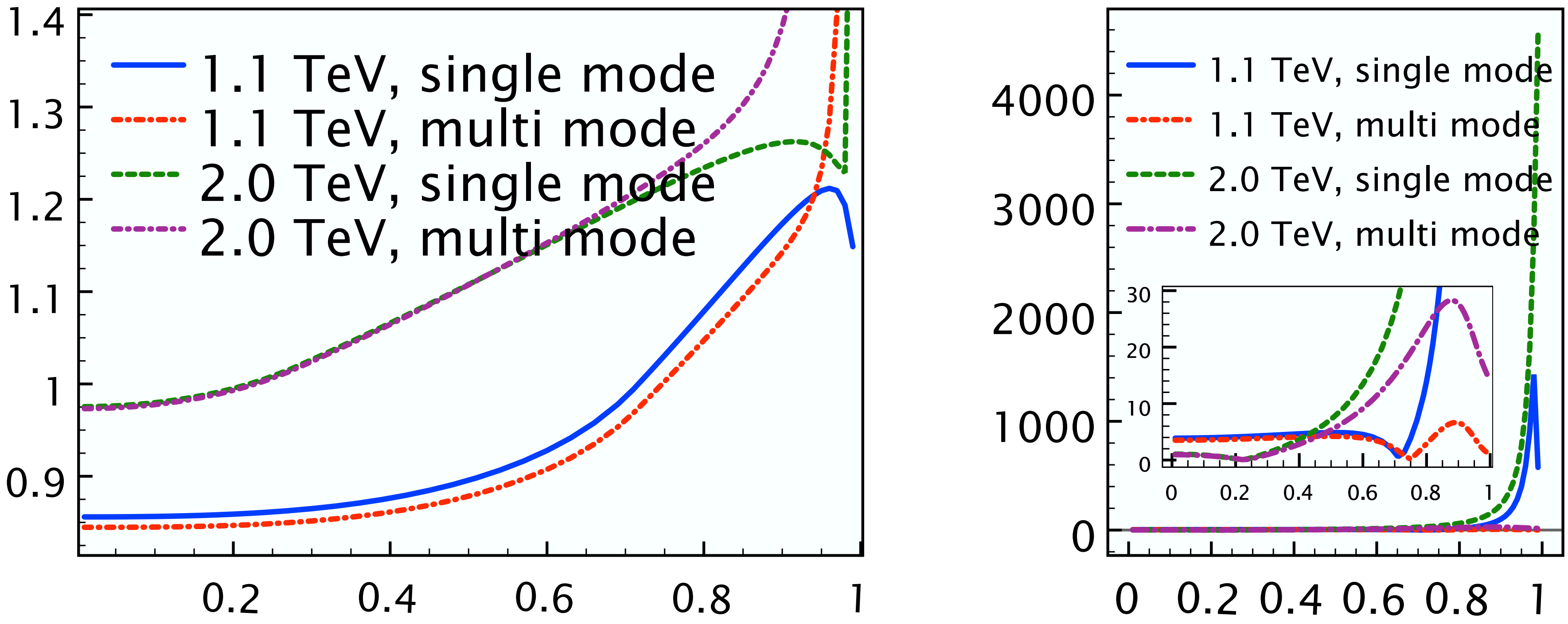
$\longrightarrow$ expect only $O(\Gamma/m)$ effects (but that can be large)

2. Multi-mode decays: $\Gamma_{tot} \longrightarrow$ const. as $m_F \longrightarrow m_S$

$\longrightarrow$ "rare" decay can receive *huge* (β_F^4) correction if $m_S - m_F$ small

S:FF exact results numerically (M.C.)

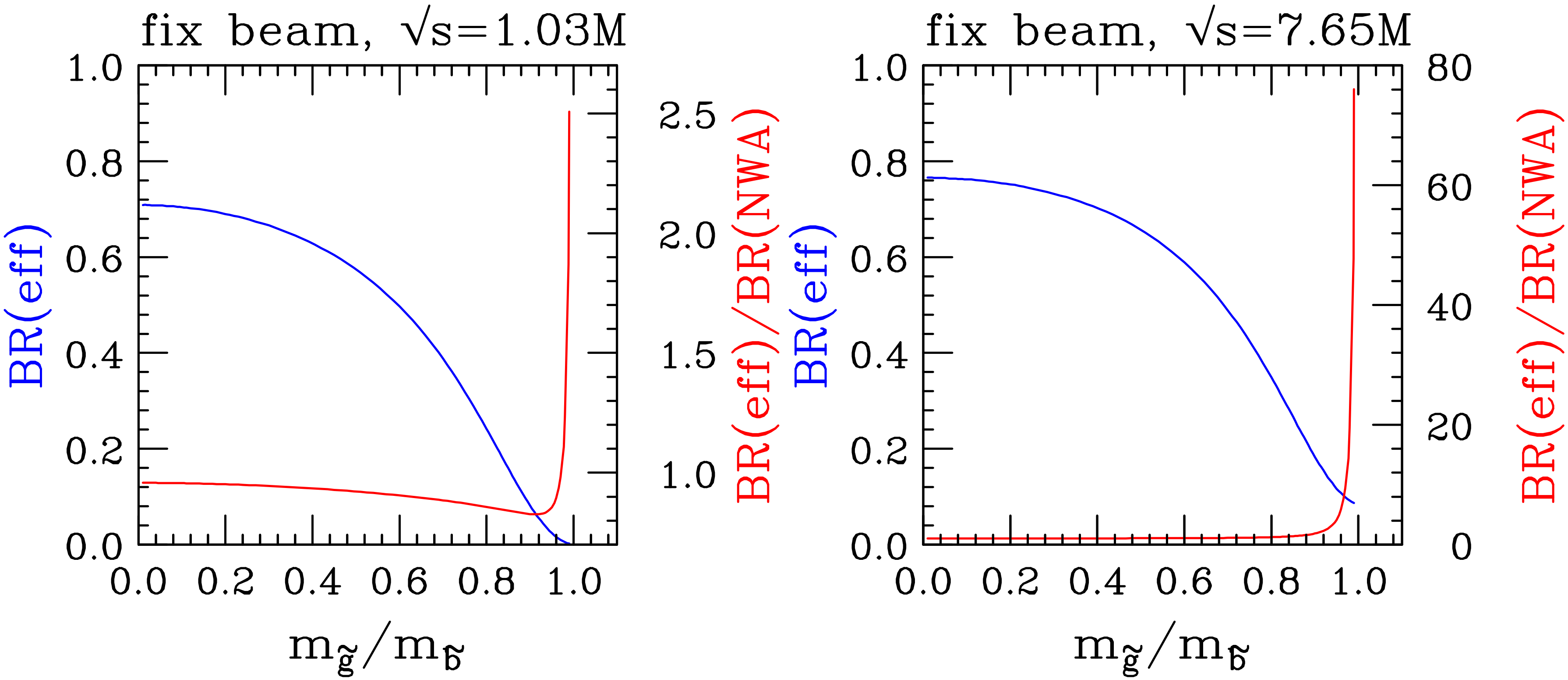
· fake fixed quark beams just to study behavior



Corrections can be 10^x times Γ/m (x large), but a small value.

S:FF exact results numerically (M.C.)

· What happens to the effective branching ratios?

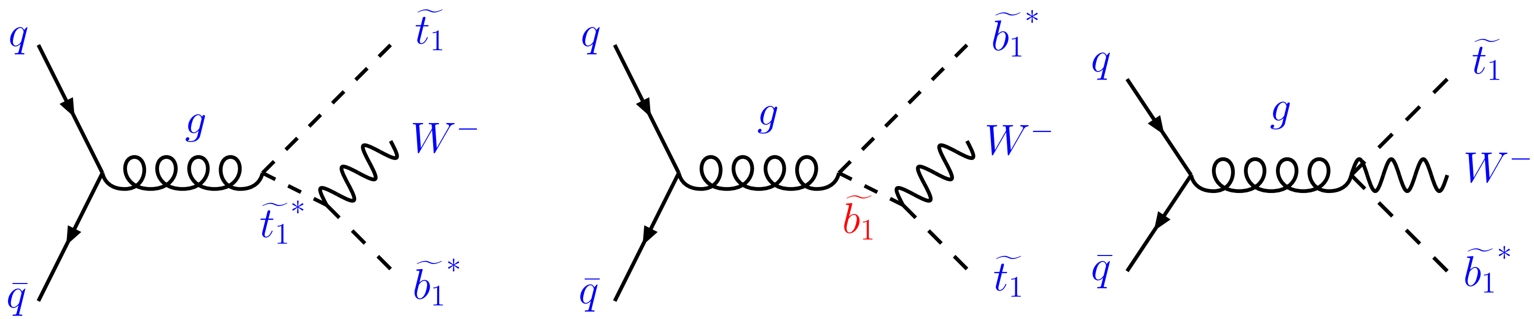


Near threshold, factors of a few for R close to 1.

Above threshold, factors of many to tens.

S:SV type decays

→ mostly relevant for stops & sbottoms, for example:

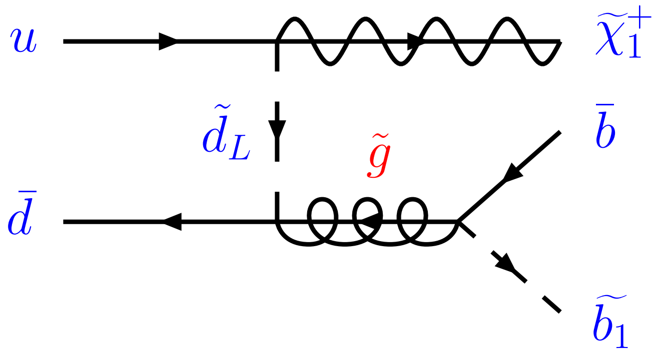


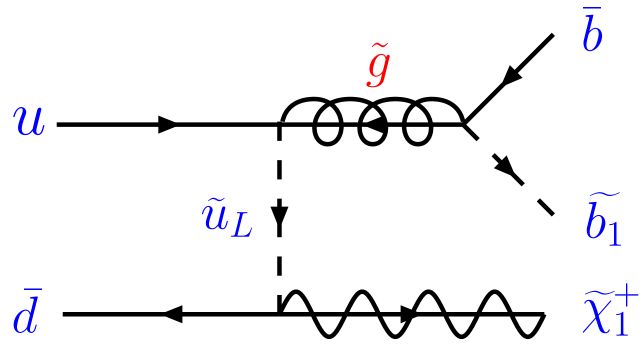
► gauge cancellations prevent any $m = 0$ limits, so more complicated (no detailed results yet, sorry)

Note: W decay ONS v. OFS is no change.

F:FS type decays

$\longrightarrow$ relevant for gluino & weak inos:





[Note: no non-resonant diagrams possible!]

▶ t-channel $\widetilde{u}_L$ and $\widetilde{d}_L$ diagrams separable, consider $\widetilde{d}_L$ only

• approximations: $m_b = m_{\widetilde{\chi}_1^+} = 0$

F:FS

Again, the full result is several pages of Mathematica output. Leading $1/s$ terms (to match NWA $1/s$ behavior) are:

$$\frac{\sigma_{OFS}}{\sigma_{NWA}} \sim \frac{m_F}{2\pi\left(m_S^2 - m_F^2\right)^2\left(m_F^2 + \Gamma_F^2\right)} \cdot$$

$$\left(m_F \left(m_S^4 + \left(m_F^2 - 2m_S^2\right)\left(m_F^2 + \Gamma_F^2\right)\right)\left(\pi - 2\cot^{-1}\left(\frac{m_F \Gamma_F}{m_S^2 - m_F^2}\right)\right)\right.$$

$$+ m_F^2 \Gamma_F \left(m_F^2 + \Gamma_F^2\right)\left(-6 + \log\left(\frac{s^4}{m_T^4\left(\left(m_S^2 - m_F^2\right)^2 + m_F^2 \Gamma_F^2\right)}\right)\right)$$

$$\left. + m_S^4 \Gamma_F \log\left(\frac{\left(m_S^2 - m_F^2\right)^2 + m_F^2 \Gamma_F^2}{m_S^4}\right)\right)$$

Note presence of t-channel sparticle mass in log!

▶ F:FS decay corrections depend on production mechanism

F:FS

SM limit in this case is again $\mathcal{O}(\Gamma/m)$, but again with a $\log(s)$ term.

$$\frac{\sigma_{OFS}}{\sigma_{NWA}} \xrightarrow{m_S \to 0} \frac{1}{2\pi} \cdot \left(\pi + 2\tan^{-1}\left(\frac{m_F}{\Gamma_F}\right) - 6\frac{\Gamma_F}{m_F} + \frac{\Gamma_F}{m_F} \log\left(\frac{s^4}{m_T^4 m_F^2 \left(m_F^2 + \Gamma_F^2\right)} \right) \right)$$

$$\xrightarrow{\Gamma \ll m} 1 + \frac{2}{\pi}\frac{\Gamma_F}{m_F}\left(-2 + \log\left(\frac{s}{m_T m_F}\right)\right)$$

F:FS

As before, let's rewrite the full result in a clearer form:

$$\frac{\sigma_{OFS}}{\sigma_{NWA}} \stackrel{m_S \to m_F}{\longrightarrow} \frac{1}{2} + \frac{1}{\pi}\tan^{-1}\left(\beta_S^2 \frac{m_F}{\Gamma_F}\right) + \frac{1}{\pi}\beta_S^{-4}\frac{\Gamma_F}{m_F}\left(-3+2\log\left(\frac{s}{\beta_F m_T m_F}\right)+2\frac{m_S^4}{m_F^4}\log\left(\frac{\beta_F m_F}{m_S}\right)\right)$$

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β_S^{-x} blows up for $m_S \to m_F$, same as S:FF case (easy in SUSY):

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- don't expand $\tan^{-1}$ yet!

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We test this by setting $\Gamma = 0$ in the M.E. and multiply outside by:

$$\frac{(q^2 - m^2)^2}{(q^2 - m^2)^2 + (m\Gamma)^2}$$

▶ identical results – no issue with finite widths

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What about logarithmic α_s *running?*

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What about logarithmic α_s *running?*

$\longrightarrow$ or, does that compensate the $\log(s)$ M.E.-dependence?

We test this by calculating $\alpha_s(q^2)$ point-by-point.

The log coefficient is diminished slightly, but overall behavior $\sigma \propto \frac{\log(s)}{s}$ remains.

That is, B-W integration and α_s running are orthogonal.

$\longrightarrow$ but what happens at higher order is still an interesting question

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What about unitarity?

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Froissart bound is actually $4\pi\frac{\log^2(s)}{s}$:
logs come from summing over all partial waves
($1/s$ behavior applies only *individual* partial wave amps)

Our results are orders of magnitude away from this.

► no problem with unitarity

Two levels of decay

In SUSY and most BSM physics, new particles cascade to an LP.
(LSP in SUSY, LPOP in Little Higgs, LKP in extra-D models, etc.)

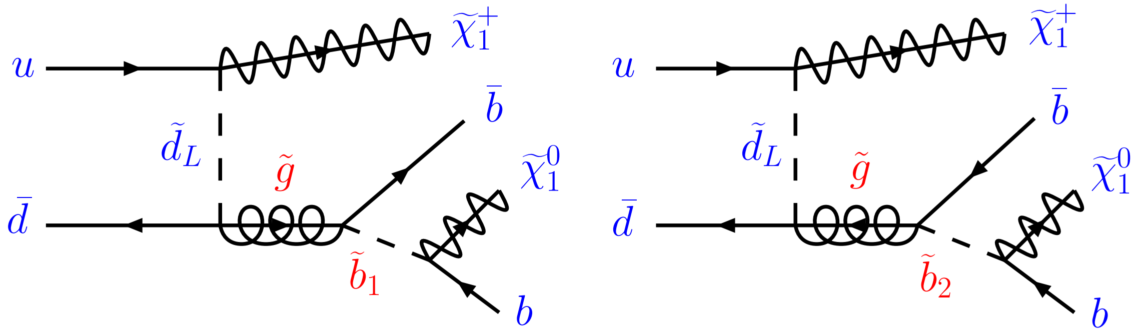
Three big questions we need to ask:

1. Does M.E. effect depend on intermediate resonance or final-state masses?
2. Does a daughter B-W introduce its own effect?
3. Do resonant and non-resonant diagrams interfere?

Two-level decays

First examine $u\bar{d} \to \bar{b}b\widetilde{\chi}_1^0\widetilde{\chi}_1^+$ numerically (various MSSM points):

(analytical form not yet feasible)



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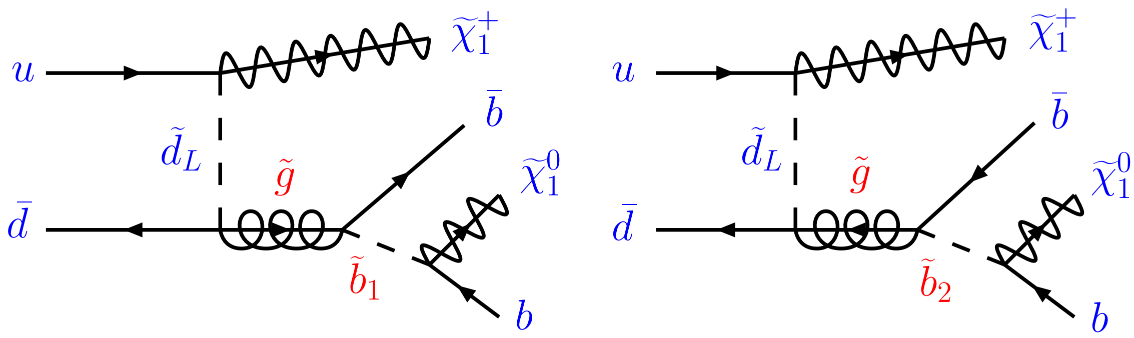

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Using fixed-energy beams we find:

M.E. enhancement from first decay still present.

- M.E. effect depends on daughter pole, not final-state masses
- Note: can have "superenhancement" for daughter very near the parent – is due to daughter's B-W

Two decay levels

Next examine $u\bar{d} \to \bar{b}W^-\widetilde{t}_1\widetilde{\chi}_1^+$ numerically (at SPS1a only):

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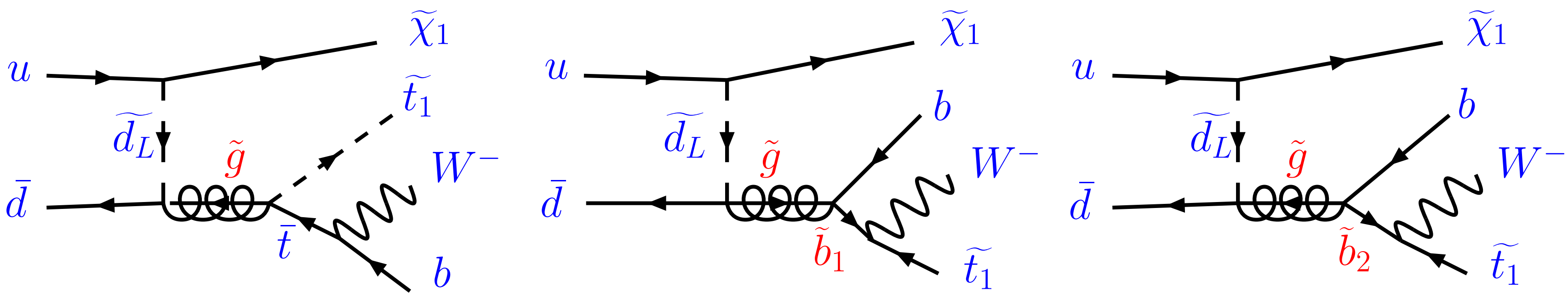

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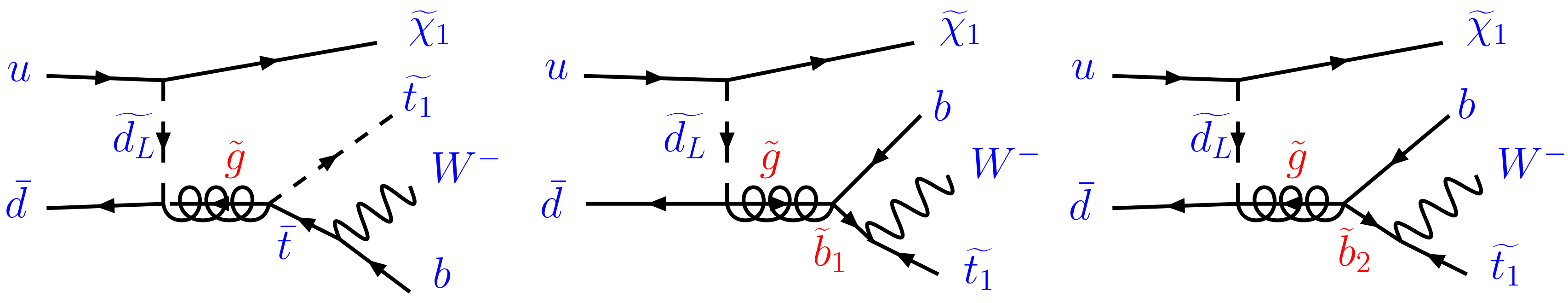

(can decouple $\widetilde{b}_2$)

Two important observations:

[1] Interference of 3 diagrams present at few-percent level.

[2] Subsequent W decay gives identical results (no M.E. effect).

CONCLUSIONS

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- Non-resonant interference can be many times Γ/m (and not straightforwardly predictable).
- Reminder: off-shell effects are *not* specific to SUSY!

NEXT STEPS

- QCD interference for heavy gluino (v. squarks).
- SS v. OS asymmetry affecting bkg-subtracted discoveries.
- Practical LHC results for 3 vertex types and mass scan: changes in effective BRs.
- Attempt to find rule of thumb for successive decays (when they need to be done off-shell).
- Impact on jet/lepton edges at LHC.