Holographic Truncated Space Model: Baryons

\[ \alpha \Pi(\zeta) \psi(\zeta) = \mathcal{M} \psi(\zeta) \]

\[ \Pi_\nu(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma_\zeta \right) \]

\[ \alpha^\dagger = \alpha, \quad \alpha^2 = 1, \]
\[ \gamma_\zeta^\dagger = \gamma_\zeta, \quad \gamma_\zeta^2 = 1, \]
\[ \{\alpha, \gamma_\zeta\} = 0. \]

\[ \begin{pmatrix} 0 & -\frac{d}{d\zeta} \\ \frac{d}{d\zeta} & 0 \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} - \begin{pmatrix} 0 & \nu + \frac{1}{2} \\ \nu + \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \mathcal{M} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}, \]

Frame-Independent LF Dirac Equation

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AdS/QCD
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Holographic Harmonic Oscillator Model: Baryons

\[(\alpha \Pi(\zeta) - \mathcal{M}) \psi(\zeta) = 0.\]

**Frame-Independent LF Dirac Equation**

\[
\Pi_\nu(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 - \kappa^2 \zeta \gamma_5 \right)
\]

\[
\Pi^\dagger_\nu(\zeta) = -i \left( \frac{d}{d\zeta} + \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 + \kappa^2 \zeta \gamma_5 \right)
\]

**Coupled Equations**

\[
\begin{pmatrix}
0 & -\frac{d}{d\zeta} \\
\frac{d}{d\zeta} & 0
\end{pmatrix}
\begin{pmatrix}
\psi_+ \\
\psi_-
\end{pmatrix}
= \begin{pmatrix}
0 & \frac{\nu + \frac{1}{2}}{\zeta} + \kappa^2 \zeta \\
\frac{\nu + \frac{1}{2}}{\zeta} + \kappa^2 \zeta & 0
\end{pmatrix}
\begin{pmatrix}
\psi_+ \\
\psi_-
\end{pmatrix}
= \mathcal{M}
\begin{pmatrix}
\psi_+ \\
\psi_-
\end{pmatrix}.
\]

\[- \frac{d}{d\zeta} \psi_+ - \frac{\nu + \frac{1}{2}}{\zeta} \psi_+ - \kappa^2 \zeta \psi_+ = \mathcal{M} \psi_+;
\]

\[
\frac{d}{d\zeta} \psi_- - \frac{\nu + \frac{1}{2}}{\zeta} \psi_- - \kappa^2 \zeta \psi_- = \mathcal{M} \psi_-.
\]

**HO due to Linear Potential!**

\[V = -\beta \kappa^2 \zeta\]
Holographic Harmonic Oscillator Model: Baryons

\[(\alpha \Pi(\zeta) - \mathcal{M}) \psi(\zeta) = 0,\]

\[
\Pi_\nu(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 - \kappa^2 \zeta \gamma_5 \right) \]

\[
\Pi^{\dagger}_\nu(\zeta) = -i \left( \frac{d}{d\zeta} + \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 + \kappa^2 \zeta \gamma_5 \right) \]

\[
(H_{LF} - \mathcal{M}^2) \psi(\zeta) = 0, \quad H_{LF} = \Pi^{\dagger} \Pi
\]

Uncoupled Schrodinger Equations

\[
\left( \frac{d^2}{d\zeta^2} + \frac{1 - 4\nu^2}{4\zeta^2} - \kappa^4 \zeta^2 - 2(\nu + 1)\kappa^2 + \mathcal{M}^2 \right) \psi_+(\zeta) = 0,
\]

\[
\left( \frac{d^2}{d\zeta^2} + \frac{1 - 4(\nu + 1)^2}{4\zeta^2} - \kappa^4 \zeta^2 - 2\nu\kappa^2 + \mathcal{M}^2 \right) \psi_-(\zeta) = 0,
\]

Solution

\[
\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^\nu(\kappa^2 \zeta^2),
\]

\[
\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu+1}(\kappa^2 \zeta^2),
\]

Same eigenvalue! \[\mathcal{M}^2 = 4\kappa^2(n + \nu + 1)\]

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Holographic Baryon Spectrum

\[ \psi(\zeta) = \kappa^{2+L} \sqrt{\frac{n!}{(n+L+2)!}} \zeta^{3+L} e^{-\kappa^2 \zeta^2/2} \left[ L_n^{L+1} (\kappa^2 \zeta^2) u_+ + \frac{\kappa \zeta}{\sqrt{n+L+2}} L_n^{L+2} (\kappa^2 \zeta^2) u_- \right] \]

Vacuum Energy Shift?

\[ \mathcal{M}^2 = 4\kappa^2(n + L + 2). \]
\[ \mathcal{M}^2 \rightarrow \mathcal{M}^2 - 4\kappa^2, \]
\[ \mathcal{M}^2 = 4\kappa^2(n + L + 1). \]

\[ J = L + 1/2 \text{ Regge trajectory} \]
\[ \kappa \simeq 0.49 \text{ GeV} \]

Same slope in \( L \) and \( n \)
Example: Evaluation of QCD Matrix Elements

Pion decay constant $f_\pi$ defined by the matrix element of EW current $J_W^+$:

$$\langle 0 | \bar{\psi}_u \gamma^+ (1 - \gamma_5) \psi_d | \pi^- \rangle = i \sqrt{2} P^+ f_\pi,$$

with

$$|\pi^-\rangle = |d\bar{u}\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left( b_c \dagger d_c u\uparrow - b_c \dagger d_c u\downarrow \right) |0\rangle.$$

Use light-cone expression:

$$f_\pi = 2 \sqrt{N_C} \int_0^1 dx \int \frac{d^2 k_\perp}{16\pi^3} \psi_{\bar{q}q/\pi}(x, k_\perp).$$

Lepage and Brodsky '80

Find:

$$f_\pi = \frac{\sqrt{3} \Lambda_{QCD}}{8 J_1(\beta_{0,1})} = 83.4 \text{ Mev},$$

for $\Lambda_{QCD} = 0.2 \text{ GeV}$ (fixed from the pion FF).

Experiment: $f_\pi = 92.4 \text{ Mev}.$

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Pion Decay Constant in HO Model

\[
f_\pi = \frac{\sqrt{3} \kappa}{8} = 86.6 \text{ MeV} \quad \kappa = 0.4 \text{ GeV.}
\]

\[
f_\pi = 92.4 \text{ MeV} \quad \text{Exp.}
\]
\[ F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} \Phi_P(z) J(Q, z) \Phi_P(z). \]

\[ \Phi(z) = \frac{\sqrt{2\kappa}}{R^{3/2}} z^2 e^{-\kappa^2 z^2/2}. \]

\[ J(Q, z) = z Q K_1(z Q). \]

\[ F(Q^2) = 1 + \frac{Q^2}{4\kappa^2} \exp \left( \frac{Q^2}{4\kappa^2} \right) Ei \left( -\frac{Q^2}{4\kappa^2} \right) \]

\[ Ei(-x) = \int_\infty^x e^{-\frac{t}{t}} \, dt. \]

**Space-like Pion Form Factor**

\( \kappa = 0.4 \text{ GeV} \)

\( \Lambda_{QCD} = 0.2 \text{ GeV}. \)

**Identical Results for both confinement models**

\[ F(Q^2) \rightarrow \frac{4\kappa^2}{Q^2} \]

High \( Q^2 \) from short distances

\[ z^2 = \zeta^2 = b_\perp^2 x (1 - x) = \mathcal{O}(\frac{1}{Q^2}) \]

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**Hadron Distribution Amplitudes**

\[ \phi(x_i, Q) \equiv \prod_{i=1}^{n-1} \int Q \, d^2 \vec{k}_\perp \, \psi_n(x_i, \vec{k}_\perp) \]

- Fundamental measure of valence wavefunction
- Gauge Invariant (includes Wilson line)
- Evolution Equations, OPE
- Conformal Expansion
- Hadronic Input in Factorization Theorems

Lepage, SJB
\begin{align*}
\phi(x, Q_0) &\propto \sqrt{x(1-x)} \\
\text{Increases PQCD leading twist prediction for} \\
F_\pi(Q^2) &\text{by factor 16/9}
\end{align*}

\textit{AdS/CFT:}

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\[ F_\pi(Q^2) = \int_0^1 dx \phi_\pi(x) \int_0^1 dy \phi_\pi(y) \frac{16\pi C_F \alpha_V(Q_V)}{(1-x)(1-y)Q^2} \]

\[ \phi(x, Q_0) \propto \sqrt{x(1-x)} \]

\[ \phi_{asymptotic} \propto x(1-x) \]

\[ Q^2 F_\pi(Q^2) \quad (\text{GeV}^2) \]

Normalized to \( f_\pi \)

**AdS/CFT:** Increases PQCD leading twist prediction for \( F_\pi(Q^2) \) by factor 16/9

**AdS/QCD**

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Neutral pair angular distribution sensitive to AdS/CFT distribution!

$$\phi^{AdS/QCD}_\pi(x) \propto [x(1-x)]^{1/2}$$

(a): $$\phi_\pi(x) \propto x(1-x)$$
(b): $$\phi_\pi(x) \propto [x(1-x)]^{1/4}$$
(c): $$\phi_\pi(x) \propto \delta(x - 1/2)$$
**Diffractive Dissociation of Pion into Quark Jets**

\[ M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp) \]

Measure Light-Front Wavefunction of Pion

**Minimal momentum transfer to nucleus**

**Nucleus left Intact!**

---

**E791 Ashery et al.**
Diffractive Dissociation of a Pion into Dijets

\[ \pi A \rightarrow \text{JetJet}A' \]

- E789 Fermilab Experiment Ashery et al
- 500 GeV pions collide on nuclei keeping it intact
- Measure momentum of two jets
- Study momentum distributions of pion LF wavefunction

\[ \psi_{q\bar{q}}^\pi(x, \vec{k}_\perp) \]

1.5 \leq k_t \leq 2.5 \text{ GeV/c}
Fluctuation of a Pion to a Compact Color Dipole State

Color-Transparent Fock State For High Transverse Momentum Di-Jets

Same Fock State Determines Weak Decay
**Key Ingredients in Ashery Experiment**

Local gauge-theory interactions measure transverse size of color dipole

$M \propto b_{\perp}$

**AdS/QCD**

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**Key Ingredients in Ashery Experiment**

Small color-dipole moment pion not absorbed; interacts with each nucleon coherently

**QCD COLOR Transparency**

\[ M_A = A \ M_N \]

\[ \frac{d\sigma}{dt} (\pi A \rightarrow q\bar{q}A') = A^2 \ \frac{d\sigma}{dt} (\pi N \rightarrow q\bar{q}N') \ F_A^2(t) \]

Target left intact

Diffraction, Rapidity gap

---

**AdS/QCD**

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- Fully coherent interactions between pion and nucleons.

- Emerging Di-Jets do not interact with nucleus.

\[ \mathcal{M}(A) = A \cdot \mathcal{M}(N) \]

\[ \frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0 \]

\[ \sigma \propto A^{4/3} \]

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Ashery E791:
Measure of pion LFWF in diffractive dijet production
Confirmation of color transparency,
gauge theory of strong interactions

Mueller, sjb; Bertsch et al; Frankfurt, Miller, Strikman

A-Dependence results: \( \sigma \propto A^\alpha \)

<table>
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<tr>
<th>( k_t ) range (GeV/c)</th>
<th>( \alpha )</th>
<th>( \alpha ) (CT)</th>
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<tr>
<td>1.25 &lt; ( k_t ) &lt; 1.5</td>
<td>1.64 ± 0.06 -0.12</td>
<td>1.25</td>
</tr>
<tr>
<td>1.5 &lt; ( k_t ) &lt; 2.0</td>
<td>1.52 ± 0.12</td>
<td>1.45</td>
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<tr>
<td>2.0 &lt; ( k_t ) &lt; 2.5</td>
<td>1.55 ± 0.16</td>
<td>1.60</td>
</tr>
</tbody>
</table>

\( \alpha \) (Incoh.) = 0.70 ± 0.1

Conventional Glauber Theory Ruled Out!

Factor of 7

AdS/QCD
Color Transparency

A. H. Mueller, sjb
Bertsch, Gunion, Goldhaber, sjb
Frankfurt, Miller, Strikman

- Fundamental test of gauge theory in hadron physics
- Small color dipole moments interact weakly in nuclei
- Complete coherence at high energies
- Clear Demonstration of CT from Diffractive Di-Jets
Key Ingredients in Ashery Experiment

Two-gluon exchange measures the second derivative of the pion light-front wavefunction

\[ M \propto \frac{\partial^2}{\partial^2 k_{\perp}} \psi_{\pi}(x, k_{\perp}) \]

AdS/QCD

Brodsky, Gunion, Frankfurt, Mueller, Strikman
Frankfurt, Miller, Strikman

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\[ \frac{d\sigma}{dk_t^2} \propto |\alpha_s(k_t^2)x_N G(u, k_t^2)|^2 \left| \frac{\partial^2}{\partial k_t^2} \psi(x, k_t) \right|^2 \]
THE $k_t$ DEPENDENCE OF DI-JETS YIELD

$$\frac{d\sigma}{dk_t^z} \propto |\alpha_s(k_t^z)G(x, k_t^z)|^2 \left| \frac{\partial^z}{\partial k_t^z} \psi(u, k_t) \right|^2$$

With $\psi \sim \frac{\phi}{k_t^2}$, weak $\phi(k_t^z)$ and $\alpha_s(k_t^z)$ dependences and $G(x, k_t^z) \sim k_t^{z/2^+}$: $\frac{d\sigma}{dk_t} \sim k_t^{-z}$

High Transverse momentum dependence consistent with PQCD, ERBL Evolution

Two Components?

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Narrowing of $x$ distribution at higher jet transverse momentum

$x$: distribution of diffractive dijets from the platinum target for $1.25 \leq k_t \leq 1.5$ GeV/$c$ (left) and for $1.5 \leq k_t \leq 2.5$ GeV/$c$ (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

Possibly two components: Nonperturbative and Perturbative (ERBL) Evolution

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Prediction from AdS/CFT: Meson LFWF

Harmonic Oscillator model

\[ \psi_M(x, k^2_{\perp}) \propto \sqrt{x(1-x)} \]

\[ \phi_M(x, Q_0) \propto \sqrt{x(1-x)} \]

\[ \kappa = 0.77 \text{GeV} \]

de Teramond, sjb

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New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT: Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances
Quark Interchange
(Spin exchange in atom-atom scattering)

\[
\frac{d\sigma}{dt} = \left| \frac{M(s,t)}{s^2} \right|^2
\]

\[M(t, u)_{\text{interchange}} \propto \frac{1}{ut^2}\]

Gluon Exchange
(Van der Waal -- Landshoff)

\[M(s, t)_{\text{gluon exchange}} \propto sF(t)\]

MIT Bag Model (de Tar), large \(N_c\), (‘t Hooft), AdS/CFT all predict dominance of quark interchange:

AdS/QCD

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AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions.

\[ M(t, u)_{\text{interchange}} \propto \frac{1}{ut^2} \]

**Non-linear Regge behavior:**

\[ \alpha_R(t) \to -1 \]
Why is quark-interchange dominant over gluon exchange?

Example: \( M(K^+p \to K^+p) \propto \frac{1}{ut^2} \)

Exchange of common \( u \) quark

\[ M_{QIM} = \int d^2k_\perp dx \, \psi_C^\dagger \psi_D^\dagger \Delta \psi_A \psi_B \]

Holographic model (Classical level):

Hadrons enter 5th dimension of \( AdS_5 \)

Quarks travel freely within cavity as long as separation \( z < z_0 = \frac{1}{\Lambda_{QCD}} \)

LFWFs obey conformal symmetry producing quark counting rules.
Comparison of Exclusive Reactions at Large $t$

B. R. Baller, (a) G. C. Blazey, (b) H. Courant, K. J. Heller, S. Heppelmann, (c) M. L. Marshak, E. A. Peterson, M. A. Shupe, and D. S. Wahl (d)

University of Minnesota, Minneapolis, Minnesota 55455

D. S. Barton, G. Bunce, A. S. Carroll, and Y. I. Makdisi

Brookhaven National Laboratory, Upton, New York 11973

and

S. Gushue (e) and J. J. Russell

Southeastern Massachusetts University, North Dartmouth, Massachusetts 02747

(Received 28 October 1987; revised manuscript received 3 February 1988)

Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.: $\pi^\pm p\rightarrow p\pi^\pm, K^\pm p\rightarrow pK^\pm, \pi^\pm p\rightarrow pp^\pm, \pi^\pm p\rightarrow \pi^+\Delta^\pm, \pi^\pm p\rightarrow K^+\Sigma^\pm, (\Lambda^0/\Sigma^0)K^0; K^\pm p\rightarrow pK^\pm; p^\pm p\rightarrow pp^\pm$. By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.
Hadron Dynamics at the Amplitude Level

- LFWFS are the universal hadronic amplitudes which underlie structure functions, GPDs, exclusive processes.

- Relation of spin, momentum, and other distributions to physics of the hadron itself.

- Connections between observables, orbital angular momentum

- Role of FSI and ISIs—Sivers effect
Some Applications of Light-Front Wavefunctions

• Exact formulae for form factors, quark and gluon distributions; vanishing anomalous gravitational moment; edm connection to anm

• Deeply Virtual Compton Scattering, generalized parton distributions, angular momentum sum rules

• Exclusive weak decay amplitudes

• Single spin asymmetries: Role if ISI and FSI

• Factorization theorems, DGLAP, BFKL, ERBL Evolution

• Quark interchange amplitude

• Relation of spin, momentum, and other distributions to physics of the hadron itself.
Weak Exclusive Decay

\[ \langle D | J^+ (0) | B \rangle \]

**Exact Formula**

Hwang, SJB

\[ \sum_n \psi_n \rightarrow B \rightarrow D^+ \]

Annihilation amplitude needed for Lorentz Invariance

\[ n = n' + 2 \]

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See also: Diehl, Feldmann, Jakob, Kroll

N=3 VALENCE QUARK ⇒ Light-cone Constituent quark model

N=5 VALENCE QUARK + QUARK SEA ⇒ Meson-Cloud model

Pasquini
The Generalized Parton Distribution $E(x, \zeta, t)$

The generalized form factors in virtual Compton scattering
\[ \gamma^*(q) + p(P) \rightarrow \gamma^*(q') + p(P') \] with $t = \Delta^2$ and
\[ \Delta = P - P' = (\zeta P^+, \Delta_\perp, (t + \Delta_\perp^2)/\zeta P^+) \], have been constructed in the light-front formalism. [Brodsky, Diehl, Hwang, 2001]

We find, under $q_\perp \rightarrow \Delta_\perp$, for $\zeta \leq x \leq 1$,

\[
\frac{E(x, \zeta, 0)}{2M} = \sum_a (\sqrt{1 - \zeta})^{1-n} \sum_j \delta(x - x_j) \int [dx][d^2k_\perp] \\
\times \psi_a^*(x'_j, k_\perp, \lambda_i) S_\perp \cdot L_\perp^{qj} \psi_a(x_i, k_\perp, \lambda_i),
\]

with $x'_j = (x_j - \zeta)/(1 - \zeta)$ for the struck parton $j$ and $x'_i = x_i/(1 - \zeta)$ for the spectator parton $i$.

The $E$ distribution function is related to a $S_\perp \cdot L_\perp^{qj}$ matrix element at finite $\zeta$ as well.
**Link to DIS and Elastic Form Factors**

**Form factors (sum rules)**

\[
\int dx \sum_q [H^q(x, \xi, t)] = F_1(t) \text{ Dirac f.f.} \\
\int dx \sum_q [E^q(x, \xi, t)] = F_2(t) \text{ Pauli f.f.} \\
\int dx \tilde{H}^q(x, \xi, t) = G_{A,q}(t), \quad \int dx \tilde{E}^q(x, \xi, t) = G_{P,q}(t)
\]

**Dis at \( \bar{\xi} = t = 0 \)**

\[
H^q(x,0,0) = q(x), \quad \tilde{q}(-x) \\
\tilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta \tilde{q}(-x)
\]

**Quark angular momentum (Ji’s sum rule)**

\[
J^q = \frac{1}{2} - J^G = \frac{1}{2} \int_{-1}^{1} x dx \left[ H^q(x, \xi, 0) + E^q(x, \xi, 0) \right]
\]

Veriﬁed using LFWFs

Diehl, Hwang, sjb

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**AdS/QCD**

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**Space-time picture of DVCS**

\[ \sigma = \frac{1}{2} x^- P^+ \]

The position of the struck quark differs by \( x^- \) in the two wave functions

Measure \( x^- \) distribution from DVCS:
Use Fourier transform of skewness, the longitudinal momentum transfer

\[ \zeta = \frac{Q^2}{2p \cdot q} \]

S. J. Brodsky\(^a\), D. Chakrabarti\(^b\), A. Harindranath\(^c\), A. Mukherjee\(^d\), J. P. Vary\(^e,a,f\)

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AdS/CFT Holographic Model

\[ \psi(\sigma, b_\perp) \]

\[ |b_\perp|(\text{GeV}^{-1}) \]

\[ \sigma = ct - z \]
\[ \tau = t + z/c \]

3-dimensional photograph:
meson LFWF at fixed LF Time

AdS/QCD
Stan Brodsky, SLAC
Hadron Optics

\[ A(\sigma, b_{\perp}) = \frac{1}{2\pi} \int d\zeta e^{i\sigma\zeta} \tilde{A}(b_{\perp}, \zeta) \]

\[ \sigma = \frac{1}{2} x - P^+ \]
\[ \zeta = \frac{Q^2}{2p \cdot q} \]

DVCS Amplitude using holographic QCD meson LFWF

\[ \Lambda_{QCD} = 0.32 \]

The Fourier Spectrum of the DVCS amplitude in \( \sigma \) space for different fixed values of \(|b_{\perp}|\). GeV units

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Features of Light-Front Formalism

- *Hidden Color* Of Nuclear Wavefunction
- *Color Transparency, Opaqueness*
- *Intrinsic glue, sea quarks, intrinsic charm*
- Simple proof of Factorization theorems for hard processes (Lepage, sjb)
- *Direct mapping to AdS/CFT* (de Teramond, sjb)
- New Effective LF Equations (de Teramond, sjb)
- Light-Front Amplitude Generator
String Theory

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

AdS/QCD

Semi-Classical QCD / Wave Equations

Conformal behavior at short distances + Confinement at large distance

Boost Invariant 3+1 Light-Front Wave Equations

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

Goal: First Approximant to QCD QCD at the Amplitude Level

Holography

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Light-Front QCD
Heisenberg Equation

\[ H_{QC}^{LC} |\psi_h\rangle = \mathcal{M}_h^2 |\psi_h\rangle \]

<table>
<thead>
<tr>
<th>Sector</th>
<th>1 ( q\bar{q} )</th>
<th>2 ( gg )</th>
<th>3 ( q\bar{q} g )</th>
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Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations

Vary, Harinandrath, sjb
**AdS/QCD**

- New initial approximation to QCD based on conformal invariance, and confinement

- Underlying principle: Semi-Classical QCD

- AdS5: Mathematical representation of conformal gauge theory

- Challenges: chiral symmetry, heavy quark masses

- Systematically improve using DLCQ

- Successes: Hadron spectra, LFWFs, dynamics

- QCD at the Amplitude Level
Outlook

- Only one scale $\Lambda_{QCD}$ determines hadronic spectrum (slightly different for mesons and baryons).
- Ratio of Nucleon to Delta trajectories determined by zeroes of Bessel functions.
- String modes dual to baryons extrapolate to three fermion fields at zero separation in the AdS boundary.
- Only dimension 3, $\frac{9}{2}$ and 4 states $\overline{q}q$, $qqq$, and $gg$ appear in the duality at the classical level!
- Non-zero orbital angular momentum and higher Fock-states require introduction of quantum fluctuations.
- Simple description of space and time-like structure of hadronic form factors.
- Dominance of quark-interchange in hard exclusive processes emerges naturally from the classical duality of the holographic model. Modified by gluonic quantum fluctuations.
- Covariant version of the bag model with confinement and conformal symmetry.
AdS/CFT and QCD
Bottom-Up Approach

• Nonperturbative derivation of dimensional counting rules of hard exclusive glueball scattering for gauge theories with mass gap dual to string theories in warped space:
  Polchinski and Strassler, hep-th/0109174.

• Deep inelastic structure functions at small \( x \):
  Polchinski and Strassler, hep-th/0209211.

• Derivation of power falloff of hadronic light-front Fock wave functions, including orbital angular momentum, matching short distance behavior with string modes at AdS boundary:
  Brodsky and de Téramond, hep-th/0310227. E. van Beveren et al.

• Low lying hadron spectra, chiral symmetry breaking and hadron couplings in AdS/QCD:
• Gluonium spectrum (top-bottom):
Csaki, Ooguri, Oz and Terning, hep-th/9806021; de Mello Kock, Jevicki, Mihailescu and Nuñez, hep-th/9806125; Csaki, Oz, Russo and Terning, hep-th/9810186; Minahan, hep-th/9811156; Brower, Mathur and Tan, hep-th/0003115, Caceres and Nuñez, hep-th/0506051.

• D3/D7 branes (top-bottom):

• Other aspects of high energy scattering in warped spaces:
Giddings, hep-th/0203004; Andreev and Siegel, hep-th/0410131; Siopsis, hep-th/0503245.

• Strongly coupled quark-gluon plasma ($\eta/s = 1/4\pi$):
A Theory of Everything Takes Place

String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists’ best hope for unifying gravity and quantum theory -- into a single coherent theory.

I thought I had discovered the Theory of Everything But everything canceled out!