Amplitude method studies of effective field theories

Karol Kampf
Charles University, Prague

Outline:
- Role model: gluon amplitudes
- Effective Field Theories
- From NLSM to Periodic Table of Scalar Theories
- Further avenues
- New soft theorem
- Summary

Davis, 11-13-2019
Introduction: amplitudes

Objective of amplitude community:

Study a priori known objects from different perspective

Example in mind: gluon amplitudes

- 1986: Parke and Taylor calculated 6-point gluon-scattering
- simplification: tree-level, no-fermions
- final result: extremely simple
- other way of calculation?
Example: gluon amplitudes

At tree level:

- colour ordering $\rightarrow$ stripped amplitude

$$M^{a_1 \ldots a_n}(p_1, \ldots, p_n) = \sum_{\sigma/Z_n} \text{Tr}(t^{a_\sigma(1)} \ldots t^{a_\sigma(n)}) M_\sigma(p_1, \ldots, p_n)$$

- $M_\sigma(p_{\sigma(1)}, \ldots, p_{\sigma(n)}) = M(p_1, \ldots, p_n) \equiv M(1, 2, \ldots n)$

- propagators $\Rightarrow$ the only poles of $M_\sigma$

- thanks to ordering the only possible poles are:

$$P_{ij}^2 = (p_i + p_{i+1} + \ldots + p_{j-1} + p_j)^2$$
Pole structure

Weinberg’s theorem (one particle unitarity): on the factorization channel

\[ \lim_{P_{1j}^2 \to 0} M(1, 2, \ldots n) = \sum_{h_l} M_L(1, 2 \ldots j, l) \times \frac{1}{P_{1j}^2} \times M_R(l, j + 1, \ldots n) \]
BCFW relations, preliminaries

[Britto, Cachazo, Feng, Witten '05]

Reconstruct the amplitude from its poles (in complex plane)

- shift in two external momenta

\[ p_i \rightarrow p_i + zq, \quad p_j \rightarrow p_j - zq \]

- keep \( p_i \) and \( p_j \) on-shell, i.e.

\[ q^2 = q \cdot p_i = q \cdot p_j = 0 \]

- amplitude becomes a meromorphic function \( A(z) \)
- only simple poles coming from propagators \( P_{ab}(z) \)
- original function is \( A(0) \)
BCFW relations: factorization channels

Cauchy’s theorem

\[ \frac{1}{2\pi i} \int \frac{dz}{z} A(z) = A(0) + \sum_k \frac{\text{Res} (A, z_k)}{z_k} \]
Cauchy’s theorem

\[ 0 = \frac{1}{2\pi i} \int \frac{dz}{z} A(z) = A(0) + \sum_k \frac{\text{Res}(A, z_k)}{z_k} \]

If \( A(z) \) vanishes for \( z \to \infty \)

\[ A = A(0) = -\sum_k \frac{\text{Res}(A, z_k)}{z_k} \]
BCFW relations

\[ P_{ab}^2(z) = 0 \]

if one and only one \( i \) (or \( j \)) in \((a, a+1, \ldots, b)\).

Suppose \( i \in (a, \ldots, b) \not\ni j \)

\[ P_{ab}^2(z) = (p_a + \ldots + p_{i-1} + p_i + zq + p_{i+1} + \ldots + p_b)^2 = \]

\[ = P_{ab}^2 + 2q \cdot P_{ab}z = 0 \]

solution

\[ z_{ab} = -\frac{P_{ab}^2}{2(q \cdot P_{ab})} \]

\[ \Rightarrow \]

\[ P_{ab}^2(z) = -\frac{P_{ab}^2}{z_{ab}}(z - z_{ab}) \]

Thus

\[ \text{Res}(A, z_{ab}) = \sum_s A_L^{-s}(z_{ab}) \times \frac{-z_{ab}}{P_{ab}^2} \times A_R^s(z_{ab}) \]

and for allowed helicities it factorizes into two subamplitudes
BCFW relations

Using Cauchy’s formula, we have finally as a result

$$A = \sum_{k,s} A_L^{-s_k}(z_k) \frac{1}{P^2_k} A_R^{s_k}(z_k)$$

- based on two-line shift (convenient choice: adjacent $i,j$)
- recursive formula (down to 3-pt amplitudes)
- number of terms small $\equiv$ number of factorization channels
- all ingredients are on shell
BCFW Example: gluon amplitudes

# od diagrams for $n$-body gluon scatterings at tree level

<table>
<thead>
<tr>
<th>$n$</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td># diagrams (inc.crossing)</td>
<td>1</td>
<td>4</td>
<td>25</td>
<td>220</td>
<td>2485</td>
<td>34300</td>
</tr>
<tr>
<td># diagrams (col.ordered)</td>
<td>1</td>
<td>3</td>
<td>10</td>
<td>38</td>
<td>154</td>
<td>654</td>
</tr>
<tr>
<td># BCFW terms</td>
<td>–</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>20</td>
</tr>
</tbody>
</table>

[C.Cheung: TASI Lectures '17]
[KK, Novotny, Trnka '13]
We have assumed that

\[ A(z) \to 0, \quad \text{for} \quad z \to \infty \]

More generally we have to include a boundary term in Cauchy’s theorem. This is intuitively clear: we can formally use the derived BCFW recursion relations to obtain any higher \( n \) amplitude starting with the leading interaction. But this does not have to be the correct answer.
BCFW recursion relations: problems

example: scalar-QED

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - |D\phi|^2 - \frac{1}{4} \lambda |\phi|^4 \]

Due to the power-counting the boundary term is proportional to

\[ B \sim 2e^2 - \lambda \]

In order to eliminate the boundary term we have to set \( \lambda = 2e^2 \), then the original BCFW works.

I.e. we needed some further information (e.g. supersymmetry) to determine the \( \lambda \) piece.
Effective field theories
Effective field theories: general picture

Now we have infinitely many unfixed \( \lambda \) terms. Schematically

\[
\mathcal{L} = \frac{1}{2} (\partial \phi)^2 + \lambda_4 (\partial^4 \phi)^4 + \lambda_6 (\partial^6 \phi)^6 + \ldots
\]

Example: 6pt scattering, Feynman diagrams

\[
\mathcal{M}_6 = \sum_{l=\text{poles}} \lambda_4^2 \cdots \frac{\lambda_6}{P_l} + \lambda_6 (\ldots)
\]

\( \lambda_6 \) part: not fixed by the pole behaviour.

Task: to find a condition in order to link these two terms
Effective field theories: introduction

Usual steps:

Symmetry $\rightarrow$ Lagrangian $\rightarrow$ Amplitudes $\rightarrow$ physical quantities

(cross-section, masses, decay constants, ...)

In our work – opposite direction:

Amplitudes $\rightarrow$ Lagrangian $\rightarrow$ Symmetry

Our aim: classification of interesting EFTs

*works done in collaborations with Clifford Cheung, Jiri Novotny, Chia-Hsien Shen, Jaroslav Trnka and Congkao Wen*
Effective field theories: scalar theories

As simple as possible: a spin-0 massless degree of freedom with a three-point interaction.

General formula for three-particle amplitude

\[ A(1^{h_1}2^{h_2}3^{h_3}) = \begin{cases} 
\langle 12 \rangle^{h_3-h_1-h_2} \langle 23 \rangle^{h_1-h_2-h_3} \langle 31 \rangle^{h_2-h_3-h_1}, & \sum h_i \leq 0 \\
[12]^{h_1+h_2-h_3} [23]^{h_2+h_3-h_1} [31]^{h_3+h_1-h_2}, & \sum h_i \geq 0 
\end{cases} \]

Used a spinor-helicity notation, e.g. \( p_i \cdot p_j \sim \langle ij \rangle [ij] \)

For scalars \( (h_i = 0) \) this is a constant - corresponding to \( \mathcal{L}_{\text{int}} = \lambda \phi^3 \).

All derivatives can be removed by equations of motions (boxes)

\[ \mathcal{L}_{\text{int}} = (\partial_\alpha \ldots \partial_\omega \phi)(\partial^\alpha \ldots \partial^\omega \phi)\phi \rightarrow \mathcal{L}_{\text{int}} = (\Box \phi)(\ldots) \]
Effective field theories: scalar theories

We start with \((m\) counts number of derivatives\)

\[
\mathcal{L}_{\text{int}} = \lambda_4 \partial^m \phi^4
\]

n.b. we want to connect this four-point vertex with the 6-point contact terms

This rules out again no-derivative terms, as the powercounting dictates:

\[
\partial^m \times \frac{1}{\partial^2} \times \partial^m \rightarrow \partial^{2m-2} \phi^6
\]
Simplest example: two derivatives, single scalar

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \lambda_4 \partial^2 \phi^4 + \lambda_6 \partial^2 \phi^6 + \ldots \]

How to connect \( \lambda_4 \) and \( \lambda_6 \)?
Well Lagrangian, an infinite series, looks complicated, but it is not the case. It represents a free theory:

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \left( 1 + \lambda_4 \phi^2 + \ldots \right) \]

\( F(\phi) \) can be removed by a field redefinition

Non-trivial simplest example:
- more derivatives
- more flavours \( (\phi \rightarrow \phi_1, \phi_2, \ldots) \)
More flavours

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi^i \partial^\mu \phi^i + \lambda_{ijkl} \partial_\mu \phi^i \partial^\mu \phi^j \phi^k \phi^l + \lambda_{i_1...i_6} \partial_\mu \phi^{i_1} \partial^\mu \phi^{i_2} \phi^{i_3} ... \phi^{i_6} + \ldots \]

- Can be used for systematic studies of two species, three species, etc.
- Very complicated generally
- Assume some simplification, organize using a group structure

\[ \phi = \phi^a T^a \]

- motivated by the ‘gluon case’: flavour ordering \cite{KK,Novotny,Trnka '13}

\[ A^{a_1...a_n} = \sum_{\text{perm}} \text{Tr}(T^{a_1} \ldots T^{a_n})A(p_1, \ldots p_n) \]
More flavours: stripped amplitude

first non-trivial case 6pt scattering:

\[ \lambda^2 \frac{p^2}{p^2} \frac{1}{p^2} p^2 + \lambda_6 p^2 \]

in order to combine the pole and contact term we need to consider some limit. The most natural candidate: We will demand soft limit, i.e.

\[ A \to 0, \quad \text{for} \quad p \to 0 \]

\[ \Rightarrow \lambda_4^2 \sim \lambda_6 \]
Standard direction(s)

Assuming the shift symmetry

$$\phi^a \rightarrow \phi^a + \epsilon^a$$

⇒ Noether current

$$A^a_\mu = \frac{\delta L}{\delta \partial^\mu \phi^a}$$

⇒ Ward identity ⇒ LSZ

$$\langle 0| A^a_\mu(x) |\phi^b(p) \rangle = iF \delta^{ab} p_\mu e^{-ipx}$$

⇒ Adler zero

$$\lim_{p \rightarrow 0} \langle f|i + \phi^a(p) \rangle = 0$$

⇒ CCWZ: non-linear sigma model

$$\mathcal{L} = \frac{F^2}{2} \text{Tr}(\partial_\mu U^\dagger \partial^\mu U), \quad U = e^{\frac{i}{F} \phi^a T^a}$$

[Weinber’66], [Ian Low ’14–’15]
Natural classification: $\sigma$ and $\rho$

Soft limit of one external leg of the tree-level amplitude

$$A(tp_1, p_2, \ldots, p_n) = \mathcal{O}(t^\sigma), \quad \text{as} \quad tp_1 \to 0$$

Interaction term

$$\mathcal{L} = \partial^m \phi^n$$

Then another natural parameter is (counts the homogeneity)

$$\rho = \frac{m - 2}{n - 2} \quad \text{“averaging number of derivatives”}$$

e.g. $\mathcal{L} = \partial^m \phi^4 + \partial^{\tilde{m}} \phi^6$

so these two diagrams can mix: $p^{2m-2} \sim p^{\tilde{m}}$
Natural classification: $\sigma$ and $\rho$

Soft limit of one external leg of the tree-level amplitude

$$A(tp_1, p_2, \ldots, p_n) = \mathcal{O}(t^\sigma), \quad \text{as} \quad tp_1 \to 0$$

Interaction term

$$\mathcal{L} = \partial^m \phi^n$$

Then another natural parameter is (counts the homogeneity)

$$\rho = \frac{m - 2}{n - 2} \quad \text{“averaging number of derivatives”}$$

e.g. $\mathcal{L} = \partial^m \phi^4 + \partial^{\tilde{m}} \phi^6$

\[
\begin{align*}
\begin{array}{c}
\text{Diagram 1} \\
\text{Diagram 2}
\end{array}
\end{align*}
\]

so these two diagrams can mix: $p^{2m-2} \sim p^{\tilde{m}}$

$$2m - 2 - 2 = \tilde{m} - 2 \Rightarrow \frac{2m-4}{4} = \frac{\tilde{m}-2}{4} \Rightarrow$$
Natural classification: $\sigma$ and $\rho$

Soft limit of one external leg of the tree-level amplitude

$$A(tp_1, p_2, \ldots, p_n) = \mathcal{O}(t^\sigma), \quad \text{as} \quad tp_1 \to 0$$

Interaction term

$$\mathcal{L} = \partial^m \phi^n$$

Then another natural parameter is (counts the homogeneity)

$$\rho = \frac{m - 2}{n - 2} \quad \text{“averaging number of derivatives”}$$

e.g. $\mathcal{L} = \partial^m \phi^4 + \partial^{\tilde{m}} \phi^6$

so these two diagrams can mix: $p^{2m-2} \sim p^{\tilde{m}}$

$$2m - 2 - 2 = \tilde{m} - 2 \Rightarrow \frac{2m-4}{4} = \frac{\tilde{m}-2}{4} \Rightarrow \rho = \tilde{\rho}$$

rho is same if they talk to each other
Non-trivial cases

for: \( \mathcal{L} = \partial^m \phi^n : \quad m < \sigma n \)

or

\[ \sigma > \frac{(n - 2)\rho + 2}{n} \]

i.e.

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( \sigma ) at least</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

i.e. non-trivial regime for \( \rho \leq \sigma \)
First case: $\rho = 0$ (i.e. two derivatives)

Schematically for single scalar case

$$\mathcal{L} = \frac{1}{2} (\partial \phi)^2 + \sum_i \lambda^i_4 (\partial^2 \phi^4) + \sum_i \lambda^i_6 (\partial^2 \phi^6) + \ldots$$

similarly for multi-flavour ($\phi_i$: $\phi_1, \phi_2, \ldots$).

non-trivial case

$$\sigma = 1$$

Outcome:

- single scalar: free theory
- multiple scalars (with flavour-ordering): non-linear sigma model

n.b. it represents a generalization of [Susskind, Frye ’70], [Ellis, Renner ’70]
Second case: $\rho = 1, \sigma = 2$ (double soft limit)

1. Focus on the lowest combination and fix the form:

$$\mathcal{L}_{\text{int}} = c_2 (\partial \phi \cdot \partial \phi)^2 + c_3 (\partial \phi \cdot \partial \phi)^3$$

condition: $c_3 = 4c_2^4$
Second case: $\rho = 1, \sigma = 2$ (double soft limit)

1. focus on the lowest combination and fix the form:

$$\mathcal{L}_{int} = c_2(\partial\phi \cdot \partial\phi)^2 + c_3(\partial\phi \cdot \partial\phi)^3$$

condition: $c_3 = 4c_2^4$

2. find the symmetry

$$\phi \rightarrow \phi - b_\rho x^\rho + b_\rho \partial^\rho \phi \phi$$  (again up to 6pt so far)
Second case: \( \rho = 1, \sigma = 2 \) (double soft limit)

1. focus on the lowest combination and fix the form:

\[
\mathcal{L}_{int} = c_2 (\partial \phi \cdot \partial \phi)^2 + c_3 (\partial \phi \cdot \partial \phi)^3 \quad \text{condition: } c_3 = 4c_2^4
\]

2. find the symmetry

\[
\phi \rightarrow \phi - b_\rho x^\rho + b_\rho \partial^\rho \phi \phi \quad \text{(again up to 6pt so far)}
\]

3. ansatz of the form

\[
c_n (\partial \phi \cdot \partial \phi)^n + c_{n+1} (\partial \phi \cdot \partial \phi)^n \partial \phi \cdot \partial \phi
\]

4. in order to cancel: \( 2(n + 1)c_{n+1} = (2n - 1)c_n \)

i.e. \( c_1 = \frac{1}{2} \Rightarrow c_2 = \frac{1}{8}, c_3 = \frac{1}{16}, c_4 = \frac{5}{128}, \ldots \)
Second case: $\rho = 1, \sigma = 2$ (double soft limit)

4. in order to cancel: $2(n + 1)c_{n+1} = (2n - 1)c_n$
   i.e. $c_1 = \frac{1}{2} \Rightarrow c_2 = \frac{1}{8}, c_3 = \frac{1}{16}, c_4 = \frac{5}{128}, \ldots$

solution:

$$\mathcal{L} = -\sqrt{1 - (\partial\phi \cdot \partial\phi)}$$

This theory known as a scalar part of the Dirac-Born-Infeld [1934] – DBI action
Scalar field can be seen as a fluctuation of a 4-dim brane in five-dim Minkowski space

Remark: soft limit and symmetry are “equivalent”
Third case: $\rho = 2$, $\sigma = 2$ (double soft limit)

Similarly to previous case we will arrive to a unique solution: the Galileon Lagrangian

$$
\mathcal{L} = \sum_{n=1}^{d+1} d_n \phi \mathcal{L}_{n-1}^{\text{der}}
$$

$$
\mathcal{L}_{n}^{\text{der}} = \varepsilon^{\mu_1 \ldots \mu_d} \varepsilon^{\nu_1 \ldots \nu_d} \prod_{i=1}^{n} \partial_{\mu_i} \partial_{\nu_i} \phi \prod_{j=n+1}^{d} \eta_{\mu_j \nu_j} = -(d - n)! \det \{ \partial_{\nu_i} \partial_{\nu_j} \phi \}.
$$

It possesses the Galilean shift symmetry

$$
\phi \rightarrow \phi + a + b_{\mu} x^{\mu}
$$

(leads to EoM of second-order in field derivatives)
Surprise: $\rho = 2, \sigma = 3$ (enhanced soft limit)

- general galileon: three parameters (in 4D)
- only two relevant (due to dualities [de Rham, Keltner, Tolley '14] [KK, Novotny '14])
Surprise: $\rho = 2, \sigma = 3$ (enhanced soft limit)

- general galileon: three parameters (in 4D)
- only two relevant (due to dualities [de Rham, Keltner, Tolley '14] [KK, Novotny '14])
- we demanded $O(\rho^3)$ behaviour
- we have verified: possible up to very high-pt order
- suggested new theory: special galileon [Cheung, KK, Novotny, Trnka 1412.4095]
Surprise: $\rho = 2, \sigma = 3$ (enhanced soft limit)

- general galileon: three parameters (in 4D)
- only two relevant (due to dualities [de Rham, Keltner, Tolley '14] [KK, Novotny '14])
- we demanded $\mathcal{O}(\rho^3)$ behaviour
- we have verified: possible up to very high-pt order
- suggested new theory: special galileon [Cheung, KK, Novotny, Trnka 1412.4095]
- symmetry explanation: hidden symmetry [K. Hinterbichler and A. Joyce 1501.07600]

\[ \phi \rightarrow \phi + s_{\mu\nu} x^\mu x^\nu - 12 \lambda_4 s_{\mu\nu} \partial_\mu \phi \partial_\nu \phi \]
New recursion for effective theories

[Cheung, KK, Novotny, Shen, Trnka 2015]
The high energy behaviour forbids a naive Cauchy formula

\[ A(z) \neq 0 \quad \text{for} \quad z \to \infty \]

Can we instead use the soft limit directly?
The high energy behaviour forbids a naive Cauchy formula

\[ A(z) \neq 0 \quad \text{for} \quad z \to \infty \]

Can we instead use the soft limit directly? yes!
The standard BCFW not applicable, we propose a special shift:

\[ p_i \to p_i(1 - za_i) \quad \text{on all external legs} \]

This leads to a modified Cauchy formula

\[
\oint \frac{dz}{z} \frac{A(z)}{\prod_i(1 - a_iz)^\sigma} = 0
\]

note there are no poles at \( z = 1/a_i \) (by construction).
Now we can continue in analogy with BCFW
Further avenues

• similarly for vector EFT:

\[ \mathcal{L}_{\text{BI}} = 1 - \sqrt{(-1)^{D-1} \det(\eta_{\mu\nu} + F_{\mu\nu})}, \]

(see [Cheung, KK, Novotny, Shen, Trnka, Wen ’18])

• so far avoided the fermionic degrees of freedom (see e.g. Elvang et al.’18)

• higher orders in NLSM (see [Bijnens,KK,Sjö 2019])

• multiple flavours – especially without flavour ordering

• only two-flavour case fully classified

• preliminary study of the mixed scalar-vector case (Galileon-BI): more promising than the pure Galileon-like BI

• spin-2: similar to Galileon-like studies – no exceptional candidate

• non-abelian Born-Infeld

• non-zero masses (technically possible)

• loop corrections – focused on the exceptional theories
BUT...

[KK, Novotny, Shifman, Trnka 2019 and in prep.]
it seems we have a powerful method to classify effective field theories
more efficient than the standard group oriented methods:
spontaneous symmetry breaking, (non-)compact groups,
(semi-)simple, CCWZ construction . . . \[\rightarrow\] complicated monomial
structure, where equivalence is not transparent

- e.g. for two flavours – two-derivative counting:
  only one non-trivial theory: \(O(3)/O(2)\)

- more problematic for three flavours – finished (work in progress with
  J.Bijnens): again only \(O(4)/O(3)\) and combinations of \(O(3)/O(2)\)
  plus one free scalar

- but what about completely broken \(O(3)\):

  \(SU(2)/1\)
it describes three GBs
- CCWZ construction
  - $\Rightarrow$ Lagrangian
  - it is neither equivalent to $O(4)/O(3)$ nor to $O(3)/O(2)$, nor any their flavour combinations
  - on top of it: the amplitudes don’t have adler zero!
- What have we missed?
General discussion

- answer is then easy – we missed “non-zero” Adler zero
- beyond the scope of our classification
- so our method is not that general
- can we extend it?
Adler zero: textbook derivation

GB couples to the associated Noether current

$$\langle 0 | J^\mu(x) | \phi(p) \rangle = -i p^\mu F e^{-i p \cdot x}$$

For the process $i \to f + \phi(p)$ we have:

$$\langle f | J^\mu(0) | i \rangle = F \frac{p^\mu}{p^2} A(f + \phi(p), i) + R^\mu(p)$$

The current conservation $p_\mu \langle f | J^\mu(0) | i \rangle = 0$ yields

$$A(f + \phi(p), i) = -\frac{1}{F} p_\mu R^\mu(p)$$

and thus finally

$$\lim_{p \to 0} A(f + \phi(p), i) = 0$$

if $R(p)$ regular in the limit.
Adler zero: loophole

When

$$\lim_{p \to 0} p_\mu R^\mu \neq 0$$

? 

Two possibilities:

- there are cubic vertices

- Noether current is quadratic in fields
SU(2)/1: new soft theorem

- n.b.: three GBs, can be rotated to: two charged $\phi^\pm$ and one neutral $\phi$.
- Simplification: charge conservation + shift symmetry in the neutral mode.
- Standard Adler zero for the neutral mode
- We can focus only on the $\phi^+$ shift. Ansatz:

$$\lim_{p_1 \to 0} \phi^+(p_1) \ldots \phi^+(p_n) \phi^-(q_1) \ldots \phi^-(q_n) \phi(k_1) \ldots \phi(k_m) =$$

$$= x \sum_{i=1}^{m} \phi^+(k_i) \ldots \phi^+(p_n) \phi^-(q_1) \ldots \phi^-(q_n) \phi(k_1) \ldots \phi(k_m)$$

$$+ y \sum_{i=1}^{n} \phi^+(p_1) \ldots \phi^+(p_n) \phi^-(q_1) \ldots \phi^-(q_n) \phi(q_i) \phi(k_1) \ldots \phi(k_m)$$
SU(2)/1: new soft theorem

\[\lim_{p_1 \to 0} \phi^+(p_1) \ldots \phi^+(p_n)\phi^-(q_1) \ldots \phi^-(q_n)\phi(k_1) \ldots \phi(k_m) =\]

\[\times \sum_{i=1}^{m} \phi^+(k_i) \ldots \phi^+(p_n)\phi^-(q_1) \ldots \phi^-(q_n)\phi(k_1) \phi(k_i) \ldots \phi(k_m)\]

\[+ y \sum_{i=1}^{n} \phi^+(p_1) \ldots \phi^+(p_n)\phi^-(q_1) \phi(q_i) \ldots \phi^-(q_n)\phi(q_i)\phi(k_1) \ldots \phi(k_m)\]

- IT WORKS!
- \( y = -x \)
- verified on amplitudes up to 7-pt
- can be proved from Lagrangian
- new generalization for the formal Lagrangian (work in progress)
Summary

- We have offered a new tool for effective field theories
- motivated by the amplitude methods employed for renormalizable theories
- analogy between gravity and soft scalar theories (Bonifacio et al. ’today)
- used for classification of scalar theories
- one new theory discovered: special galileon
- one exceptional theory for spin-1 particles: BI
- generalization of Adler zero
- work in progress: classification can be extended for generalized Adler zero
Summary

- We have offered a new tool for effective field theories
- motivated by the amplitude methods employed for renormalizable theories
- analogy between gravity and soft scalar theories (Bonifacio et al. ’today)
- used for classification of scalar theories
- one new theory discovered: special galileon
- one exceptional theory for spin-1 particles: BI
- generalization of Adler zero
- work in progress: classification can be extended for generalized Adler zero

Thank you!