Reviving chaotic inflation with fermion production: a SUGRA model

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PART I: MOTIVATION
Fact

The structure we inhabit (Galaxies, clusters…) originates from the growth of a primordial spectrum of perturbations on the top of a homogeneous, isotropic distribution of matter.

What are the observed properties of this primordial spectrum?
The primordial power spectrum

- Is quasi scale invariant, \( n_s - 1 = -1/30 \pm 10\% \)
- No observed running of spectral index
- Gaussian to 1 part in \( \sim 10^4 \)
- Isocurvature modes are below \( \sim 5\% \)
- No observed tensor modes, \( r < .04 \)

All properties (+ flatness of spatial slices of Universe) that are in agreement with simple predictions of inflation
very early Universe filled by scalar field $\phi$, the \textit{inflaton}, with potential $V(\phi) > 0$

to give enough inflation, $V(\phi)$ must be flat
Inflation requires $|V'(\phi)| << V(\phi)/M_P, |V''(\phi)| << V(\phi)/M_P^2$

A simple (the simplest?) way of obtaining this: monomial potential, with $\phi$ large enough

Famous example: quadratic potential (chaotic inflation)

$Linde$ $1983$

$V(\phi) = m^2 \phi^2 / 2$

Amplitude of perturbations produced during inflation

$m \sim 10^{13} \text{ GeV}$
A MODEL OF NATURAL QUADRATIC INFLATION...
Let me introduce you the 4-form…

(Higher rank relative of the electromagnetic field)

\[ S_{4\text{form}} = - \frac{1}{48} \int F_{\mu \nu \rho \lambda} F^{\mu \nu \rho \lambda} \, d^4x \]

\[ F_{\mu \nu \rho \lambda} = \partial_{[\mu} A_{\nu \rho \lambda]} \]

tensor structure in 4d \Rightarrow F_{\mu \nu \rho \lambda} = q(x^\alpha) \, \varepsilon_{\mu \nu \rho \lambda}

equations of motion \quad D^\mu F_{\mu \nu \rho \lambda} = 0 \Rightarrow q(x^\alpha) = \text{constant}

(this is why particle physicists do not care about 4-forms: trivial dynamics)
Sources for the 4-form: membranes

\[ S_{brane} \equiv \frac{e}{6} \int d^3 \xi \sqrt{-g} e^{abc} \partial_a x^\mu \partial_b x^\nu \partial_c x^\lambda A_{\mu\nu\lambda} \]

[ \( x^a(\xi^a) \) = membrane worldvolume]

\( e = \) charge per unit membrane surface

\( q(x^\alpha) \) jumps by \( e \) across a membrane

\( q(x^\alpha) \) is locally constant and quantized in units of \( e \)
Let us couple the 4-form to a pseudoscalar action invariant under shift symmetry:

\[ S_{\text{bulk}} = \int d^4 x \sqrt{g} \left( \frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{48} F_{\mu\nu\lambda\sigma}^2 + \frac{\mu \phi}{24} \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}} F_{\mu\nu\lambda\sigma} + \ldots \right) \]

Action invariant under shift symmetry:

under \( \phi \rightarrow \phi + c \), \( \mathcal{L} \rightarrow \mathcal{L} + c \mu \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu\rho\lambda}/24 \)
Let us couple the 4-form to a pseudoscalar

\[ S_{\text{bulk}} = \int d^4x \sqrt{g} \left( \frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{48} F_{\mu\nu\lambda\sigma}^2 + \frac{\mu\phi}{24} \epsilon^{\mu\nu\lambda\sigma} \sqrt{g} F_{\mu\nu\lambda\sigma} + \ldots \right) \]

Action invariant under shift symmetry:

under \( \phi \rightarrow \phi + c, \mathcal{L} \rightarrow \mathcal{L} + c \mu \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu\rho\lambda}/24 \)

total derivative! \((F=dA)\)
Equations of motion (away from branes)

Variation of the action

\[ \nabla^\mu \left( F_{\mu\nu\sigma\lambda} - \mu \varepsilon_{\mu\nu\sigma\lambda} \phi \right) = 0 \]
\[ \nabla^2 \phi + \mu \varepsilon_{\mu\nu\sigma\lambda} F_{\mu\nu\sigma\lambda}/24 = 0 \]

After simple manipulations

\[ F_{\mu\nu\sigma\lambda} = \varepsilon_{\mu\nu\sigma\lambda} (q + \mu \phi) \]
\[ \nabla^2 \phi - \mu^2 (\phi + q/\mu) = 0 \]

\( q = \) integration constant
• $(\mu/24) \phi \varepsilon^{\mu \nu \rho \lambda} F_{\mu \nu \rho \lambda}$ is actually a mass term!

• The theory is massive while retaining the shift symmetry!

• No contributions $\propto \phi^4, \phi^6, \phi^8...$ to potential.

• The symmetry is broken spontaneously when a solution is picked

• $q$ changes by $e$ across branes $\Rightarrow q$ is quantized
HOW ABOUT DATA?
Simple is beautiful...

...and quadratic inflation is in agreement with all the observed properties of the power spectrum (including the spectral index), but is ruled out by non observation of tensors!

(figure from the BICEP-Keck 2021 paper)
Simple is beautiful…

…and quadratic inflation is in agreement with all the observed properties of the power spectrum (including the spectral index), but is ruled out by non observation of tensors!

These lines computed assuming metric perturbations generated by amplification of vacuum fluctuations

(figure from the BICEP-Keck 2021 paper)
Let us look more in detail into the models of quadratic inflation to see how robust this conclusion is...

Disclaimer: I will make heavy use of this one theorist’s prejudice.
I- Supersymmetry

Even if we do not see SUSY at the $TeV$ scale, it might be there at the $\sim 10^{16} \text{GeV}$ inflationary scale...

A simple superpotential

$$W = \frac{\mu}{2} \Phi^2 \implies V = \frac{\mu^2}{2} |\phi|^2$$

works great...

...but since the inflaton takes values $> M_P$, must use full supergravity
Given superpotential $W(\Phi_i)$ and Kähler potential $K(\Phi_i, \Phi_i^*)$,

$$V = e^{K/M_P^2} \left[ \sum_i \left| \frac{\partial W}{\partial \Phi_i} + \frac{\partial K}{\partial \Phi_i} \frac{W}{M_P^2} \right|^2 - 3 \frac{|W|^2}{M_P^2} \right]$$

makes $V$ steep at large $\phi$ (“$\eta$ problem”) and typically dominate at large $\phi$.

only term surviving in global SUSY
Given superpotential $W(\Phi_i)$ and Kähler potential $K(\Phi_i, \Phi_i^*)$

$$V = e^{K/M_P^2} \left[ \sum_i \left| \frac{\partial W}{\partial \Phi_i} + \frac{\partial K}{\partial \Phi_i} \frac{W}{M_P^2} \right|^2 - 3 \frac{|W|^2}{M_P^2} \right]$$

typically dominate at large $\phi$

Problem solved in **stabilizer models**: $W=W(S,\Phi)=S f(\Phi)$

where the stabilizer $S=0$ during inflation, thanks to $S$-dependence of $K$
III- Shift symmetry

Given superpotential $W(\Phi_i)$ and Kähler potential $K(\Phi_i, \Phi_i^*)$

$$V = e^{K/M_P^2} \left[ \sum_i \left| \frac{\partial W}{\partial \Phi_i} + \frac{\partial K}{\partial \Phi_i} \frac{W}{M_P^2} \right|^2 - 3 \left| \frac{W}{M_P^2} \right|^2 \right]$$

makes $V$ steep at large $\phi$

(“$\eta$ problem”)

Problem solved in **shift-symmetric models**: $K=K(\Phi+\Phi^*; S, S^*)$

If inflaton=$\text{Im}(\Phi)$, then Kähler does not contribute to $V$
SUGRA models of inflation

More complicated theory, contains fermions and new interactions

INTERESTING PHENOMENOLOGY?
PART II: PHENOMENOLOGY OF FERMION PRODUCTION IN AXION INFLATION
A rolling \textit{pseudoscalar, shift symmetric inflaton} $\phi$ interacts with a fermion field $Y$ of mass $m_\psi$ via

$$\bar{Y} \left[ i \gamma^\mu \partial_\mu - m_\psi a - \frac{1}{f} \gamma^\mu \gamma^5 \partial_\mu \phi \right] Y$$

($f$=constant with dimensions of a mass)
A useful field redefinition

\[ Y = e^{-i\gamma^5 \phi / f} \psi \]

allows to write the fermion Lagrangian as

\[
\bar{\psi} \left\{ i \gamma^\mu \partial_\mu - m_\psi \left[ \cos \left( \frac{2\phi}{f} \right) - i\gamma^5 \sin \left( \frac{2\phi}{f} \right) \right] \right\} \psi
\]

oscillating effective mass with amplitude \( m_\psi \) and frequency \( 2\dot{\phi}/f \)

resonant production of fermions up to momenta \( \sim \dot{\phi}/f \)
Time-dependent $\phi \rightarrow$ fermion generation

Assume $d\phi/dt=$constant,

$$\psi = \int \frac{d^3k}{(2\pi)^{3/2}} e^{ik \cdot x} \sum_{r=\pm} \left[ U_r(k, \tau) a_r(k) + V_r(-k, \tau) b_r^\dagger(-k) \right]$$

\begin{align*}
U_r(k, \tau) &= \frac{1}{\sqrt{2}} \left( \chi_r(k) u_r(x) \right), \\
V_r(k) &= C U_r(k)^T, \\
\chi_r(k) &= \frac{(k + r \sigma \cdot \mathbf{k})}{\sqrt{2k(k+k\bar{3})}} \bar{\chi}_r, \\
\bar{\chi}_+ &= \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \bar{\chi}_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\
u_r(x) &= \frac{1}{\sqrt{2x}} \left[ e^{i r \phi(x)} s_r(x) + e^{-i r \phi(x)} d_r(x) \right], \\
u_r(x) &= \frac{1}{\sqrt{2x}} \left[ e^{i r \phi(x)} s_r(x) - e^{-i r \phi(x)} d_r(x) \right].
\end{align*}

\begin{align*}
s_r(x) &= e^{-\pi r \xi} W_{\frac{1}{2} + 2i r \xi, i \sqrt{\mu^2 + 4 \xi^2}}(-2i x), \\
d_r(x) &= -i \tilde{\mu} e^{-\pi r \xi} W_{-\frac{1}{2} + 2i r \xi, i \sqrt{\tilde{\mu}^2 + 4 \xi^2}}(-2i x), \\
x &\equiv -k \tau, \quad x_{\text{in}} \equiv -k \tau_{\text{in}}, \\
\tilde{\mu} &\equiv \frac{m_\psi}{H}, \quad \xi \equiv \frac{\dot{\phi}_0}{2fH}.\end{align*}
Time-dependent $\phi \rightarrow$ fermion generation

**Occupation numbers of fermions**

Different helicities $\Rightarrow$ different occupation $\#s$ (parity violation)

(can be used for leptogenesis)

For $m_\psi \rightarrow 0$, neither helicity is produced

*Adshead and Sfakianakis 15*
Time-dependent $\phi \rightarrow$ fermion generation

Scalings, for $\xi \gg 1$, $\tilde{\mu} \leq 1$

+ helicity: $N \sim 1$ for $k < am_\psi$, $N \sim 0$ for $k > am_\psi$

- helicity: $N \sim 1$ for $k < am_\psi$, $N \sim \mu^2 / \xi$ for $am_\psi < k < 2aH\xi$, $N \sim 0$ for $k > 2aH\xi$

Total number density of -helicity $\sim \tilde{\mu}^2 \xi^2 H^3$, can be $>> H^3$!
Time-dependent $\phi \rightarrow$ fermion generation

Occupation numbers of fermions

Even heavy $m_\psi \gg H$ fermions copiously produced!
Effects of these fermions on CMB power spectrum

Using in-in formalism

$$\delta P_\zeta (\tau, k) \bigg|_{-k \tau \ll 1} = \frac{k^3}{2\pi^2} \frac{H^2}{\phi_0^2} \sum_{N=1}^{\infty} (-i)^N \int^\tau d\tau_1 \cdots \int^{\tau_{N-1}} d\tau_N \times \left\langle \left[ \cdots \left[ \delta \phi^{(0)} (\tau, k) \delta \phi^{(0)} (\tau, k'), H_{\text{int}} (\tau_1) \right], \cdots \right], H_{\text{int}} (\tau_N) \right\rangle'$$

two leading order contributions

dominant, and can be computed analytically!
Effects of these fermions on CMB power spectrum

The full result of the first diagram

\[
\frac{\delta P_\zeta}{P_0} \approx \frac{4mH}{3\pi^2 f^2} \ln(x) \int dy \, y \sum_r \text{Re}[s_r(y)d^*_r(y)]
\]

\[
\sum_r \int dy \, y \text{Re}(s_r(y)d^*_r(y)) = \tilde{\mu} \left[ \frac{1}{2} \left( 2\Lambda^2 + \frac{1}{4} \right) \left(-8(\log(2\Lambda) + \gamma_E) (\mu^2 - 8\xi^2 + 1) + \tilde{\mu}^4 - 7\tilde{\mu}^2 + 12 \right) \right. \\
+ \frac{1}{4} (\mu^2 - 8\xi - 6i\xi + 1) \left[ H_{-i} \left( 2\xi + \sqrt{\mu^2 + 4\xi^2} \right) \left( \sinh(4\pi\xi) \text{csch} \left( 2\pi \sqrt{\mu^2 + 4\xi^2} \right) + 1 \right) \right. \\
+ \left. \left. H_i \left( \sqrt{\mu^2 + 4\xi^2} - 2\xi \right) \left( 1 - \sinh(4\pi\xi) \text{csch} \left( 2\pi \sqrt{\mu^2 + 4\xi^2} \right) \right) \right] \right]
\]

\[
+ \frac{1}{4} (\mu^2 - 8\xi^2 + 6i\xi + 1) \left[ H_i \left( 2\xi + \sqrt{\mu^2 + 4\xi^2} \right) \left( \sinh(4\pi\xi) \text{csch} \left( 2\pi \sqrt{\mu^2 + 4\xi^2} \right) + 1 \right) \right. \\
+ \left. \left. H_{-i} \left( \sqrt{\mu^2 + 4\xi^2} - 2\xi \right) \left( 1 - \sinh(4\pi\xi) \text{csch} \left( 2\pi \sqrt{\mu^2 + 4\xi^2} \right) \right) \right] \right]
\]

\[
+ 6\xi \sqrt{\mu^2 + 4\xi^2} \sinh(4\pi\xi) \text{csch} \left( 2\pi \sqrt{\mu^2 + 4\xi^2} \right) - \frac{\tilde{\mu}^4}{8} + \frac{11\tilde{\mu}^2}{8} - 12\xi^2 \right],
\]

(D.7)
In the limit $\zeta \gg 1$, $\bar{\mu} \simeq 1$

$$\delta P_\zeta(k) \bigg|_{\text{end of inflation}} \approx P_\zeta^{(0)} \frac{32 m_\psi^2 \xi^2 \log \xi}{3\pi^2 f^2} \log \left( \frac{H}{k} \right)$$
In the limit $\zeta \gg 1$, $\tilde{\mu} \approx 1$

$$\delta P_\zeta(k) \bigg|_{\text{end of inflation}} \approx P_\zeta^{(0)} \frac{32 m_\psi^2 \xi^2 \log \xi}{3\pi^2 f^2} \log \left(\frac{H}{k}\right)$$

$$n_s - 1 = -3\epsilon - \frac{1}{N} + \frac{2\epsilon - \eta}{\log \xi}$$
Effects of these fermions on CMB bispectrum

Three leading order contributions

Figure 4. The three diagrams that contribute at leading order to the three-point function of .

5 Non-Gaussianity

As we saw above, the calculation of the fermionic contribution to the two-point function of the inflaton is challenging, and for the cubic diagram we could only obtain what we consider to be a reasonable estimate. As one can expect, the calculation of the three-point function is even more challenging. There is a new operator, besides the cubic and the quartic interaction Hamiltonians $H^{(3)}$ and $H^{(4)}$ given in eq. (4.1) above, that contributes to the three-point function. It is a quintic interaction Hamiltonian

$$H^{(5)} = 4m a^3 f^3 \bar{\phi}^4 \sin^2 \theta + i 5 \cos^2 \theta,$$

(5.1)

which leads to a new $\bar{\phi}^3$ vertex. Using the vertices generated from $H^{(3)}$, $H^{(4)}$, and $H^{(5)}$, we obtain, at leading order in $1/f$, the three diagrams of figure 4.

5.1 The quintic diagram

As we argue below, the first of these diagrams gives the leading contribution to the bispectrum. Fortunately, this contribution can be calculated analytically; after some long calculations that we report in appendix G, we find the following expression

$$h(k_1, \tau_1)(k_2, \tau_2)(k_3, \tau_3)i = \frac{3 H^6 m^2 f^3}{Z \tau_1} a(\tau_1)f(k_1, k_2, k_3, \tau_1)\xi Z^{3/2} \frac{1}{X_r^3} \{ d \xi^3 (p \tau_1) r \}.$$

(5.2)

where

$$f(k_1, k_2, k_3, \tau_1) = \frac{1}{k_3^3} (k_3^3 2 k_3^3 3 k_3^3 \cdot \xi \tau_1 \xi k_1 k_2 k_3 \cos(\tau_1 (k_1 + k_2 + k_3)))$$

(5.3)

Since most of the dynamics occurs at momenta $k_\tau \sim \sim \sim 1$, which is well within the horizon, we expect the non-Gaussianities to be of equilateral shape. Therefore we estimate the magnitude of the bispectrum by setting $k_1 = k_2 = k_3 = \tau k$. As in the two-point functions, dominant, and can be computed analytically!
Effects of these fermions on CMB bispectrum

Since source of perturbations is sub horizon, expect equilateral bispectrum
Effects of these fermions on CMB bispectrum

Since source of perturbations is sub horizon, expect equilateral bispectrum

Main message: can have spectrum dominated by sourced component and small $f_{\text{NL}}$
(Planck constrains $f_{\text{NL}}^\text{eq} \lesssim 40$)

Surprising: source quadratic in gaussian field $\psi$, so nongaussian

...but many many modes contribute\(\rightarrow\)central limit\(\rightarrow\)gaussian
So...

The system has a regime where Planck measures

\[ \frac{H^2 m^2}{f^4} \]

instead of the usual

\[ \frac{H^4}{\dot{\phi}^2} \]
How about the tensors?

Computed them in Adshead, Pearce, Peloso, LS, Roberts 19:

The component sourced by the fermions _always subdominant_ with respect to the standard one

so we keep the standard expression

\[ P_t = \frac{2}{\pi^2} \frac{H^2}{M_P^2} \]
PART III: REVIVING CHAOTIC INFLATION
General study: equations of motion for fermions in models of inflation with stabilizer

General $N=1$, $d=4$ SUGRA with two superfields $S$, $\Phi$ with $W = S f(\Phi)$, $K = K(\Phi, \Phi^*) + g(S S^*)$

Stabilizer condition $S=0 \implies W=0$

$$V = e^{K/M_P^2} |f(\phi)|^2$$
Two matter fermions (one is goldstino, can be set to zero in unitary gauge) + helicity-1/2 part of gravitino.
So two coupled fermions $\theta$ and $\Upsilon$ in the end

$$
\mathcal{L}_f = -\frac{\alpha a^3}{4k^2} \hat{\theta} \left[ \left( \gamma^0 \hat{\partial}_0 + i \gamma^i k_i \hat{A} + \gamma^0 \hat{B} \right) \theta - \frac{4k^2}{a\alpha} \gamma^0 \Upsilon \right] +
- \frac{4a}{\alpha \Delta^2} \tilde{\Upsilon} \left[ \left( \gamma^0 \hat{\partial}_0 - i \gamma^i k_i \hat{A} + \gamma^0 \hat{B}^\dagger + a \gamma^0 \hat{F} + 2\hat{a} \gamma^0 + \frac{a}{M_P^2} \gamma^0 \mathbf{m} \gamma^0 \right) \Upsilon + \frac{1}{4} a\alpha \Delta^2 \gamma^0 \theta \right],
$$

(helicity-3/2 gravitino is decoupled and irrelevant here)

formulae from Kallosh, Kofman, Linde and Van Proeyen 00
does not look simple…

\[
m = e^{\frac{\kappa}{2M_P^2}} W, \\
m^i = \left( \partial^i + \frac{1}{2M_P^2} \partial^i K \right) m, \\
\hat{\partial}_0 = \partial_0 - \frac{i}{2} A_0^B \gamma^5, \\
\hat{H}^2 = \frac{1}{3M_P^2} \left( |\phi|^2 + V \right), \\
\alpha = 3M_P^2 \left( H^2 + \frac{|m|^2}{M_P^4} \right), \\
\hat{A} = \frac{1}{\alpha} (\alpha_1 - \gamma^0 \alpha_2), \\
\xi_i = m_i - \gamma^0 g_i^j \phi_j, \\
\mathbf{m} = \Re\{m\} - i \Im\{m\} \gamma^5, \\
m^{ij} = \left( \partial^i + \frac{1}{2M_P^2} \partial^i K \right) m^j - \Gamma_k^{ij} m^k, \\
A_0^B = \frac{i}{2M_P^2} (\phi'^i \partial_i K - \phi'_i \partial_i K), \\
V = m_i (g^{-1})^j_i m^j - 3 \frac{|m|^2}{M_P^2}, \\
|\phi|^2 \equiv g_j^i \phi_i \phi^j, \\
\alpha_1 = -3M_P^2 \left( H^2 + \frac{2}{3} \dot{H} + \frac{|m|^2}{M_P^4} \right), \\
\alpha_2 = 2 \mathbf{m}^\dagger, \\
\dot{B} = -\frac{3}{2} \dot{a} \dot{\hat{A}} + \frac{a}{2M_P^2} \mathbf{m} \gamma^0 (1 + 3 \hat{A}), \\
\Delta = 2 \frac{\sqrt{V} |\phi|}{\alpha},
\]
Fermions in models of inflation with stabilizer
Making a number of field redefinitions...

\[ \mathcal{L} = (\bar{\chi}_1, \bar{\chi}_2) \left[ -\gamma^0 \partial_0 + i \gamma \cdot k + a \begin{pmatrix} M_1 + iM_2\gamma^5 & 0 \\ 0 & -M_1 - iM_2\gamma^5 \end{pmatrix} \right] \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \]

...where \( M_1 \) and \( M_2 \) are reasonably complicated functions of the background fields

\( (M_2=0 \text{ for } Im[\Phi]=0) \)
Specializing to our system: quadratic inflaton plus small instanton corrections

\[ f(\Phi) = \mu \Phi + \hat{\Lambda}^2 e^{-\frac{\sqrt{2} \Phi}{F}}, \quad K(\Phi, \Phi') = \frac{1}{2}(\Phi + \Phi')^2 \]

- gives dominant quadratic potential
- small “instanton” correction (we'll require these to be negligible in \( V! \))
- no \( \eta \) problem for inflaton in imaginary part of \( \Phi \)

To fix ideas...

\[ \mu = \mathcal{O}(10^{13}) \text{ GeV} \quad \hat{\Lambda} = \mathcal{O}(10^{14}) \text{ GeV} \quad F = \mathcal{O}(10^{15}) \text{ GeV} \]
Specializing to our system: quadratic inflaton plus small instanton corrections

Full scalar potential

\[ V = e^{\frac{\rho^2}{M_P^2}} \left[ \frac{\mu^2}{2} (\rho^2 + \varphi^2) + \sqrt{2} \mu \hat{\Lambda}^2 e^{-\frac{\rho}{F}} \left( \rho \cos \frac{\varphi}{F} - \varphi \sin \frac{\varphi}{F} \right) + \hat{\Lambda}^4 e^{-\frac{2\rho}{F}} \right] \]

\[ \phi = \frac{1}{\sqrt{2}} (\rho + i \varphi) \]

...but \( \rho \ll F \), so we are left with

\[ V \approx \frac{\mu^2}{2} \varphi^2 - \sqrt{2} \mu \hat{\Lambda}^2 \varphi \sin \frac{\varphi}{F} \]

quadratic plus (small) wiggles
Specializing to our system: quadratic inflation plus small instanton corrections

And for the fermions, remind that we had

\[ \mathcal{L} = (\tilde{\chi}_1, \tilde{\chi}_2) \left[ -\gamma^0 \partial_0 + i \gamma \cdot k + a \left( \begin{array}{cc} M_1 + i M_2 \gamma^5 & 0 \\ 0 & -M_1 - i M_2 \gamma^5 \end{array} \right) \right] \left( \begin{array}{c} \chi_1 \\ \chi_2 \end{array} \right) \]

in this regime

\[ M_1 + i M_2 \gamma^5 \simeq \mu \left( \frac{2M_P^2}{\phi^2} - \sqrt{2} \frac{\hat{\Lambda}^2}{\mu F} \left( \cos(\varphi/F) + i \sin(\varphi/F)\gamma^5 \right) \right) \]

negligible

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**FIG. 2:** Results of exact numerical integration (solid, blue) and analytical approximations, eq. (54), (dashed, orange) for the quantities \( M_1(t) \) and \( M_2(t) \). The parameters used for these plots are \( \mu = 5 \times 10^{-6} M_P, F = 5 \times 10^{-4} M_P \). At these times \( \varphi \simeq 13.9 M_P \).
Specializing to our system: quadratic inflaton plus small instanton corrections

And for the fermions, remind that we had

\[ \mathcal{L} = (\bar{\chi}_1, \bar{\chi}_2) \left[ -\gamma^0 \partial_0 + i \gamma \cdot k + \alpha \left( \begin{array}{cc} M_1 + i M_2 \gamma^5 & 0 \\ 0 & -M_1 - i M_2 \gamma^5 \end{array} \right) \right] \left( \begin{array}{c} \chi_1 \\ \chi_2 \end{array} \right) \]

in this regime

\[ M_1 + i M_2 \gamma^5 \simeq \mu \left( \frac{2 M_P^2}{\phi^2} - \sqrt{2} \frac{\Lambda^2}{\mu F} \left( \cos(\varphi/F) + i \sin(\varphi/F)\gamma^5 \right) \right) \]

negligible

...equivalent to the system discussed in part II!

\[ \overline{\psi} \left\{ i \gamma^\mu \partial_\mu - m_\psi \left[ \cos \left( \frac{2 \phi}{f} \right) - i \gamma^5 \sin \left( \frac{2 \phi}{f} \right) \right] \right\} \psi \]
Importing the results from part II…

Imposing constraints:

- Monotonicity of potential
- Energies below $4\pi F$ cutoff
- $r < 0.04$
- Negligible backreaction of fermions on background
- No oscillations in scalar power spectrum
- No nongaussianities
- Scalar spectral index
Importing the results from part II...

Three parameters.
Eliminate $\Lambda$ with normalization of scalar spectrum

ALLOWED!
Summing up

- Natural generalization of quadratic potential to sugra, with inclusion of instantons, in agreement with all existing data.
- Lower bound on $r \gtrsim 0.004$, to be probed in next $O(10)$ years.
- Analysis easily generalizable to monomial potentials (monodromy).
- Oscillations in scalar power spectrum in monodromy models: do they survive in sugra models?
Conclusion

Monomial inflation is beautiful…
…but in its simplest form is ruled out by non observation of tensors

“Natural” embedding in supergravity can revive it…
…and at least for a few years