Motivation

The Model

Dark Matter

Minimal Little Higgs Model and Dark Matter

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Outline

1 Motivation
   - Little Hierarchy Problem
   - Little Higgs Model
   - One Example

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   - Bosonic Sector
   - Fermionic Sector: $Z_2$ broken
   - Fermionic Sector: $Z_2$ unbroken

3 Dark Matter
   - Dark Matter
   - Relic Abundance
   - Direct Detection
Explore New Physics beyond the Standard Model

- **The Electroweak Symmetry Breaking:**
  - **Higgs Mechanism**: described in the standard model and predicts the Higgs Boson.
  - **Dynamical Symmetry Breaking**: Technicolor, Topcolor or Walking Technicolor model.

- **The Dark Matter:**
  - What is the particle content of it? Scalars, Fermions or Gauge Bosons?
  - How does it couple to ordinary particles?

- **Other Puzzles:**
  - Hierarchies of fermion masses and their mixings?
  - The large mixings in the lepton sector versus the small mixings in the quark sector?
  - . . . . .
Radiative Corrections to the Higgs Boson Mass

- The mass of the Higgs field is not stable against radiative corrections:

\[
- \frac{3\lambda_t^2}{8\pi^2} \Lambda^2
\]

\[\Lambda = 10 \text{ TeV} \sim (2 \text{ TeV})^2\]

\[
\frac{3(3g^2+g_Y^2)}{64\pi^2} \Lambda^2
\]

\[\sim (700 \text{ GeV})^2\]

\[
\frac{\lambda^2}{16\pi^2} \Lambda^2
\]

\[\sim (500 \text{ GeV})^2\]

- Large hierarchy problem: SUSY, Technicolor, RS, ADD, · · ·
- Little hierarchy problem [LEP paradox] [Barbieri and Strumia, 2000]:
  - The mass of Higgs boson is less than 250 GeV.
  - The cutoff \(\Lambda\) of relevant higher-dimensional operators must be greater than 5-10 TeV.
Identify the Higgs doublet as a pseudo-Nambu-Goldstone boson (PNGB) of a spontaneously broken global symmetry.

Collective Symmetry Breaking: two or more couplings are needed to explicitly break the global symmetry.

Consequence: only logarithmically divergent potentials of the Higgs doublet are generated at one-loop level. The weak scale can be protected up to 5-10 TeV.
One Example

The VEV of a triplet spontaneously breaks $U(3)$ to $U(2)$:

$$U(3) \xrightarrow{\langle \phi \rangle = (0,0,f)^T} U(2).$$

5 Goldstone Bosons: one doublet and one singlet of $U(2)$:

$$\phi = \exp \left[ \frac{i}{f} \begin{pmatrix} h \end{pmatrix} \begin{pmatrix} 0 \\ f \end{pmatrix} \right] = \begin{pmatrix} h \\ f - \frac{h^2}{2f} \end{pmatrix} + \cdots$$

Using Yukawa couplings to explicitly break the global symmetry:

$$y_1 \bar{Q}_L \phi t_R + y_2 f \bar{\psi}_L \psi_R + h.c.$$  

$$= y_1 \bar{q}_L h t_R - \frac{y_1}{2f} \bar{\psi}_L h^2 t_R + y_1 f \bar{\psi}_L t_R + y_2 f \bar{\psi}_L \psi_R + h.c.$$

with $\bar{Q}_L \equiv (\bar{q}_L, \bar{\psi}_L)$, a triplet of $U(3)$.
The cutoff-squared terms are cancelled:

\[
\begin{align*}
m_f^2 &= m_f m_f^\dagger = \\
&= \begin{pmatrix}
y_1^2 f^2 \sin^2 \frac{h}{f} & y_1^2 f^2 \sin \frac{h}{f} \cos \frac{h}{f} \\
y_1^2 f^2 \sin \frac{h}{f} \cos \frac{h}{f} & y_1^2 f^2 \cos^2 \frac{h}{f} + y_2^2 f^2
\end{pmatrix}
\end{align*}
\]

\[
V_{CW} = -\frac{3}{16\pi^2} \Lambda^2 \text{Tr}[m_f^2] + \frac{3}{16\pi^2} \text{Tr}[m_f^4 \log \left( \frac{\Lambda^2}{m_f^2} + \frac{3}{2} \right)]
\]

One-Loop Effective Potential [Coleman and Weinberg, 1973]
Cancellations in the gauge sector:

\[ W, Z, \gamma \]

h  h

\[ W', Z' \]

h  h

Gauge symmetries in various little Higgs models \([SU(3)_c \text{ is not included}]\):

- The minimal moose model: \(SU(3) \times SU(2) \times U(1)\).
- The littlest Higgs model: \([SU(2) \times U(1)]^2\).
- The simplest little Higgs model: \(SU(3) \times U(1)\).

Predict \(Z', W'; t'\) and partners of other light quarks; extra scalars including triplets and singlets.
How about the most minimal extension of the standard model gauge group: $SU(2) \times U(1) \times U(1)$?

What is the symmetry for this cancellation?
Linear Realization

- The field content under the gauge symmetry:

<table>
<thead>
<tr>
<th></th>
<th>$SU(2)_w$</th>
<th>$U(1)_1$</th>
<th>$U(1)_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>2</td>
<td>$1/2$</td>
<td>$1/2$</td>
</tr>
<tr>
<td>$S$</td>
<td>1</td>
<td>$5/3$</td>
<td>$-5/3$</td>
</tr>
</tbody>
</table>

- The kinetic terms of scalars:

$$|(\partial_\mu + ig^a t^a W^a_\mu + i\frac{g'}{2\sqrt{2}}(B_{1\mu} + B_{2\mu}))H|^2 + |(\partial_\mu + i\frac{5g'}{3\sqrt{2}}(B_{1\mu} - B_{2\mu}))S|^2$$

- A $Z_2$ interchanging symmetry: $g_1 = g_2 = \sqrt{2}g'$

  $g'$ is the gauge coupling of $U(1)_Y$; $g$ is the gauge coupling of $SU(2)_w$.

- The $\Lambda^2$ contributions to scalar masses from gauge bosons are:

  $$V_g = \frac{3\Lambda^2}{64\pi^2} \left[ (3g^2 + g'^2)HH^\dagger + \frac{100}{9} g'^2 SS^\dagger \right] + \cdots ,$$

  $$\approx \frac{25g'^2\Lambda^2}{48\pi^2} \left[ HH^\dagger + SS^\dagger \right] \propto \phi \phi^\dagger \Rightarrow \text{Approximate } U(3) \text{ global symmetry}$$

  $$\sin^2 \theta_w = g'^2 / (g^2 + g'^2) \approx 0.23$$

[Chacko, Goh and Harnik, 2005]
Nonlinear Realization

- Write $H$ and $S$ together as a triplet of $U(3)$: $\phi = (H, S)^T$
- $\langle \phi \rangle = (0, 0, f)^T$ from underlying dynamics

  - global symmetry: $U(3) \rightarrow U(2)$
  - gauge symmetry: $SU(2)_w \times U(1)_1 \times U(1)_2 \rightarrow SU(2)_w \times U(1)_Y$

- Below the cutoff $\Lambda \approx 4\pi f$, the EFT contains 9-4=5 GB’s.
  - One is eaten by the massive neutral gauge boson: $B' \equiv (B_1 - B_2)/\sqrt{2}$
  - The other 4 become PNGB’s and identified as the Higgs doublet: $h$

  $$\phi^T = f(\frac{ih}{\langle h \rangle} \sin \frac{\langle h \rangle}{f}, \cos \frac{\langle h \rangle}{f}) = (ih, f - \frac{\langle h \rangle^2}{2f}) + \cdots .$$
The field dependent masses of gauge bosons are:

\[ M_W^2(h) = c_w^2 M_Z^2(h) = \frac{1}{2} g^2 f^2 \sin^2 \frac{\langle h \rangle}{f} \quad M_{B'}^2(h) = \frac{50}{9} g'^2 f^2 \cos^2 \frac{\langle h \rangle}{f} \]

Calculate the one-loop effective potential

\[ V_{CW} = \frac{3}{32 \pi^2} \Lambda^2 \text{Tr}[M_g^2] - \frac{3}{64 \pi^2} \text{Tr}[M_g^4 \log (\frac{\Lambda^2}{M_g^2} + \frac{3}{2})] \]

The Higgs mass contributions from the gauge sector:

\[ m_h^2 |_{g} = \frac{3 g'^2 \Lambda^2}{32 \pi^2} \left( \frac{27 - 118 s_w^2}{9 s_w^2} \right) + \frac{3 M_{B'}^4}{32 \pi^2 f^2} \left( \log \frac{\Lambda^2}{M_{B'}^2} + 1 \right) \]

\[ M_{B'} = 5 \sqrt{2} g' f / 3 \approx 0.8 f \]

For \( s_w^2 \text{ around 0.23} \), the \( \Lambda^2 \) term is even smaller than \( \log \Lambda \) term.

\[ m_h^2 |_{g} \approx -(87 \text{ GeV})^2 + (116 \text{ GeV})^2, \]

for \( f = 800 \text{ Gev} \), \( \Lambda = 10 \text{ TeV} \), \( s_w^2 = 0.23 \).
Z\textsubscript{2} Broken Model

- The field content under the gauge symmetry:

<table>
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<tr>
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</tr>
<tr>
<td>S</td>
<td>1</td>
<td>1</td>
<td>5/3</td>
<td>-5/3</td>
</tr>
<tr>
<td>q\textsubscript{L}</td>
<td>3</td>
<td>2</td>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>t\textsubscript{R}</td>
<td>3</td>
<td>1</td>
<td>2/3</td>
<td>2/3</td>
</tr>
<tr>
<td>b\textsubscript{R}</td>
<td>3</td>
<td>1</td>
<td>-1/3</td>
<td>-1/3</td>
</tr>
<tr>
<td>ψ\textsubscript{L}</td>
<td>3</td>
<td>1</td>
<td>7/3</td>
<td>-1</td>
</tr>
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</tr>
</tbody>
</table>

Only a colored vector-like quark added; gauge anomalies are still cancelled.

- The Yukawa couplings in the top sector are:

\[ \mathcal{L}_t = y_1 (\bar{q}_L, \bar{\psi}_L) \phi t_R + y_2 f \bar{\psi}_L \psi_R = y_1 (\bar{q}_L \bar{H} + \bar{\psi}_L S) t_R + y_2 f \bar{\psi}_L \psi_R + h.c. \]

Z\textsubscript{2} symmetry is manifestly broken
**Z\textsubscript{2} Broken Model**

- Higgs boson masses from the top sector:

  \[ m^2_h|t| = -\frac{3}{8\pi^2} y_t^2 m_t (\log \frac{\Lambda^2}{m_t^2} + 1) \]

  No \( \Lambda^2 \) contribution: collective breaking mechanism protects it.

- Spontaneously electroweak symmetry breaking:

  \[ m^2_h = m^2_h|g| + m^2_h|t| < 0 \]

  Minimizing the full potential, we get a light Higgs boson below 200 GeV.

- Spectrum:

  \[ m_t = y_t \langle h \rangle \quad y_t = \frac{y_1 y_2}{\sqrt{y_1^2 + y_2^2}} \quad m_{t'} = \sqrt{y_1^2 + y_2^2} \]

  \[ t_{L,m} \approx t_L \quad t_{R,m} \approx (y_2 t_R - y_1 \psi_R)/\sqrt{y_1^2 + y_2^2} \]

  \[ t'_L \approx \psi_L \quad t'_R \approx (y_1 t_R + y_2 \psi_R)/\sqrt{y_1^2 + y_2^2} \]

- Large mixing between the right-handed parts of \( t \) and \( t' \) quarks.

- Both \( Z \) and \( B' \) couple to \( t_R \) and \( t'_R \) with order one couplings.
Electroweak Precision Test

- At tree level, only the experimentally unmeasured top quark couplings to $W$ and $Z$ bosons are changed.

- At one-loop level, the strongest constraint comes from the $T$ parameter:

$$\alpha T = \frac{3y_1^2 y_2^2 m_t^2}{16\pi^2 y_2^2 m_t^2} \left( \log \frac{m_t'}{m_t^2} - 1 + \frac{y_1^2}{2y_2^2} \right)$$

[From PDG, $\alpha T < 1.2 \times 10^{-3}$ at 95% confidence level for $m_h < 300$ GeV.]

- For $y_1/y_2 < 3/4$, there is no bound on the symmetry breaking scale $f$. Hence, $f$ can be as low as 400 GeV (to have the cutoff $\Lambda$ above 5 TeV).
Signatures of the $Z_2$ Broken Model

- Two new parameters: $y_2$ and $f$ ($y_1$ is determined by $y_2$ and $y_t$).
- Predicts two new particles: $B'$ and $t'$.

\[ M_{B'} = 5\sqrt{2}g'f/3 \approx 0.8f \quad \quad m_{t'} = \sqrt{y_1^2 + y_2^2f} \geq 2f \]

- For $f \geq 400$ GeV, $M_{B'} \geq 300$ GeV. This possible light neutral gauge boson only couples to top quarks (nonuniversal).
- $B'$ can mainly be produced through loop diagrams at Hadron Colliders like:

- $B'$ decays to two top quarks. Mainly look for $t\bar{t} + 1$ jet.
To have a cold dark matter candidate, we need to keep this $Z_2$ to be unbroken to have stable particles. [Low and Cheng, 2003]

Introduce two more vector-like quarks:

$$
\begin{array}{c|cccc}
   & SU(3)_c & SU(2)_w & U(1)_1 & U(1)_2 \\
\hline
H & 1 & 2 & 1/2 & 1/2 \\
S & 1 & 1 & 5/3 & -5/3 \\
q^1_L & 3 & 2 & 1/6 & 1/6 \\
t^R & 3 & 1 & 2/3 & 2/3 \\
b^R & 3 & 1 & -1/3 & -1/3 \\
\psi^1_L & 3 & 1 & 7/3 & -1 \\
\psi^1_R & 3 & 1 & 7/3 & -1 \\
\psi^2_L & 3 & 1 & -1 & 7/3 \\
\psi^2_R & 3 & 1 & -1 & 7/3 \\
q^2_L & 3 & 2 & 1/6 & 1/6 \\
q^R & 3 & 2 & 1/6 & 1/6 \\
\end{array}
$$

Gauge anomalies are cancelled.
Z\textsubscript{2} invariant

\[\mathcal{L}_t = \frac{y_1}{\sqrt{2}} (\bar{q}_1 \mathcal{H} + \bar{\psi}_1 S) t_R + \frac{\sqrt{2}}{y_2} \psi_1 \psi_1^c \]

\[+ \frac{y_1}{\sqrt{2}} (\bar{q}_2 \mathcal{H} + \bar{\psi}_2 S^\dagger) t_R + \frac{\sqrt{2}}{y_2} \psi_2 \psi_2^c \]

\[+ \frac{y_3}{\sqrt{2}} f (\bar{q}_1 - \bar{q}_2) q_R^c + h.c. \]

Under the Z\textsubscript{2} transformation, we have

\[\text{Z}_2 : \quad q_1 \leftrightarrow q_2, \quad \psi_1 \leftrightarrow \psi_2, \quad q_R \rightarrow -q_R^c, \quad B_1 \leftrightarrow B_2, \quad S \leftrightarrow S^\dagger \]

and all other fields are invariant.
Mass Spectrum

$Z_2$ is exact; all particles are $Z_2$ eigenstates.

- **$Z_2$ even particles:**

  \[
  t: \quad t_{L,m} \approx t_L \quad t_{R,m} \approx \frac{y_2 t_R - y_1 (\psi_1^R + \psi_2^R)}{\sqrt{y_1^2 + y_2^2}} / \sqrt{y_1^2 + y_2^2} \quad m_t = \frac{y_1 y_2}{\sqrt{y_1^2 + y_2^2}} \langle h \rangle \\
  t': \quad t'_{+L} \approx \frac{\psi_{1L} + \psi_{2L}}{\sqrt{2}} \quad t'_{+R} \approx \frac{y_1 t_R + y_2 (\psi_1^R + \psi_2^R)}{\sqrt{y_1^2 + y_2^2}} m_{t'} \approx \sqrt{y_1^2 + y_2^2} f \geq 2 f
  \]

  The $\Lambda^2$ contribution to the Higgs mass from $t$ is cancelled by $t'_{+}$.

  All other standard model particles are also $Z_2$ even.

- **$Z_2$ odd particles:**

  \[
  t': \quad t'_{-L} \approx \frac{\psi_{1L} - \psi_{2L}}{\sqrt{2}} \quad t'_{-R} \approx \frac{\psi_1^R - \psi_2^R}{\sqrt{2}} m_{t'} = y_2 f \\\n  q': \quad q'_{-L} \approx \frac{q_1^L - q_2^L}{\sqrt{2}} \quad q'_{-R} \approx q_R' m_{q'} = y_3 f \\\n  B': \quad (B_1 - B_2)/2 \quad M_{B'} \approx 0.8 f
  \]

  For $y_2, y_3 \geq 1$, $B'$ is the lightest $Z_2$ odd particle and a potential dark matter candidate in this model.
From WMAP, the relic abundance of the dark matter is:

\[0.098 < \Omega_{dm} h^2 < 0.122 \, (2\sigma)\]

In the non-relativistic limit, \(\Omega_{dm} h^2\) is relating to sum of the quantities, \(a(X) = v_r \sigma(B'B' \rightarrow X)\), as

\[\Omega_{dm} h^2 \approx \frac{1.04 \times 10^9 \text{ GeV}^{-1}}{M_{pl}} \frac{x_F}{\sqrt{g^*}} \frac{1}{a_{tot}}\]

Approximately, only need to calculate \(a_{tot}\) and require:

\[a_{tot} \approx 0.81 \pm 0.09 \, \text{pb}\]
Couplings of $B'$ to Higgs Boson

Minimal Little Higgs Model

$$\frac{50}{9} g'^2 v$$

Hypercharge-like Gauge Boson [LHT and UED]

$$\frac{1}{2} g'^2 v$$
The leading processes for $B^\prime \ B^\prime$ annihilation into SM particles:

$$a(WW) = 2a(ZZ) = 2a(h^0 h^0) = \frac{2\pi\alpha^2}{3\cos^4\theta_w} \frac{5^4}{3^4} \frac{1}{M_{B^\prime}^2}$$

$$a(\bar{t}t) = \frac{16\pi\alpha^2}{3\cos^4\theta_w} \frac{5^4}{3^4} \frac{y_t^4}{y_2^4} \frac{M_{B^\prime}^2}{(M_{B^\prime}^2 + m_{t^\prime}^2)^2}$$

For $y_2 \gg 1$, $a(\bar{t}t)$ is negligible.

$[ M_{B^\prime} \approx 0.8 \ f \ m_{t^\prime} = y_2 \ f ] \ \ \ \ 0.098 < \Omega_{dm} h^2 < 0.122 \ (2\sigma) \ \Rightarrow \ a_{tot} \approx 0.81 \pm 0.09 \ pb$
Direct Detection

- Measure the recoil energy in the elastic scattering of dark matter particles with nuclei.

\[ \sigma_{SI} \approx \frac{0.35^2}{16\pi} \frac{g'^4}{M'_{B^0}} \frac{10^4}{3^4} \frac{m_p^4}{m_h^4} \approx 1.6 \times 10^{-44} \text{cm}^2 \left( \frac{1 \text{ TeV}}{M'_{B^0}} \right)^2 \left( \frac{100 \text{ GeV}}{m_h} \right)^4 \]

- Only contribute to spin-independent cross section

- Using the matrix element of quarks and gluons in a nucleon state: [Ellis, Olive, Santoso, Spanos, 2005]

- Box diagrams with the top quark propagating in the the loop also contribute to spin-dependent cross section.
Direct Detection

![Graph showing the cross-section (σ) in cm² as a function of the dark matter mass (M_{\chi}) in GeV. The graph includes lines for different experiments and models, such as CDMS II 2006, XENON10 2007, CDMS II Feb. 2008, Super-CDMS 25kg, Model m_{\chi}=120 GeV, and Model m_{\chi}=160 GeV.](image-url)
Summary

- A very simple little Higgs model has been constructed based on the $SU(2)_w \times U(1)^2$ gauge symmetry.

- A $Z_2$ interchanging symmetry is introduced between these two $U(1)$'s.

- For $Z_2$ broken case: only $B'$ and $t'$ appears in the EFT. The mass of $B'$ can be as light as 300 GeV.

- For $Z_2$ unbroken case:
  - $B'$ is a stable particle and can serve as a dark matter candidate.
  - The direct detection of this $B'$ dark matter is promising.
  - The $\sigma_{SI}(B'N)$ is two order of magnitude larger than a hypercharge-like neutral gauge boson dark matter candidate.