Observing the Dimensionality of Our Parent Vacuum

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with

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Inspiration

Why is the universe 3 dimensional?

What is the overall shape and structure of the universe?
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Do we have a landscape of vacua, extra dimensions?

Does this explain properties of our universe (e.g. the Cosmological Constant)?

Did we have a period of eternal inflation in our past?

Is our universe a vast, inhomogeneous multiverse?
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How will we know?
Outline

1. Motivation

2. The Anisotropic Universe

3. Observables
Lower Dimensions

Lower dimensional vacua seem generic

Even SM has a “landscape” of lower dimensional vacua

More ways to compactify more dimensions

We assume our universe came from a lower dimensional vacuum

Possibly all dimensions began compact?

Dimensions tend to decompactify

Arkani-Hamed et. al. (2008)

Brandenberger & Vafa (1989)

Giddings & Myers (2004)
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More ways to compactify more dimensions  

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Possibly all dimensions began compact?  
Dimensions tend to decompactify  

Creates large initial anisotropy, diluted by slow-roll inflation

We will look for residual signs of this special direction
Landscape Signals

For signals to be observable, inflation must not have lasted too long.

Inflation needs tuning. Few e-folds may be generic.

Many landscape signals require this

e.g. curvature, bubble collisions

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- e.g. curvature, bubble collisions


These also assume other vacua are 3+1 dimensional

What if we relax this assumption?

These signals could reveal our history of decompactification

(see also Blanco-Pillado and Salem, 2010)
The Anisotropic Universe
Initial Transition

If the parent vacuum is 2+1 dimensional

Coleman - De Luccia tunneling creates a bubble of 3+1 dimensional space

Could be radion tunneling (or change in fluxes, etc.)
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spatial dimensions = $\mathbb{R}^2 \times S^1$
Initial Transition

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Could be radion tunneling (or change in fluxes, etc.)

spatial dimensions = $\mathbb{R}^2 \times S^1$

creates an infinite, open FRW universe, \textit{in 2 dimensions}

negative curvature only in 2 dimensions, third dimension flat
Initial Transition

Alternatively, if the parent vacuum is 1+1 dimensional:

The single uncompactified dimension is flat

Other two may be any compact 2-manifold with geometry $S^2$, $E^2$, or $H^2$

Generic compactifications have large curvature
Initial Transition

Alternatively, if the parent vacuum is $1+1$ dimensional:

The single uncompactified dimension is flat

Other two may be any compact 2-manifold with geometry $S^2$, $E^2$, or $H^2$

Generic compactifications have large curvature

We won’t consider the $0+1$ dimensional case.

In general, expect anisotropic curvature after transition
We assume after the transition:

\[ ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\phi^2 \right) - b(t)^2 dz^2 \]

\( k = \pm 1 \)
After the Transition

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“FRW” equations:

\[ \frac{\dot{a}^2}{a^2} + 2\frac{\dot{a}}{a} \frac{\dot{b}}{b} + \frac{k}{a^2} = 8\pi G\rho \]

\[ \frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a} \frac{\dot{b}}{b} = -8\pi G\rho_r \]

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z-dimension is flat ⇒ anisotropic curvature

anisotropic pressure
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“FRW” equations:

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\begin{align*}
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\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a} \frac{\dot{b}}{b} &= -8\pi G p_r \\
2 \frac{\dddot{a}}{a} + \frac{\dddot{a}}{a^2} + \frac{k}{a^2} &= -8\pi G p_z
\end{align*}
\]

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normal FRW eqn

anisotropc curvature \( \Rightarrow \) anisotropic expansion: \( H_a \equiv \frac{\dot{a}}{a} \neq H_b \equiv \frac{\dot{b}}{b} \)
Assume immediately after the tunneling $\dot{b} \approx 0$

$$a(t) \sim t \left(1 + O \left(G \Lambda t^2\right) \right)$$

$$b(t) \sim b_0 \left(1 + O \left(G \Lambda t^2\right) \right)$$

so $b(t)$ is frozen $H_b \approx 0$

but $H_a$ is large
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$a(t)$ will expand, diluting curvature until $t^2 \sim G \Lambda$ when $\Omega_k < \Omega_\Lambda$

slow-roll inflation takes over and drives all dimensions to expand $H_b \rightarrow H_a$
Evolution of the Anisotropic Universe

normal FRW eqn: \[ 2\dot{H}_a + 3H^2_a + \frac{k}{a^2} = -8\pi G p_z \]

our universe is approximately isotropic: \[ \Delta H \equiv H_a - H_b \ll H \quad \text{and} \quad \Omega_k \ll 1 \]
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\[ \frac{d}{dt} \Delta H + 3H_a \Delta H + \frac{k}{a^2} = 8\pi G (p_r - p_z) \]
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thermal equilibrium \(\Rightarrow\) isotropic pressure \[\frac{\Delta p}{p} \sim \frac{\Delta H}{H}\]
and during MD pressure is small enough
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eqn for \(b(t)\): \[ 2\dot{H}_b + 3H_b^2 - \frac{k}{a^2} = -8\pi G p \]

\(a(t)\) expands normally, \(b(t)\) expands as if curvature was opposite sign
Return of Curvature

\[
\frac{d}{dt} \Delta H + 3H_a \Delta H + \frac{k}{a^2} = 0 \quad \Omega_k = \frac{k}{a^2 H^2}
\]

inhomogeneous solutions are:

Inflation \quad \frac{\Delta H}{H_a} = -\Omega_k

RD \quad \frac{\Delta H}{H_a} = -\frac{1}{3} \Omega_k

MD \quad \frac{\Delta H}{H_a} = -\frac{2}{5} \Omega_k

homogeneous solutions sourced at every transition but die off quickly
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\~ 1 \ in curvature dominance

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\~ e^{-120} \ after inflation

**MD**
\[ \frac{\Delta H}{H_a} = -\frac{2}{5} \Omega_k \]
\~ 10^{-5} \ at recombination
\~ 10^{-2} \ today

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inhomogeneous solutions are:

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  \]
  \~ 10^{-5} \quad \text{at recombination}

  \~ 10^{-2} \quad \text{today}

homogeneous solutions sourced at every transition but die off quickly

we need the full solutions during MD:

\[
a(t) \propto t^{\frac{2}{3}} \left(1 - \frac{\Omega_k}{5}\right)
\]

\[
b(t) \propto t^{\frac{2}{3}} \left(1 + \frac{\Omega_k}{5}\right)
\]
Observables
Measuring Curvature

Sound horizon at recombination provides a “standard ruler”
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Curvature is measured by observing angular size of ruler ~ 1°, a high-l observable
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Sound horizon at recombination provides a “standard ruler”

Curvature is measured by observing angular size of ruler $\sim 1^\circ$, a high-l observable

What does anisotropic curvature look like?
Standard Rulers

\[ ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\phi^2 \right) - b(t)^2 dz^2 \]

universe roughly flat before recombination ⇒ rulers are fixed physical length \( ds \)
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transform to locally flat frame
\( \Rightarrow \) observable angle is

\[ \tan(\theta) = \left( \frac{a(t)}{b(t)} \frac{dr}{dz} \right) + O \left( \frac{1 \text{ m}}{28 \text{ Gpc}} \right) \]
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\[ \Delta \theta \left( 1 + \frac{1}{5} \Omega_k \right) \]

\[ \Delta \theta \left( 1 + \frac{3}{5} \Omega_k \right) \]

\[ \Delta \theta \left( 1 - \frac{1}{5} \Omega_k \right) \]

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Should be easier to measure than isotropic curvature
Effect of Geometric Warp
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CMB Flux today =

\[ \Phi_0 = \frac{dN_0}{d\Omega_0 dA_0 dt_0 dE_0} \]
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Anisotropic Curvature:
1. Non-sphericity of LSS
2. Bending of photon path
3. Angle dependent redshift

Late time effect acts on all multipoles
CMB Flux today =
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\[
\Phi_0 (E_0, \theta_0) = \frac{E_0^2}{\exp \left( \frac{E_0}{T_{\text{LSS}}(\theta_0 + \delta\theta)} (1 + \Omega_{k0} Y_{20} (\theta_0)) \right) - 1}
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#1 #2 #3
The Quadrupole

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\[ T = \bar{T} + \sum_{lm} a_{lm} Y_{lm} \]
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contribution to CMB quadrupole anisotropy:

\[ a_{20} \approx -\frac{8}{15} \sqrt{\frac{\pi}{5}} \Omega_{k_0} \overline{T} \]

tuning ⇒ likely range \( \sim 10^{-4} \gtrsim \Omega_{k_0} \gtrsim 10^{-5} \) \( \sim \) cosmic variance

low-l multipoles have high cosmic variance

local ISW effect may raise quadrupole

Francis & Peacock (2009), WMAP7 (2010)
Angular Correlations

\[ a_{lm} = \text{size of temperature fluctuation in } Y_{lm} \text{ mode} \]

statistical isotropy \( \Rightarrow \) \[ \langle a_{l_1 m_1} a_{l_2 m_2}^* \rangle = 0 \]

except \[ \langle a_{lm} a_{lm}^* \rangle \sim C_l \]
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statistical isotropy \(\Rightarrow\) \(\langle a_{l_1 m_1} a_{l_2 m_2}^{*} \rangle = 0\)

except

\(\langle a_{lm} a_{lm}^{*} \rangle \sim C_l \left(1 + \Omega_{k_0} \# l m\right)\)

\(\langle a_{lm} a_{l-2,m}^{*} \rangle \sim \Omega_{k_0} \left(C_{l-2} \# l m + C_l \# l m\right)\)
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a good measure of anisotropy:

\[
A_{ll'}^{LM} = \sum_{mm'} \langle a_{lm} a_{l' m'}^* \rangle (-1)^{m'} C_{l,m,l',-m'}^{LM} = 0 \text{ for isotropic}
\]
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\( \langle a_{lm} a^*_{l-2,m} \rangle \sim \Omega_{k_0} (C_{l-2} \#_{lm} + C_l \#_{lm}) \)

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anisotropic curvature gives:

\[
A_{ll}^{20} \sim \Omega_{k_0} C_l \sqrt{l}
\]

\[
A_{l,l-2}^{20} \sim \Omega_{k_0} (l (C_l - C_{l-2}) + C_l) \sqrt{l}
\]

These are our high-l observables - low cosmic variance
WMAP Anomaly

WMAP sees only two nonzero: $A_{ll}^{20}$ and $A_{l,l-2}^{20}$

More precision than isotropic curvature, no degeneracy with scale factor expansion history

possibly due to instrumental systematics

Planck should improve measurement
Is it just another anisotropy?
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Symmetries of Bubble Nucleation $\Rightarrow$ Specific initial geometry

$$ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\phi^2 \right) - b(t)^2 dz^2$$
Is it just another anisotropy?

Symmetries of Bubble Nucleation => Specific initial geometry

\[ ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\phi^2 \right) - b(t)^2 dz^2 \]

Power in just one linearly independent harmonic e.g. \( Y_{20} \)
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Power in just one linearly independent harmonic e.g. $Y_{20}$

Expect power in all harmonics for generic anisotropy.

Symmetries valid in thin wall regime.
Thick wall?
Signals of Compact Topology

Eternal inflation seems to imply space is infinite

But we’re led to finite, compact topology in at least one dimension
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Observe matched circles in the sky

Current limit = 24 Gpc may get to ~ 28 Gpc diameter of our universe

Cornish, Spergel & Starkman (1996)
Signals of Compact Topology

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2+1 dimensional parent: curvature and topology are in different directions
1+1 dimensional: same directions

Cornish, Spergel & Starkman (1996)
Other Measurements

CMB is a snapshot - only 2 dimensional information

3D info can directly distinguish anisotropy from inhomogeneity

21 cm and galaxy surveys

21 cm can observe curvature to $\Omega_k \sim 10^{-4}$
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Quadrupole from anisotropy generates correlated E-mode polarization.

Anisotropic curvature also causes differential Hubble expansion $\Delta H \sim \Omega_k H$

Visible directly in Hubble measurements

Current limits $\sim$ few %

May improve to $< 10^{-2}$ with e.g. GW sirens

Schutz (2001)
Conclusions + Future Directions

• Have high-l, low cosmic variance, observables of dimension changing transitions
  • Due to late time effect of anisotropic curvature
  • Not statistical predictions, though provide evidence for landscape/eternal inflation

• Can test an observation of curvature for isotropy
  • Anisotropy implies lower dimensional parent vacuum
  • Isotropy is evidence for 3+1 dimensional parent vacuum

• Interesting to explore dimension changing transitions
  • Other observables, e.g. bubble collisions, gravitational waves?
  • Does the landscape provide a reason for 3+1 dimensions?