Lepton and Quark masses from Top loops

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to appear...
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Loopy masses for leptons and quarks

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Standard Model Higgs

Responsible for $W, Z$ mass and (charged) fermion masses

Associated hierarchies:

Gauge hierarchy

$$m_W \ll M_{pl}$$

Yukawa hierarchy

$$y_e \ll y_t$$
Yukawa hierarchy

Technically natural but would still like an explanation

Symmetries (Froggatt Nielsen Models)

\[ Y_{ij} \left( \frac{\phi}{M} \right)^{q_i + q_j + q_H} H \bar{\psi}_i \psi_j \]

\[ Y_{ij}^{SM} = Y_{ij} \epsilon^{q_i + q_j + q_H} \quad \epsilon = \frac{\langle \phi \rangle}{M} \]

Charge the SM fermions differently
Geography  (Extra dimensional models)

\[ Y_{ij}^{SM} = \int dx_5 \psi_i(x_5) \psi_j(x_5) h(x_5) \]

Place the SM fermions in different places
Quantum mechanics

• The SM is coupled to a strongly coupled CFT
• SM fields get large anomalous dimensions
• Enters approximate fixed point at scale $\mu$ and leaves at scale $\mu_0$

$$Y_{ij}^{SM}(\mu) = Y_{ij}(\mu_0) \left( \frac{\mu}{\mu_0} \right)^{\frac{1}{2}(\gamma_i + \gamma_j + \gamma_H)}$$

SM fermions have different couplings
• Many clever mechanisms exist but must treat SM fermions separately.

• Convert small differences to large differences

• Example where SM fermions all charged the same way but get differences in Yukawas?
Quantum mechanics

Masses are generated through quantum effects

Electron mass from muon mass? Georgi and Glashow, `73

Work in the `80’s, mainly one and two loop mass generation

Babu and Ma, `89
Quantum mechanics

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Electron mass from muon mass?  
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Work in the `80’s, mainly one and two loop mass generation  
Babu and Ma, `89

Naively all masses at approximately the same loop order
More ambitious attempt

Loop-level where mass is generated
Loop-level where mass is generated
More likely to fail...?

Loop-level where mass is generated
More likely to fail...?

Loop-level where mass is generated

Lepton and Quark masses at 1 TeV
Top is clearly special

So,

assume only the top has a tree level Yukawa

\[ y_t H \bar{u}_R^3 Q_L^3 \]
Top is clearly special

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\[ y_t H \bar{u}^3_R Q^3_L \]

Charge the top?
Top is clearly special

So,

assume only the top has a tree level Yukawa

\[ y_t H \bar{u}^3_R Q^3_L \]

Charge the top?
Instead charge Higgs under an extra $U(1)_H$

$U(1)_H$ broken by the vev of a SM singlet $\phi$ of charge -1

Introduce a vector like pair of fermions with quantum numbers of left handed quarks, also charged under $U(1)_H$
Yukawas:

\[ \phi(-1) \rightarrow \tilde{c}^i \rightarrow \psi_R(-1) \rightarrow \psi_L(-1) \rightarrow c^j \rightarrow H(-1) \]

\[ m_{ij} \propto \tilde{c}_i c_j \]

But lh top and rh top only appear *linearly* in couplings.
Redefine couplings so only one lh and one rh couple.
Call these the top.

Mass matrix is rank 1.

Only the top gets a tree level mass.
Chiral symmetries

\[ y_t \neq 0 \]
\[ U(3)_Q \times U(3)_u \times U(3)_d \rightarrow U(1)_t \times U(2)_Q \times U(2)_u \times U(3)_d \]

Need to break remaining chiral symmetries

Introduce a scalar leptoquark \[ r : (3, 2, +7/6) \]
Chiral symmetries

$y_t \neq 0$

$U(3)_Q \times U(3)_u \times U(3)_d \rightarrow U(1)_t \times U(2)_Q \times U(2)_u \times U(3)_d$

Need to break remaining chiral symmetries

Introduce a scalar leptoquark $r : (3, 2, +7/6)$

(charge 0 under extra $U(1)$)
Chiral symmetries

\[ y_t \neq 0 \]

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Need to break remaining chiral symmetries

Introduce a scalar leptoquark \( r : (3, 2, +7/6) \) (charge 0 under extra \( U(1) \))

Most general interactions:

\[ \lambda_{ij} r \overline{u}^i_R L^j_L + \lambda'_{ij} r \overline{Q}^i_L e^j_R + \text{H.c.} \]
\[
U(3)_Q \times U(3)_u \times U(3)_d \rightarrow U(1)_t \times U(2)_Q \times U(2)_u \times U(3)_d
\]

\[
y_t \neq 0
\]

\[
\lambda \neq 0
\]

\[
\lambda' \neq 0
\]

\[
\rightarrow U(1)_u \times U(3)_d
\]

\[
U(3)_L \times U(3)_e \rightarrow U(1)_L
\]

With this breaking of chiral symmetries up type quarks and charged leptons can get a mass at some loop order.
\[ y_t \neq 0 \]

\[ U(3)_Q \times U(3)_u \times U(3)_d \rightarrow U(1)_t \times U(2)_Q \times U(2)_u \times U(3)_d \]

\[ \lambda \neq 0 \]
\[ \lambda' \neq 0 \]

\[ \rightarrow U(1)_u \times U(3)_d \]

\[ \lambda \neq 0 \]
\[ \lambda' \neq 0 \]

\[ U(3)_L \times U(3)_e \rightarrow U(1)_L \]

With this breaking of chiral symmetries up type quarks and charged leptons can get a mass at some loop order

But what loop order?
\[ \lambda_{ij} r \bar{u}^i_R L^j_L + \lambda'_{ij} r \bar{Q}^i_L e^j_R + H.c. \]

Linear couplings

Redefine fields:

\[
\begin{pmatrix}
\lambda_{11} & \lambda_{12} & \lambda_{13} \\
\lambda_{21} & \lambda_{22} & \lambda_{23} \\
\lambda_{31} & \lambda_{32} & \lambda_{33}
\end{pmatrix}
\]
\[ \lambda_{ij} \, r \, \overline{u}_R^i \, L_L^j + \lambda'_{ij} \, r \, \overline{Q}_L^i \, e_R^j + \text{H.c.} \]

Linear couplings

Redefine fields:
\[
\lambda_{ij} \, r \, \bar{u}^i_R \, L^j_L + \lambda'_{ij} \, r \, \bar{Q}^i_L \, e^j_R + \text{H.c.}
\]

Linear couplings

Redefine fields:

- Define \( L_3 \) so it only couples only to \( u_3 \)

\[
\begin{pmatrix}
\lambda_{11} & \lambda_{12} & \lambda_{13} \\
\lambda_{21} & \lambda_{22} & \lambda_{23} \\
0 & 0 & \lambda_{33}
\end{pmatrix}
\]
\[
\lambda_{ij} r \overline{u}^i_R L^j_L + \lambda'_{ij} r \overline{Q}^i_L e^j_R + \text{H.c.}
\]

Linear couplings

Redefine fields:

• Define \( L_3 \) so it only couples only to \( u_3 \)
• \( u_2 \) couples only to \( L_2 \) and \( L_3 \)

\[
\begin{pmatrix}
\lambda_{11} & \lambda_{12} & \lambda_{13} \\
0 & \lambda_{22} & \lambda_{23} \\
0 & 0 & \lambda_{33}
\end{pmatrix}
\]
\[
\lambda_{ij} r \overline{u}_R^i L_L^j + \lambda'_{ij} r \overline{Q}_L^i e_R^j + \text{H.c.}
\]

Linear couplings

Redefine fields:

- Define \( L_3 \) so it only couples only to \( u_3 \)
- \( u_2 \) couples only to \( L_2 \) and \( L_3 \)
- Rotation of \( u_1 \) and \( u_2 \)

\[
\begin{pmatrix}
\lambda_{11} & \lambda_{12} & 0 \\
0 & \lambda_{22} & \lambda_{23} \\
0 & 0 & \lambda_{33}
\end{pmatrix}
\]
\[
\lambda_{ij} r \overline{u}_R^i L_L^j + \lambda'_{ij} r \overline{Q}_L^i e_R^j + \text{H.c.}
\]

Linear couplings

Redefine fields:

- Define \( L_3 \) so it only couples only to \( u_3 \)
- \( u_2 \) couples only to \( L_2 \) and \( L_3 \)
- Rotation of \( u_1 \) and \( u_2 \)

\[
\begin{pmatrix}
\lambda_{11} & \lambda_{12} & 0 \\
0 & \lambda_{22} & \lambda_{23} \\
0 & 0 & \lambda_{33}
\end{pmatrix}
\]

\( \lambda_{ij}, \lambda'_{ij} \) can be made real and positive
One loop tau mass

\[ m_\tau \simeq \lambda_{33} \lambda'_{33} m_t \frac{N_c}{16\pi^2} \ln \left( \frac{\Lambda^2}{M_{\tilde{r}}^2} \right) \]

\[ \approx 0.09 \text{ for } \Lambda \approx 10M_{\tilde{r}} \]

\[ \lambda_{33} \lambda'_{33} \approx (0.36)^2 \text{ for correct } m_\tau / m_t \text{ ratio} \]
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Two loop charm mass - a "rainbow" diagram

\[
M_u[\tilde{r}\tilde{r}] = \begin{pmatrix}
0 & 0 & 0 \\
0 & \lambda'_{23} \lambda_{23} & \lambda'_{33} \lambda_{23} \\
0 & \lambda'_{23} \lambda_{33} & \lambda'_{33} \lambda_{33}
\end{pmatrix} \lambda'_{33} \lambda_{33} m_t \epsilon^{(2)}_{\tilde{r}}
\]

\[
m_c = \lambda'_{23} \lambda_{23} m_\tau \frac{1}{16\pi^2} \log \frac{\Lambda^2}{M^2_{\tilde{r}}}
\]

\[
\lambda_{23} \lambda'_{23} \approx (3.3)^2 \text{ for correct } m_c/m_\tau \text{ ratio}
\]
Two loop charm mass - a “rainbow” diagram

\[ M_u[\tilde{r}\tilde{r}] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda'_{23}\lambda_{23} & \lambda'_{33}\lambda_{23} \\ 0 & \lambda'_{23}\lambda_{33} & \lambda'_{33}\lambda_{33} \end{pmatrix} \lambda'_{33}\lambda_{33} m_t \epsilon^{(2)}_\tilde{r} \]

\[ m_c = \lambda'_{23}\lambda_{23} m_\tau \frac{1}{16\pi^2} \log \frac{\Lambda^2}{M_{\tilde{r}}^2} \]

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\end{pmatrix} \lambda'_{33}\lambda_{33} m_t \epsilon_{\tilde{r}}^{(2)}
\]

\[
m_c = \lambda'_{23}\lambda_{23} m_\tau \frac{1}{16\pi^2} \log \frac{\Lambda^2}{M_{\tilde{r}}^2}
\]

\[
\lambda_{23}\lambda'_{23} \approx (3.3)^2 \text{ for correct } m_c/m_\tau \text{ ratio}
\]
Loop-level where mass is generated

Rinse and repeat?
Three loop muon mass

Rainbow $\sim N_C^2$

The diagram with no name $\sim N_C$
Three loop muon mass

Rainbow \sim N_C^2

Two loop charm mass

The diagram with no name \sim N_C
Three loop muon mass

\[
\begin{pmatrix}
0 & 0 & 0 \\
0 & \lambda' & 22\lambda' \\
0 & 23\lambda & (\lambda^2 + \lambda'^2)
\end{pmatrix}
\]

\[m_\mu \approx \lambda'_2 \lambda_2 m_c (1 + x) \frac{N_c}{16\pi^2} \log \frac{\Lambda^2}{M^2_{\tilde{r}}}
\]

\[\lambda_2 \lambda'_2 (1 + x) \approx (1.5)^2 \text{ for correct } m_\mu / m_c \text{ ratio}\]
Three loop muon mass

\[
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & \lambda \\
0 & \lambda & 0
\end{pmatrix}
\]

\[
m_\mu \approx \lambda'_2 \lambda_{22} m_c (1 + x) \frac{N_c}{16\pi^2} \log \frac{\Lambda^2}{M_{\tilde{r}}^2}
\]

\[
\lambda_{22} \lambda'_{22} (1 + x) \approx (1.5)^2 \text{ for correct } m_\mu/m_c \text{ ratio}
\]
Four loop up quark mass

Muon mass implies: \#\lambda_{12}\lambda'_{12} \approx (0.6)^2.
Four loop up quark mass

Three loop muon mass

Muon mass implies: \[ \# \lambda_{12} \lambda'_{12} \approx (0.6)^2. \]
Five loop electron mass

If only source of electron mass will determine $\lambda_{11} \lambda'_{11}$

Only input: $r : (3, 2, +7/6)$

$$\lambda_{ij} r \bar{u}_R^i L_L^j + \lambda'_{ij} r \bar{Q}_L^i e_R^j + \text{H.c.}$$

Loop-level where mass is generated

Lepton and Quark masses at 1 TeV

$\text{e}$ $\text{u}$ $\text{c}$ $\text{t}$ $\text{d}$ $\text{s}$ $\text{b}$ $\text{f}$

Fermilab
Down quark masses

Need to break the remaining chiral symmetries

\[ U(3)_d \times U(1)_u \times U(1)_L \]

Have choices diquarks, leptoquarks...

\[ H_8 : (8, 2, -1/2) \]
\[ \tilde{q} : (3, 2, 1/6) \]
\[ \tilde{d}_6 : (\bar{6}, 1, -1/3) \]
\[ \tilde{d} : (3, 1, -1/3) \]
### New field content

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<td>0</td>
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<td>1</td>
<td>1</td>
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</tr>
</tbody>
</table>

- **Up quarks and leptons**
- **Down quarks**
Most general couplings

\[ \kappa_i \Phi_8 \overline{u}^i_R \Psi_L + \kappa' \Phi'_8 \overline{d}^3_R \Psi_L \]

\[ \eta_{ij} \Phi_3 \overline{d}^i_R \overline{L}^j_L + \text{h.c.} \]

break the remaining chiral symmetries

\[ U(3)_d \times U(1)_u \times U(1)_L \rightarrow U(1)_L \times U(1)_Q \]
Most general couplings

\[ \kappa_i \Phi_8 \overline{u}_R^i \Psi_L + \kappa' \Phi'_8 \overline{d}_R^3 \Psi_L \]

Only couples to b

\[ \eta_{ij} \Phi_3 \overline{d}_R^i L_L^j + \text{h.c.} \]

break the remaining chiral symmetries

\[ U(3)_d \times U(1)_u \times U(1)_L \rightarrow U(1)_L \times U(1)_Q \]
Without altering up type and leptons have the freedom to rotate such that,

\[ \eta = \begin{pmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{21} & \eta_{22} & \eta_{23} \\ 0 & \eta_{32} & \eta_{33} \end{pmatrix} \]

\[ \kappa = (\kappa_1, \kappa_2, \kappa_3) \]
Without altering up type and leptons have the freedom to rotate such that,

\[ \eta = \begin{pmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{21} & \eta_{22} & \eta_{23} \\ 0 & \eta_{32} & \eta_{33} \end{pmatrix} \]

Diagonal entries can be made real and positive

\[ \kappa = (\kappa_1, \kappa_2, \kappa_3) \]
One loop bottom mass

\[ m_b \approx N_c \kappa_3 \kappa' c m_t \left( \frac{\langle \phi \rangle}{M_\Psi} \right)^2 \frac{1}{16\pi^2} \log \left( \frac{M_\Psi^2}{M_8^2} \right) \]
One loop bottom mass

\[ m_b \approx N_c \kappa_3 \kappa' c m_t \left( \frac{\langle \phi \rangle}{M_\Psi} \right)^2 \frac{1}{16\pi^2} \log \left( \frac{M_\Psi^2}{M_8^2} \right) \]
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Three loop strange mass

\[ \Phi_3 r H H \left( \frac{\phi}{m_{r'}} \right)^2 \]

Integrate out \( r' \)

\[ \begin{array}{cccc}
Q^3_L & \tau_R & Q^3_L & \tau_R \\
\Phi_3 & H & H & \Phi_3 \\
L^3_L & s_R & L^3_L & s_R
\end{array} \]
Four loop down masses

The down has a 4 loop mixed diagram (exercise for reader)
Figure 10: Down-quark mass induced at four loops.

\[
\Phi' = \begin{pmatrix}
1 & \epsilon^2 & 0 \\
-\epsilon^2 & 1 & \epsilon^2 \\
0 & -\epsilon^2 & 1
\end{pmatrix}
\]

\[
R_d = \begin{pmatrix}
1 & -\epsilon^2 & \epsilon - \epsilon^3 & \epsilon^3 \\
-\epsilon + \epsilon^3 & 1 & -\epsilon^2 & \epsilon^2 \\
\epsilon^3 & -\epsilon^2 & 1
\end{pmatrix}
\] (4.14)
“Cross Talk”

There are also corrections to some of the states that have mass:

Charm gets a two loop correction

Up gets a four loop correction

Muon gets a three loop correction

Electron gets a five loop correction
Charm gets a two loop correction

• Different parameter dependence
• Different number of logs
• Changes (lowers) certain couplings $\lambda_2 r_3 \approx (3.3)^2$

Doesn’t change loop counting
Lepton and Quark masses at 1 TeV

Five loop
Lepton and Quark masses at 1 TeV

Tree level

Five loop
Lepton and Quark masses at 1 TeV

- Tree level
- One loop
- Five loop
Lepton and Quark masses at 1 TeV

- Tree level
- One loop
- Two loop
- Five loop
Lepton and Quark masses at 1 TeV

Tree level

One loop

Two loop

Three loop

Five loop
Lepton and Quark masses at 1 TeV

Tree level

One loop

Two loop

Three loop

Four loop

Five loop

GeV

Lepton and Quark masses at 1 TeV

Tree level

One loop

Two loop

Three loop

Four loop

Five loop

GeV
CKM

\[ m_u \approx m_t \begin{pmatrix} \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix} \quad m_d \approx m_t \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^4 \\ \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^4 & \epsilon^3 & \epsilon \end{pmatrix} \]

Resulting in

\[ V_{CKM} \approx \begin{pmatrix} 1 - \epsilon^2 & \epsilon & \epsilon^3 \\ -\epsilon & 1 - \epsilon^2 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \]

Still to think about phases...
The model contains extra fermions and scalar **Leptoquarks**

(Alternative realisation contains diquarks - easier to see at LHC than TeVatron)
Mass scales

\[ m_f \approx \text{parameters} \times m_t \times \left[ \frac{1}{16\pi^2} \log \left( \frac{M^2}{M'^2} \right) \right]^n \]

Only determines ratio of masses

Works at all scales, what is the lowest?
Constraints

Tree level exchange of leptoquark can lead to flavour changing processes e.g.

\[
K^+ \rightarrow \mu^+ e^- \pi^+ \quad BR < 10^{-11}
\]

\[
\tau^+ \rightarrow K^0 e^+
\]

\[
\pi^+ \rightarrow e^+ \nu \text{ versus } \pi^+ \rightarrow \mu^+ \nu
\]

\[
\mu \rightarrow e \text{ conversion}
\]

\[
M^* \gtrsim 5 - 50 \text{ TeV}
\]
Dipole moments

Usually loop suppressed

\[ \sim \frac{1}{16\pi^2} \frac{m_f}{M^2} e \]

But for us mass is already a loop effect so no additional loop suppression

\[ \sim \frac{m_f}{M^2} e \]
Dipole moments

Usually loop suppressed

\[ \sim \frac{1}{16\pi^2} \frac{m_f}{M^2} e \]

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\[ \sim \frac{m_f}{M^2} e \quad M > \text{a few TeV} \]
Dipole moments

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\[ \sim \frac{1}{16\pi^2} \frac{m_f}{M^2} e \]

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\[ \sim \frac{m_f}{M^2} e \]

\( M > \text{a few TeV} \)
Conclusions

• Fermions have complicated mass hierarchy
• Many attempts exist to explain it
• Top is probably special, perhaps only top mass has a tree level Yukawa
• With extra scalars coupling to fermions top mass is communicated at loop level
• Interesting structure of fermion mass spectrum arises
• Predicts flavour changing processes
Conclusions

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• Predicts flavour changing processes
• Project X?