A (THE?) HIGGS VACUUM INSTABILITY DURING INFLATION

with thanks to William East, Anson Hook, Bibhushan Shakya, Hojin Yoo and Kathryn Zurek


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THE STANDARD MODEL HIGGS POTENTIAL HAS AN UNSTABLE ELECTROWEAK VACUUM!

Tunneling today?

\[ \Gamma^{-1}_{\text{EW Vacuum}} > \Gamma^{-1}_{\text{Age of Universe}} \]

Sher [e.g., hep-ph/9307342]

Isidori et al. [0712.0242]

\( \Rightarrow \) EW Vacuum metastable

But what about inflation? In other words, how did we end up in this vacuum in the first place?
1. How do Higgs fluctuations evolve during inflation?

2. How does a large (super-barrier) fluctuation impact the surrounding spacetime?
Assuming:

• SM valid to high energies
• Inflation started “ideally”
• Minimally-Coupled Higgs
• Neglect (subleading) mass correction
EVOLUTION OF HIGGS FIELD DURING INFLATION
CONTRIBUTIONS TO HIGGS EVOLUTION

(I) Stochastic evolution
- Freeze out of mode fluctuations $\delta h_k \sim H/2\pi$ leads to local field value that is sum over superhorizon modes (as for massless fields)
- Higgs field undergoes “random walk” within patch with each subsequent mode crossing

(II) Higgs Potential
- Drives net evolution depending on $V'(h)$. 
MODELLING BOTH: FOKKER-PLANCK

Treats Higgs as a “test particle” in “thermal” background

\[ \frac{\partial P}{\partial t} = \frac{\partial}{\partial h} \left( \frac{V'(h)}{3H} P \right) + \frac{H^2}{8\pi^2} \frac{\partial P}{\partial h} \]

\[ P(h,t) \equiv \text{Probability to find a patch of size } \sim H^{-1} \\
\text{with local field value } h \text{ at time } t \]

First applied to Higgs by Espinosa, Giudice, Riotto [0710.2484]
CHOOSING THE CORRECT $V(H)$
AN EXERCISE IN WILSONIAN EFT

1. Identify the correct degrees of freedom

   Fokker-Planck describes superhorizon modes.
   Mode functions of fermions, gauge bosons decay rapidly outside the horizon.
   So, potential contains Higgs only, $V(h) = \frac{1}{4} \lambda h^4$. Not, e.g., one-loop effective $V_{CW}$.

2. Identify the correct input parameters/couplings

   Fermions & gauge bosons do contribute in UV/subhorizon (which looks flat)
   Renormalize quartic coupling as in Minkowski space
Wilsonian Approach: run SM down from UV as in Minkowski space, integrating out non-scalar states at scale where mode functions become suppressed.

\[ V(h) = \frac{\lambda}{4} h^4 \quad \text{with} \quad \lambda \left( \mu \approx \sqrt{H^2 + h^2} \right) \]

Consistency checks:

- \( h << H \): fermions and gauge bosons renormalizing quartic decouple at horizon scale \( \sim H \).
- \( h >> H \): states decouple at “mass threshold,” \( m_f = yh \), \( m_V = gh \).

Details in JK, Yoo, Zurek [1503.05193]
Verified by explicit computation of \( V_{\text{eff}} \) in dS [1407.3141]
FP allows calculation of coincident correlators: $\langle h^n \rangle = \int dh \ h^n \ P(h, N)$

Scalar modes in (toy) $h^4$ theory give IR and UV contributions, e.g.,

$$3\lambda \int_{a_0 H}^{a\Lambda} \frac{d^3 k}{(2\pi)^3} |h_k(t)|^2 = 3\lambda \left\{ \frac{\Lambda^2}{8\pi^2} + \frac{H^2}{8\pi^2} \log \left( \frac{a\Lambda}{a_0 H} \right) \right\} \rightarrow \frac{3\lambda(\mu)H^2}{8\pi^2} \left( 2N + \log \frac{\mu^2}{H^2} \right)$$

Fermions and gauge bosons contribute from $k = aH$ to $a\Lambda$. So (UV) contribution to logarithms, but no (leading) IR contribution.
NUMERICAL RESULTS FOR PROBABILITY DISTRIBUTION
FP SOLUTION WITH $H/\Lambda = 0.07 \ (\Lambda/H = 14.3)$
PRODUCTION OF LARGE FLUCTUATIONS

- $P(h,t)$ exhibits “long tails:” distribution spreads out at $h > \Lambda$ due to unstable potential.

- As inflation produces $e^{3N}$ patches, regions exhibiting fluctuations beyond the barrier can still appear, even for $\Lambda/H >> 1$.

- This leads to the next question: what happens to these patches? In particular, is their formation consistent with the inflationary history of our Universe?
FATE OF LARGE FLUCTUATIONS
**PHASES OF HIGGS FLUCTUATION EVOLUTION**

**WHAT DO WE MEAN BY “LARGE?”**

<table>
<thead>
<tr>
<th>Regime</th>
<th>Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h \approx \Lambda )</td>
<td>Grows due to inflationary fluctuations, stabilized by positive quartic (assuming ( H &lt; \Lambda ))</td>
</tr>
<tr>
<td>( h \gtrsim \Lambda )</td>
<td>Growth accelerated by negative quartic...but spacetime evolution still dominated by inflationary background</td>
</tr>
<tr>
<td>( h \gtrsim V'(h)/3H^2 )</td>
<td>Slow-roll violation! Fluctuation grows rapidly...</td>
</tr>
<tr>
<td>( h \gtrsim (H M_P)^{1/2} )</td>
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THE GROWTH OF A LARGE FLUCTUATION
• From slow-roll breakdown to true vacuum takes $\approx 1$ e-fold
  
  In particular, $h \gtrsim h_{\text{sr}}$ cannot be stabilized by, e.g., efficient reheating

• “Not your grandmother’s bubble nucleation”

  Not “thin-wall” CdL bubble: broad Hubble-sized (Hawking-Moss-like) fluctuation, dynamical (not $cc > 0$ outside, $cc < 0$ inside).

  Details differ from bubble approx employed by Espinosa et al [1505.04825]

• Contraction $\Rightarrow$ blue-shifting of (rolling) Higgs energy density $\Rightarrow$
  formation of apparent horizon/black hole @ center of fluctuation. Compensated by surrounding shell of $\rho < 0$. 
BUT THE MAIN RESULT...
AT LEAST, FROM THE STANDPOINT OF OUR UNIVERSE

Fluctuation and true vacuum region continue to grow throughout inflation, and even in Minkowski limit, in spite of local contraction due to negative energy density...

In other words, once born, these regions never die.

In agreement with 1505.04825
Initial true vacuum region growth can be *spacelike*

- Region REALLY not a bubble causally sweeping outwards.
- Grows because adjacent points are falling to true vacuum...so quickly in fact that their behavior is causally disconnected from adjacent points doing the same.
- So, growth is insensitive to behavior of interior (including crunching, details of $V_{\text{min}}$).

Also, observe violation of Hoop Conjecture (Thorne)
IMPLICATIONS FOR OUR UNIVERSE
The initial patch that inflated to give rise to our observable universe must have undergone $N_e \gtrsim 50-60$ e-folds of inflation.

Present horizon must have been in causal contact at some point.

Regions *re-entering* causal contact during RD or MD *left* during inflation.

**THE NECESSARY INGREDIENTS?**

- Comoving horizon expansion from end of inflation to now $\leq$ Comoving horizon contraction during inflation until end

Leach, Liddle [astro-ph/0305263]
• Minimal assumption: There exists a patch in the EW vacuum that underwent the necessary $N_e$ to give rise to our universe.

$$P(h, 0) = \delta(h)$$

• But, if any large fluctuations subsequently form, they will continue to grow and persist throughout inflation.

Then, once inflation ends, these true vacuum regions will expand and destroy surrounding space in the EW vacuum.

• So, no large fluctuations can have formed in our past lightcone during inflation/during the growth of this patch

$$P(|h| \gtrsim h_{srb}, N_e)e^{3N_e} \lesssim 1$$
• Bound on inflationary scale

\[ \frac{H}{\Lambda} \lesssim 0.07 \]

• Interestingly, due to long tails of distribution, similar bound obtained by requiring

\[ P (|h| > \Lambda) e^{3N_e} \lesssim 1 \]
BEYOND THE MINIMAL STORY
What if the Higgs has a Hubble-scale mass during inflation?

\[ V(h) \supset \frac{1}{2} c_1 H^2 h^2 \]

For instance, could arise due to Planck-suppressed operator

\[ V \supset k h^2 \left( \frac{V_{\text{inf}}}{M_P^2} \right) \]

c_1 \gtrsim \frac{1}{2} \Rightarrow \text{EW vacuum stable throughout required inflation!}
AFTER INFLATION: PREHEATING?
RELEVANT IN CASE OF H-INFLATON COUPLING

• Large oscillations in inflaton could induce large $\delta h$ via “parametric resonance”
  e.g., Kofman, Linde, Starobinsky [hep-ph/970445]

• Constrains $h$-inf coupling
  Ema, Mukaida, Nakayama [1602.00483]
  Kohri, Matsui [1602.02100]
  Enqvist et al [1608.08848]

• But constraints mild, e.g.,
  \[ V \supset k \ h^2 \left( V_{inf}/M_P^2 \right) \Rightarrow k \approx 10^3 \]
AFTER INFLATION: REHEATING?

Thermal effects potentially drive $h > \Lambda$ back to EW vacuum

- In principle, relaxes bounds
- In practice, effect marginal (due to long tails of FP distribution)

Requires reheating be efficient

Espinosa et al [1505.04825]
BEYOND $N_e = 60$: FRACTURING SPACETIME?

Consider proportion of “true vacuum” regions formed each e-fold

$$f_N \equiv \frac{\int_{-h_{srb}}^{h_{srb}} dh \{P(h, N) - P(h, N - 1)\}}{\int_{-h_{srb}}^{h_{srb}} dh \ P(h, N - 1)}$$

- Eventually, start “sloughing off” certain proportion of patches
  See JK, Yoo, Zurek [1503.05193]

- What if, overall, > 50% of space transitions to true vacuum?

- One possibility: all spacetime becomes unstable, crunches.
  e.g., Sekino, Shenker, Susskind [1003.1347]

- Could imply bound on total $N_e$

Constant $\lambda = -0.005$ (red) or -0.01 (blue)
TO CONCLUDE
Takeaways:  
- $h > h_{srb}$ $\Rightarrow$ rapid divergence to true vacuum  
- Such fluctuations form **expanding** shells of negative $\rho$ surrounding black holes.  
- Formation of such a region in our past lightcone likely unless $H/\Lambda \approx 0.07$.

Implications:  
- $\exists$ incompatible $(m_h,m_t)$ and $r$. Measurement could be indicative of BSM physics?  
- (Additional) challenge for inflationary models?  
- Simple reconciliation? $h$-inflaton coupling

Future Directions:  
- New physics, dynamical evolution, similar systems (relaxions?),
THANK YOU!