

(results from the)  
IPMU Mass and Spin  
Determination  
Workshop

M. E. Peskin  
HEFTI MET Workshop  
April 2009

Several of us here attended the recent

“IPMU Focus week on Determination of Masses and Spins of New Particles at the LHC” March 16-20

organized by Mihoko Nojiri and Bryan Webber.

The conference agenda and most of the transparencies are posted at: <http://www.ipmu.jp/seminars/20090316-focusweek.html>

There is clearly an overlap of topics between that meeting and this one. Hsin-Chia asked me to summarize what was discussed.

The talks covered a number of interesting topics.

I group these into four categories:

1. Talks by “friends of  $m_{T2}$ ” :

Barr, Lester, and Gripaos, Choi and Cho, Kong,  
Nojiri, Sakurai, and Takeuchi

2. Talks on related topics that we will hear at this workshop

geometrical understanding of equations for kinematic  
endpoints - Myeonghun Park

general theory of spin asymmetries with missing particles  
- K. C. Kong

how MADGRAPH can rule the world (and why  
that is a Good Thing) - Johan Alwall / Claude Duhr

3. Talks on other topics in BSM phenomenology, e.g.,

Jet angular correlation in Vector Boson Fusion

- Kentaro Mawatari

How to discover  $h^0 \rightarrow a^0 a^0$  with  $m(a^0) < m_b$

- Mariangela Lisanti

An extra-dimension model with the phenomenology of  
gauge-mediated SUSY - Csaba Csaki

Threshold region for  $gg \rightarrow \tilde{g}\tilde{g}$  - Hiroshi Yokoya

4. Experimental summaries from jaded world travellers

- (e.g. Albert de Roeck)

In this summary, I will concentrate on applications of  $m_{T2}$  .

To begin, let me place  $m_{T2}$  in a scheme for analyzing the results of new physics measurements:

anomalies w. resp. to SM  $\longleftrightarrow$  precise kinematic endpoints

OSETs

max. kinematic region  
(UC Davis method)



$\longleftarrow$  utility for low lumi  $\text{—————}$

$\text{—————}$  theoretical precision  $\longrightarrow$

$\text{—————}$  method depends on the actual spectrum  $\longrightarrow$

Begin with transverse mass  $m_T$  :

The **transverse energy** of a particle A is  $E_{TA} = (m_A^2 + p_T^2)^{1/2}$

$$E_A = E_{TA} \cosh \eta_A \quad p_A^3 = E_{TA} \sinh \eta_A$$

$$E_A E_B - p_A^3 p_B^3 = E_{TA} E_{TB} \cosh(\eta_A - \eta_B)$$

then, if  $C \rightarrow A + B$

$$\begin{aligned} m_C^2 &= m_A^2 + m_B^2 + 2(E_A E_B - p_A^3 p_B^3 - \vec{p}_{TA} \cdot \vec{p}_{TB}) \\ &\geq m_A^2 + m_B^2 + 2(E_{TA} E_{TB} - \vec{p}_{TA} \cdot \vec{p}_{TB}) \end{aligned}$$

Thus, definite the **transverse mass** of C as

$$m_{TC}^2 = m_A^2 + m_B^2 + 2(E_{TA} E_{TB} - \vec{p}_{TA} \cdot \vec{p}_{TB})$$

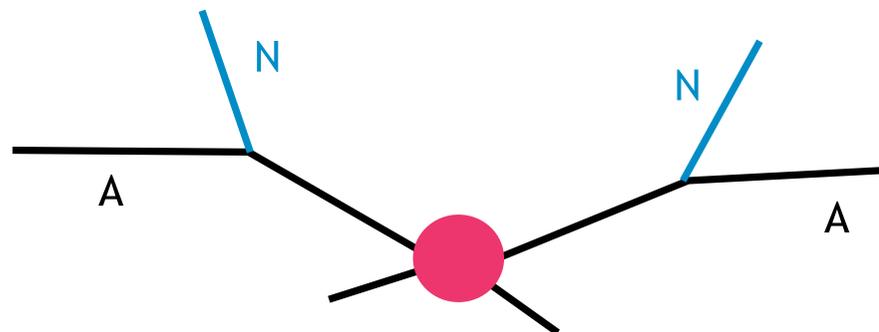
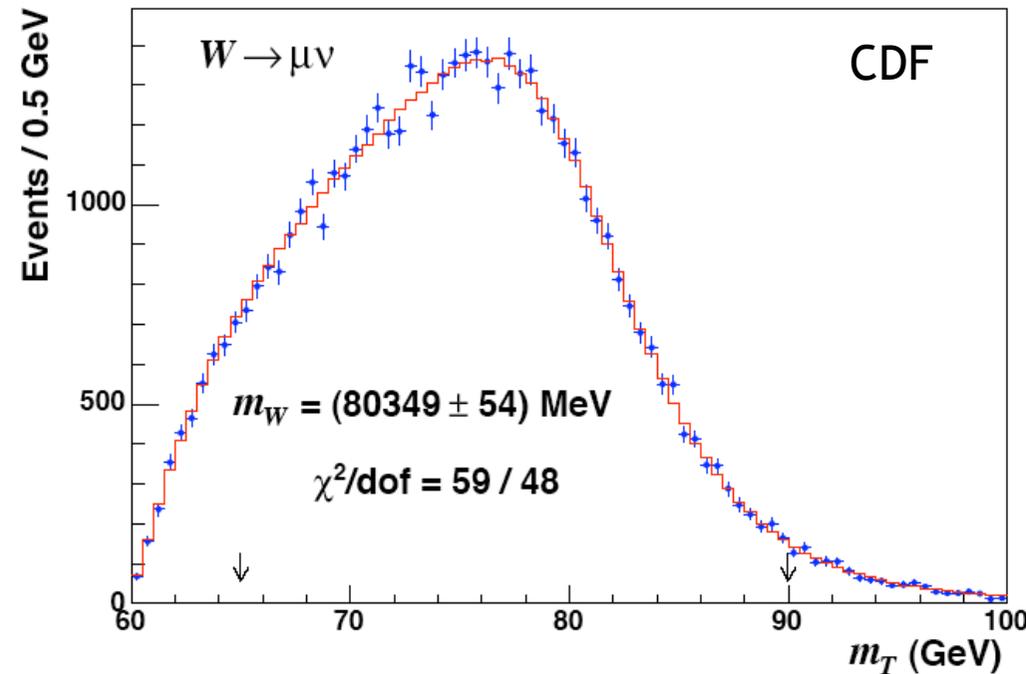
Then  $m_{TC} \leq m_C$  ; **equality if**  $\eta_A = \eta_B$  .

For example, in  $W^+ \rightarrow \mu^+ \nu$  only the (missing) transverse momentum of the  $\nu$  is observable. The transverse mass of the W is

$$m_{TW}^2 = 2E_{T\ell}E_{T\nu} - \vec{p}_{T\ell} \cdot \vec{p}_{T\nu}$$

In an ideal measurement, this cuts off sharply at  $m_W$ . Even in practice, this is an excellent way to determine the mass of the W.

But, what if there are two missing particles in the final state, as would be typical for SUSY?



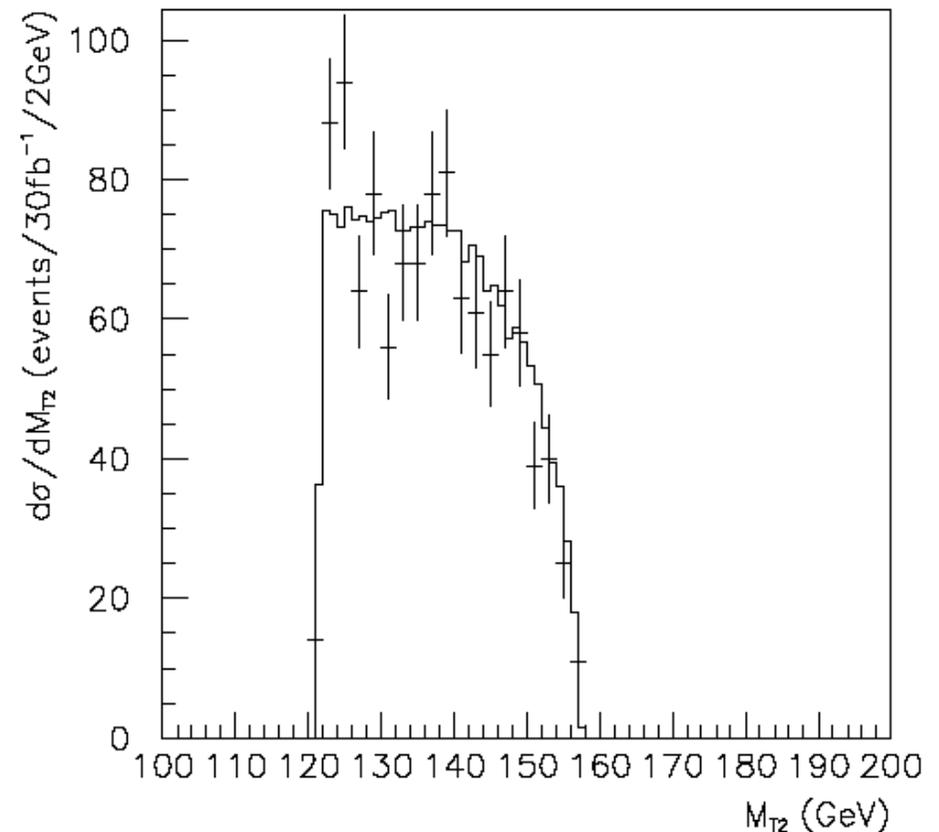
Lester and Summers:

Divide the missing  $p_T$  into two pieces in an arbitrary way. Define

$$m_{T2}^2 = \min_{p_{T,miss} = p_T^1 + p_T^2} \left\{ \max_{i=1,2} \left[ m_T^2(A_i + p_{Ti}) \right] \right\}$$

Some split must be the right one. For this choice, both values of  $m_T$  are lower bounds to the decaying SUSY particle mass.

Example of slepton pair production and decay:



The philosophy of  $m_T$  is that there is an advantage in working with a quantity that is a rigorous bound.

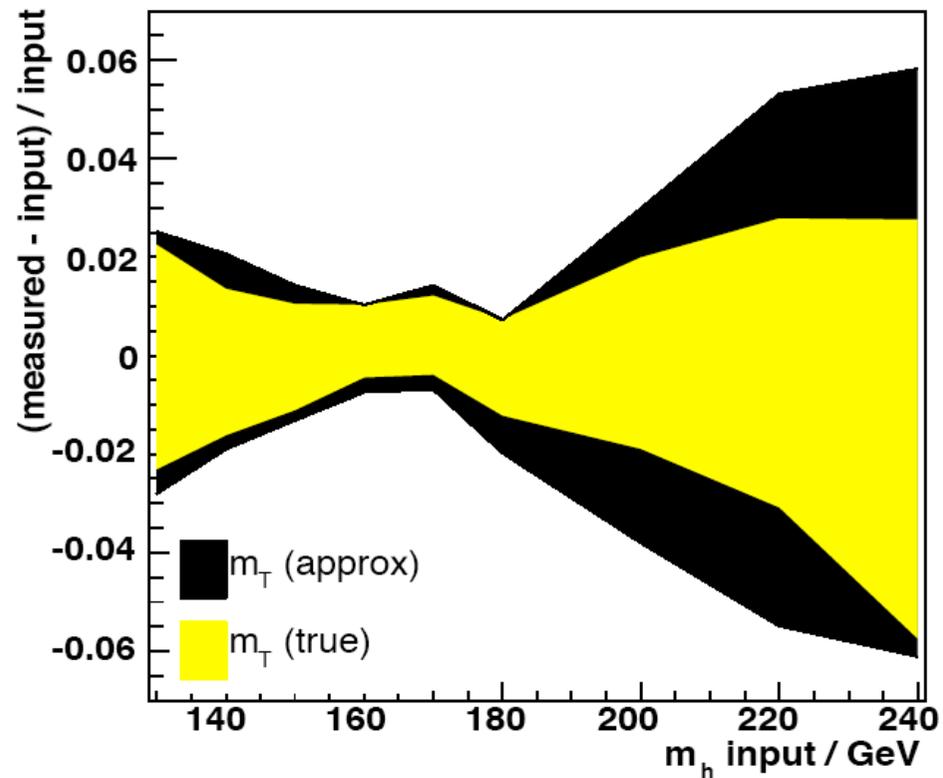
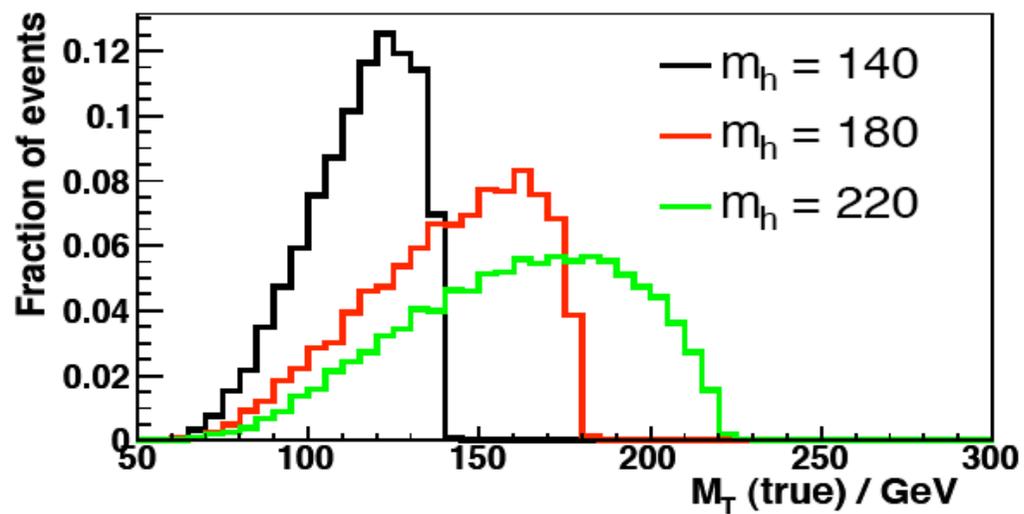
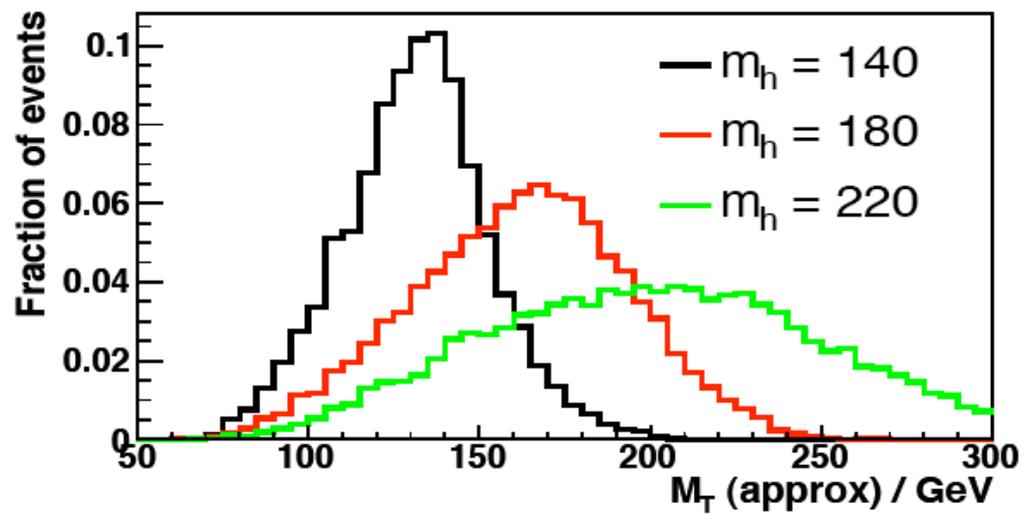
An example (Barr, Grapaio, and Lester):

$$h^0 \rightarrow WW^* \rightarrow \ell^+ \ell^- \nu \bar{\nu}$$

Consider the 2-lepton systems to be A; its mass can be measured. Consider the 2-neutrino system to be the invisible decay product; its mass is unknown and varies from event to event.

$$\begin{aligned} m_T^2 &= m_{\ell\ell}^2 + m_{\nu\bar{\nu}}^2 + 2(E_{\ell\ell}E_{\nu\bar{\nu}} + \vec{p}_{T\ell\ell} \cdot \vec{p}_{T\nu\bar{\nu}}) \\ &\geq m_T(m_{\nu\bar{\nu}}^2 = 0) \end{aligned}$$

Rainwater and Zeppenfeld had proposed a similar analysis in which  $m_{\nu\bar{\nu}}$  is estimated by  $m_{\ell\ell}$  in the same event. Setting  $m_{\nu\bar{\nu}} = 0$  works better, producing a sharper endpoint and a better estimate of  $m_h$ .



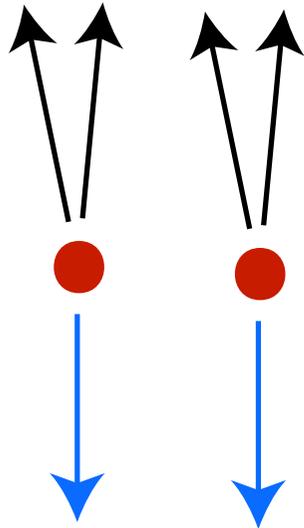
We have seen that, when we do not know the mass of the invisible particle, we can usefully consider  $m_T$  or  $m_{T2}$  as a function of the unknown mass.

An important property is that  $m_{T2}$  is **monotonically increasing** as a function of the unknown mass.

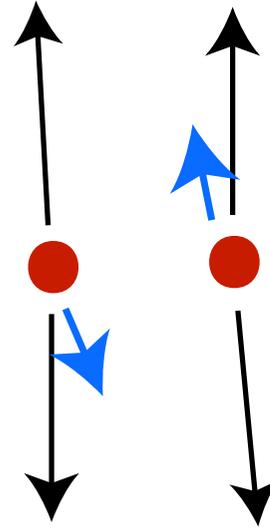
Here is an application: (Cho, Choi, Kim, and Park):

$$gg \rightarrow \tilde{g}\tilde{g} \quad \text{with the 3-body decay} \quad \tilde{g} \rightarrow q\bar{q}N$$

Consider the decay configurations:



The best solution for  $p_{T1}$  has a large value of the momentum;  $m_{T2}$  is relatively insensitive to the assumed value of  $m_N$

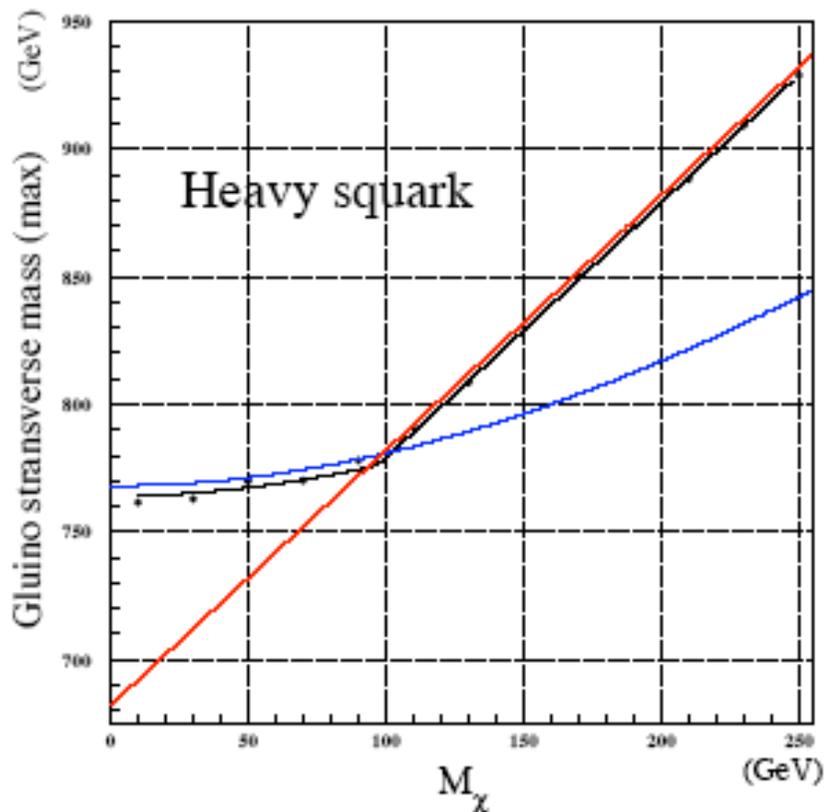


The best solution for  $p_{T1}$  has  $p_{T1} = 0$ ;  $m_{T2}$  varies linearly with the assumed value of  $m_N$

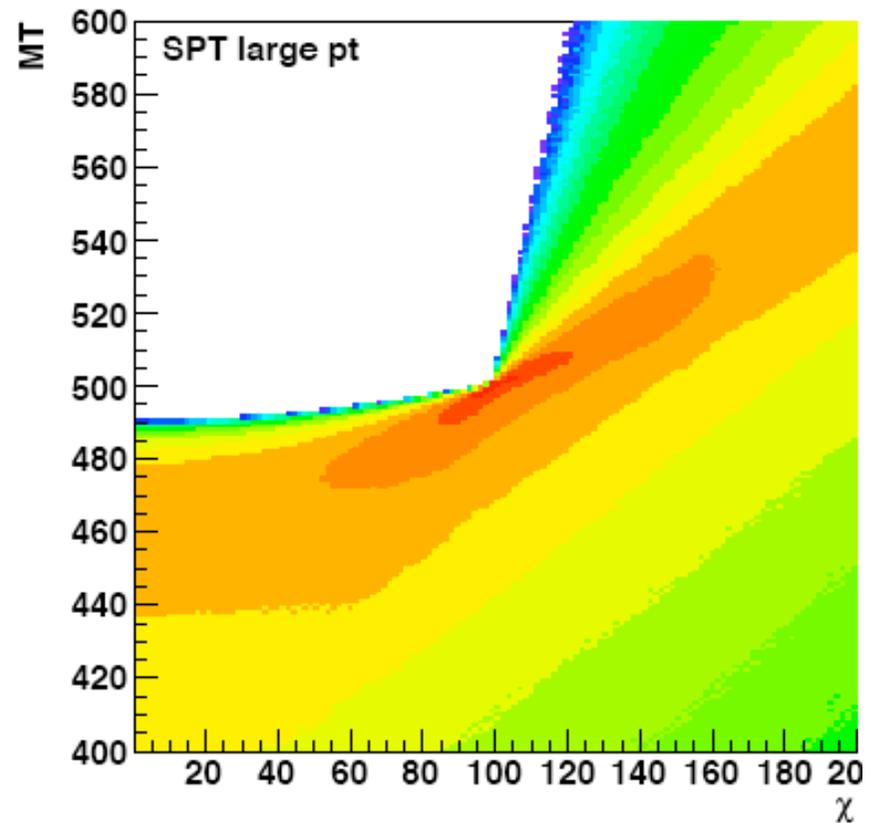
But, at the true value of  $m_N$ ,  $m_{T2}$  should equal the gluino mass regardless of the kinematic configuration, as long as  $\eta_{qq} = \eta_N$ .

This means that there will be a crossover, with one configuration giving a stronger lower bound for small  $m_N$  and the other for large  $m_N$ .

The resulting lower bounds, as a function of  $m_N$ , have a kink at the correct value!



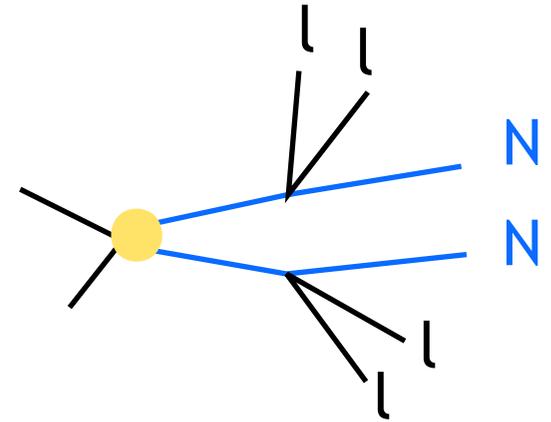
original Cho et al.



Barr, Gripaios, Lester

It is also possible to combine  $m_{T2}$  constraints with other kinematic constraints in the problem (Barr)

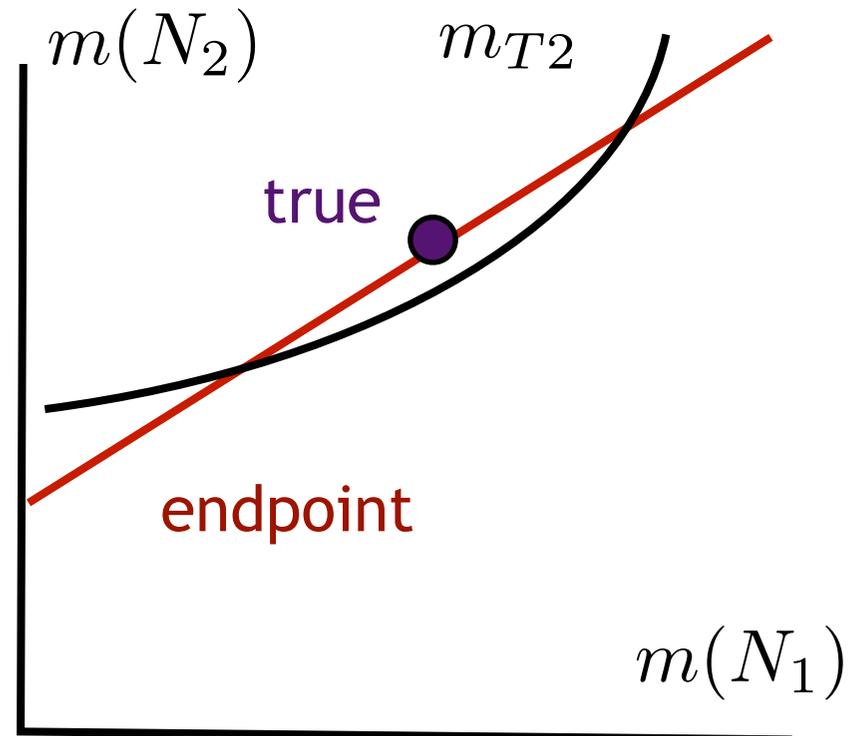
e.g. the  $N_2 \rightarrow \ell^+ \ell^- N_1$  decay cascade



The usual endpoint constraint gives the mass splitting between  $N_2$  and  $N_1$ .

The  $m_{T2}$  constraint gives a lower bound on  $m(N_2)$  which is a function of  $m(N_1)$ .

Combining these, we obtain upper and lower bounds on  $m(N_2)$  and  $m(N_1)$ .



Use of the  $m_{T2}$  vectors (Cho, Choi, Kim, Park):

Having determined the masses, use the solution for  $p_{T1}$ , with  $\eta_N$  equal to the  $\eta$  of the visible decay products, as an estimator of the **missing momentum 3-vector**.

This is called  $m_{T2}$  - **assisted on-shell (MAOS)** reconstruction.

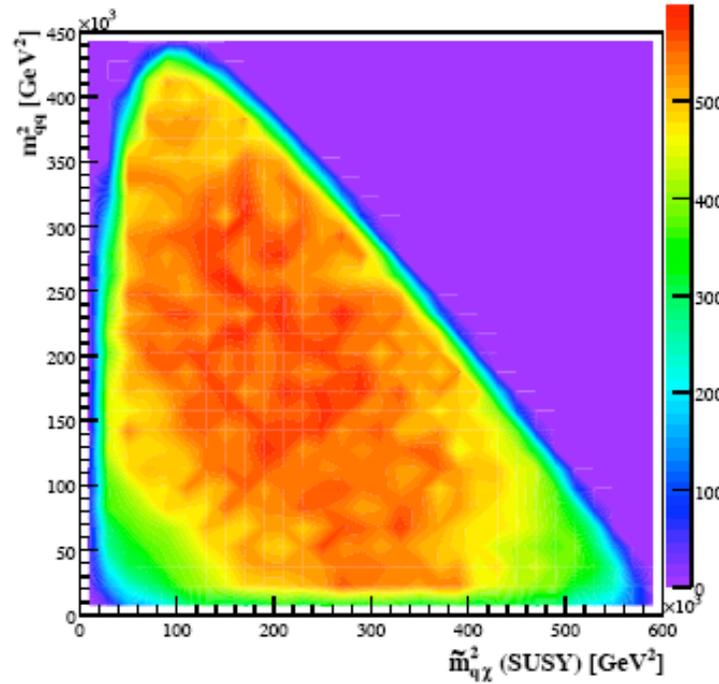
The MAOS vectors can be used to construct production and decay angles for spin determination.

The method can be improved systematically: The MAOS momentum is more accurate if we select events closer to the  $m_{T2}$  endpoint.

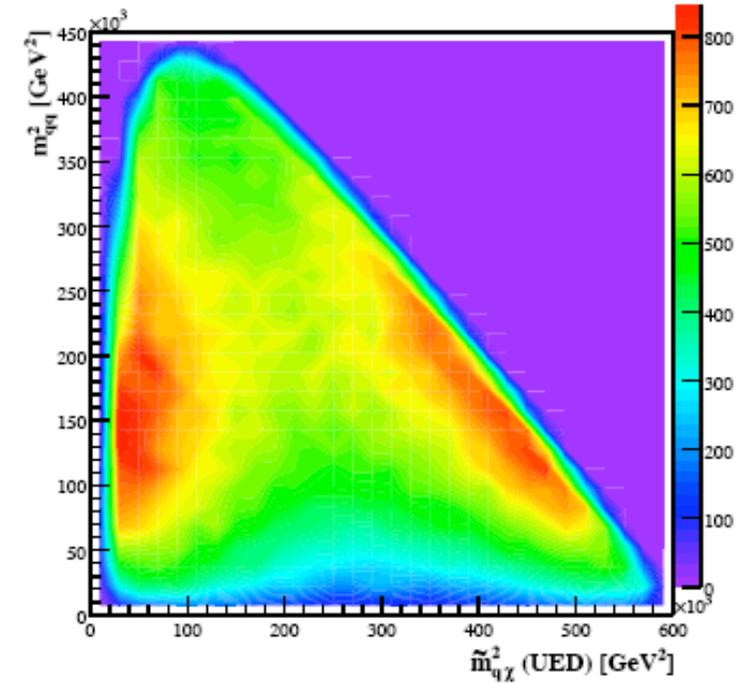
3-body  
qqN decay

true  
partons

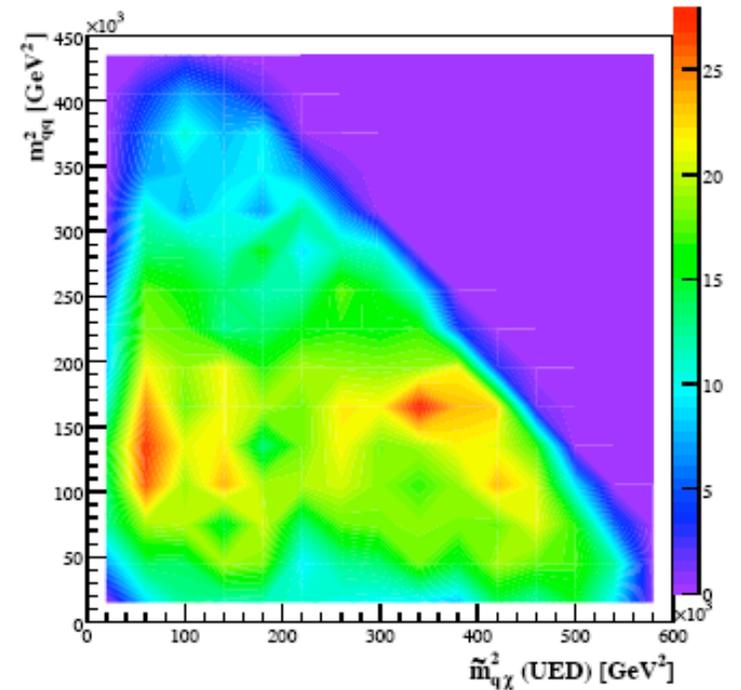
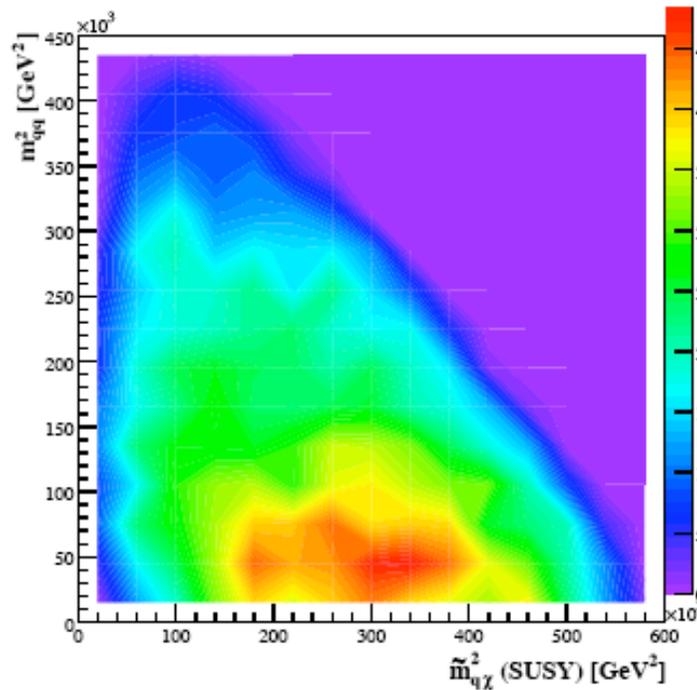
SUSY



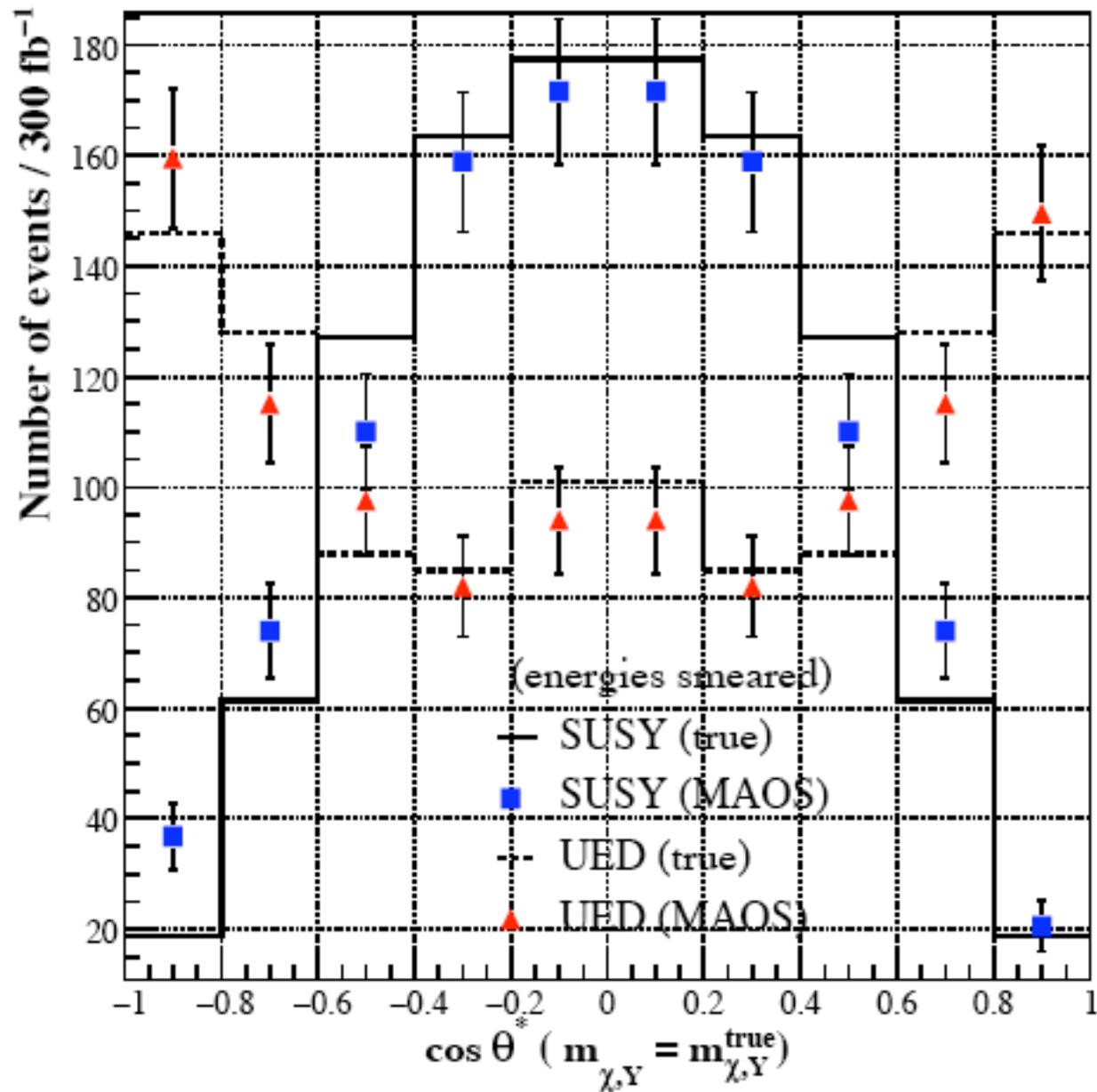
UED



MAOS



production  
angular  
distribution



Several significant problems remain, even for the conceptual parton-level analyses that I am discussing.

First, if there are many final-state jets, there is a combinatoric ambiguity in associating them to parents. This is especially true if the production of SUSY particles contains associated production of  $\tilde{q} + \tilde{g}$  with, e.g.,  $\tilde{q} \rightarrow q + \tilde{g}$  .

Nojiri and her students have been exploring methods to deal with this problem: hemisphere  $m_{T2}$ , subsystem  $m_{T2}$  . (Burns, Kong, Matchev, and Park have also explored these ideas.)

Find two hard objects in the event (e.g. the two leading- $p_T$  jets). Use these as the basis of a division into hemispheres (Moortgat and Pape). Sort other objects in the event into one hemisphere or the other according to, e.g., distance from the two reference vectors in the  $(\eta, \phi)$  plane.

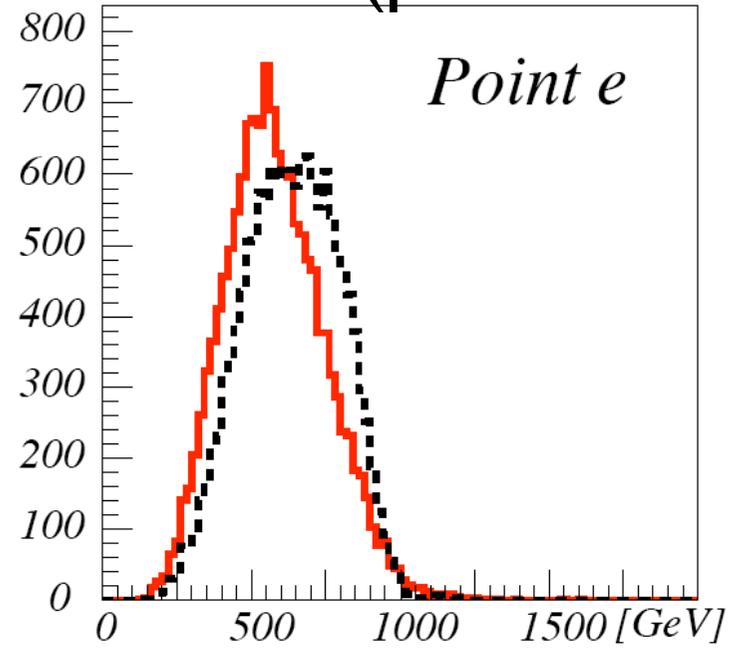
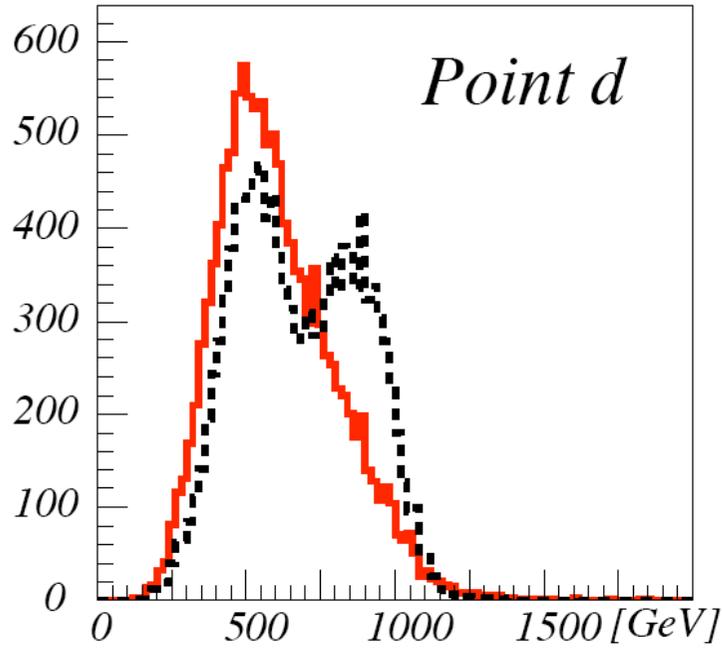
For hemisphere  $m_{T2}$ , assign the jets in each hemisphere to one parent particle and use the two sets of jets to define  $m_{T2}$ .

For subsystem  $m_{T2}$  relevant to  $\tilde{q} + \tilde{g}$  associated production, **throw out the hardest jet in the event** and associate the remaining jets into two groups by hemisphere.

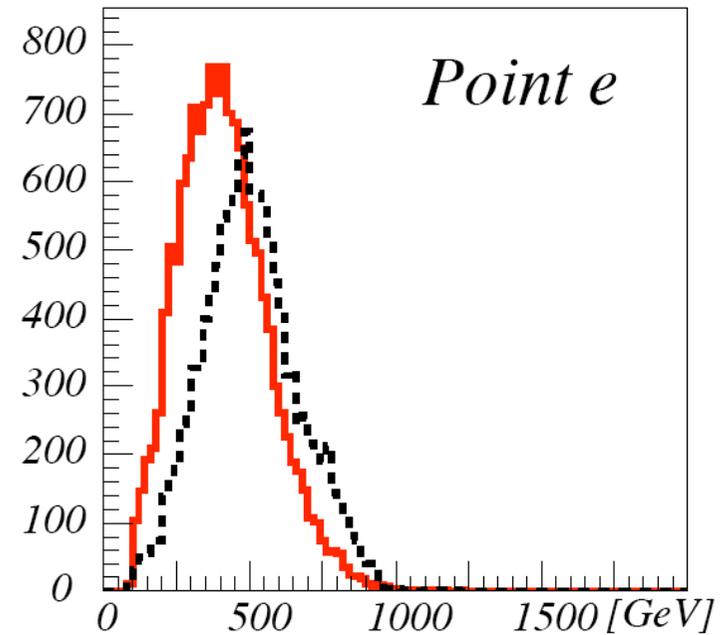
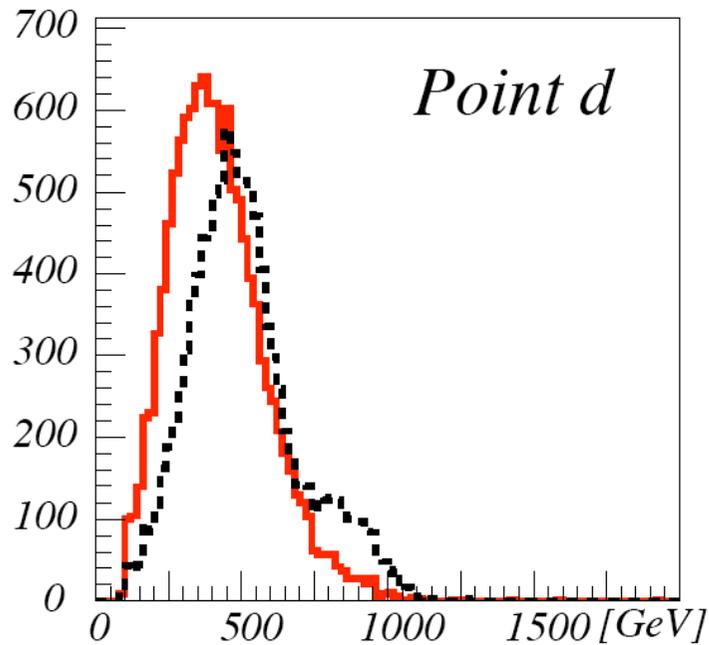
Nojiri, Sakurai, Shimizu, and Takeuchi

computed/true  
(parton level)

$m_{T2}$



subsystem  
 $m_{T2}$



Another important problem is **initial- and final-state QCD radiation**. We idealize that a SUSY production event contains only the jets from SUSY particle decays.

But, at a hadron collider, **we can have as many jets as we want by going to lower and lower  $p_T$** . Often, the extra jets come to us when we do not want them !

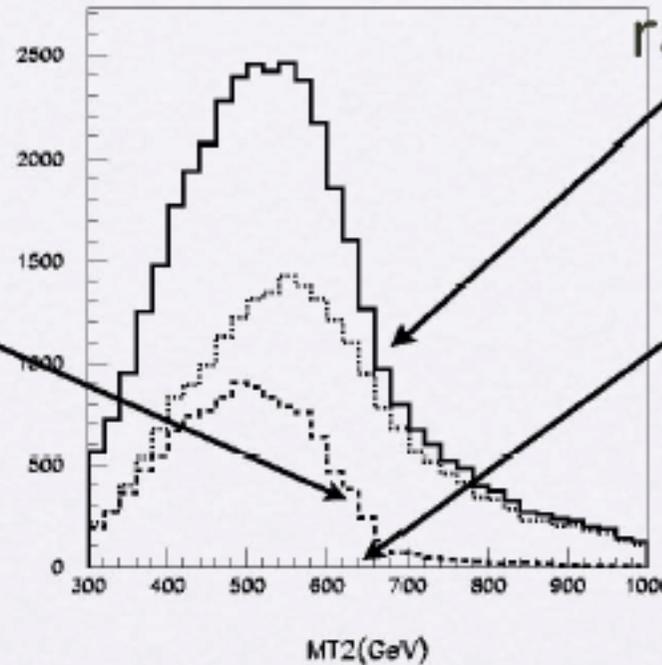
With Alwall, Hiramatsu, and Shimizu, Nojiri has been studying this question by generating SUSY pair + 1 jet events using MADGRAPH.

model with  $gg \rightarrow \tilde{g}\tilde{g}$

all events passing SUSY cuts

Events without  
Initial radiation  
after matching  
“exclusive”

select events  
with exactly 4 jets



Events with Initial state  
radiation “inclusive”

input gluino mass

total cross section 3pb  
50000 events generated  
no SUSY cut

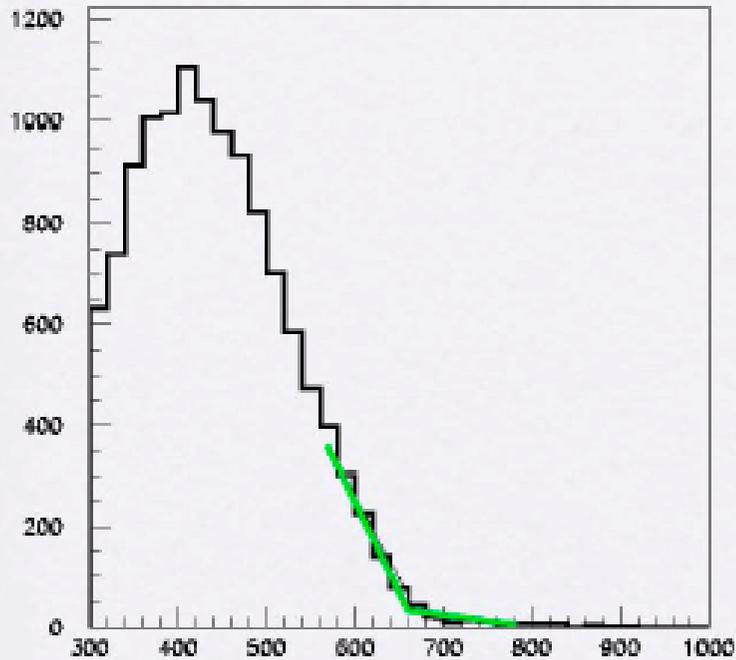
A better strategy:

Look at the 5 highest  $p_T$  jets in the event.

Remove one jet. Associate the others in pairs.  
Compute  $m_{T2}$ .

Take the minimum of the  $m_{T2}$  values obtained.  
This should have an endpoint at  $m(\tilde{g})$ . The  
excluded jet is likely to be an ISR jet.

parton level distribution

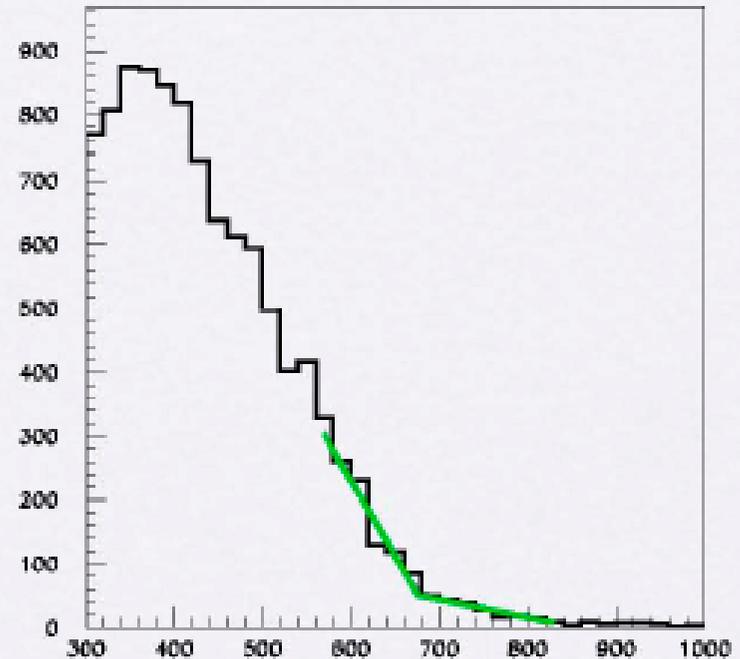


the minimum parton level  $MT_2$ (GeV)

$673.9 \pm 2.5$  GeV

MADGRAPH

jet level



minimum  $MT_2$ (GeV)

$675.4 \pm 6.4$  (imin. ge.3 )

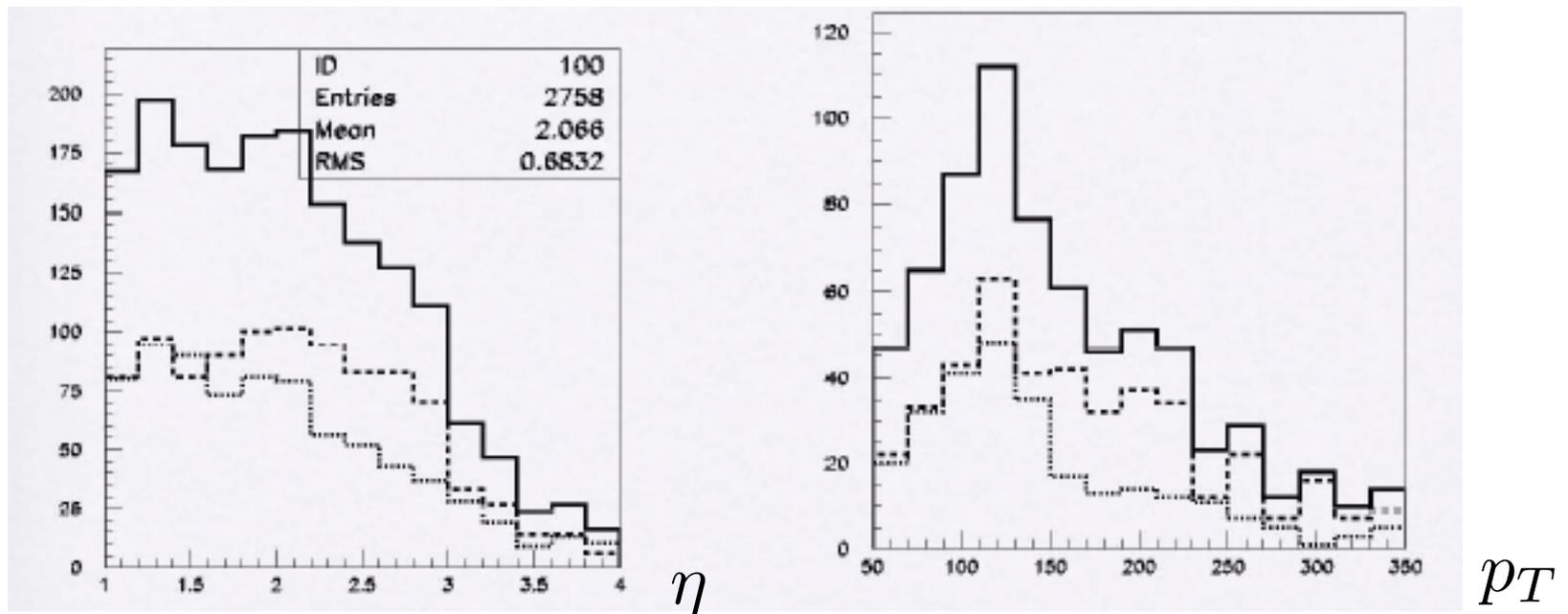
$672.7 \pm 3.5$  (for all)

MADGRAPH + PYTHIA + ACERDet

There is an interesting sanity check on this idea:

An initial-state gluon comes from initial-state quark splitting. If the gluino mass is large, we need a hard initial-state gluon. This would come from radiation from a valence quark. Many of these radiations occur at relative high  $p_T$  .

The recoil quark would then show up in the event, as an extra jet at high  $\eta$  .



Conclusion:

$m_{T2}$  is a powerful object to add to our kinematic toolbox.

It is useful as a preformed method,  
but more useful as a **way of thinking**.

The vectors that solve the  $m_{T2}$  constraint can themselves be useful in kinematic analyses.

More uses of  $m_{T2}$  are out there. Can you find a new one ?