N-flationary magnetic fields

Lorenzo Sorbo

UMass Amherst

UC Davis

01/22/2007
Plan of the talk

• Inflation in QFT and in String Theory
• pNGBs and inflation
• cosmological magnetic fields
• pNGB inflation produces cosmological magnetic fields
The success of inflation

Cosmological observations strongly support (at the % level)

- A spatially flat Universe
- A (quasi) scale invariant spectrum of primordial perturbations

All of this in excellent agreement with the predictions of inflation
But what are the properties of inflation?

...and in particular, when did it take place?

...or, equivalently, at what energy scales did it take place?
What is the energy scale of inflation?

An upper bound from CMB:

\[ 10^{-5} \sim \left( \frac{\delta T}{T} \right)_{\text{CMB}} \sim \frac{\sqrt{\rho_{\text{infl}}}}{M_{\text{Pl}}^2} \frac{1}{\sqrt{\varepsilon}} \]

Where \[ \varepsilon \sim \frac{\rho_{\text{infl}} + p_{\text{infl}}}{\rho_{\text{infl}}} \] has to be <1 during inflation

\[ \rho_{\text{infl}}^{1/4} < 10^{16} \text{ GeV} \]
Unless the inflationary dynamics is very finely tuned, $\epsilon$ is not “too small”

The inflationary scale might be just a factor of $\sim 1000$ smaller than the gravity scale

Inflation can be a probe of the physics of the fundamental theory of gravity

Strings
“Inflation in String Theory”

...a challenge!

At “low” energies String Theory must reduce to Quantum Field Theory...

...and finding good models of inflation in QFT is very difficult

...it is even more difficult to find which of those models can come from string theory

so let us start by looking at inflation in QFT...
Requirements for Inflation

In simplest models, inflation is driven by a scalar field $\phi$ with potential $V(\phi)$.

Requirements on $V(\phi)$:

$$\varepsilon = \frac{M_P^2}{2} \frac{V'(\phi)^2}{V(\phi)^2} \simeq 10^{-2}$$
$$\eta = M_P^2 \frac{V''(\phi)}{V(\phi)} \simeq 10^{-2}$$

$\Rightarrow V(\phi)$ has to be flat
The enemy: radiative corrections

Quantum effects bring couplings to be $O(1)$ in units of the cutoff of the theory ($\Rightarrow M_P$)

Spoil flatness of $V(\phi)$

Our ally: symmetries

Supersymmetry is an option...

...but supergravity corrections generate

$\text{mass}^2 = O(V/M_P^2)$

$\eta = O(1)$ (\textit{\eta problem})
A field $\phi$ has a *shift symmetry* if the theory that describes it is invariant under the transformation 

$$\phi \rightarrow \phi + c$$

If this symmetry is exact, the only possible potential for $\phi$ is $V(\phi) = \text{constant}$ (i.e. a cosmological constant...)

*pNGBs and inflation*
now let us break the shift symmetry a little bit...

the potential for $\varphi$ changes to

$$V(\varphi) = \mu^4 \left[ \cos(\varphi/f) + 1 \right]$$

$f$ measures the breaking of the symmetry

in the limit $f \to \infty$ the symmetry is restored

Freese et al 1990
The cosine potential: where does it come from?

- Theory with a spontaneously broken global U(1)

\[ \mathcal{L} = \partial_\mu H^* \partial^\mu H - \lambda \left( |H|^2 - v^2 \right)^2 \]

- Decompose \( H = (v + \delta H) e^{i\phi/v} \)

where \( \delta H \) is massive and \( \phi \) is a massless Goldstone boson

- The global U(1) is broken e.g. by gravitational interactions

\[ \delta \mathcal{L} = \frac{1}{M_P} (H + H^*)^5 \]

- A potential is generated:

\[ \delta V(\varphi) \simeq \frac{v^5}{M_P} \cos \left( \frac{5 \varphi}{v} \right) \]
Because of its radiative stability,

A pNGB is an excellent candidate for inflation in Quantum Field Theory

(Natural Inflation)
What about String Theory?

😊 String Theory contains a lot of pNGBs  
(many inflaton candidates)

☺ Not every pNGB can come from String Theory  
(“swampland”):

String theory appears to require

\[ 0 < f \lesssim M_p \]

Banks, Dine, Fox and Gorbatov 2003
The data: $f > 3.5 M_P$

In contradiction with the requirement from String Theory!

from Savage et al, 2006
Way out: use extra fields (i)

Racetrack inflation:

Blanco-Pillado et al 2004

inflaton is mixture of a pNGB and a modulus
Way out: use extra fields (ii)

With two pNGBs:

\[ V = \Lambda_1^4 \left[ 1 - \cos \left( \frac{\theta}{f_1} + \frac{\rho}{g_1} \right) \right] + \Lambda_2^4 \left[ 1 - \cos \left( \frac{\theta}{f_2} + \frac{\rho}{g_2} \right) \right] \]

Kim, Nilles and Peloso 2004
Way out: use extra fields (iii)

Use *several* pNGBs

N-flation

(assisted inflation with pNGBs)
Start from $N$ pNGBs:

$$\mathcal{L} = -\sqrt{-g} \sum_{i=1}^{N} \left\{ \frac{1}{2} (\partial \phi_i)^2 + \Lambda_i^4 \left[ 1 \cos(\phi_i/f_i) \right] \right\}$$

Assume that all the $\phi_i$, all the $f_i$ and all the $\Lambda_i$ are equal:

$$\mathcal{L} = -\sqrt{-g} \left\{ \frac{N}{2} (\partial \phi)^2 + \Lambda^4 \left[ 1 \cos(\phi/f) \right] \right\}$$

Canonically normalized field $\Phi = \sqrt{N} \phi$

$$\mathcal{L} = -\sqrt{-g} \left\{ \frac{1}{2} (\partial \Phi)^2 + \Lambda \left[ 1 \cos\left(\frac{\Phi}{\sqrt{N} f} \right) \right] \right\}$$

Can be $> M_P$ even if $f < M_P$!
How many pNGBs can String Theory have?

In principle, up to $10^5$

...but if we want to keep radiative corrections to $M_P$ under control,

$N \lesssim 200$

is needed
How many pNGBs do we need?

**Dimopoulos et al 2005**

Assuming $\varphi_1 = \varphi_2 = \ldots = \varphi_N \ll f$:

$$N \sim 200$$

(marginally compatible with radiative stability requirements) (!)

**Liddle & Kim 2006**

Assuming $\varphi_1, \varphi_2, \ldots, \varphi_N \ll f$,

$\varphi_1, \varphi_2, \ldots, \varphi_N$ homogeneously distributed

$$N \sim 600$$

(Things get worse)
However things are not so bad if we drop the requirement $\phi_i << f$...

This requirement corresponds to approximating

$$1 - \cos(x) \sim x^2/2...$$

...that corresponds to requiring a large effective $f$

...that corresponds to requiring a large $N$

If we drop this requirement $N$ as small as 50 is enough to have enough N-flation

VERY PRELIMINARY!
In any case, pNGBs seem to play a role in many models of inflation in String Theory...
pNGBs are coupled to the electromagnetic field

Magnetic fields can be produced by the rolling pNGBs at inflation

\[ \mathcal{L} \supset \sum_{i=1}^{N} \alpha \frac{\phi_i}{4M_p} F_{\mu \nu} \tilde{F}^{\mu \nu} \]

M. Anber, LS 2006
Cosmological magnetic fields

Observed with a number of techniques

- Zeeman splitting
- Faraday rotation
- Synchrotron emission

- In the Galaxy (~kpc), solid evidence of $B \approx 2-4 \, \mu G$

- In clusters (~10-100 kpc), some evidence of $B \approx 1 \, \mu G$
  (but people consider also $B \sim nG$)

- At larger scales, situation more confused
...and their origin is unknown!

Main question: 
primordial or astrophysical?

- Difficult to produce
  - Constraints from CMB on primordial B amplitude

- Smaller coherence lengths
  - Easier to obtain: dynamo mechanism

cosmological magnetic fields
The dynamo

Uses differential rotation of plasma in galaxies to amplify an existing “seed” B field

\[ \frac{\partial B}{\partial t} = \nabla \times (v \times B) + \frac{1}{4\pi\sigma} \nabla^2 B. \]

velocity of plasma

conductivity

How large a seed field is needed?

cosmological magnetic fields
Cosmological magnetic fields

$|B|$ doubles at every galaxy rotation

Exponential amplification

Exponential uncertainties:

Need $10^{-23}$ G at 1 Mpc

Giovannini

Enough $10^{-30}$ G at 10 kpc ✓

Davis

Very difficult to produce even such weak fields...
As in Dimopoulos et al 2005, simplify analysis by assuming
\[ |\varphi_1| = |\varphi_2| = ... = \Phi / \sqrt{N} \]

Electromagnetic field coupled to the sum of the pNGBs

the direction of rolling of the pNGBs matters:

\[ \gamma = (N_+ - N_-) / N \]

where

\[ N_+ = \# \text{ of pNGBs with } \varphi > 0 \quad N_- = \# \text{ of pNGBs with } \varphi < 0 \]

\[ [-1 < \gamma < 1] \]
Main equation:

\[ \frac{\partial^2 F_\pm}{\partial \tau^2} + \left( k^2 \pm \frac{\alpha \gamma \sqrt{N}}{M_P} \frac{d\Phi}{d\tau} k \right) F_\pm = 0 \]

\( F_\pm = \text{>ve and <ve helicity comoving modes of the magnetic field} \) 

\( (\tau = \text{conformal time}) \)

One of the two modes has a \textit{negative, time dependent} “mass term”

\textbf{Exponential amplification of one helicity mode}
The result depends only on one combination of parameters

\[ \xi \equiv |\alpha \gamma| \sqrt{N \varepsilon/2} \]

where \( \varepsilon \) is the slow-roll parameter

\[ \varepsilon \sim \dot{\phi}^2/V(\phi) \]
Our result

\[ F(\tau, \vec{k}) \approx \sqrt{\frac{k}{2}} \left( \frac{k}{2\xi aH} \right)^{1/4} e^{-2\sqrt{2\xi k/aH}} e^{\pi \xi} \]
(Comoving) Energy density in magnetic modes

$$\rho_{\text{magn}} \sim k^3 F^2$$

$$\rho_{\text{magn}} \sim H^4 \left(\frac{k}{H}\right)^{4.5} e^{2\pi \xi} \quad \text{for} \quad k \lesssim H$$

Power is concentrated in short wavelength modes
A Constraint

The energy in the magnetic field should not exceed the energy in the inflaton condensate!

If insist on COBE normalization ($H \sim 10^{13}\text{GeV}$),

If require just $H > 10^{-3}\text{eV}$,
...implication for model building:

In models of pNGB inflation in String Theory

If $\alpha \gamma \sqrt{N} \geq 100$

the backreaction of the magnetic field 
*during inflation*

*cannot be neglected!*

(difficult to tell what happens - nonlinearities)
Can we start the dynamo with these fields?

Garretson, Field and Carroll 1992

If we obey the constraints above

AND

If after inflation the magnetic field does not evolve
(apart from effects related to expansion of the Universe)

THEN

The resulting magnetic field today is too weak
to be the one we observe
Evolving the field in the cosmic plasma

The magnetic field produced has *maximal helicity*

\[ \mathcal{H} \equiv \int_V d^3x \mathbf{B} \cdot \mathbf{A} \]

and helicity is (almost) conserved for large conductivities

\[ \frac{d\mathcal{H}}{dt} = -\frac{1}{4\pi\sigma} \int_V d^3x \mathbf{B} \cdot (\nabla \times \mathbf{B}) \approx 0 \]

Dissipative processes suppress power at small scales

In order to conserve helicity, power has to go to larger scales:

*Inverse cascade*
Numerical solutions

Evolution of the comoving magnetic field:

without helicity

with helicity

From Jedamzik and Banerjee 2004

pNGB inflation $\Rightarrow$ magnetic fields
Scalings:

- Coherence length $\propto \tau^{2/3}$
- Magnetic field strength $\propto \tau^{-2/3}$
- Spectral index for scales $>\text{coherence length}$: constant

(property of self-similarity)
Final value of the magnetic field (before the dynamo)

\[ B \simeq 10^{-33} \frac{e^{\pi \xi}}{\xi^{17/12}} \left( \frac{T_{RH}}{10^9 \text{GeV}} \right)^{11/36} \left( \frac{l_{\text{phys}}}{10 \text{ kpc}} \right)^{-9/4} \text{ G} \]

\[ \xi \geq 2 \]

is sufficient to initiate the dynamo
In terms of the original parameters

\[ \alpha \gamma \sqrt{N} \geq 10 \]

Enough magnetic field for \( \alpha \) and/or \( \gamma \sqrt{N} \) of \( O(\text{few}) \)!
One obvious possibility: $N = \text{few, } \alpha \sim 10$

More difficult: insist on $\alpha = 1$

e.g. for $N = 600$ (as required by Liddle and Kim 2006),
need $N_+ \sim 420$ and $N_- \sim 180$...

...rather improbable, if the theory is exactly symmetric wrt $\phi_i \rightarrow -\phi_i$

Probability $\propto \exp\{-\frac{(N_+ - N_-)^2}{2N}\} \approx 10^{-20}$!
...but an asymmetry can exist:

(V(\phi) = \Lambda_1^4 \cos a\phi + \Lambda_2^4 \cos b\phi + \Lambda_3^4 \cos (a - b) \phi

(Blanco-Pillado et al 2004)
Conclusions

• pNGBs are very well motivated candidates for inflation

• By taking into account MHD effects, they can lead to the production of the observed cosmic magnetic fields

• To do: effects of pNGB perturbations on the magnetic fields (flat spectrum)