Magnetic inflation: realizing natural inflation on a steep potential

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5 INTERESTING FACTS ABOUT THE UNIVERSE

- it is old and very large
- in first approximation it is homogeneous and isotropic
- it is approximately flat
- structure grew out of small, scale invariant perturbations
- spectrum of primordial perturbations was gaussian
All these facts can be explained by

**INFLATION**

:= period of accelerated expansion in the very early Universe

\( a = \) scale factor of the Universe. Obeys

\[
H^2 = \frac{8\pi G}{3} \frac{\rho}{\rho_\text{dilu}}
\]

\[
H = \frac{\dot{a}}{a}
\]

during inflation require \( H \sim \text{constant} \)

(not so easy, since \( \rho \) dilutes away for ordinary matter...)
How to get some “slowly diluting” matter?

✓ very early Universe filled by scalar field $\phi$, potential $V(\phi)>0$

✓ to induce acceleration, $V(\phi)$ must be flat

$|V'(\phi)| \ll V(\phi)/M_P$

✓ to have long enough inflation, $V(\phi)$ must stay flat for long enough

$|V''(\phi)| \ll V(\phi)/M_P^2$
Simple way of realizing $|V'(\phi)| < |V(\phi)/M_P|$, $|V''(\phi)| < |V(\phi)/M_P^2|$: monomial potential, with $\phi$ large enough.

Most famous example: quadratic potential (chaotic inflation) 

$Linde 1983$

$V(\phi) = \frac{m^2 \phi^2}{2}$

Amplitude of perturbations produced during inflation $\Rightarrow m \sim 10^{13} \text{ GeV}$
...but, in general, quantum loops will contribute to $V'$ and $V''$ (and $V'''$ etc...)}
Radiative corrections can disrupt the inflationary potential in two ways

1- affect the functional form of $V(\phi)$

2- affect value of the parameters that appear in $V(\phi)$

Chaotic inflation example

1- adds terms $\propto \phi^n$, $n=4, 6, ...$

2- push $m$ to larger values (e.g. $M_P$ - cf EW hierarchy pbm)

How to make sure that radiative effects are under control?
The situation is actually not so horrible...

If we have a theory where $\phi$ interacts only with gravity then quantum corrections are not a problem!

Indeed: for potential $V(\phi)$, quantum gravity effects are

$$\mathcal{O}(1) \frac{V(\phi)^2}{M_P^4}$$

and

$$\mathcal{O}(1) \frac{V''(\phi) V(\phi)}{M_P^2}$$

negligible during inflation

however, in general there will coupling to other fields

reheating
How to make sure that radiative effects are under control?

A very well-known system that contains “controllably small” quantities is the **Standard Model**: “small” quantities are protected against radiative effects by *symmetries*

If a model has a symmetry, quantum effects cannot violate it (unless the symmetry is anomalous...)

If the symmetry is broken, quantum effects cannot make the breaking much larger (ie the breaking parameter is controllably small)
A field $\phi$ has a *shift symmetry* if the theory that describes it is invariant under the transformation

$$\phi \rightarrow \phi + c$$

($c=$ arbitrary constant)

If this symmetry is exact, the only possible potential for $\phi$ is $V(\phi)=$constant

(i.e. a cosmological constant)

*an exact shift symmetry is an overkill...*  
...but we can break the symmetry a bit and generate a potential
An (important) example

If $\phi$ is a phase, then shift symmetry $\Leftrightarrow$ global U(1)

- Theory with a spontaneously broken global U(1)

$$\mathcal{L} = \partial_\mu H^* \partial^\mu H - \lambda (|H|^2 - v^2)^2$$

- Decompose $H = (v + \delta H) e^{i\phi/v}$

  where $\delta H$ is massive and $\phi$ is a massless Goldstone boson (pseudoscalar)

- The global U(1) is broken e.g. by gravitational instantons

  $$\delta \mathcal{L} = e^{-S} M_P^3 (H + H^*) + \ldots$$

  ($S = \text{instanton action, } \propto M_P^n$)

- A potential is generated:

  $$\delta V \sim e^{-S} M_P^3 v \cos (\phi/v)$$
Natural inflation

$V(\phi) = \mu^4 \left[ \cos(\phi/f) + 1 \right]$
Because of its radiative stability,

A pNGB gives an extremely well motivated model of inflation from the point of view of effective field theory.
What about data?

from Savage et al, 2006

\[ f > 3.5 \, M_P \]
Stringy models of natural inflation?

**YES, in principle**
(string theory contains a plethora of pNGBs)

**However**

String Theory appears to require $f < M_p$

$n$-instanton actions contribute $\propto e^{-(n M_p / f)} \cos(n \phi / f)$ to pNGB potential

↓

first $f / M_p$ harmonics in $V(\phi)$ matter

Banks, Dine, Fox and Gorbatov 03
Ways out?

- Use two pNGBs

\[ V = \Lambda_1^4 \left[ 1 - \cos \left( \frac{\theta}{f_1} + \frac{\rho}{g_1} \right) \right] + \Lambda_2^4 \left[ 1 - \cos \left( \frac{\theta}{f_2} + \frac{\rho}{g_2} \right) \right] \]

- Use pNGBs and moduli

- Use many pNGBs

\[ \mathcal{L} = -\sqrt{-g} \sum_{i=1}^{N} \left\{ \frac{1}{2} (\partial \phi_i)^2 + \Lambda_i^4 [1 + \cos(\phi_i/f_i)] \right\} \]

- ...

...all based on multi field dynamics to generate a flat effective potential
A different way of approaching the problem...

The inflaton can be slowed down (even on a steep potential!) if it *dissipates* its kinetic energy.

e.g. particle production associated to motion of \( \phi \) rate depends on \( \phi \)
In the early ‘70s (pre-inflation), try to explain isotropy from initial anisotropy by particle production.

Today, chaotic inflation paradigm allows to ignore primordial anisotropy problem—but still need flat potential.

Particle production can help mitigate the requirement of flat potential.
Idea: field $\chi$ with mass $m_\chi(\phi(t))$

At some time $t_0$, $m_\chi(t_0) = 0$, with $m_\chi(t_0) \neq 0$.

$\Rightarrow$ Heisenberg inequality $\hbar \geq \Delta E \Delta t \sim m_\chi (m_\chi/m_\chi)$ violated

Concept of number of quanta of $\chi$ not well defined

Quanta of $\chi$ are produced
Particles created at expenses of inflaton kinetic energy (the only useful energy available)

Inflaton rolling is slowed down for \( \sim 1 \) efold

To get 60 efolds, need many production events

\[
\frac{1}{2} g^2 \sum_i (\phi - \phi_i)^2 \chi_i^2 .
\]

depending on parameters, 1 to \( 10^{12} \) events per efold are needed

this structure can be present in some stringy constructions

Chung et al 1999

Green et al 2009
A mechanism analogous to trapping is built in natural inflation

Idea: pNGB driving natural inflation is “naturally” coupled to gauge fields

\[ \mathcal{L}_\phi F_{\mu\nu} = \alpha \frac{\phi}{4 f} \epsilon_{\mu\nu\rho\lambda} F^{\mu\nu} F^{\rho\lambda} \]

\( \alpha = \text{dimensionless constant} \)
Equation for the $U(1)$ field in the presence of $\phi(t)$:

$$\frac{\partial^2 A_{\pm}}{\partial t^2} + \left( \frac{k^2}{a^2} \mp \frac{\alpha}{f} \frac{d\Phi}{dt} \frac{|k|}{a} \right) A_{\pm} = 0$$

$A_{\pm}$ = $>ve$ and $<ve$ helicity comoving modes of the vector potential

One of the two modes has a **negative, time dependent** “mass term”

**Exponential** amplification of one helicity mode
Equation for $A_\pm$ can be solved by assuming $\dot{\phi}$, $H=$constant

Modes with $k/a < \alpha \phi /f$
feel tachyonic mass until $k=aH$

amplification by

$\sim \exp[\alpha \phi /fH]$
more precisely...

\[ A_+ (\tau, \vec{k}) \simeq \frac{1}{\sqrt{2|\vec{k}|}} \left( \frac{|\vec{k}|}{2 \xi a H} \right)^{1/4} e^{-2 \sqrt{2 \xi |\vec{k}| / a H + \pi \xi}} \]

Exponential amplification term!

\[ \xi \equiv \frac{\alpha \dot{\phi}}{2 f H} \]
Slowing down the inflaton

backreaction equation

\[ \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = -\frac{\alpha}{f} \langle \vec{E} \cdot \vec{B} \rangle \]

with

\[ \langle \vec{E} \cdot \vec{B} \rangle \propto \exp\{\pi\alpha\phi/fH\} \]

As \( \phi \) starts increasing under the effect of the steep potential, the backreaction term gets important, slowing it down.

Slow roll equation⇒

\[ V'(\phi) \approx -\frac{\alpha}{f} \langle \vec{E} \cdot \vec{B} \rangle \]
the slow roll solution...

\[ \dot{\phi} \approx \frac{f H}{\pi \alpha} \log \left[ \frac{M_P^4 f V'(\phi)}{V(\phi)^2} \right] \]

...and a constraint on the model:

\[ \Delta \phi = \pi f \text{ from top to bottom of potential} \]

\[ \text{total # efoldings} \sim H \frac{\Delta \phi}{\phi} \sim \alpha \log \left[ \frac{M_P^4 f V'(\phi)}{V(\phi)^2} \right]^{-1} \]

\[ \alpha \gtrsim 100 \]
Consistency

We have assumed that the contribution to the electromagnetic modes to the Hubble parameter is negligible.

True?

YES!

\[ Q_{EM} \sim \vec{E}^2 + \vec{B}^2 \sim \vec{E} \cdot \vec{B} \sim fV'(\phi)/\alpha \]

\[ Q_{EM}/V(\phi) \sim fV'(\phi)/(\alpha V(\phi)) \]

negligible, for \( \alpha \gg 1 \), unless at the bottom of \( V(\phi) \)

Reheating?
...but... is it really inflation?

Need the slow roll parameters $\varepsilon$ and $\eta << 1$:

$$\varepsilon = \frac{\dot{\Phi}^2}{2H^2M_P^2} \approx \frac{2f^2}{\alpha^2M_P^3}$$

$$\eta = 2\varepsilon - \frac{f}{\pi \alpha} \left( \frac{V''(\Phi)}{V'(\Phi)} - 2 \frac{V'(\Phi)}{V(\Phi)} \right)$$

Bottom line - background evolution

For $\alpha > O(100)$, possible to get $\sim 60$ efolds of inflationary expansion
How to get such a large $\alpha$?

One example

Two axions in $E_8 \times E_8$

\[ L_{\text{axions}} = \frac{1}{2} (\partial_\mu a_1)^2 + \frac{1}{2} (\partial_\mu a_2)^2 \]

\[ - \frac{1}{2} \left( \frac{a_1}{M_1} + \frac{a_2}{M_2} \right) F_{\mu \nu}^i \tilde{F}_\mu^i \]

\[ - \frac{1}{2} \left( \frac{a_1}{M_1} - \frac{a_2}{M_2} \right) F_{\mu \nu}^i \tilde{F}_\mu^i \]

\[ - \frac{1}{2} (\bar{\theta}_\mu a_1)^2 + \frac{1}{2} (\bar{\theta}_\mu a_2')^2 - \frac{a'}{2M} \tilde{F} \]

\[ - \frac{a'}{2M} \left( \frac{M_1^2 + M_2^2}{M_1^2 - M_2^2} \frac{M_1^2 + M_2^2}{M_1^2 - M_2^2} \tilde{F} \right) \]

\[ a = \frac{M_1 a_1 + M_2 a_2}{(M_1^2 + M_2^2)^{1/2}} \]

\[ a' = \frac{M_2 a_1 - M_1 a_2}{(M_1^2 + M_2^2)^{1/2}} \]

\[ M = \frac{1}{2} (M_1^2 + M_2^2)^{1/2} \]

\[ M' = M_1 M_2 (M_1^2 + M_2^2)^{1/2} / (M_1^2 - M_2^2) \]

Choi and Kim 85
Equation for perturbations

\[ \ddot{\delta \phi} + 3H \dot{\delta \phi} + (\nabla^2 + V''(\phi)) \delta \phi = -\frac{\alpha}{f} \delta \left[ \vec{E} \cdot \vec{B} \right] \]

\[ \gg H^2 \]

Two contributions to \( \delta \left[ \vec{E} \cdot \vec{B} \right] \):

\[ \delta \left[ \vec{E} \cdot \vec{B} \right] \simeq \left[ \vec{E} \cdot \vec{B} - \langle \vec{E} \cdot \vec{B} \rangle \right]_{\delta \phi = 0} + \frac{\partial \langle \vec{E} \cdot \vec{B} \rangle}{\partial \phi} \delta \phi \]
Perturbations (II)

Fourier-transformed effective equation for perturbations

$$\ddot{\delta \phi_p} + H \left( 3 + \frac{\pi \alpha V'(\Phi_0)}{3 f H^2} \right) \dot{\delta \phi_p} + \left( \frac{p^2}{a^2} + V''(\Phi_0) \right) \phi_p = -\frac{\alpha}{f} \delta_{E\bar{E}}(p)$$

Solution (using Green function method)

$$\delta \phi_{\vec{p}}(t) = -\frac{\alpha}{f} \int dt' G(t, t') \delta_{E\bar{E}}(t', \vec{p})$$

two point function of inflaton perturbations

$$\langle 0|\delta \phi_{\vec{p}} \delta \phi_{\vec{p'}}|0\rangle = \frac{\alpha^2}{f^2} \int dt' G(t, t') \int dt'' G(t, t'') \langle 0|\delta_{E\bar{E}}(t', \vec{p}) \delta_{E\bar{E}}(t'', \vec{p'})|0\rangle$$

Barnaby et al 09
Perturbations (III)

Inflaton power spectrum related to two point function of $\phi$ by

$$\mathcal{P}_R(p) = \frac{p^3 H^2 \langle \phi_{\vec{p}} \phi_{\vec{p}'} \rangle}{2 \pi^2 \dot{\phi}^2 \delta^3(\vec{p} + \vec{p}')}$$

Spectrum of metric perturbations

$$\langle \mathcal{R} \mathcal{R} \rangle \approx \frac{4N}{\log \left[ M_P^4 f V'/V^2 \right]^2} \left( \frac{p}{\alpha H} \right) \frac{2f V''/\pi \alpha V'}{\text{quasi scale invariant for large } \alpha}$$

for $N$ gauge fields
Amplitude $\sim 0.05 \log\left(\frac{M_P}{E_{infl}}\right)^{-2} N^{-1}$

for $E_{infl} \sim \text{TeV}$, need $N \sim 10^5$

but...

did we use the true value of $\delta_{E\cdot B}$?
More dissipation?

Large electromagnetic fields on subhorizon scales

Light particles charged under the $U(1)$s copiously produced

Amplitude of $\delta_{\vec{E}, \vec{B}}$ reduced

Since perturbations in $\phi$ are sensitive only to $\delta_{\vec{E}, \vec{B}}$, amplitude of perturbations reduced

Nota bene: this does not affect the slowing down of the zero mode of the inflaton, that is just based on energy conservation
Cosmological magnetic fields

The model comes with a bonus!

If one of the $U(1)$s is $U(1)_{EM}$, then magnetic fields of cosmological interest generated

(note: the origin of galactic and cluster magnetic fields is still mysterious)

And with a signature!

The magnetic fields generated this way should be maximally helical
Conclusions

• We do not have to be very creative to realize natural inflation on a steep potential (but we need to be fine tuned at $\sim 1\%$...)

• Spectrum of perturbations is quasi-scale invariant

• Amplitude tends to be large, but there are ways out (in progress)