What Does Data Tell Us About the Theory of Inflation?

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Introduction

Holographic Space-time-HST
A General Framework for Slow Roll and HST
Generic Predictions
The Tensor 3 Pt. Function (I Wish)
Dynamics of HST Inflation Model, if there’s time
HST: A Fully Quantum Model of Cosmology

- Universe evolves from a maximal entropy \( p = \rho \) state, to one in which different identical maximal entropy systems are coupled together in an \( SO(1, 4) \) invariant way, giving localized fluctuations.
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- Zero vorticity fluid so can be brought to co-moving gauge.
General Properties of Slow Roll and HST

- In co-moving gauge, gauge invariant fluctuations around approximately dS flat FRW are encoded in the metric.

\[ h_{ij} = a^2(t)[(1 + 2\zeta)\delta_{ij} + \gamma_{ij}], \]

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- Fluctuations of \( \delta H \) and \( \gamma \), can be computed as expectation values

\[ \langle \zeta_1 \ldots \zeta_n \gamma_1 \ldots \gamma_k \rangle = \text{Tr} \left[ \rho S_1 \ldots S_n T_1 \ldots T_k \right]. \]

\( S \) and \( T \) are commuting operator valued functions on the 3-sphere, transforming in representations of \( SO(1,4) \). \( \rho \) is approximately invariant density matrix.
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- True in slow roll (Maldacena) as well as HST.
General Properties

- Maldacena’s squeezed limit theorem says 3-point functions involving zero momentum scalar are smaller by a slow roll factor. $SO(1, 4)$ invariance determines general form of all 3 point functions (Maldacena, Pimentel, McFadden, Skenderis, Trivedi, Mata, Raju, Shiraishi, Nitta, Yokoyama, Garriga, Vilenkin, Soda, Kodama, Nozawa, ) so any 3 pt function involving scalars too small to be measured.
Contrasts Between Slow Roll and HST

- In slow roll, $S$ and $T$ are modeled as free fields in adiabatic Bunch Davies vacuum of FRW space-time, $H(t)$. Normalization of two point functions fixed $\propto \left( \frac{H(t)}{m_P} \right)^2$. $SO(1,4)$ reps. picked at extreme limits of rep. theory. Normalization of $k$–point functions $\sim \left( \frac{H(t)}{m_P} \right)^k$, but coefficients depend on terms in QUEFT. Choice of Bunch-Davies is massive fine tuning of initial conditions.
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Representation theory of $T$ the same in HST as slow roll, but $S$ can be anything in scalar complementary series of unitary irreps, $\Delta_S \in [0, 3/2]$. Density matrix plausibly invariant for generic initial conditions, but not Bunch Davies. Order of magnitude normalizations $\sim (\frac{H}{m_P})^k$ where $\pi \frac{M_P^4}{H^2}$ is the entropy of the individual microsystems that are combined to make the $SO(1,4)$ invariant theories. Notice that, unlike the slow roll case, this factor is time independent and won’t contribute to the tilt. In particular, the tensor tilt is predicted to be zero in HST models.
Contrast Between Slow Roll and HST

- The scalar tilt gets contributions from the geometric pre-factor $\frac{H^2}{H}$, as well as from $\Delta S$ if $\Delta S \neq 0$. 
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- The scalar tilt gets contributions from the geometric pre-factor $\frac{H^2}{H}$, as well as from $\Delta_S$ if $\Delta_S \neq 0$.
- Unfortunately, we can choose different geometries $H_{\text{slow roll}}(t)$, $H_{\text{HST}}(t)$ which can partially mask these differences. Only the absence of tensor tilt differentiates between the two models decisively. But this is, as yet, unmeasured.
The Tensor Three Point Function

- Three functional forms allowed by $SO(1, 4)$, one violates parity. Lowest order derivative expansion gives only one of these. Nominally, the second should be down by $(H/m_P)$ because it comes from higher order terms in the derivative expansion of the bulk QUEFT.
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- Unfortunately, the prospect of measuring the tensor 3-point function seems remote.
Conclusions

- The existing data are compatible with a huge class of models, obeying only a few symmetry postulates and general rules of cosmological perturbation theory.
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- The existing data are compatible with a huge class of models, obeying only a few symmetry postulates and general rules of cosmological perturbation theory.
- Differences between Slow Roll and HST models, which are of a deep conceptual nature, can be masked by the flexibility of choosing the background FRW in both frameworks.
- Absence of tensor tilt, the only clear signal for HST in two point functions. Tensor 3 pt. function has crucial information, but good luck in measuring it this century :-).
Dynamics of HST Cosmology

- An HST cosmology is an infinite collection of quantum systems, labeled by a lattice on the initial value surface of a Big Bang universe. Nearest neighbors represent time-like trajectories, which maintain a space-like separation of one Planck unit throughout history. The rest of the geometry is determined by dynamical overlap conditions: at any time the intersection of the causal diamonds on two trajectories, will contain a maximal size causal diamond, corresponding to a Hilbert space of dimension $e^{\frac{A}{4L_P^2}}$. Dynamics in proper time of each trajectory must give a compatible density matrix in the overlap.
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- There exists a complete model obeying these rules, which has a coarse grained space-time interpretation as a flat FRW model with an equation of state that is the sum of a $p = \rho$ component and a cosmological constant. At late times, the local model has a time independent Hamiltonian, on a Hilbert space of finite dimension, $e^{\pi \left(\frac{M_P}{H}\right)^2}$. 

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- Model of inflation takes $e^{3N_e}$ of these uncorrelated systems and couples them together gradually, starting with a “central observer” and moving out. The Hamiltonian is built to become the generator of $SO(1, 4)$ in the limit $N_e$ goes to infinity. Density matrix of the whole system plausibly becomes invariant. Corrections $e^{-N_e}$ for correlation functions of a small number of observables.

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By the rules of HST, horizon radius at the end of inflation is $e^{\frac{3N_e}{2}H^{-1}}$. This must have enough entropy to account for entropy in fluctuations of CMB $\sim \delta TT^2 R^3$. This gives about 56 e-folds for unification scale inflation, which is what is indicated by the size of fluctuations. On the other hand $N_e \leq \frac{2}{3}\ln (RH)$ where $R$ is our cosmological horizon. So $56 \leq N_e \leq 88$ in HST models. Probably on low end to account for post-inflationary expansion of the horizon.
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We don’t understand particle physics in HST language well enough to build an HST model of reheating, radiation and matter dominated eras.
Meta-Cosmological Problems

- HST inflation model has two parameters $n, N$: essentially the inflationary and the final Hubble radii, in Planck units. We chose them to fit data. How does nature choose them?

- There is a fully consistent HST model in which a ball of radius $R$, evolves from $p = \rho$ to dS with radius $R = NL_P$, while the region outside it remains in the $p = \rho$ model.

- Jacobsonian geometry is black hole with dS interior embedded in $p = \rho$ FRW.

- Einstein's equations (the hydrodynamics of space-time) have solutions with many widely separated black holes, with various radii, $N_i$, and relative initial velocities. Black holes may or may not collide.

- IF our inflationary model exists, we can supply each of these black holes with its own inflationary $1 \ll n_i \ll N_i$. Thus, we can have environmental selection of “good” values of ($n, N$) for our universe.
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- In HST $N$ is connected to SUSY breaking via $m_{3/2} \sim 10N^{-1/2}M_P$. Changing c.c. drastically affects both strong and weak interactions, and physics of nuclei, atoms, stars. $N$ plausibly determined by constraint that particle physics more or less as it is.
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- Given this, Weinberg’s bound becomes relatively strong constraint on $n$: $n < C\left(\frac{\rho_0}{\Lambda}\right)^{1/3}$, where $\rho_0$ is the dark matter density at the beginning of matter domination, and is plausibly fixed once $N$ is fixed. Given the observed values, and the a priori constraint $n \gg 1$, this is pretty close to the observed value.

- By a not particularly tuned choice of initial distribution of positions and velocities of black holes in the meta-model, we can arrange that our own little universe lives as long as we observe it to but a much shorter time than a Boltzmann brain recurrence time, before colliding with another island universe. The collision leads to drastic thermalization, followed by a much lower value of the c.c., for which no brains can survive.
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