Two universal extra dimensions at the LHC

Bogdan Dobrescu (*Fermilab*)

work with Gustavo Burdman and Eduardo Ponton: hep-ph/0601186
KC Kong and Rakhi Mahbubani: hep-ph/0703nnn

Fundamental symmetries

gauge	spacetime	global	discrete
$SU(3) \times SU(2) \times U(1)$;	$SO(3,1)$;	$U(1)_B$;	CPT

Fermions:

$$\left.\begin{array}{rl} q_L: & (3,2,\ +1/6) \\ u_R: & (3,1,\ +2/3) \\ d_R: & (3,1,\ -1/3) \\ l_L: & (1,2,\ -1/2) \\ e_R: & (1,1,\ -1) \end{array}\right\} \times 3$$

Fundamental symmetries

gauge	spacetime	global	discrete
$SU(3) \times SU(2) \times U(1)$;	$SO(3,1)$;	$U(1)_B$;	CPT

$$SU(5) \subset SO(10)$$

Fermions:

$$\left.\begin{array}{rl} q_L : & (3,2,\ +1/6) \\ u_R : & (3,1,\ +2/3) \\ d_R : & (3,1,\ -1/3) \\ l_L : & (1,2,\ -1/2) \\ e_R : & (1,1,\ -1) \end{array}\right\} \times 3 = (10 + \bar{5}) \times 3 \subset 16 \times 3$$

Fundamental symmetries

gauge	spacetime	global	discrete
$SU(3) \times SU(2) \times U(1)$;	$SO(3,1)$;	$U(1)_B$;	CPT

$SO(5,1)$

6D Lorentz symmetry

Fermions:

$$\left.\begin{array}{rl} q_L: & (3,2,\ +1/6) \\ u_R: & (3,1,\ +2/3) \\ d_R: & (3,1,\ -1/3) \\ l_L: & (1,2,\ -1/2) \\ e_R: & (1,1,\ -1) \end{array}\right\} \times 3$$

required by global $SU(2)_W$ anomaly cancellation in 6D

Universal Extra Dimensions

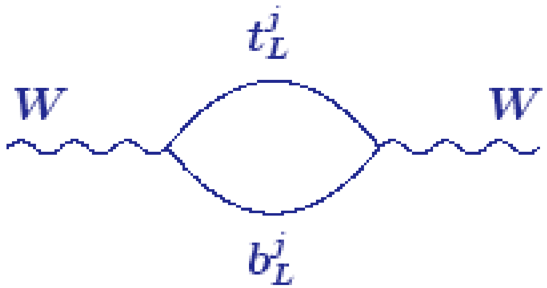
T. Appelquist, H.-C. Cheng, B. Dobrescu, Phys.Rev.D64 (2001)

All Standard Model particles propagate in $D \geq 5$ dimensions.

Kaluza-Klein modes: states of definite momentum along the compact dimensions.

Momentum conservation $\longrightarrow$ KK-number conservation at tree level (exact KK parity)

Bounds from one-loop shifts in M_W, M_Z, and other observables:



$$\frac{1}{R} \gtrsim 300 - 500 \text{ GeV}$$

Lightest KK particle

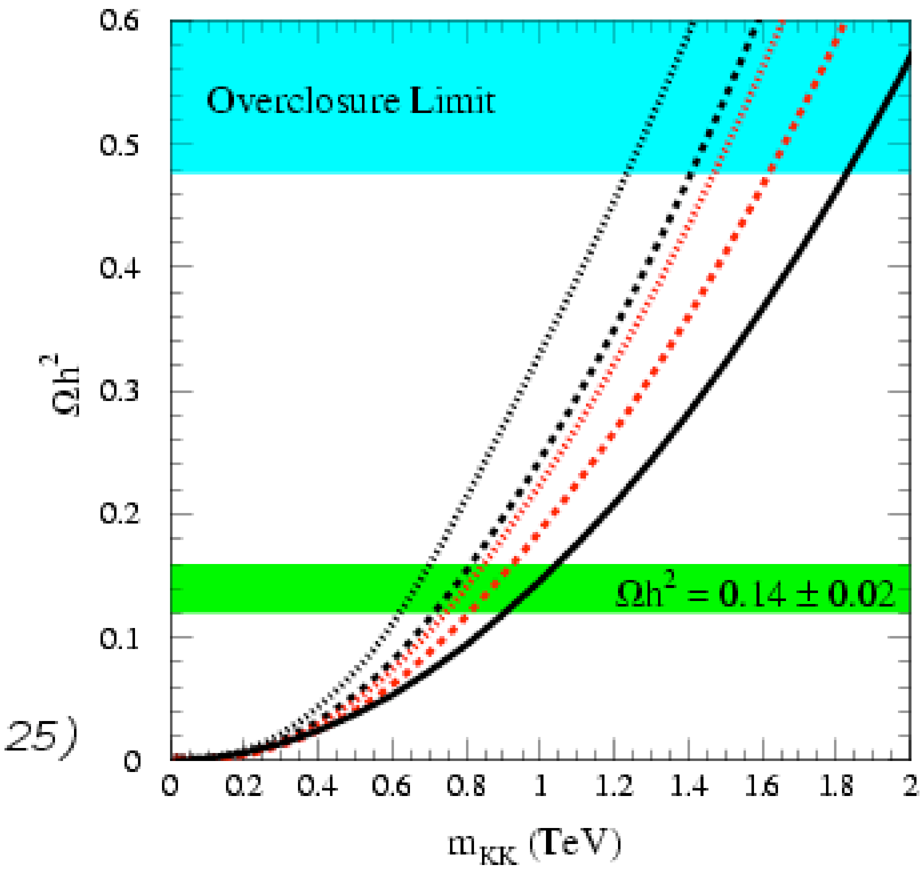
is stable in UED:

$\gamma^{(1)}$ is a viable dark

matter candidate

(from Servant, Tait, hep-ph/0206071

Cheng, Feng, Matchev, hep-ph/0207125)



Six-Dimensional Field Theory

6D is special...

Fermions in $D = 4 + n$ dimensions: chiral representations of the Lorentz group $SO(3 + n, 1)$ exist only for even n.

Properties of chirality in $D = 6$ mod 4 are different than in $D = 4$ mod 4.

A chiral fermion in $D = 6$ has 4 degrees of freedom.
6D chirality projection operators:

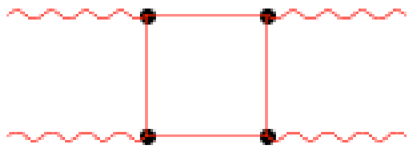
$$P_{\pm} = \frac{1}{2}\left(1 \pm \mathbf{\Gamma}^0 ... \mathbf{\Gamma}^5\right)$$

Six-Dimensional Standard Model

$n_\pm \equiv$ # of spin–1/2 fields with chirality $\pm$

Local gravitational anomaly in 6D $\propto n_+ - n_-$

$\Rightarrow$ one right-handed neutrino per generation!



Two possible chirality assignments for the 6D quarks and leptons:

$$\mathcal{Q}_+ \; , \; \mathcal{U}_- \; , \; \mathcal{D}_- \; , \; \begin{cases} \mathcal{L}_+ \; , \;\; \mathcal{E}_- \; , \;\; \mathcal{N}_- \\ \qquad\quad \text{or} \\ \mathcal{L}_- \; , \;\; \mathcal{E}_+ \; , \;\; \mathcal{N}_+ \end{cases}$$

Global $SU(2)_W$ anomaly

B. Dobrescu, E. Poppitz: PRL 87, 031801 (2001)

Homotopy group in 6D: $\pi_6\,(SU(2)) = Z_{12}$

Anomaly cancellation condition:

(Bershadsky, Vafa, 1997)

$$n(2_+) - n(2_-) = 0 \bmod 6$$

One generation: $Q_+ \Rightarrow n_Q(2_+) = 3$

$L_\pm \Rightarrow n_L(2_\pm) = 1$

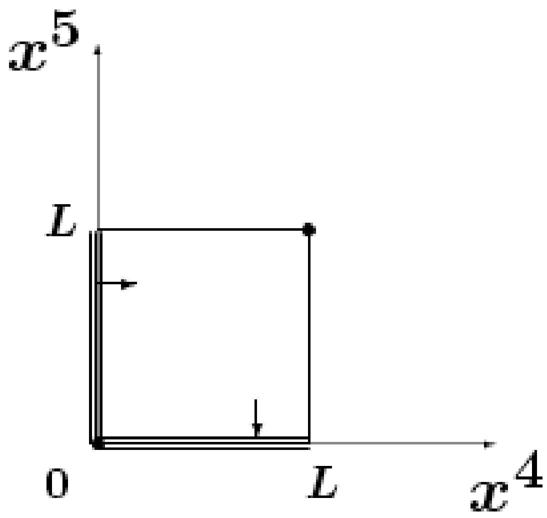
Need more than one generation of quarks and leptons:

$$(3 \pm 1)\, n_{\text{gen}} = 0 \bmod 6 \quad \Rightarrow \quad \underline{n_{\text{gen}} = 3 \bmod 3}$$

Chiral boundary conditions on a square

(Dobrescu, Ponton, hep-ph/0401032)

Identify pairs of adjacent sides:



$$\mathcal{L}(x^\mu, y, 0) = \mathcal{L}(x^\mu, 0, y)$$

$$\mathcal{L}(x^\mu, y, L) = \mathcal{L}(x^\mu, L, y)$$

$\Phi(y, 0) = e^{i\theta}\Phi(0, y), \ \ldots$

$\Rightarrow \ \theta = n\pi/2$

$$\partial_5 \Phi|_{(x^4,x^5)=(y,0)} = -e^{in\pi/2}\, \partial_4 \Phi|_{(x^4,x^5)=(0,y)}$$

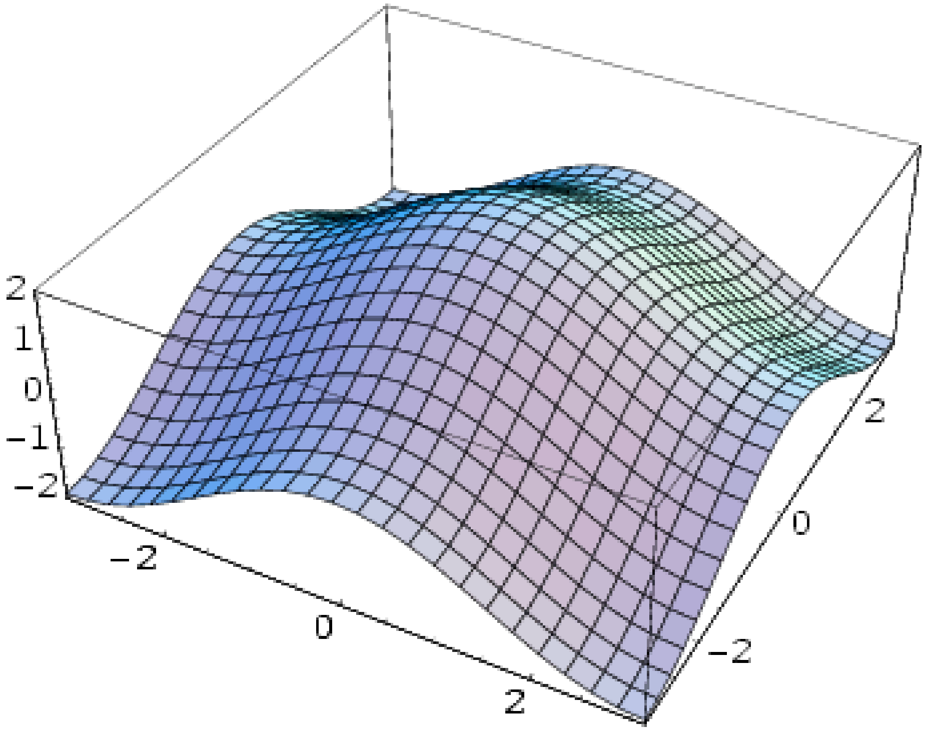
Complete sets of functions satisfying the boundary conditions:

$$f_{0,2}^{(j,k)}(x^4, x^5) = \frac{1}{1+\delta_{j,0}}\left[\cos\left(\frac{jx^4+kx^5}{R}\right) \pm \cos\left(\frac{kx^4-jx^5}{R}\right)\right]$$

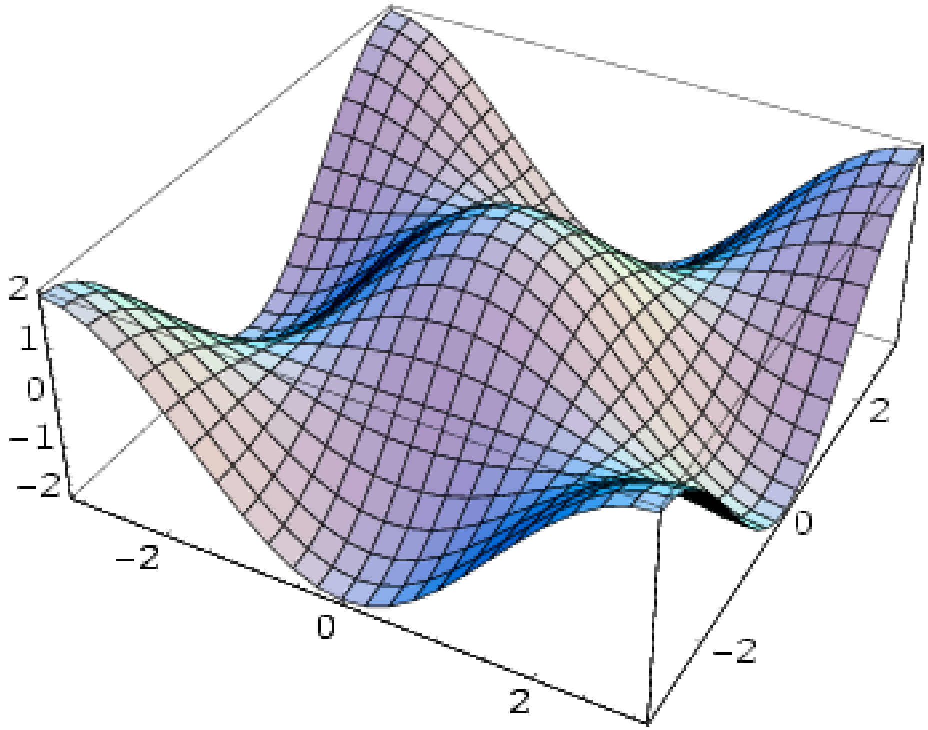
$$f_{1,3}^{(j,k)}(x^4, x^5) = i\sin\left(\frac{jx^4+kx^5}{R}\right) \mp \sin\left(\frac{kx^4-jx^5}{R}\right)$$

Spectrum of KK modes:

(j,k)	(1,0)	(1,1)	(2,0)	$(2,1)$ $(1,2)$	(2,2)	(3,0)	$(3,1)$ $(1,3)$
$M_{j,k}R$	1	$\sqrt{2}$	2	$\sqrt{5}$	$2\sqrt{2}$	3	$\sqrt{10}$



$f_0^{(1,0)}(x^4, x^5)$



$f_0^{(1,1)}(x^4, x^5)$

KK decomposition of the gauge fields:

$$A_\mu(x^\nu, x^4, x^5) = \frac{1}{L}\left[A_\mu^{(0,0)}(x^\nu) + \sum_{j\geq 1}\sum_{k\geq 0} f_0^{(j,k)}(x^4, x^5) A_\mu^{(j,k)}(x^\nu)\right]$$

$$A_4 + iA_5 \equiv A_+(x^\nu, x^4, x^5) = -\frac{1}{L}\sum_{j\geq 1}\sum_{k\geq 0} f_3^{(j,k)}(x^4, x^5) A_+^{(j,k)}(x^\nu)$$

$$A_4 - iA_5 \equiv A_-(x^\nu, x^4, x^5) = \frac{1}{L}\sum_{j\geq 1}\sum_{k\geq 0} f_1^{(j,k)}(x^4, x^5) A_-^{(j,k)}(x^\nu)$$

Physical degrees of freedom:

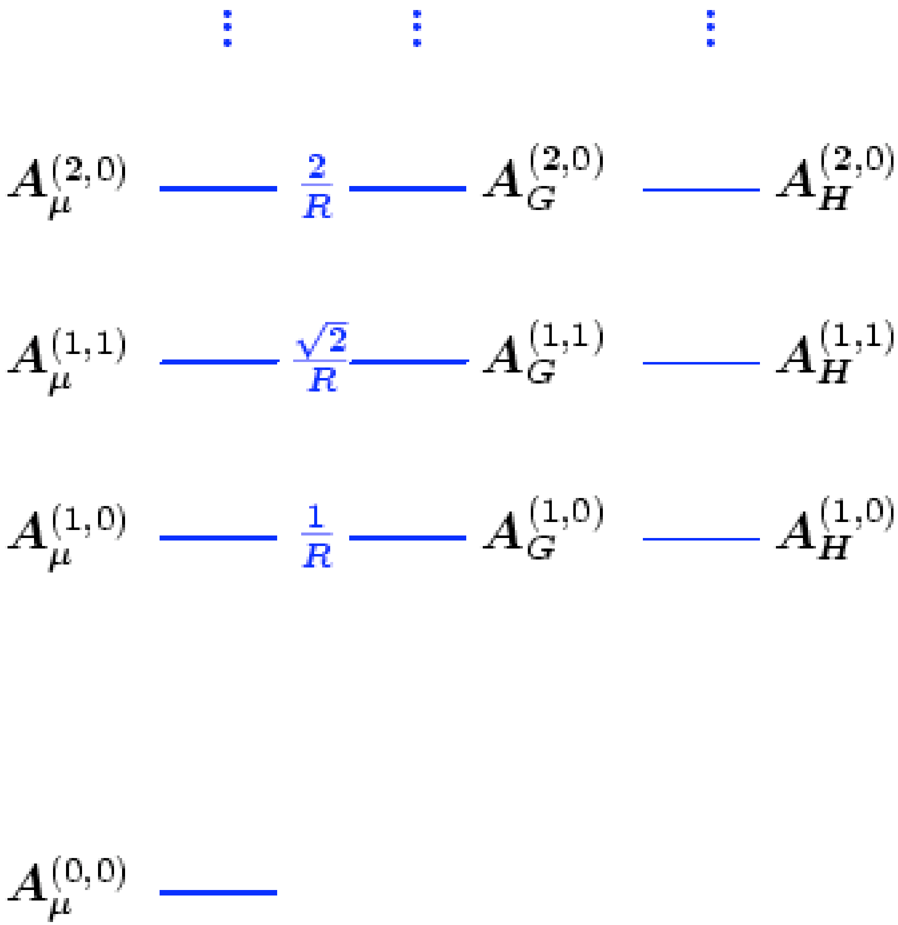
$$A_\pm^{(j,k)} = \frac{j + ik}{\sqrt{j^2 + k^2}}\left(A_H^{(j,k)} \mp iA_G^{(j,k)}\right)$$

$A_G^{(j,k)}$ is the longitudinal polarization of $A_\mu^{(j,k)}$

$A_H^{(j,k)}$ is a real scalar field ("spinless adjoint")

Kaluza-Klein spectrum of gauge bosons

$A_G^{(j,k)}(x^\nu)$ becomes the longitudinal degree of freedom of the spin-1 KK mode $A_\mu^{(j,k)}(x^\nu)$.



The KK expansions of a 6D fermion of + chirality with a left-handed zero mode:

$$\Psi_{+L} = \frac{1}{L}\left[\Psi_{+L}^{(0,0)}(x^\nu) + \sum_{j\geq 1}\sum_{k\geq 0} f_0^{(j,k)}(x^4,x^5)\Psi_{+L}^{(j,k)}(x^\nu)\right]\otimes\begin{pmatrix}1\\0\end{pmatrix}$$

$$\Psi_{+R} = -\frac{i}{L}\sum_{j\geq 1}\sum_{k\geq 0}\frac{j+ik}{\sqrt{j^2+k^2}} f_3^{(j,k)}(x^4,x^5)\Psi_{+R}^{(j,k)}(x^\nu)\otimes\begin{pmatrix}0\\1\end{pmatrix}$$

satisfies the 2D Dirac equation:

$$(\partial_4 - i\partial_5)\, f_{+R}^{(j,k)} = M_{j,k} f_{+L}^{(j,k)}$$

$$(\partial_4 + i\partial_5)\, f_{+L}^{(j,k)} = -M_{j,k} f_{+R}^{(j,k)}$$

Similar decompositions for fermions having a zero mode of the type Ψ_{+R}, Ψ_{-L} or Ψ_{+L}.

Kaluza-Klein spectrum of quarks and leptons

$(t_L^{(2,0)}, b_L^{(2,0)})$	$\frac{2}{R}$	$(T_R^{(2,0)}, B_R^{(2,0)})$	$T_L^{(2,0)}$	$\frac{2}{R}$	$t_R^{(2,0)}$
$(t_L^{(1,1)}, b_L^{(1,1)})$	$\frac{\sqrt{2}}{R}$	$(T_R^{(1,1)}, B_R^{(1,1)})$	$T_L^{(1,1)}$	$\frac{\sqrt{2}}{R}$	$t_R^{(1,1)}$
$(t_L^{(1,0)}, b_L^{(1,0)})$	$\frac{1}{R}$	$(T_R^{(1,0)}, B_R^{(1,0)})$	$T_L^{(1,0)}$	$\frac{1}{R}$	$t_R^{(1,0)}$
(t_L, b_L)					t_R

Symmetries of the "Chiral Square"

Scalars:

$$\prod_{i=1}^{p} (\Phi_i)^{m_i} (\Phi_i^\dagger)^{m_i'} \ ,$$

equality of the Lagrangians at $(y, 0)$ and $(0, y)$ $\Rightarrow$

$$\sum_{i=1}^{p} n_i (m_i - m_i') = 0 \bmod 4 \ .$$

invariance under Z_4 transformations

$$\Phi_i(x^\mu, x^4, x^5) \mapsto e^{-i n_i \pi/2} \Phi_i(x^\mu, x^4, x^5) \ .$$

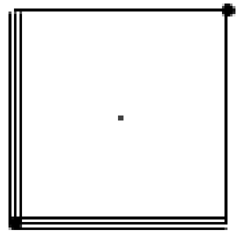
Fermions:

$$i\overline{\Psi}^i_{+L} \left(\Gamma^4 + i\Gamma^5\right) \left(\partial_4 - i\partial_5\right) \Psi^i_{+R}$$

$\Rightarrow n^+_{Ri} - n^+_{Li} = -1 \ \Rightarrow$ invariance under Z_8 transformations $\Rightarrow$

$$\begin{aligned} \Psi^i_{\pm R}(x^\mu, x^4, x^5) &\mapsto e^{-i(\pm 1/2 + n^\pm_{Ri})\pi/2} \Psi^i_{\pm R}(x^\mu, x^4, x^5) \\ \Psi^i_{\pm L}(x^\mu, x^4, x^5) &\mapsto e^{-i(\mp 1/2 + n^\pm_{Li})\pi/2} \Psi^i_{\pm L}(x^\mu, x^4, x^5) \end{aligned}$$

Kaluza-Klein parity



Reflections about the center of the square $(L/2, L/2)$,

$$(x^4, x^5) \mapsto (L - x^4, L - x^5)$$

6D Lagrangian is invariant under

$$\Phi(x^\mu, x^4, x^5) \;\mapsto\; \Phi(x^\mu, L - x^4, L - x^5) \qquad \text{(scalars)} \ ,$$
$$\Psi_\pm(x^\mu, x^4, x^5) \;\mapsto\; e^{-i\pi\Sigma_{45}/2}\, \Psi_\pm(x^\mu, L - x^4, L - x^5)$$

$\Rightarrow$ invariance under Z_2 transformation

$$\Upsilon^{(j,k)}(x^\mu) \mapsto (-1)^{j+k}\, \Upsilon^{(j,k)}(x^\mu)$$

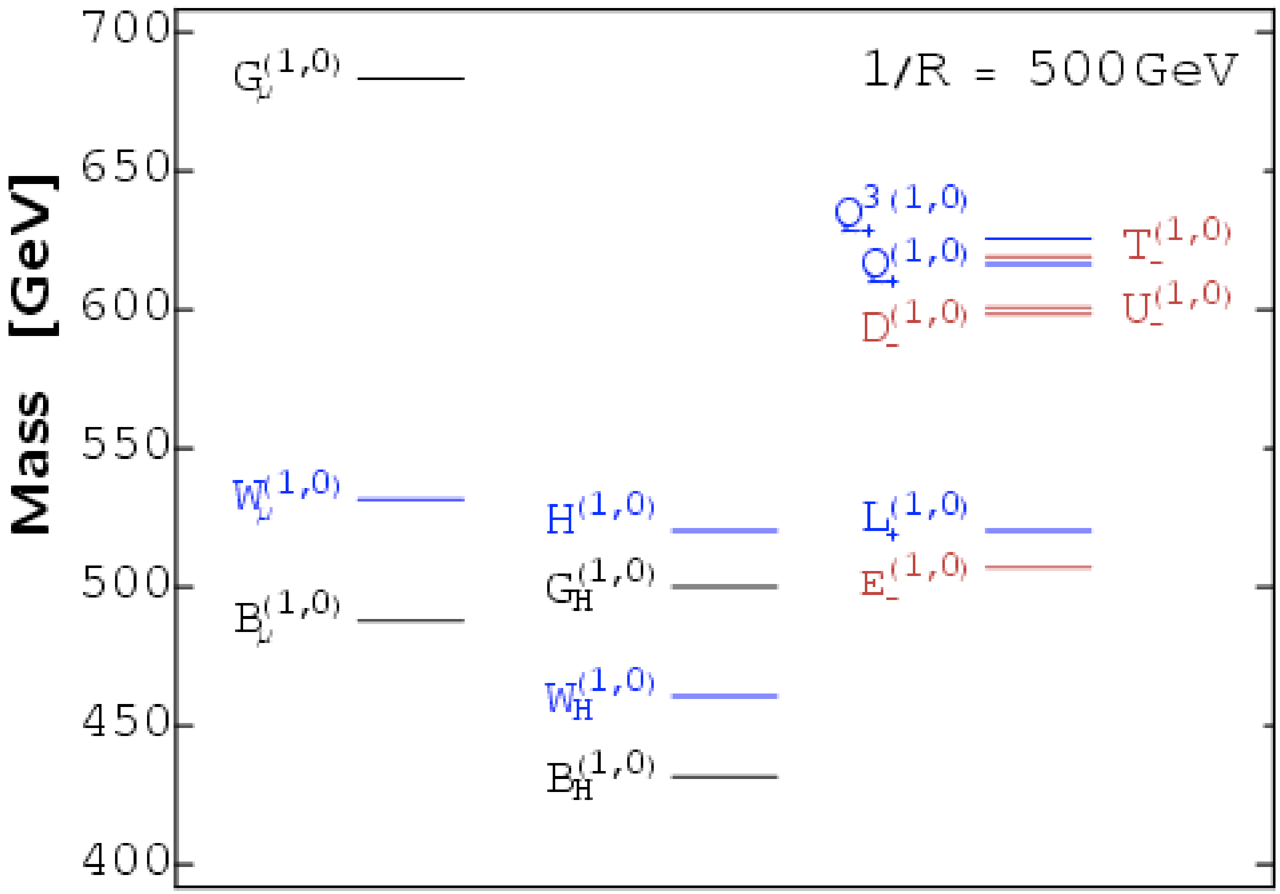
Field theory on the chiral square has $Z_8 \times Z_2$ symmetry

(1,0) modes have a tree-level mass of $1/R$, and KK parity $-$.

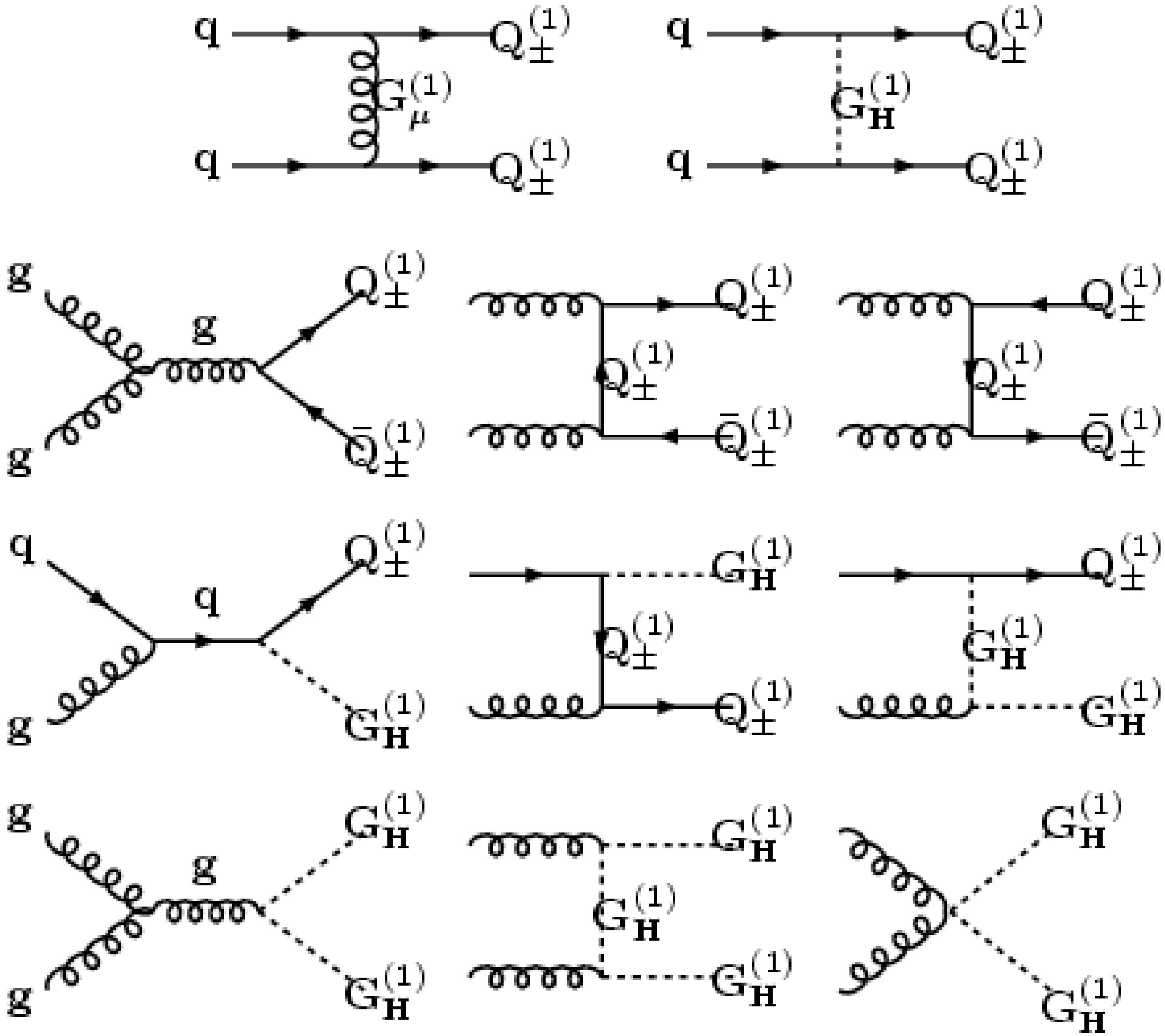
One-loop contributions and EWSB split the spectrum

(Cheng, Matchev, Schmaltz, hep-ph/0204342 ; Ponton, Wang, hep-ph/0512304)

Mass spectrum of the (1,0) level:

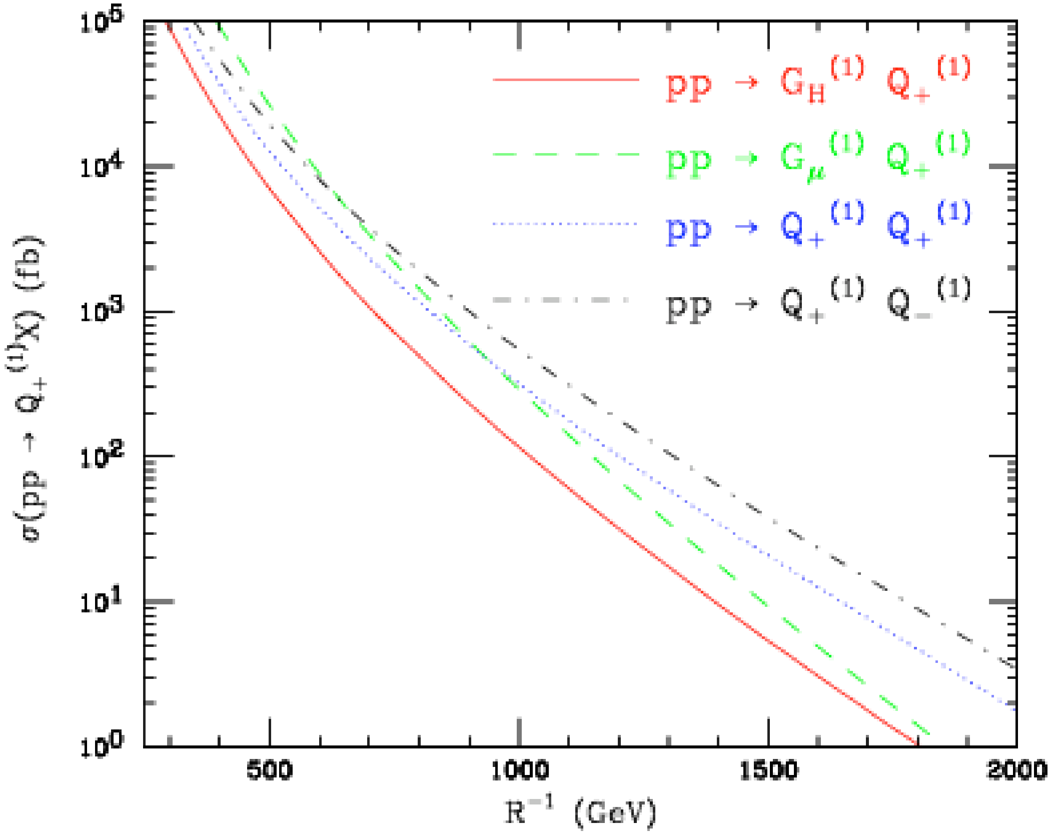


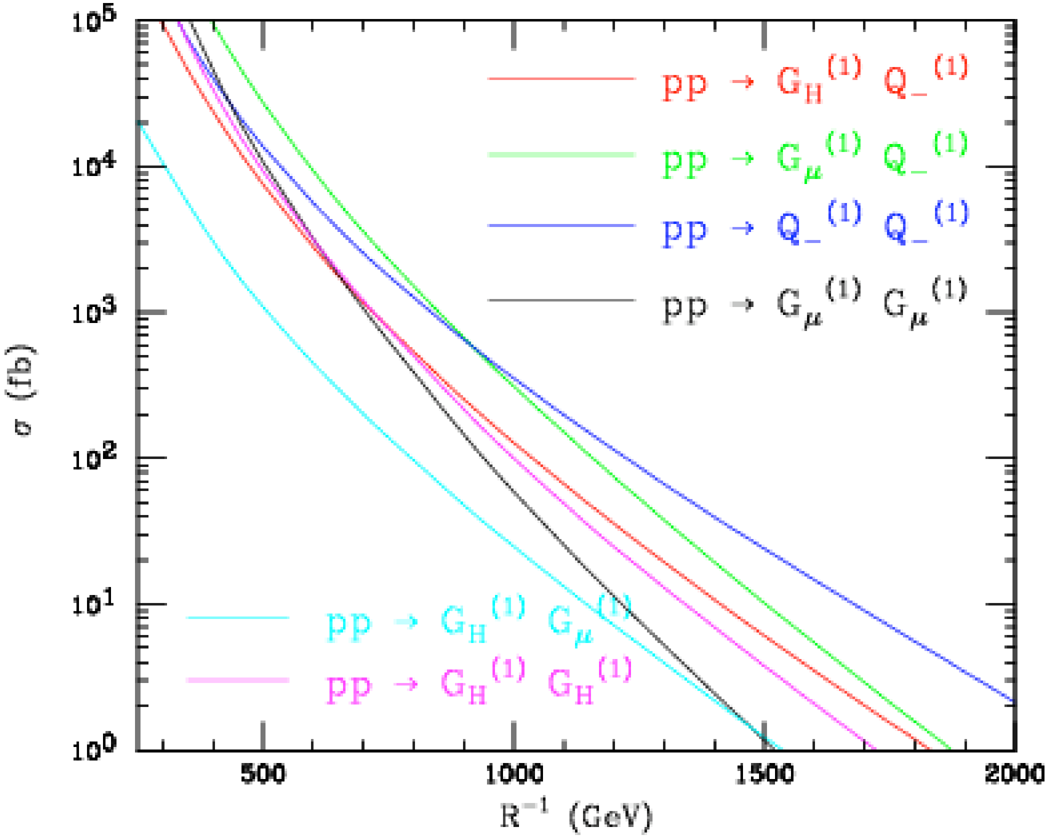
Production of (1,0) particles at the LHC



Use CalcHEP to compute cross section for (1,0) pair production.

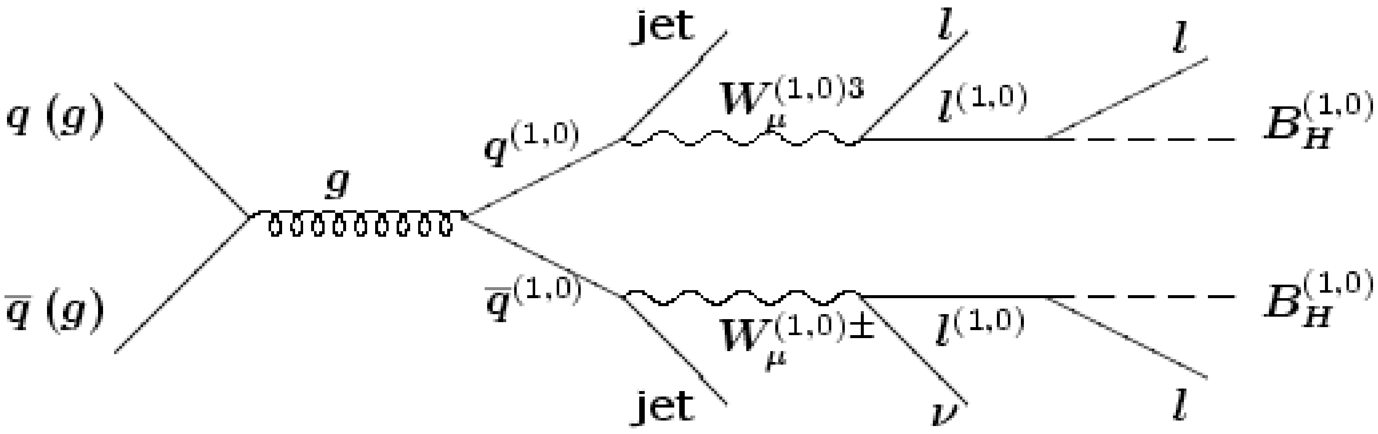
At $\sqrt{s} = 14$ TeV:





Signals at hadron colliders:

1. Pair production of (1,0) modes:



Look for: **2 hard leptons (~ 100 GeV)**

+ 1 soft lepton (~ 10 GeV)

+ 2 jets (~ 50 GeV)

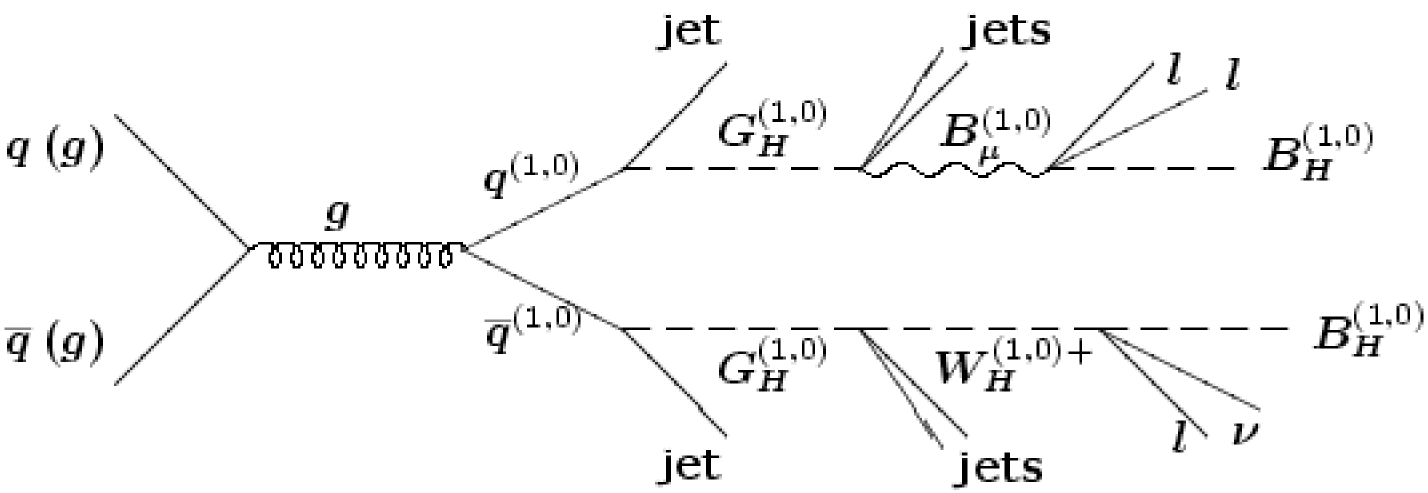
+ $\not{E}_T$

Similar to 1 UED (Cheng, Matchev, Schmaltz, hep-ph/0205314; ...)

but with smaller branching fractions.

Signals at hadron colliders:

2. Pair production of (1,0) modes, followed by cascade decays through spinless adjoints:



Signal: **leptons + jets +** $\not{E}_T$

Work with K.C. Kong and Rakhi Mahbubani.

At one loop:

$$\frac{c}{R^{-1}} B_H^{(1,0)} B_{\mu\nu}^{(1,0)} \tilde{F}^{\mu\nu}$$

Competition between 1-loop induced 2-body decays and tree-level 3-body decays of the (1,0) bosons.

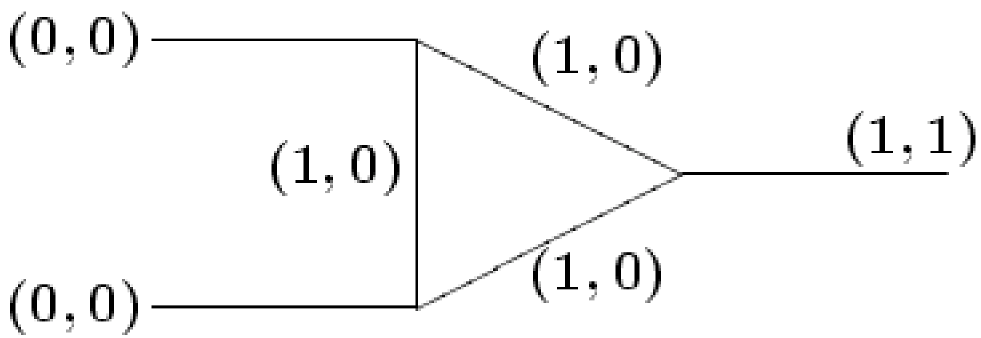
$$Br\left(B_\mu^{(1,0)} \to B_H^{(1,0)} \gamma\right) \approx 30\%$$

$$Br\left(B_\mu^{(1,0)} \to B_H^{(1,0)} \ell^+ \ell^-\right) \approx 23\%$$

$\rightarrow$ Events with leptons, photons and missing E_T.

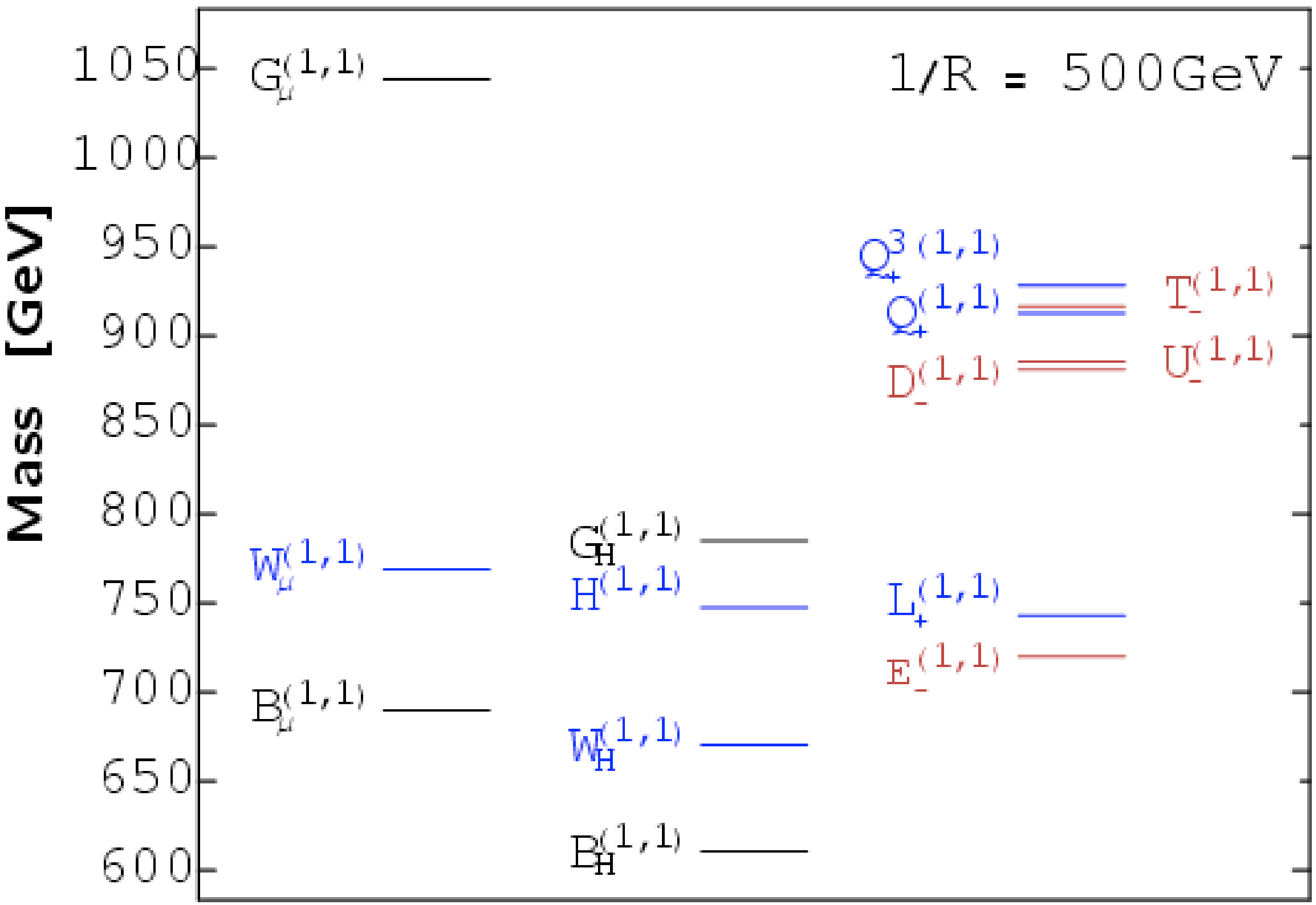
KK parity is conserved: $(-1)^{j+k}$

At colliders: s-channel production of the even-modes at 1-loop



(1,1) modes have a tree-level mass of $\sqrt{2}/\boldsymbol{R}$, and KK parity $+$.

Mass spectrum of the (1,1) level for $1/R$ = 500 GeV:



Spinless adjoints interact with the zero-mode fermions only via dimension-5 or higher operators:

$$\frac{g_s \tilde{C}^{qG}_{j,k}}{M_{j,k}} \left(\overline{q}\gamma^\mu T^a q\right) \, \partial_\mu G_H^{(j,k)a}$$

$\tilde{C}^{qG}_{j,k}$ are real dimensionless parameters.

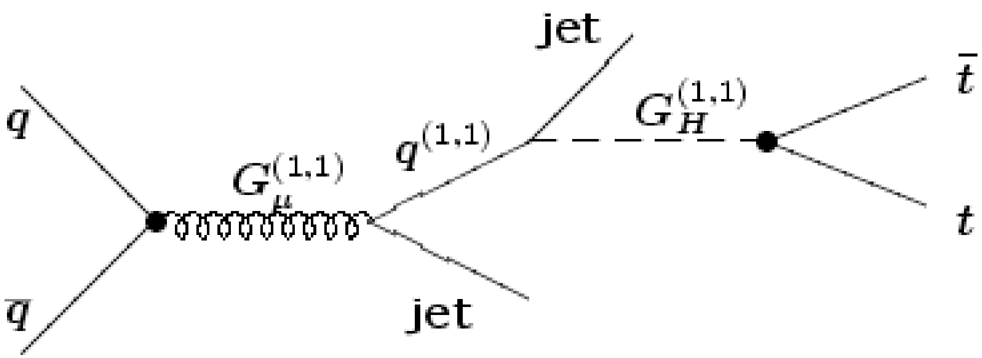
$\Rightarrow$ G_H, W_H and B_H couple to usual quarks and leptons proportional to the fermion mass!

$\Rightarrow$ KK-number violating couplings of the spinless adjoints are large only in the case of the top quark.

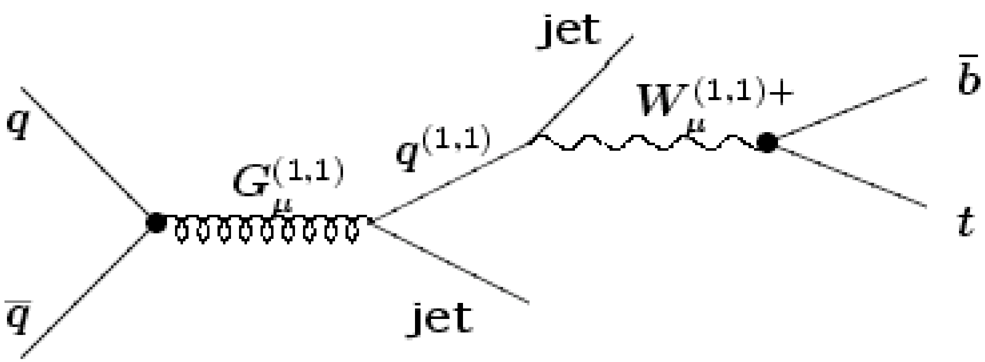
Signals at hadron colliders:

3. s-channel production of a (1,1) gluon of mass $\sim \sqrt{2}/R\,(1+\alpha_s)$

$\rightarrow$ $t\bar{t}$ resonance + 2 jets ($\sim 50 - 100$ GeV):



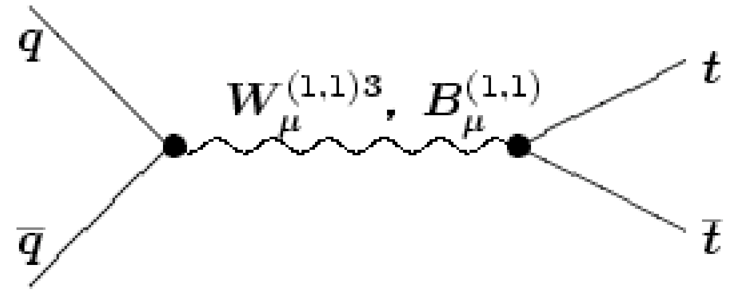
$\rightarrow$ $t\bar{b}$ resonance + 2 jets ($\sim 50 - 100$ GeV):



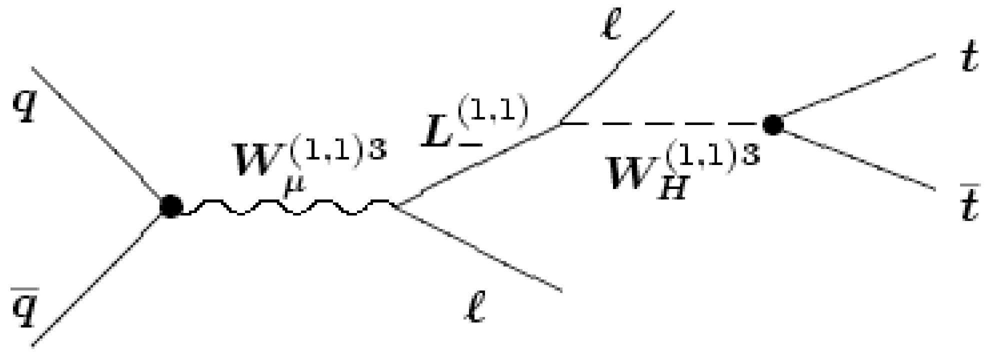
Possible signals at hadron colliders:

4. s-channel production of a (1,1) electroweak gauge boson

$\rightarrow$ $t\bar{t}$ resonance:



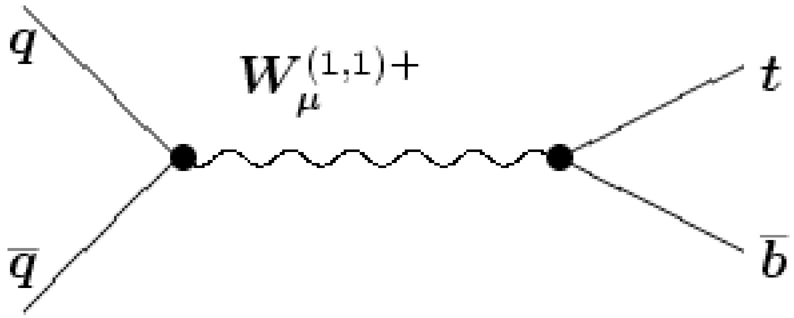
$\rightarrow$ $t\bar{t}$ resonance + 1 lepton ~ 70 GeV + 1 lepton ~ 20 GeV:



Possible signals at hadron colliders:

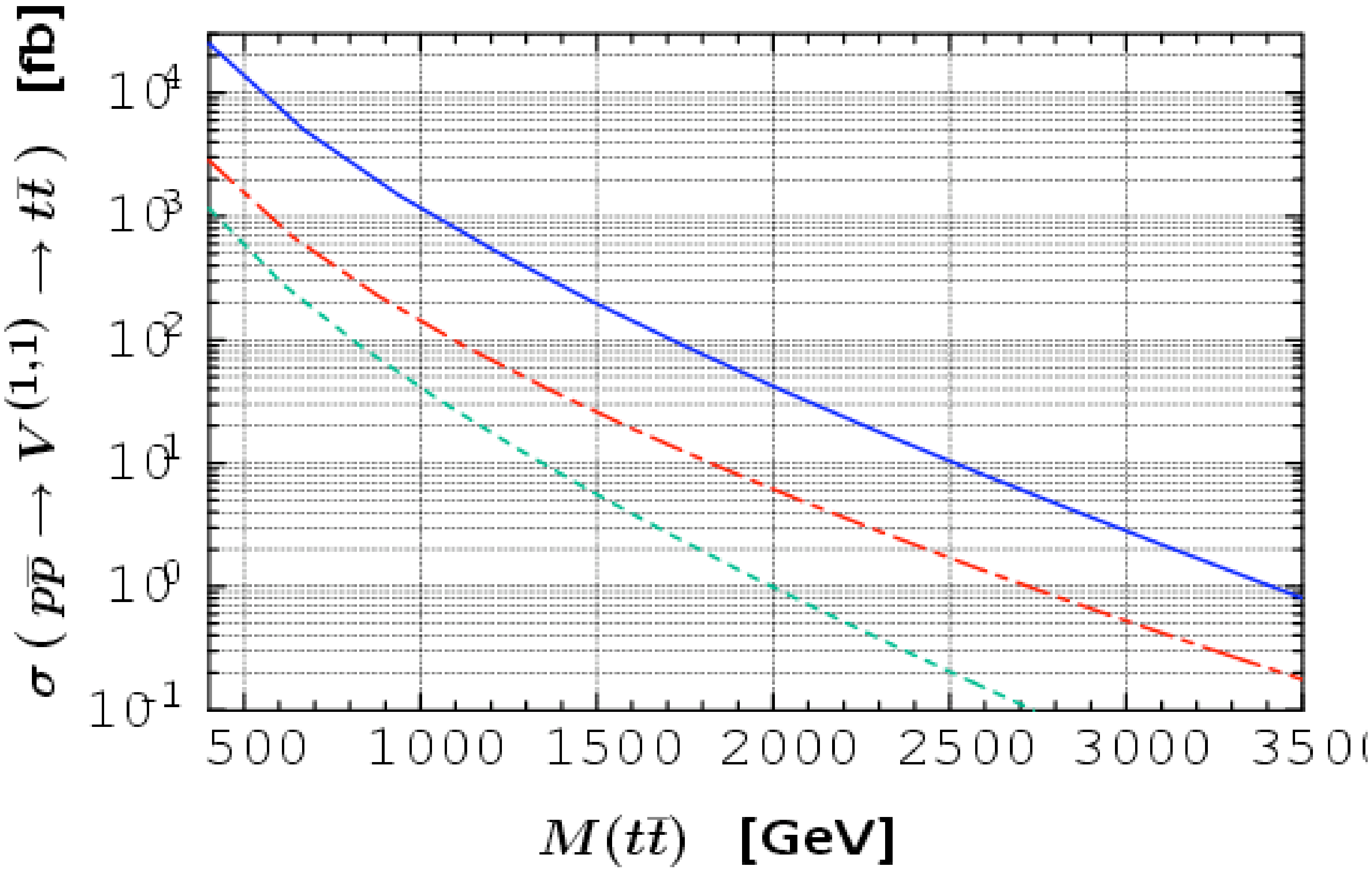
5. s-channel production of a (1,1) $W_\mu^\pm$

$\longrightarrow$ $t\bar{b}$ resonance:



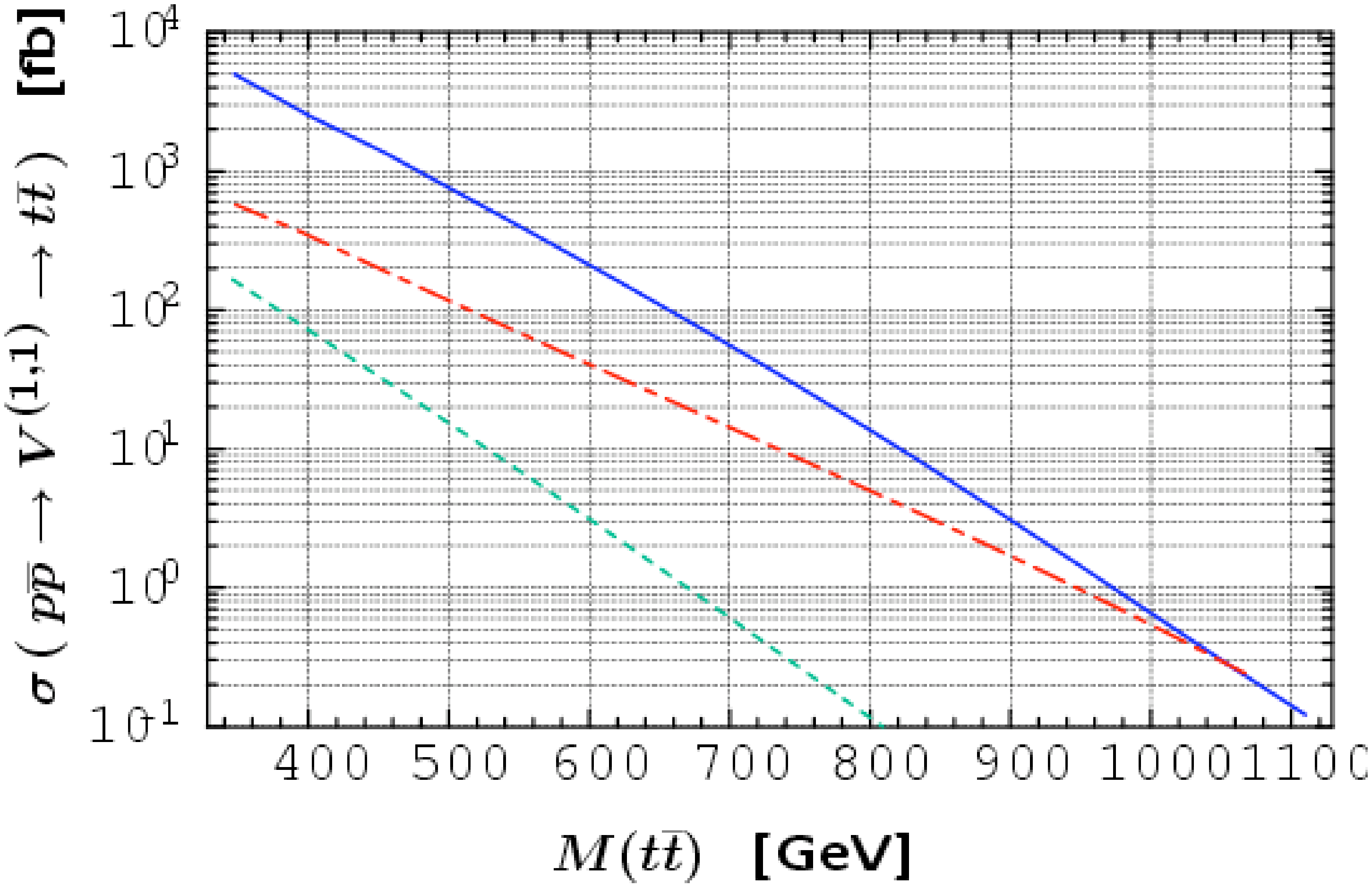
Production of $t\bar{t}$ pairs at the LHC from mass peaks at:

- $G_H^{(1,1)} + W_\mu^{(1,1)3}$ ——— $M_{t\bar{t}} \simeq 1.10\,\sqrt{2}/R$
- $W_H^{(1,1)3} + B_\mu^{(1,1)}$ – – – $M_{t\bar{t}} \simeq 0.96\,\sqrt{2}/R$
- $B_H^{(1,1)}$ – – – $M_{t\bar{t}} \simeq 0.87\,\sqrt{2}/R$



Production of $t\bar{t}$ pairs at the Tevatron from mass peaks at:

- $G_H^{(1,1)} + W_\mu^{(1,1)3}$ (blue solid line) $M_{t\bar{t}} \simeq 1.10\,\sqrt{2}/R$
- $W_H^{(1,1)3} + B_\mu^{(1,1)}$ (red dashed line) $M_{t\bar{t}} \simeq 0.96\,\sqrt{2}/R$
- $B_H^{(1,1)}$ (green dashed line) $M_{t\bar{t}} \simeq 0.87\,\sqrt{2}/R$



Conclusions

- 6-Dimensional Standard Model

– 3 generations of quarks and leptons are required for global $SU(2)_W$ anomaly cancellation

– proton is long-lived due to 6D Lorentz invariance

– neutrinos are special

- *At colliders, look for:*
  - $t\bar{t}$ and $t\bar{b}$ resonances
  - many leptons + jets + missing E_T
  - leptons + photons + jets + missing E_T
  - other signatures of Kaluza-Klein modes