Black hole analogue of triple point and reentrant phase transition

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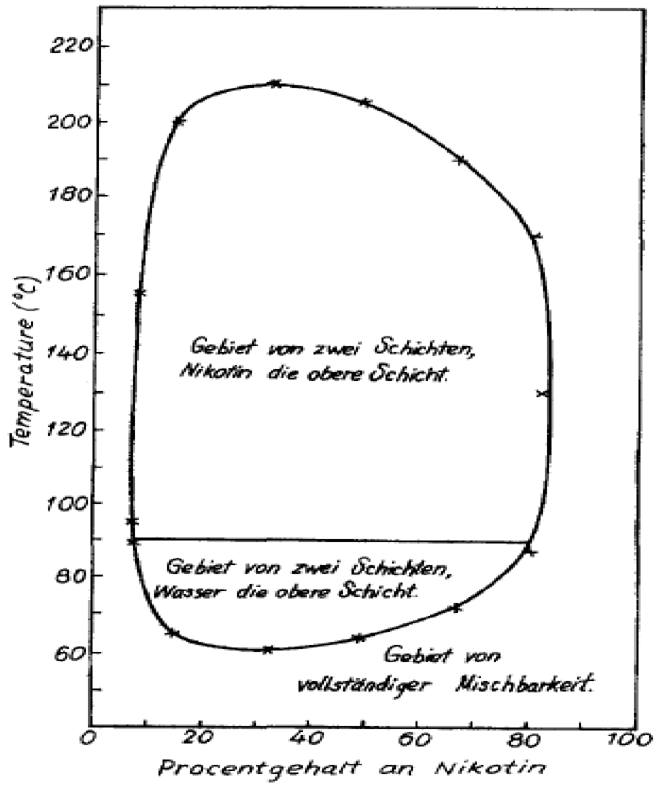


High Energy and Joint Theory Seminar
University of California, Davis, California, USA
November 4, 2013

Plan of the talk

I. Prelude: Van der Waals fluid and critical phenomena

II. Black holes as thermodynamic objects

III. AdS black holes: overview of interesting features
   I. Global AdS and its (in)stability
   II. Hawking-Page transition
   III. Scalar field condensate: holographic superconductors
   IV. VdW behavior of charged AdS black holes: analogy 1

IV. Λ as pressure and thermodynamic volume
   I. Cosmological constant and its conjugate variable
   II. VdW analogy now complete

V. Everyday thermodynamics: black hole analogue
   I. Reentrant phase transitions
   II. Triple point

VI. Conclusions

Friends: N. Altamirano, S. Gunasekaran, R. B. Mann, Z. Sherkatghanad

I) Prelude:

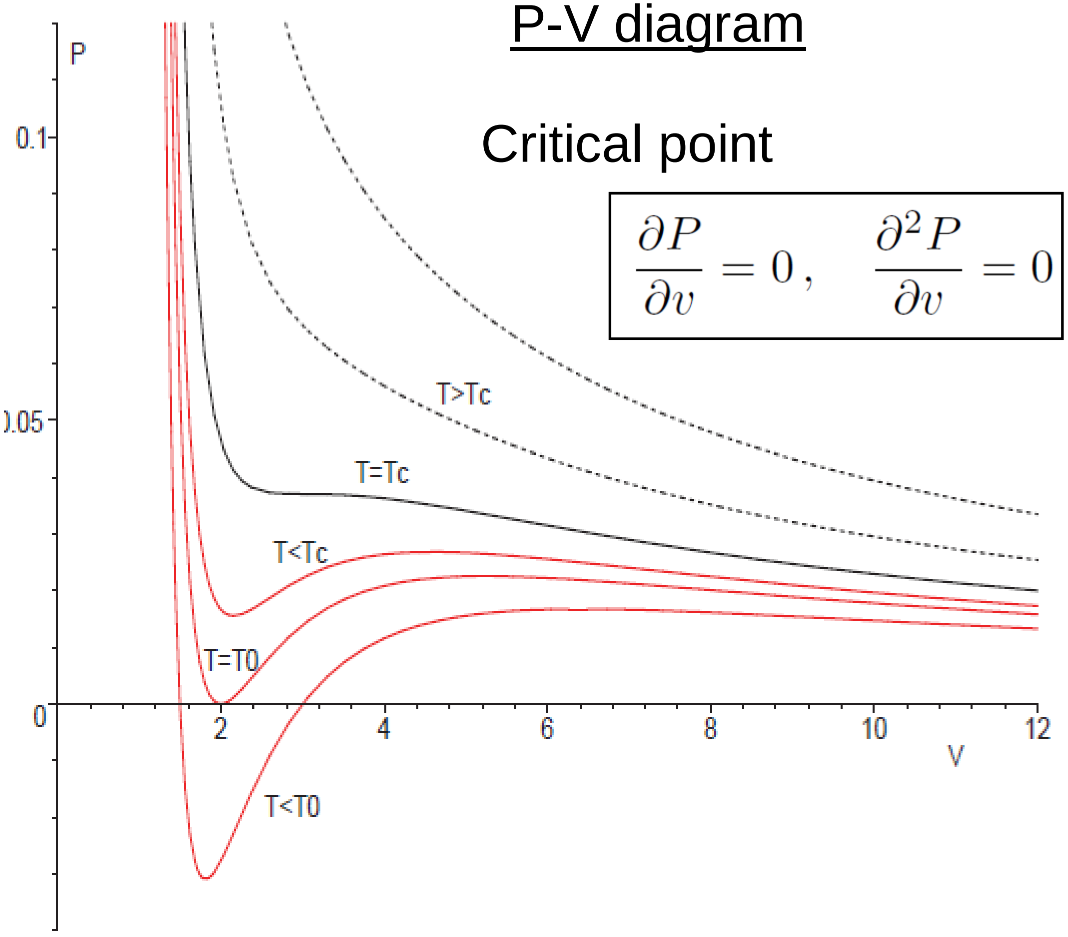
Van der Waals fluid & critical phenomena

Johannes Diderik van der Waals (1837-1923)

Van der Waals fluid

$$\left(P + \frac{a}{v^2}\right)(v - b) = T$$

Parameters a and b characterize the fluid. a measures the **attraction** between particles (a>0) and b corresponds to "**volume of fluid particles**".



Critical point

$$\frac{\partial P}{\partial v} = 0\,, \qquad \frac{\partial^2 P}{\partial v} = 0$$

Critical point:

$$T_c = \frac{8a}{27b} \qquad v_c = 3b$$

$$P_c = \frac{a}{27b^2}$$

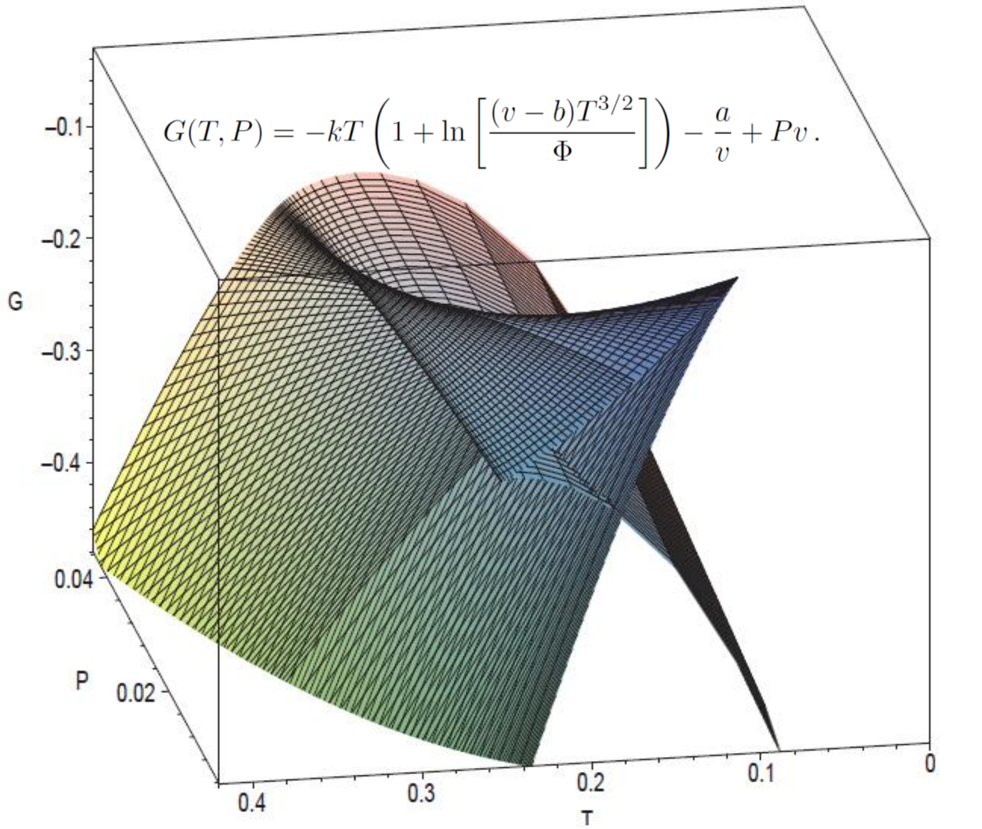
Universal critical ratio

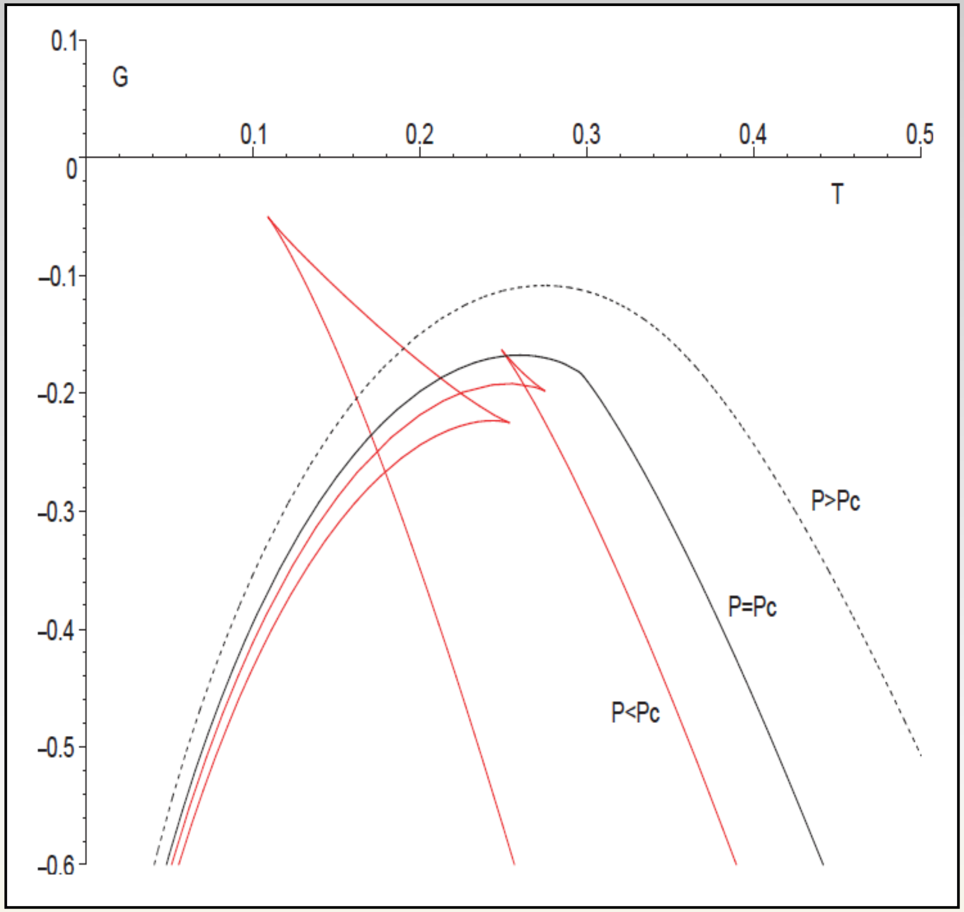
$$\rho_c = \frac{P_c v_c}{T_c} = \frac{3}{8}$$

Gibbs free energy

$$dG = -SdT + vdP$$

Swallow tail behavior





System wants to minimize Gibbs free energy → Phase transitions

Maxwell's equal area law & phase diagrams

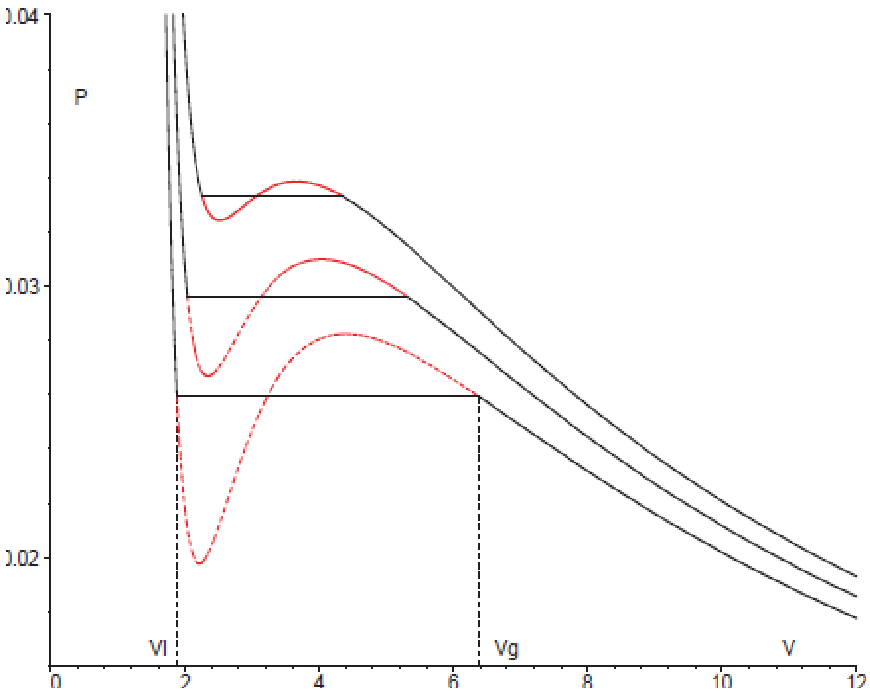


FIG. 2. **Maxwell's equal area law.** The 'oscillating' (dashed) part of the isotherm $T < T_c$ is replaced by an isobar, such that the areas above and below the isobar are equal one another.

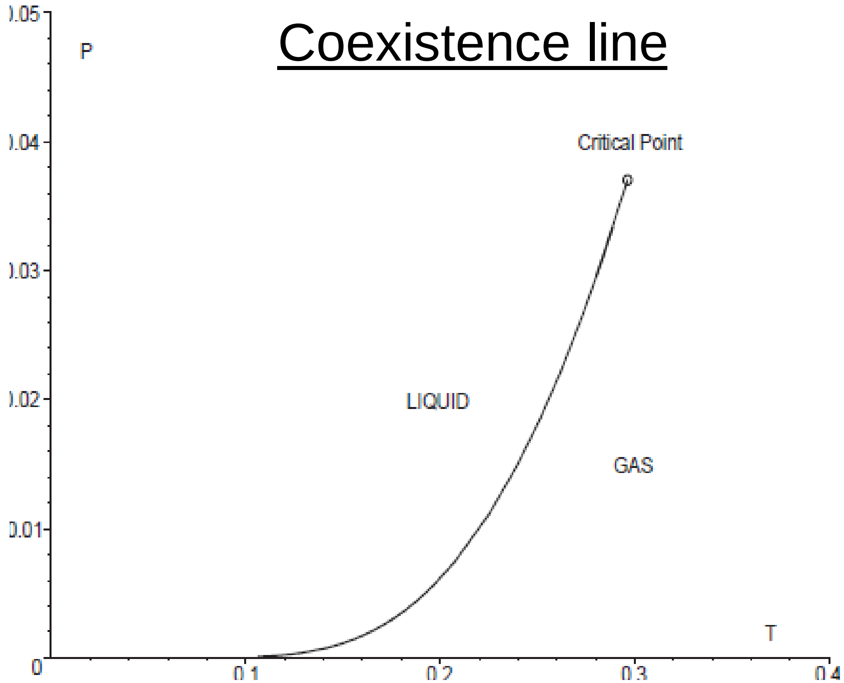


FIG. 5. **Coexistence line of liquid–gas phase.** Fig. displays the coexistence line of liquid and gas phases of the Van der Waals fluid in (P, T)-plane. The critical point is highlighted by a small circle at the end of the coexistence curve.

$$dG = -SdT + vdP$$

Both phases have the same Gibbs free energy ⇒

$$\oint vdP = 0$$

Critical exponents

- Describe the behaviour of physical quantities near the critical point
- Universal (depend on dimensionality and/or range of interactions)
- In d ≥ 4 dimensions they can be calculated using MFT (each dof couples to the average of the other dof).

For example, for a fluid with critical Tc, vc, and Pc.

- Exponent α governs the behaviour of the specific heat at constant volume,

$$C_v = T\frac{\partial S}{\partial T}\bigg|_v \propto |t|^{-\alpha}\,. \tag{A2}$$

- Exponent β describes the behaviour of the *order parameter* $\eta = v_g - v_l$ (the difference of the volume of the gas v_g phase and the volume of the liquid phase v_l) on the given isotherm

$$\eta = v_g - v_l \propto |t|^{\beta}\,. \tag{A3}$$

- Exponent γ determines the behaviour of the *isothermal compressibility* κ_T,

$$\kappa_T = -\frac{1}{v}\frac{\partial v}{\partial P}\bigg|_T \propto |t|^{-\gamma}\,. \tag{A4}$$

- Exponent δ governs the following behaviour on the critical isotherm $T = T_c$:

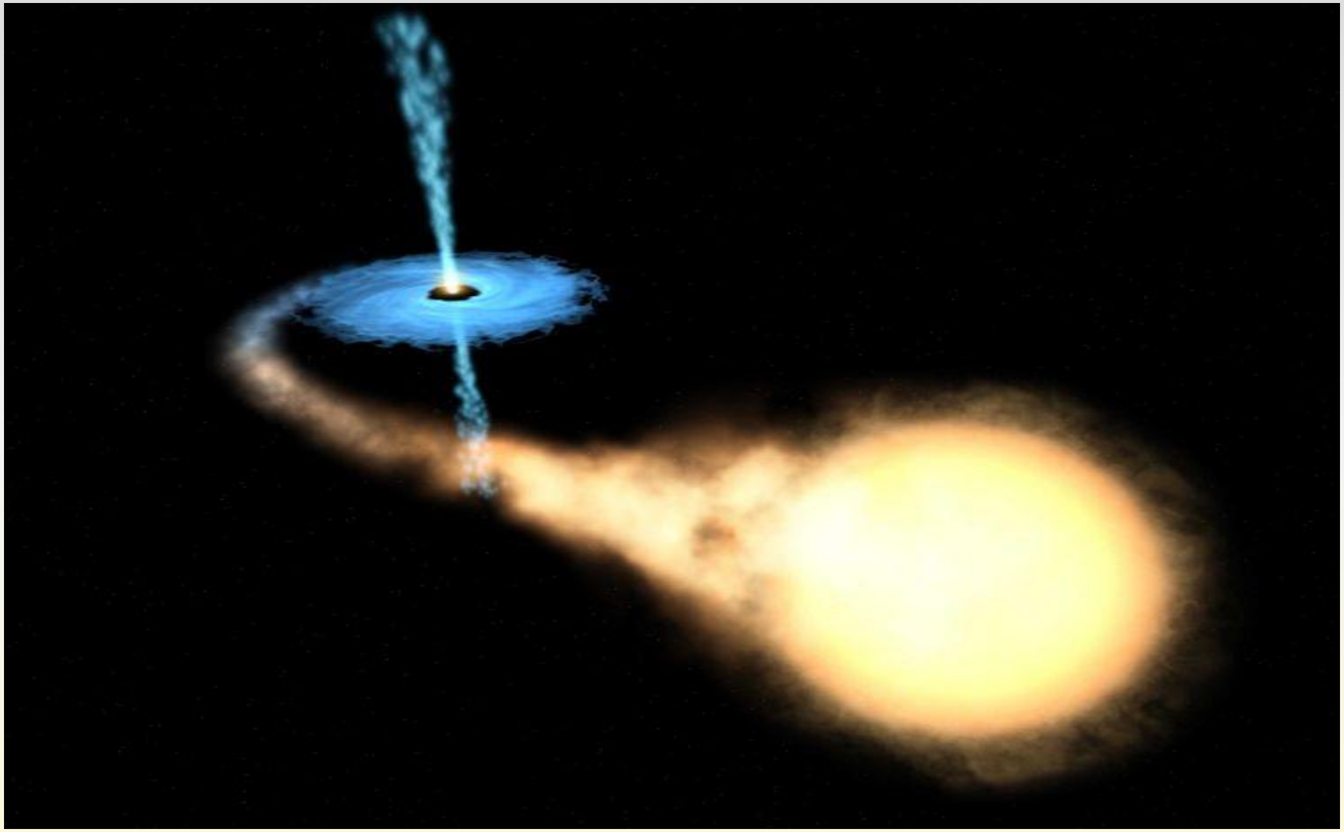
$$|P - P_c| \propto |v - v_c|^{\delta}\,. \tag{A5}$$

$$\alpha = 0\,, \quad \beta = \frac{1}{2}\,, \quad \gamma = 1\,, \quad \delta = 3$$

The same critical exponents derived for ferromagnets, superfluidity,..

Problem: MFT neglects fluctuations, to go beyond one needs to use RG techniques

II) Black Holes as Thermodynamic Objects



Black holes and their characteristics

Schwarzschild black hole:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega^2$$

- asymptotic mass (total energy)

$$M = -\frac{1}{8\pi}\int_{S_\infty} *dk\,, \quad k^a = (\partial_t)^a$$

- black hole horizon: (radius rh=2M)

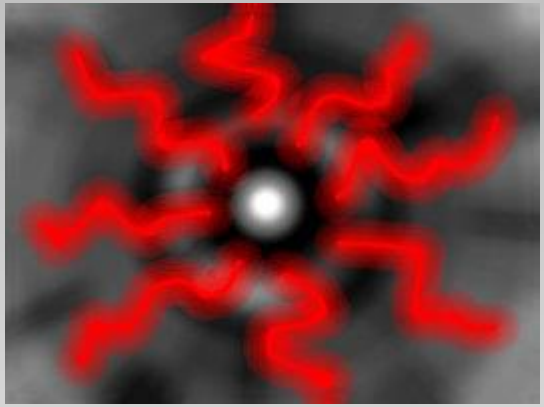
surface gravity $$(k^b \nabla_b k^a)_{|H} = \kappa k^a_{|H}$$ ⇒ $$\kappa = \frac{1}{4M}$$

surface area $$A = 4\pi r_h^2$$ never decreases

$$dM = \kappa dA$$ ⟺ Bekenstein? $$dE = TdS$$

Hawking (1974):

$$T = \frac{\kappa}{2\pi}\,, \qquad S = \frac{A}{4}$$



derivation used QFT in curved spacetime

Other approaches:

- Euclidean path integral approach (Gibbons & Hawking-1977)

$$Z = \int D[g_{ab}] e^{-S_E[g]} \approx e^{-S_E[g_c]}$$

$$F = -\frac{1}{\beta}\log Z \quad \Rightarrow \quad S = -\frac{\partial F}{\partial T} = \frac{A}{4}$$

Euclidean manifold non-singular if the imaginary time τ identified with a certain period $\Delta\tau$. In QFT this corresponds to a finite temperature

$$T = \frac{1}{\beta}\,, \qquad \beta = \Delta\tau$$

- Tunneling approach, LQG, String theory, ….

Black hole thermodynamics

- First law of black hole thermodynamics:

$$\delta M = T\delta S + \sum_i \Omega_i \delta J_i + \Phi \delta Q \quad \boxed{+ V\delta P}$$

- Smarr-Gibbs-Duhem relation:

$$\frac{d-3}{d-2} M = TS + \sum_i \Omega_i J_i + \frac{d-3}{d-2}\Phi Q \quad \boxed{- \frac{2}{d-2} VP}$$

- Specific heat of AF Schwarzschild BH is negative (cannot have thermal equilibrium)

Where is the PdV term?

- AdS black holes can be in thermal equilibrium

- Natural to identify $P = -\frac{1}{8\pi}\Lambda$

Consistency between 1st law and Smarr relation (scaling argument)

Euler's theorem:

$$f(\alpha^p x, \alpha^q y) = \alpha^r f(x, y)$$ ⇒ $$r f(x,y) = p\left(\frac{\partial f}{\partial x}\right) x + q\left(\frac{\partial f}{\partial y}\right) y.$$

Mass of black hole: $M = M(A, P)$

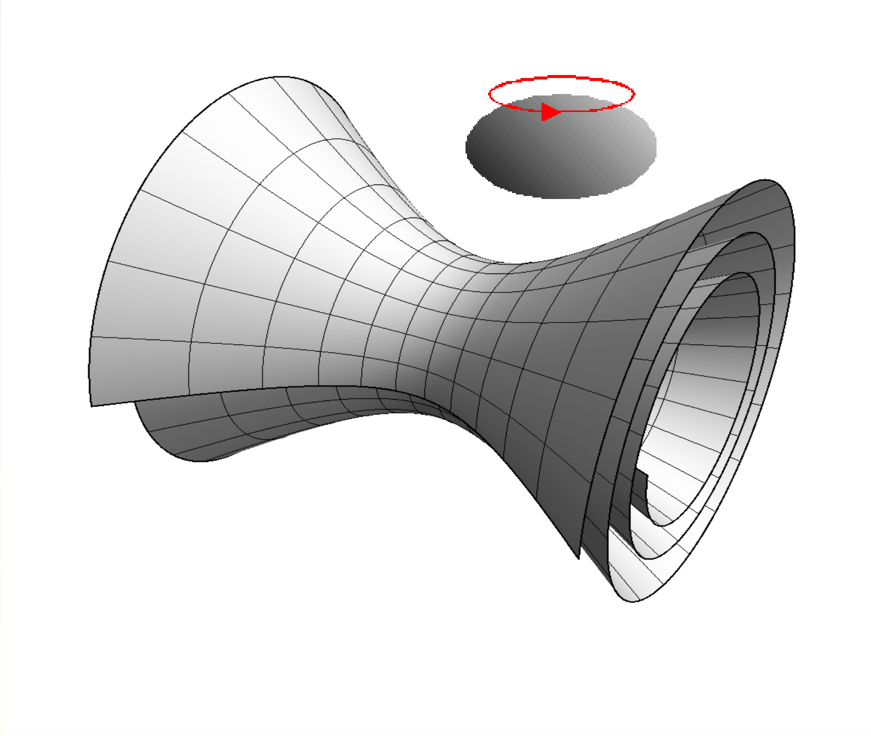
since $[P] = L^{-2}, \quad [A] = L^2, \quad [M] = L$ ⇒

$$M = 2A\left(\frac{\partial M}{\partial A}\right) - 2P\left(\frac{\partial M}{\partial \Lambda}\right)$$ + $$dM = \kappa dA + V dP$$

Smarr relation: $M = 2(TS - VP)$

Mass plays the role of **enthalpy** rather than internal energy

III) AdS Black Holes: overview of interesting features

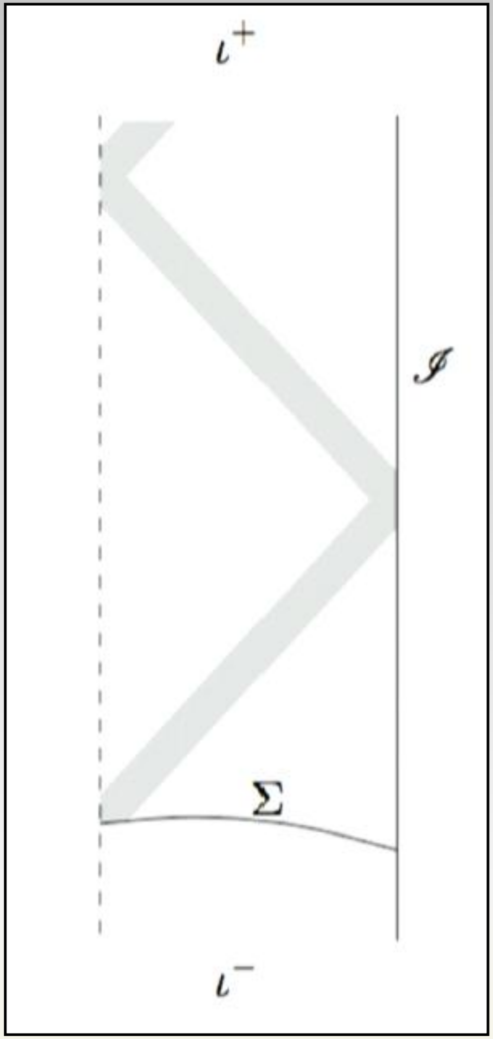


a) Global AdS and its (in)stability

Global AdS: (vacuum with Λ<0)

$$ds^2 = -\left(1 + \frac{r^2}{l^2}\right) dt^2 + \frac{dr^2}{1 + \frac{r^2}{l^2}} + r^2 d\Omega^2$$

AdS acts like a confining box due to timelike conformal boundary (nonlinearities do not decay)



a) Global AdS and its (in)stability

Global AdS: (vacuum with Λ<0)

$$ds^2 = -\left(1+\frac{r^2}{l^2}\right)dt^2 + \frac{dr^2}{1+\frac{r^2}{l^2}} + r^2 d\Omega^2$$

Conjectured to be **nonlinearly unstable** (Dafermos, Anderson -2006)

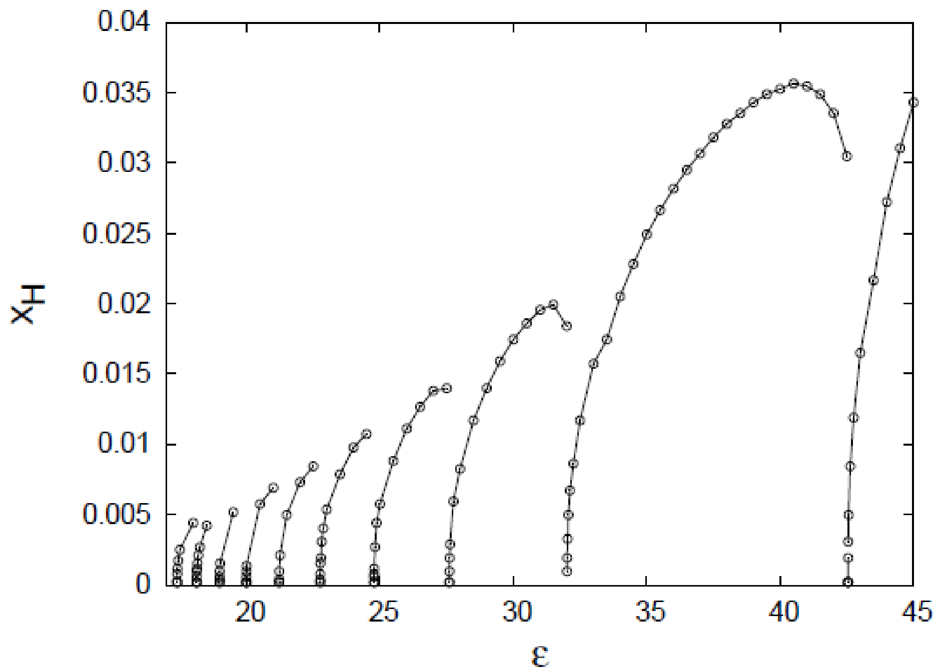
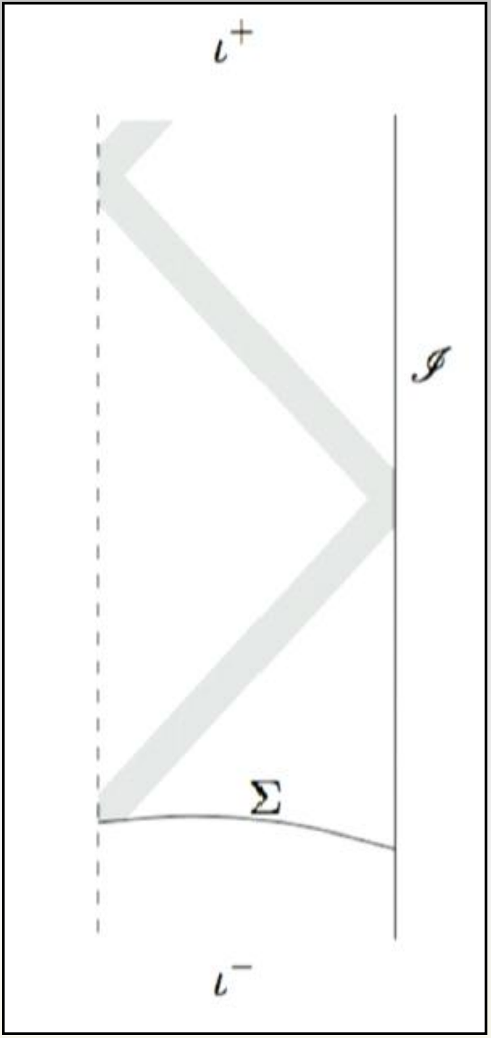


FIG. 1: Horizon radius vs amplitude for initial data (9). The number of reflections off the AdS boundary before collapse varies from zero to nine (from right to left).



Bizon and Rostworowski, *On weakly turbulent instability of anti-de Sitter space*, Phys. Rev. Lett. 107, 031102 (2011).

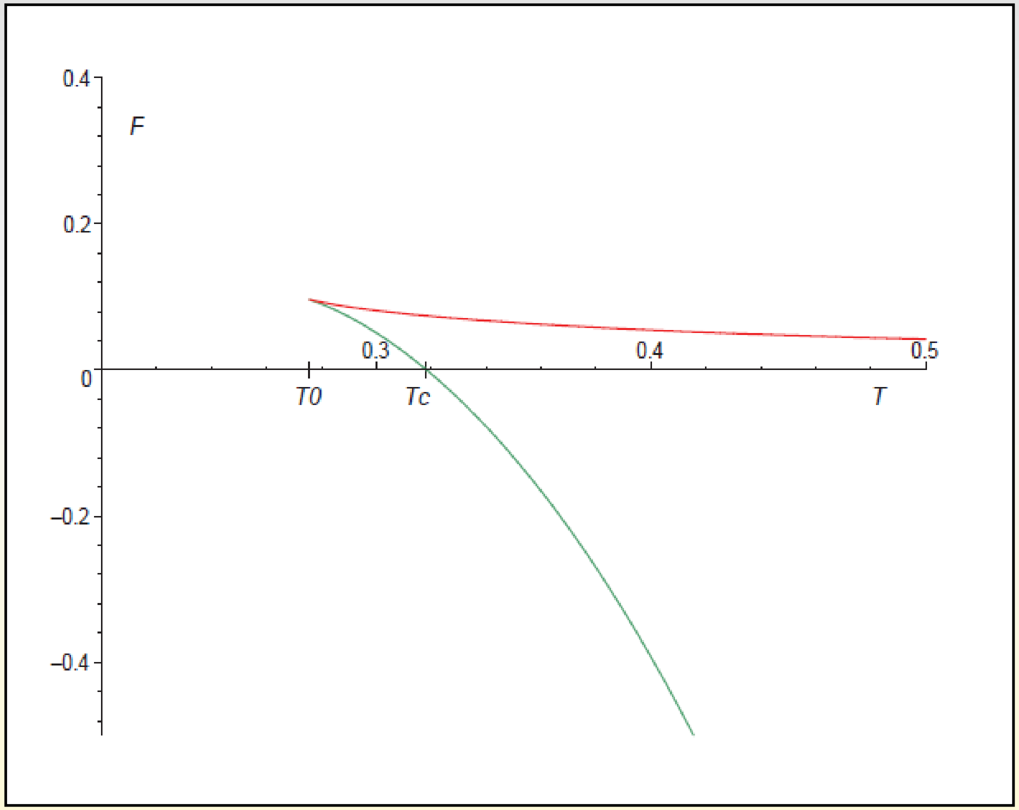
b) Hawking-Page transition

S.W. Hawking & D.N. Page, *Thermodynamics of black holes in anti-de-Sitter space*, Commun. Math. Phys. 87, 577 (1983).

Schwarzschild-AdS black hole:

$$ds^2 = V d\tau^2 + \frac{dr^2}{V} + r^2 d\Omega^2\,, \quad V = 1 - \frac{2M}{r} + \frac{r^2}{l^2}$$

AF black holes evaporate by Hawking radiation. AdS has constant negative curvature which acts like a confining box, there are static black holes in thermal equilibrium.



• Black holes have minimal temperature T=T0~1/l. For T<T0 gas of particles in AdS.

• Large black holes have positive specific heat, equilibrium configuration is stable.

• There is a **1st order transition** between gas of particles and large black holes at Tc

• Witten (1998): phase **transition in dual CFT** (quark-gluon plasma)

c) Scalar field condensate: holographic superconductors

No-hair theorems: "All black holes" uniquely characterized by four asymptotic charges (mass, electric and magnetic charges, and angular momentum). All other information "disappears".

Wheeler in 70's **"Black holes have no hair"**

possible:

not possible:

AdS: holographic superconductors: need a black hole that has hair at low temperatures but no hair at low temperatures (Gubser 2008-charged scalar field in charged AdS BH)

H^3 Theory (Hartnoll, Herzog, Horowitz, "Building a Holographic Superconductor," Phys. Rev. Lett. 101, 031601, 2008)

d) Van der Waals and charged AdS BHs: analogy 1

- Chamblin, Emparan, Johnson, Myers, *Charged AdS black holes and catastrophic holography*, Phys.Rev. D60 (1999) 064018, [hep-th/9902170];
- Chamblin, Emparan, Johnson, Myers, *Holography, thermodynamics and fluctuations of charged AdS black holes*, Phys.Rev. D60 (1999) 104026, [hep-th/9904197].

Charged AdS black hole:

$$ds^2 = -V dt^2 + \frac{dr^2}{V} + r^2 d\Omega_2^2 \,,$$

$$F = dA \,, \quad A = -\frac{Q}{r} dt \,.$$

$$V = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}$$

characteristics: (temperature, electric potential, entropy)

$$T = \frac{1}{\beta} = \frac{V'(r_+)}{4\pi} = \frac{1}{4\pi r_+}\left(1 + \frac{3r_+^2}{l^2} - \frac{Q^2}{r_+^2}\right)$$

$$\Phi = \frac{Q}{r_+}$$

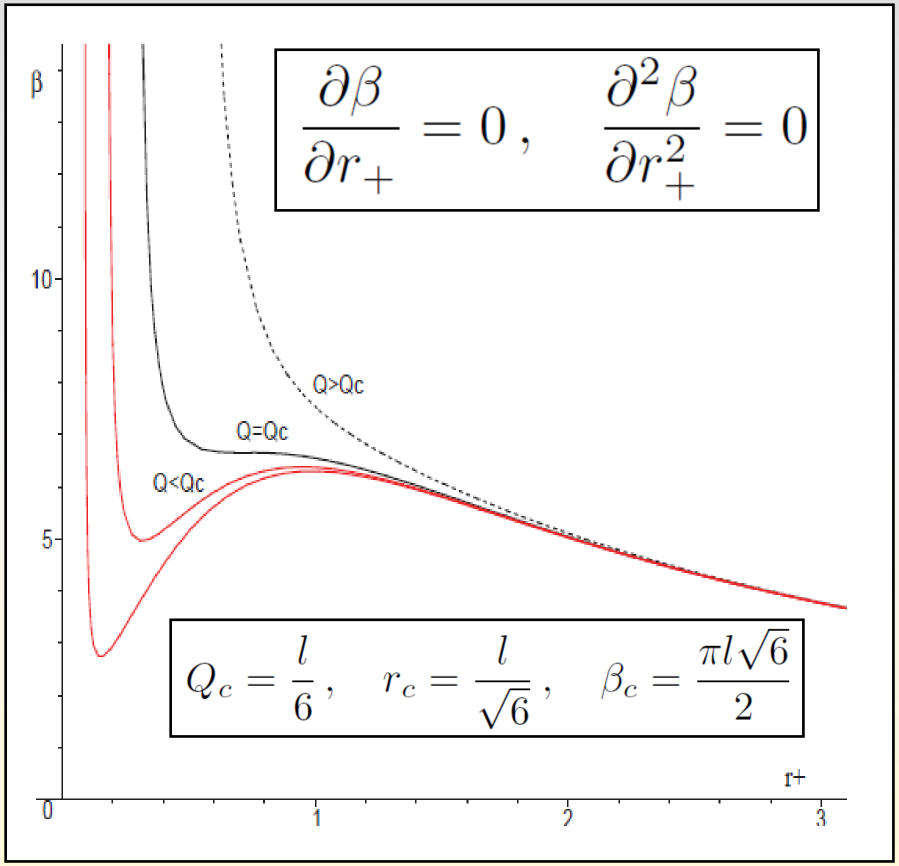
$$S = \frac{A}{4} \,, \quad A = 4\pi r_+^2$$

$$dM = TdS + \Phi dQ$$

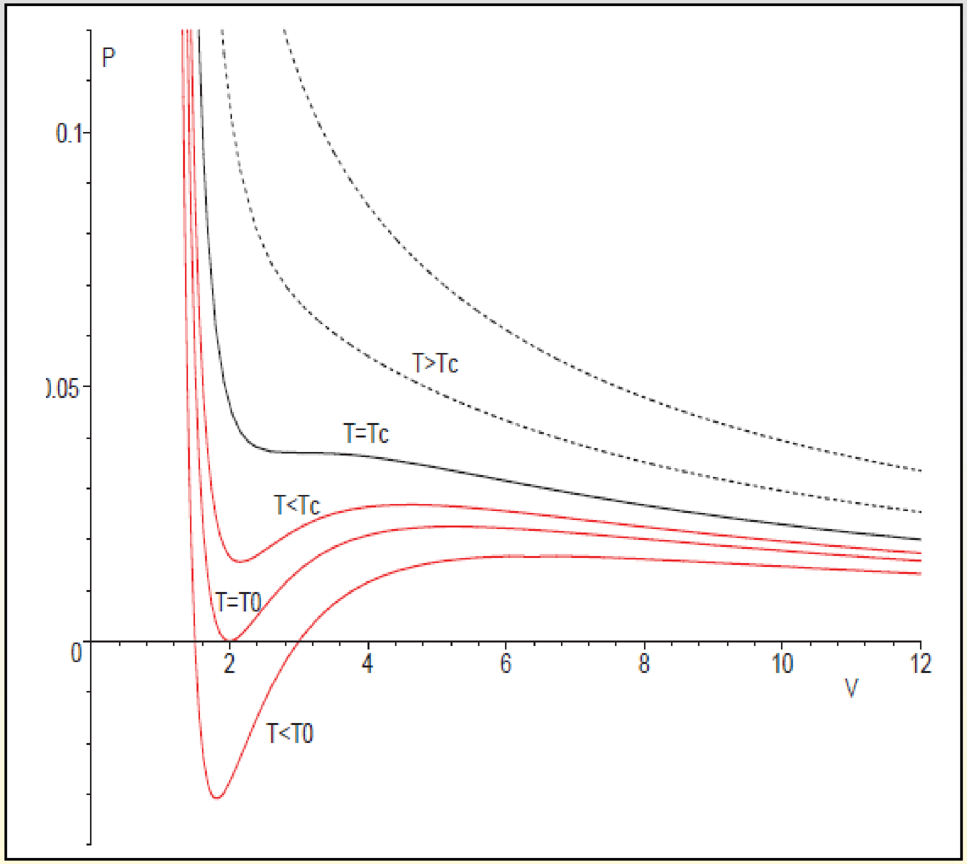
Equation of state:

$$\beta = \beta(r_+, Q) = \frac{4\pi r_+}{1 + \frac{3r_+^2}{l^2} - \frac{Q^2}{r_+^2}}$$

Analogy 1	
fluid	AdS black hole
temperature	Q
pressure	β
volume	r_+



vs.



Free energy: in canonical (fixed Q) ensemble
(calculated by Euclidean method)

$$F = F(Q,\beta) = \frac{1}{4l^2}\left(l^2 r_+ - r_+^3 + \frac{3l^2Q^2}{r_+}\right)$$

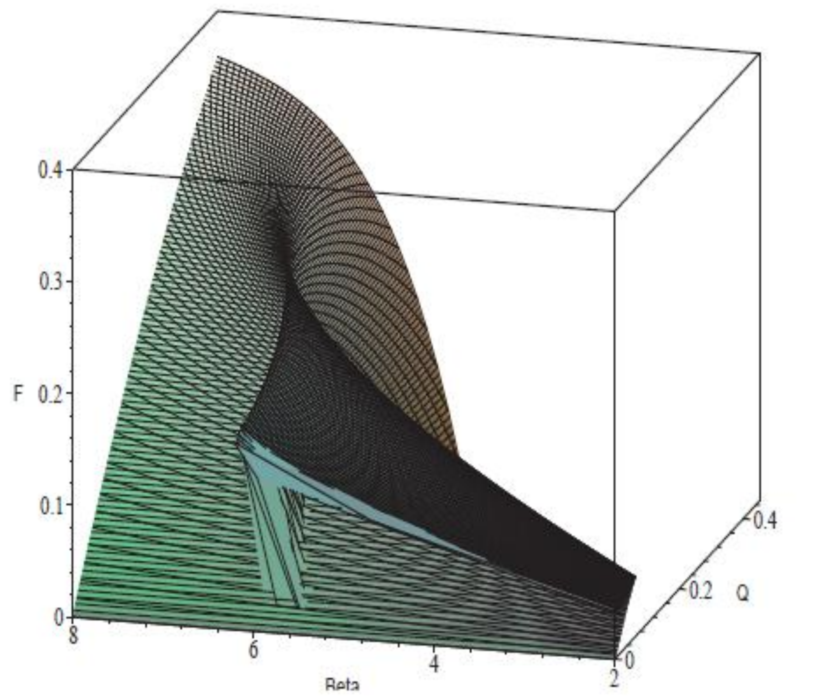


FIG. 10. **Thermodynamic potential** F. The thermodynamic potential F is depicted as a function of charge and inverse temperature for fixed $l = \sqrt{3}$. The characteristic swallowtail behaviour corresponds to the presence of the first-order phase transition in the canonical ensemble. Such features are absent in the grand canonical (fixed Φ) ensemble.

vs.

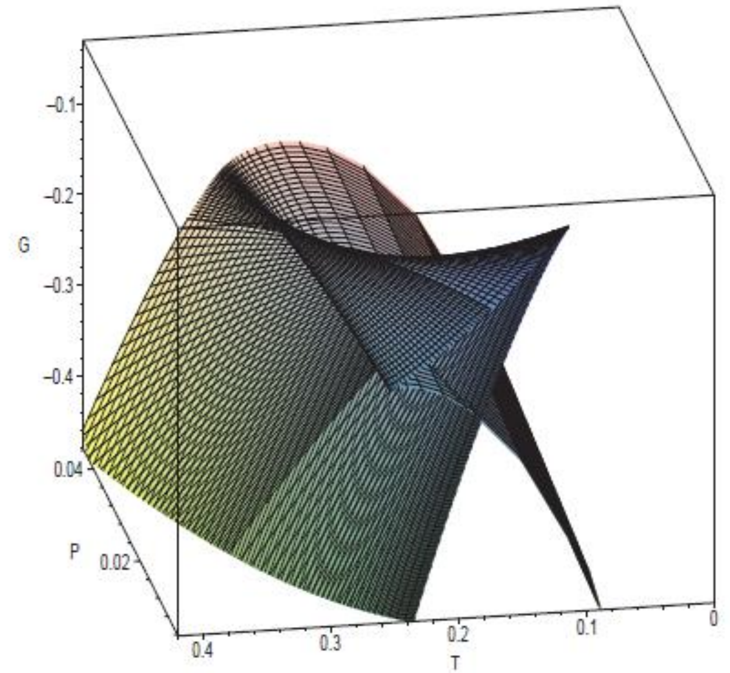
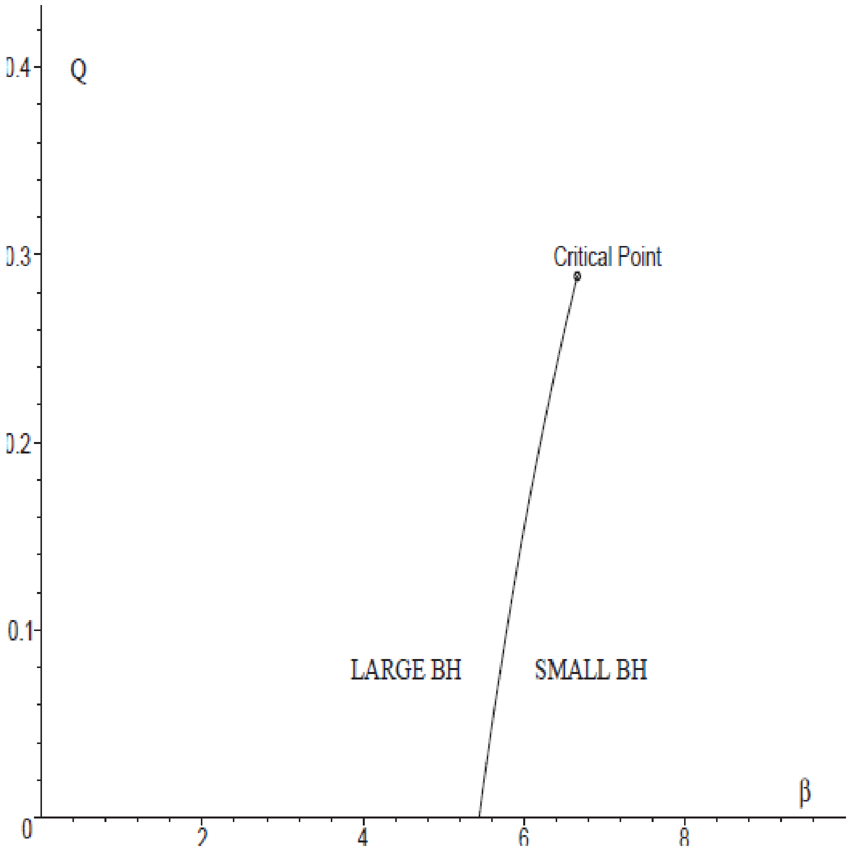


FIG. 3. **Gibbs free energy of Van der Waals fluid.** This picture shows the characteristic swallowtail behaviour of the Gibbs free energy as a function of pressure and temperature. This corresponds to a first-order liquid–gas phase transition which occurs at the intersection of G surfaces. The corresponding curve is called the coexistence line. We have set $\Phi = 1$.

Coexistence line



vs.

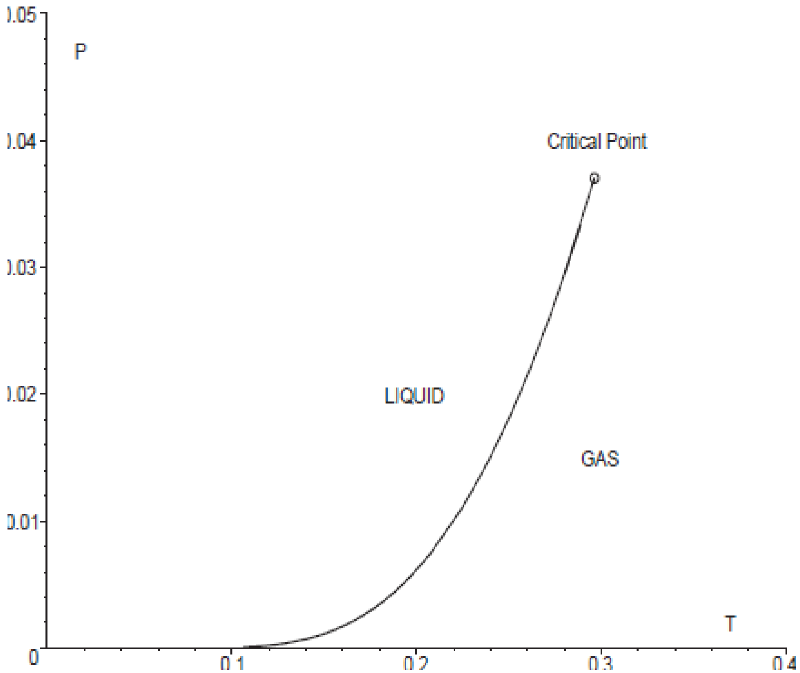
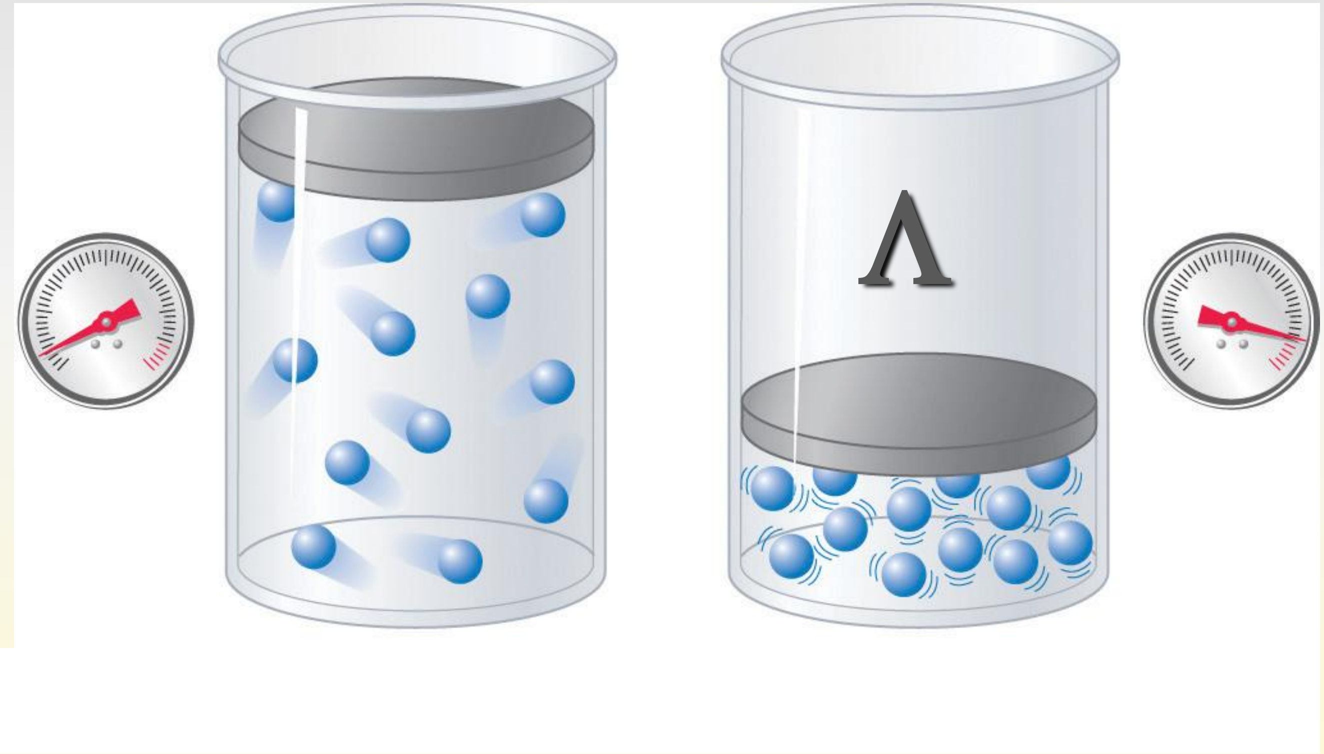


FIG. 5. **Coexistence line of liquid–gas phase.** Fig. displays the coexistence line of liquid and gas phases of the Van der Waals fluid in (P,T)-plane. The critical point is highlighted by a small circle at the end of the coexistence curve.

Critical exponents

$$\alpha = 0\,, \quad \beta = \frac{1}{2}\,, \quad \gamma = 1\,, \quad \delta = 3$$

IV) Λ as thermodynamic pressure & thermodynamic volume



a) Cosmological constant and its conjugate variable

Kastor, Ray, and Traschen, *Enthalpy and the Mechanics of AdS Black Holes*, Class. Quant. Grav. 26 (2009) 195011, [arXiv:0904.2765].

- **Identify**: the cosmological constant with a thermodynamic pressure

$$P = -\frac{1}{8\pi}\Lambda = \frac{3}{8\pi}\frac{1}{l^2}$$

and its conjugate quantity with the "**thermodynamic volume**" of the black hole

- **Calculate V**: using the extended first law (Smarr formula):

$$\delta M = T\delta S + V\delta P + \ldots$$

(mass interpreted as enthalpy rather than energy)

for example, for Schwarzschild: $V = \frac{4}{3}\pi r_+^3$

Some implications:

- **Effective theory** of non-constant Λ (inflation, quantum fluctuations)
- **Isoperimetric Inequalities** (analogue of Penrose inequalities, arXiv:1012.2888)

$$\mathcal{R} = \left(\frac{(d-1)\mathcal{V}}{\omega_{d-2}}\right)^{\frac{1}{d-1}} \left(\frac{\omega_{d-2}}{\mathcal{A}}\right)^{\frac{1}{d-2}}$$

$$\omega_d = \frac{2\pi^{\frac{d+1}{2}}}{\Gamma\left(\frac{d+1}{2}\right)}$$

Conjecture: for any AdS black hole $\mathcal{R} \geq 1$

"For a black hole of given thermodynamic volume V, the entropy is maximised for Schwarzschild-AdS"

- **Consistency** between first law and Smarr formula

Thermodynamic machinery

- **Study**: charged and rotating AdS black holes in a canonical (fixed Q or J) ensemble. **Not an analogy!** (compare same quantities)

- The corresponding thermodynamic potential is **Gibbs free energy**

$$G = M - TS = G(P, T, J_1, \ldots, J_N, Q)\,.$$

equilibrium state corresponds to the **global minimum** of G**.**

- **Local thermodynamic stability**: positivity of the specific heat

$$C_P \equiv C_{P,J_1,\ldots,J_N,Q} = T\left(\frac{\partial S}{\partial T}\right)_{P,J_1,\ldots,J_N,Q}$$

- **Phase diagrams**: P-T diagrams

- **Critical points:** calculate critical exponents,….

b) Van der Waals analogy now complete

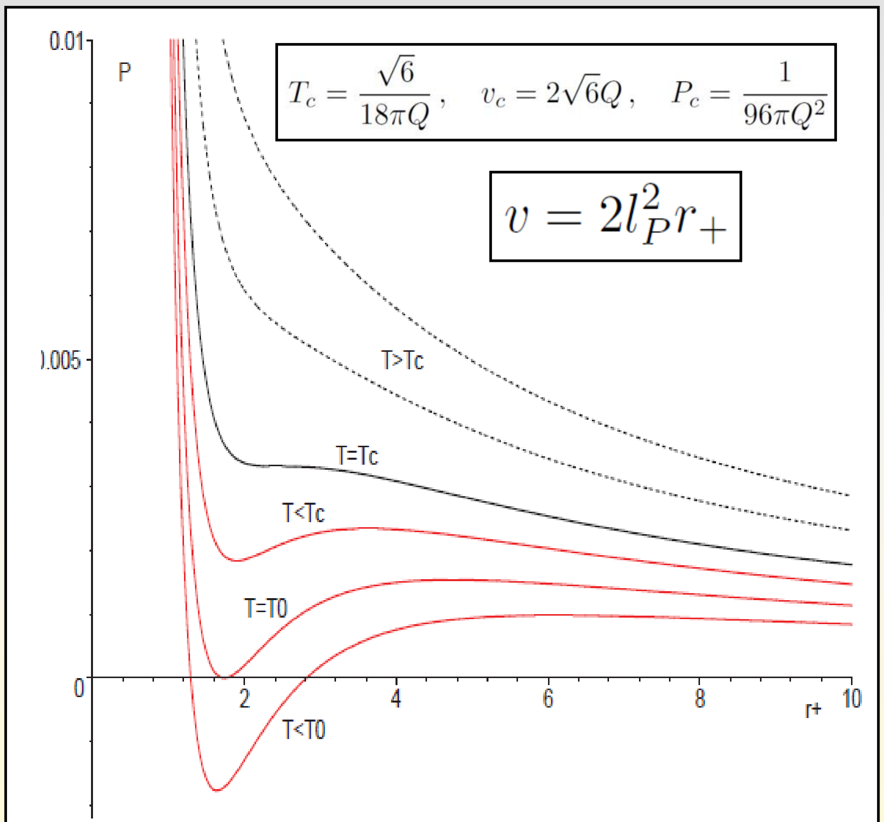
DK, R.B. Mann, *P-V criticality of charged AdS black holes*, JHEP 1207 (2012) 033;
S. Gunasekharan, DK, R.B. Mann, *JHEP 1211 (2012) 110.*

Equation of state:

$$T = \frac{1}{4\pi r_+}\left(1 + \frac{3r_+^2}{l^2} - \frac{Q^2}{r_+^2}\right)$$

translates to

$$P = \frac{T}{2r_+} - \frac{1}{8\pi r_+^2} + \frac{Q^2}{8\pi r_+^4}\,, \quad r_+ = \left(\frac{3V}{4\pi}\right)^{1/3}$$



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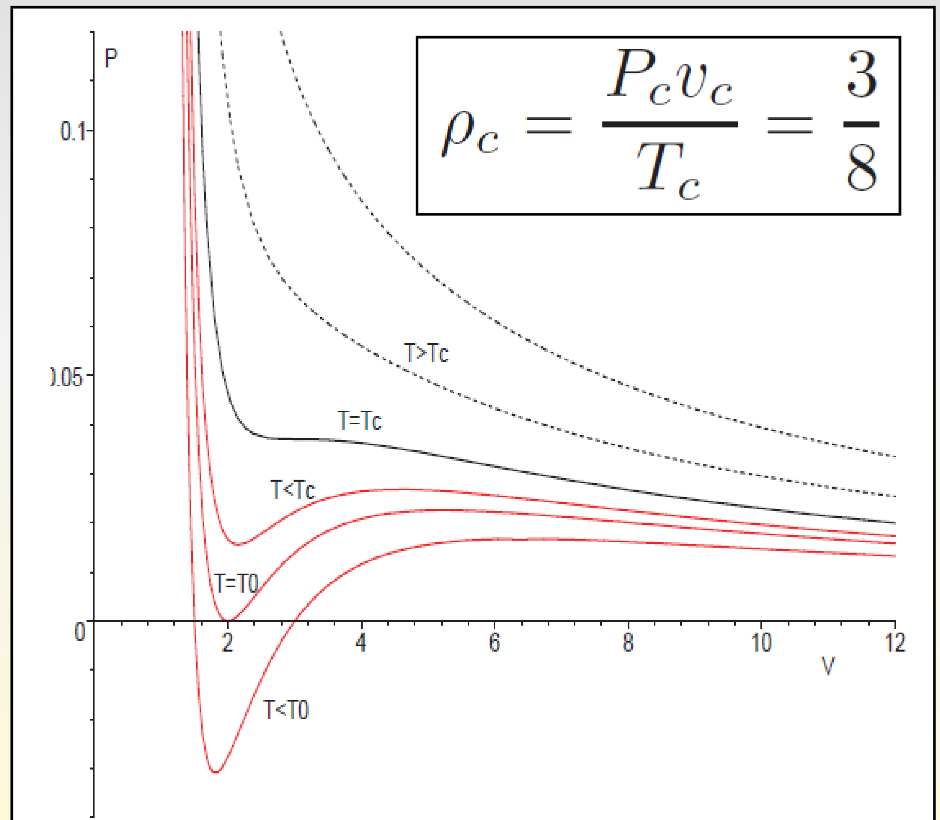


Gibbs free energy: in canonical (fixed Q) ensemble (calculated by Euclidean method)

$$G = G(T, P) = \frac{1}{4}\left(r_+ - \frac{8\pi}{3}P r_+^3 + \frac{3Q^2}{r_+}\right)$$

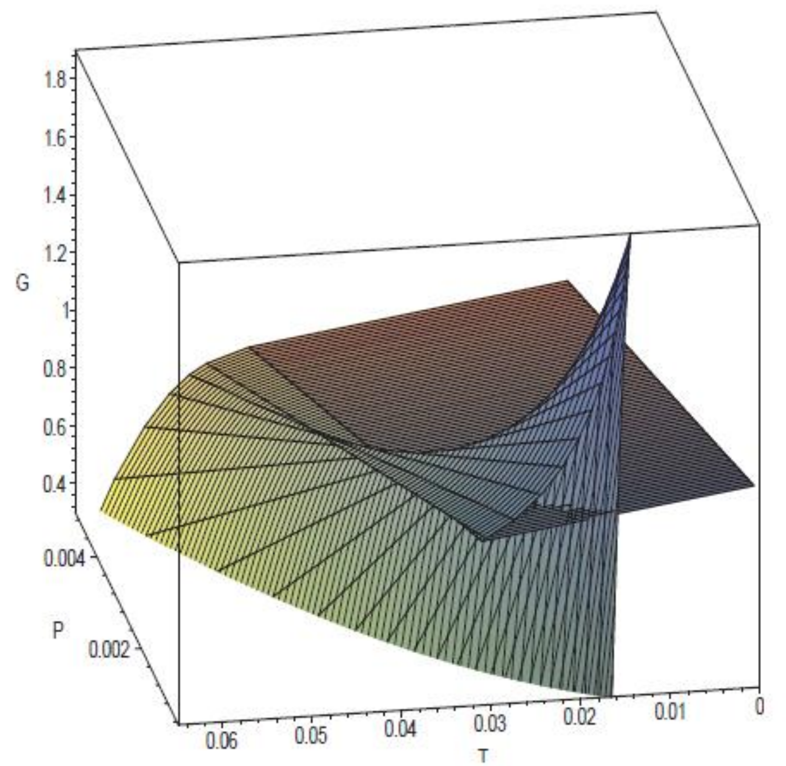
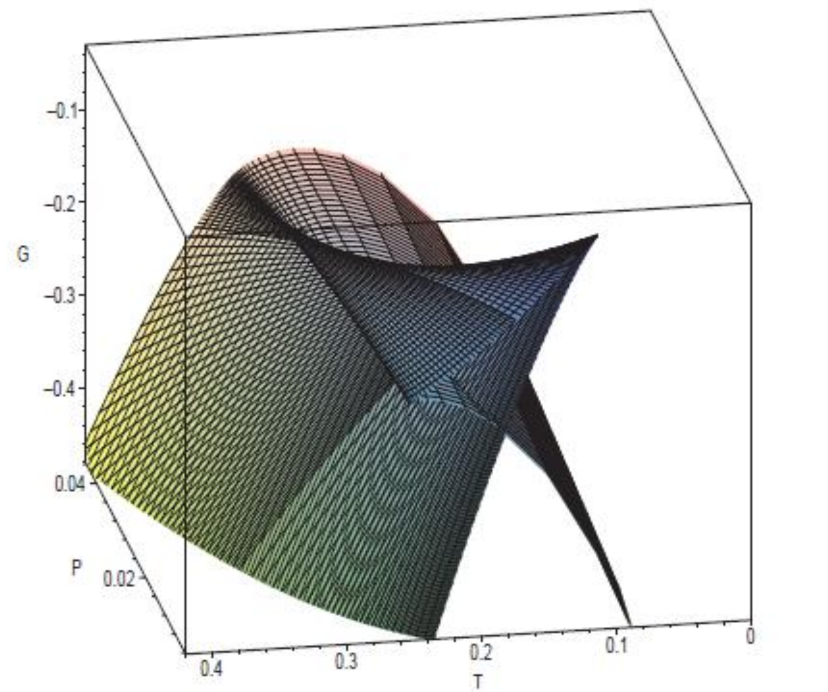


FIG. 7. **Gibbs free energy of charged AdS black holes.** Figure shows characteristic swallowtail behaviour of the Gibbs free energy as a function of pressure and temperature for a fixed $Q = 1$. The first-order small–large black hole transition occurs at a curve which is an intersection of G surfaces.

vs.



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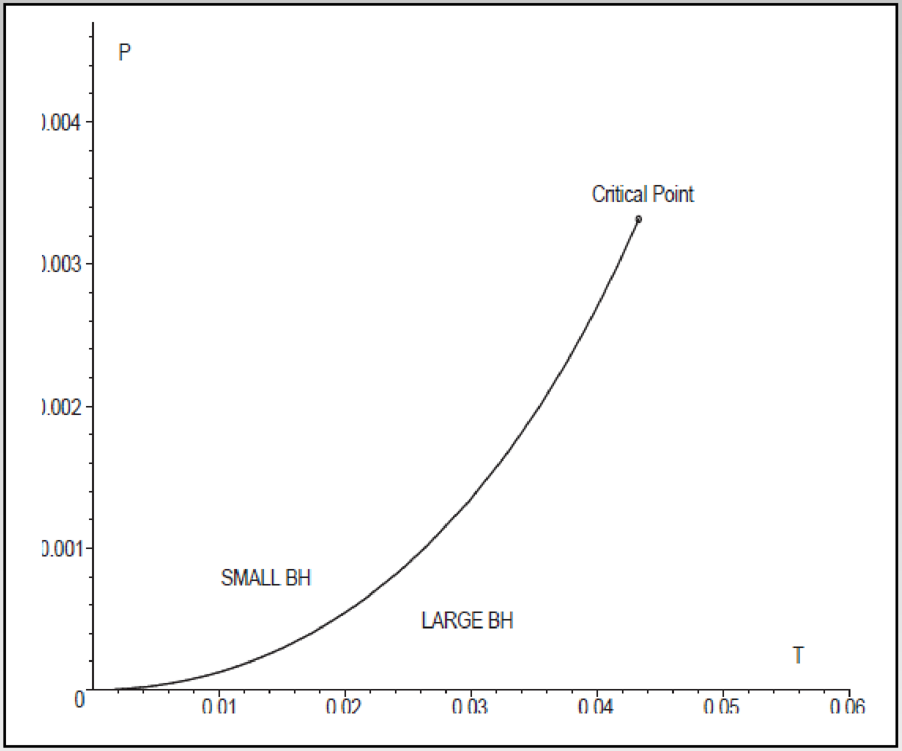
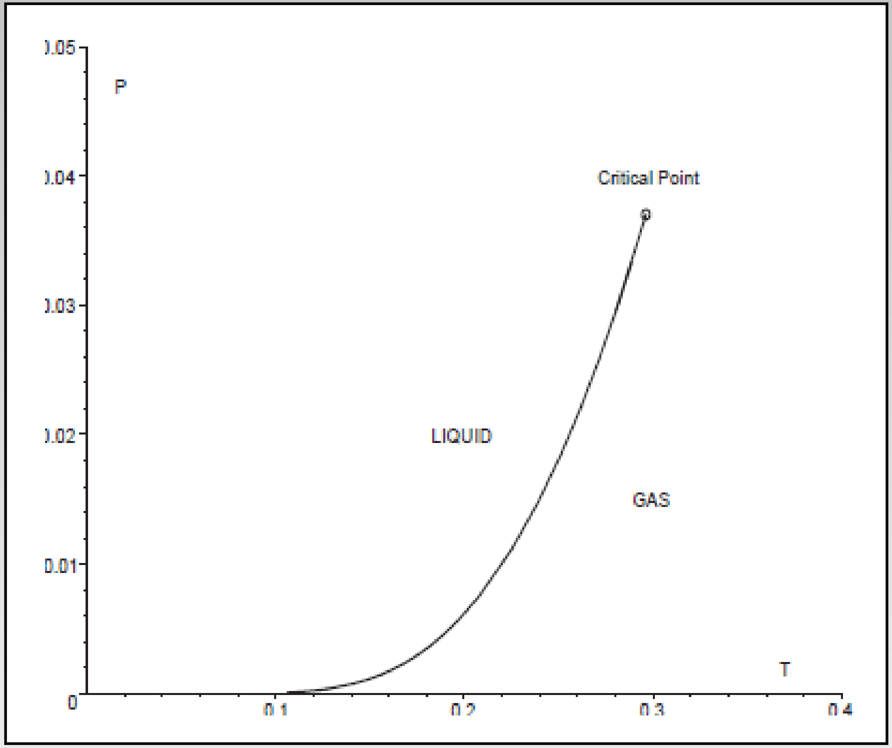
Coexistence line



vs.



Some remarks:

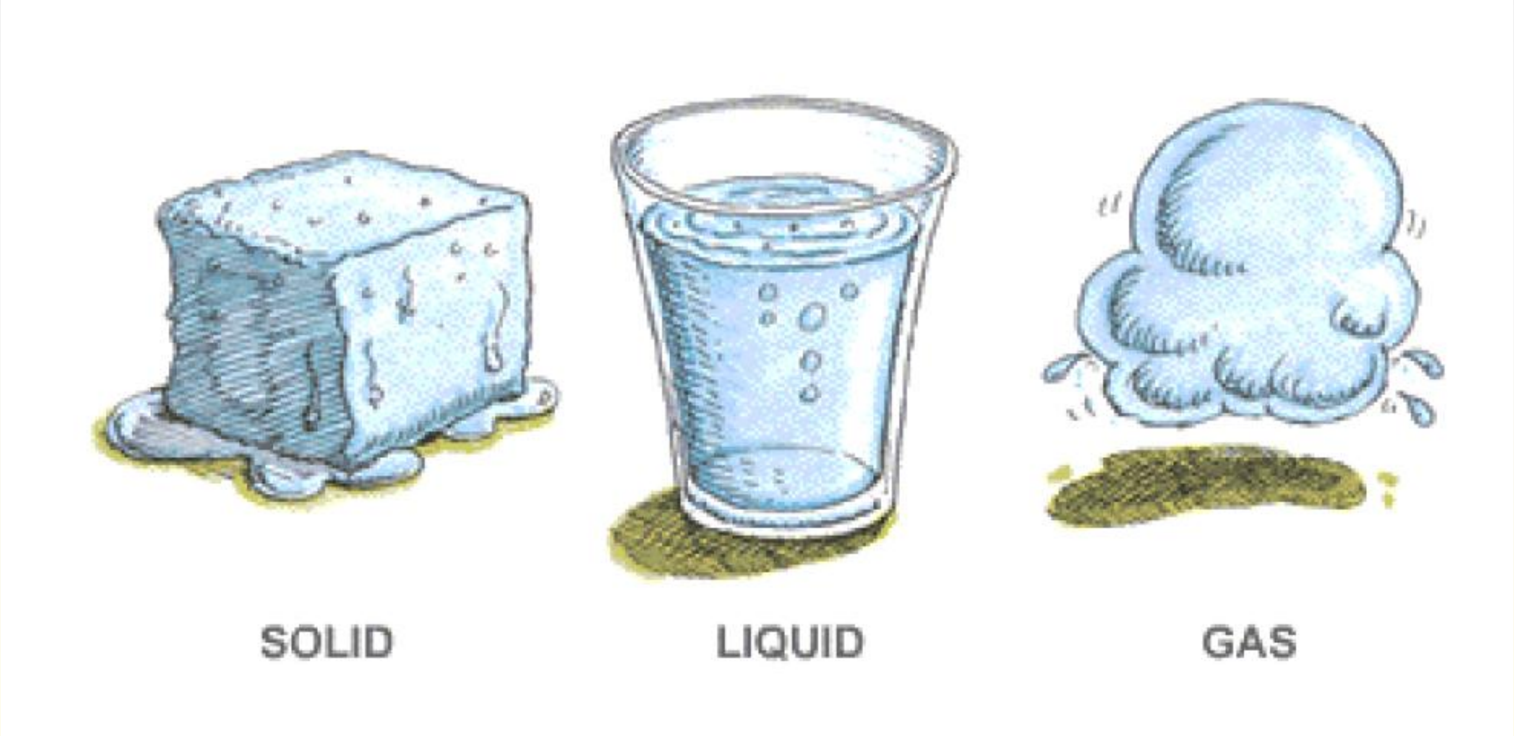
- **MFT critical exponents**

$$\alpha = 0\,, \quad \beta = \frac{1}{2}\,, \quad \gamma = 1\,, \quad \delta = 3$$

- Can do with rotation, in BI theory, in higher dimensions (no criticality in 3D), in Gauss-Bonnett gravity-all with the **same results**

- R. Emparan: geometrical description-effective FT, cannot go beyond MFT?

V) AdS analogue of "everyday thermodynamics of simple substances"



a) Reentrant phase transition

A system undergoes an RPT if a **monotonic** variation of any thermodynamic quantity results in two (or more) phase transitions such that the **final state is macroscopically similar** to the initial state.

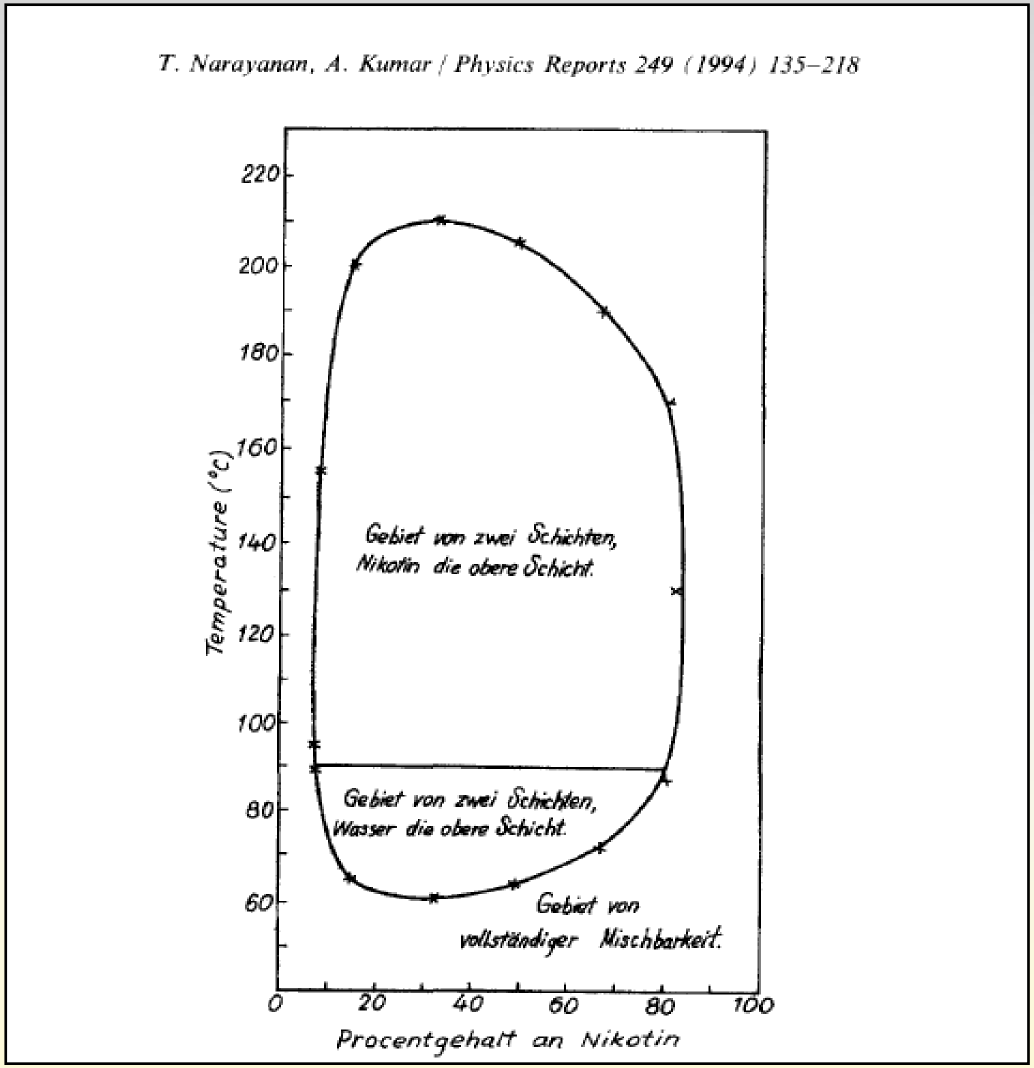
First observed by Hudson (1904) in a nicotine/water mixture

Z. Phys. Chem. 47 (1904) 113.

Since then:

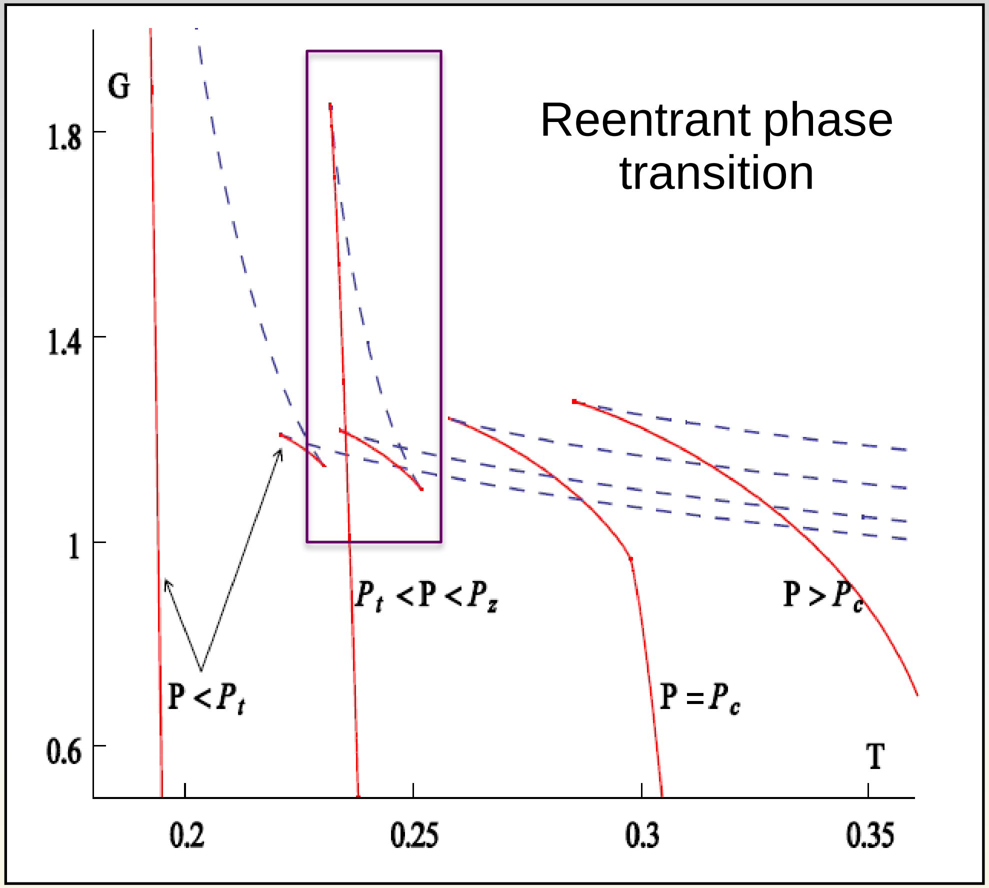
multicomponent fluid systems, gels, ferroelectrics, liquid crystals, and binary gases

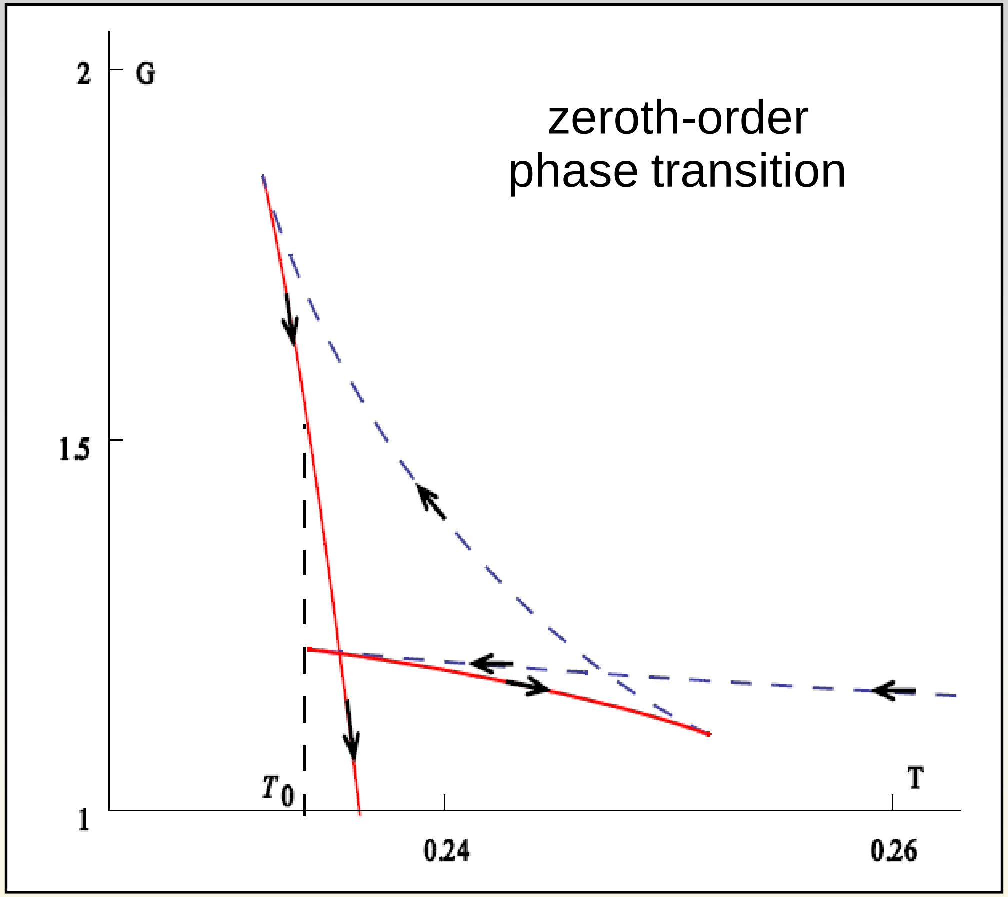
T. Narayanan and A. Kumar, Reentrant phase transitions in multicomponent liquid mixtures, Physics Reports 249 (1994) 135–218.



AdS analogue: large/small/large black hole phase transition in singly spinning Kerr-AdS BH in 6 dimensions

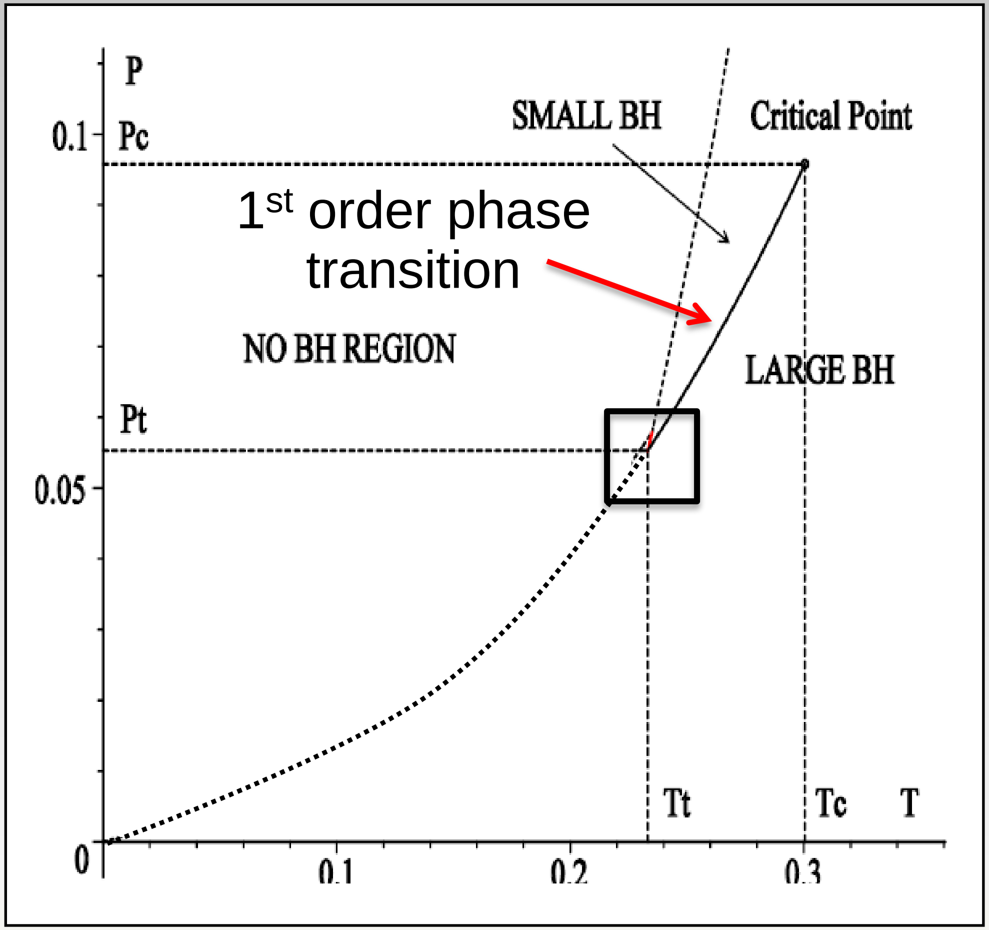
N.Altamirano, DK, R.B. Mann, *Reentrant phase transitions in rotating AdS black holes*, arXiv:1306.5756 (2013).

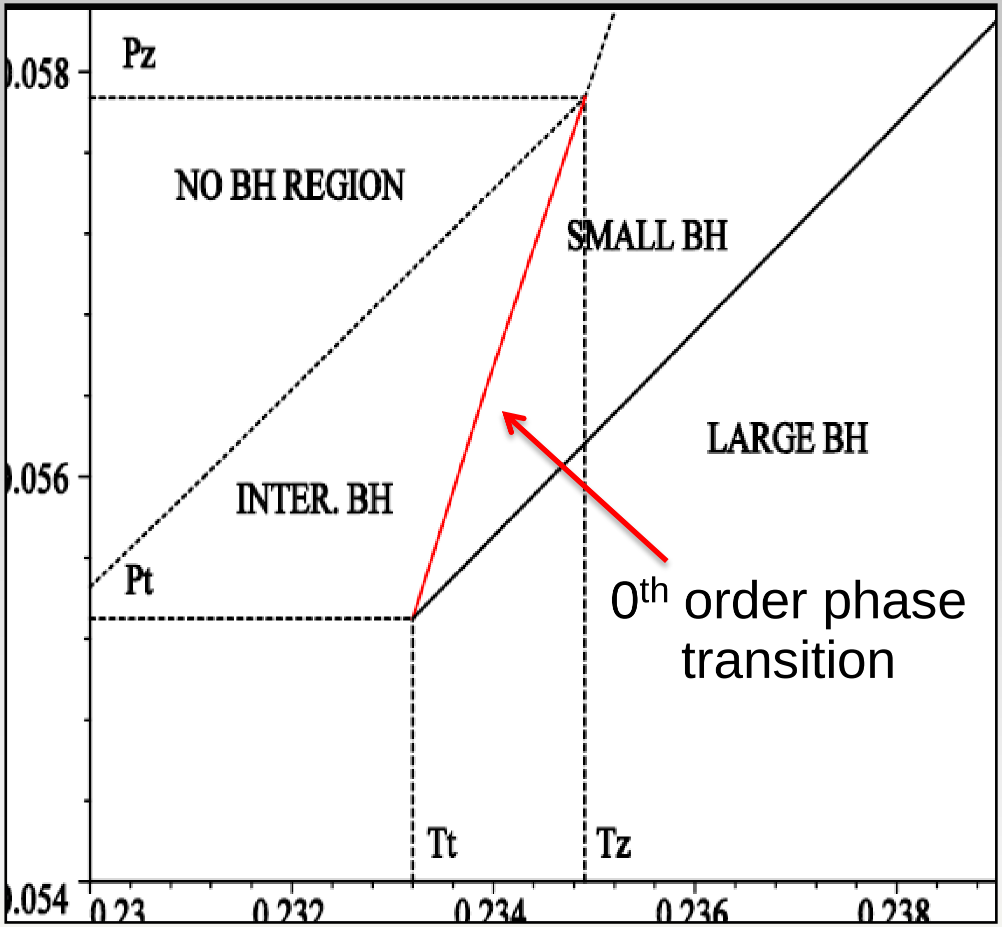




accompanied by a peculiar zeroth-order phase transition

P-T phase diagram

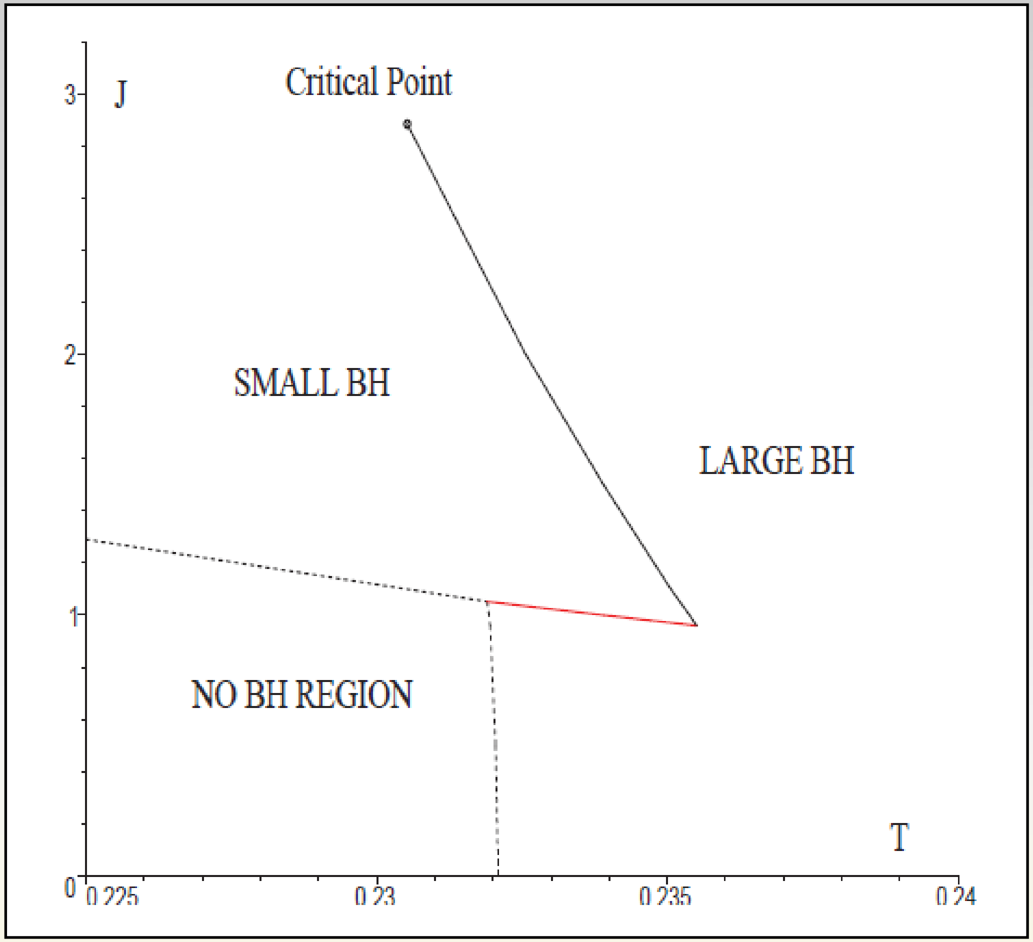




Low T	Medium T	High T
mixed	water/nicotine	mixed
intermediate BH	small BH	large BH

J-T phase diagram (analogy 1)

The discovered RPT does not require variable Λ!

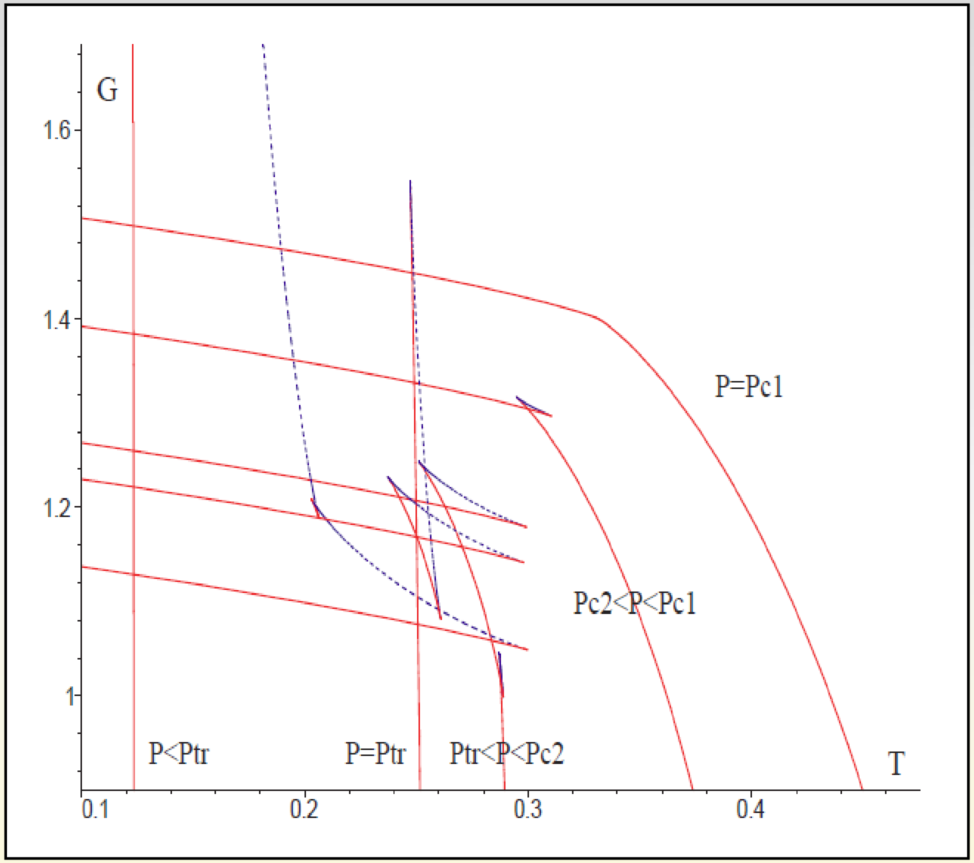


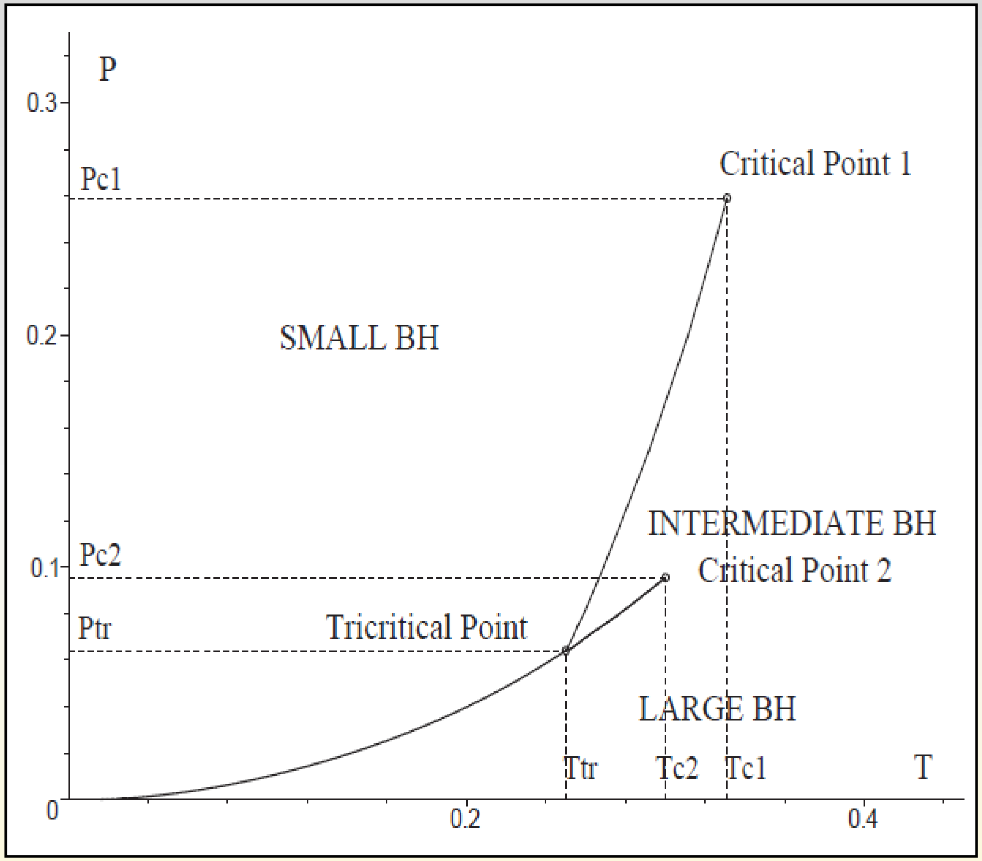
Occurs in any d>6: “two components”: BH vs. Black brane?

b) Triple point and solid/liquid/gas analogue

large/small/large black hole phase transition and a triple point in multiply spinning Kerr-AdS BH in 6 dimensions with certain ratio q of the two angular momenta.

N.Altamirano, DK, R.B. Mann, Z. Sherkatghanad, *Kerr-Ads analogue of tricritical point and solid/liquid/gas phase transition,* arXiv:1308.2672 (2013).





Conclusions

1) Thermodynamics is a **governing principle**, black holes are not an exception!
2) There are **4 interesting features** regarding the (asymptotically) AdS spacetimes: nonlinear instability of pure AdS, Hawking-Page transition, hairy black holes, Van der Waals behaviour and other "everyday phenomena"
3) Recently people have been playing with the idea of identifying the **cosmological constant with the dynamical pressure**. This gives a way of defining the volume of black holes.
4) **VdW analogy then complete!**
5) One also gains other **useful properties** (Isoperimetric inequalities, consistency with the Smarr relation, compressibility,....?).
6) Can also be extended to **dS black hole spacetimes** (arXiv:1301.5926).
7) Is there an interpretation in **AdS/CFT correspondence**? Is it possible to go **beyond MFT**?