Nongaussianity from Tachyonic Preheating in Hybrid Inflation

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Outline

1. Inflation, Fluctuations and Gaussianity
2. Hybrid Inflation and Tachyonic Preheating
3. Cosmological Perturbation Theory
4. Nongaussianity and Constraints
5. Implications for Brane Inflation
Introduction

- Hybrid inflation\textsuperscript{a} (and Inverted Hybrid Inflation\textsuperscript{b}) models are attractive from particle physics perspective.

- Appear easy to embed into SUSY, string theory (F-, D-, P-term inflation, KKLMMT)

- Inflation ends with nonperturbative amplification of fluctuations called tachyonic preheating.

- Preheating may generate large scale curvature perturbations without violation of causality.\textsuperscript{c}

- Nonadiabatic pressures at second order may give rise to large nongaussianity.\textsuperscript{d}

- Nongaussianity can be a powerful tool to discriminate between (or constrain) models of inflation.

\begin{itemize}
  \item \textsuperscript{a} Linde, Phys. Rev. D \textbf{49}, 748 (1994).
  \item \textsuperscript{b} Lyth & Stewart, Phys. Rev. D \textbf{54}, 7186 (1996).
  \item \textsuperscript{c} Brandenberger & Finelli, Phys. Rev. Lett. \textbf{82}, 1362 (1999).
\end{itemize}
Part 1: Inflation, Fluctuations and Gaussianity

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Background and Fluctuations

$$\varphi(t, \vec{x}) = \varphi_0(t) + \delta \varphi(t, \vec{x})$$

**Classical Background:** $\varphi_0$

- **Slow Roll:** $\dot{\varphi}_0 \ll H \varphi_0$
  
  $$\Rightarrow ds^2 \simeq -dt^2 + e^{2Ht} d\vec{x}^2$$

- **Requires flat potentials:**
  
  $$\epsilon \simeq \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2 \ll 1$$

  $$\eta \simeq M_p^2 \frac{V''}{V} = \frac{m_\varphi^2}{3H^2} \ll 1$$

- **We require** $\epsilon, \eta \ll 1$ **for** $Ht \simeq 60$ e-folds.

**Quantum Fluctuations:** $\delta \varphi$

- **Vacuum fluctuations of** $\delta \varphi$ **generated on small scales** $k \gg aH$.

- **Redshifted** by the expansion
  
  $$k_{\text{phys}} = \frac{k}{a} \sim k e^{-Ht}.$$  

- **Become classical at horizon crossing** $k = aH$.

- **Fluctuations re-enter horizon after reheating.**
Evolution of Scales During Inflation

Quantum Fields in deSitter

\[ ds^2 = -dt^2 + e^{2Ht} dx^2 \]

\[ \chi(t, x) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[ a_k \chi_k(t) e^{ikx} + a_k^* \chi_k^*(t) e^{-ikx} \right] \]

★ Mode functions satisfy KG equation:

\[ \ddot{\chi}_k + 3H \dot{\chi}_k + \left[ \frac{k^2}{a^2} + m^2 \right] \chi_k = 0 \]

★ Initial data fixes the vacuum \( a_k |0\rangle = 0 \).

★ Bunch-Davies vacuum choice corresponds to small scale Minkowski space fluctuations:

\[ \chi_k \approx e^{-ikt} \sqrt{2k} \quad \text{for} \quad k \gg aH \]

★ Large scale behaviour depends crucially on \( m/H \).
Heavy and Light Fields in deSitter

★ On large scales $k \ll aH$ have:

$$|\chi_k(t)| \approx \begin{cases} \frac{H}{\sqrt{2k}} & \text{if } m \ll H; \\ \frac{2}{a^{-3/2}} \sqrt{m} & \text{if } m \gg H. \end{cases}$$

★ Inflaton fluctuations are light ($\eta \ll 1$) so have scale invariant large scale fluctuations:

$$\langle (\delta \varphi)^2 \rangle = \int \frac{d^3k}{(2\pi)^3} |\chi_k(t)|^2 \approx \int d\ln k \left( \frac{H}{2\pi} \right)^2 = P_\varphi(k).$$

★ Heavy fields have exponentially damped ($\sim e^{-3Ht/2}$) large scale fluctuations.
Curvature Perturbation

★ Quantum matter fluctuations induce metric fluctuation:

\[ \varphi(t, \vec{x}) = \varphi_0(t) + \delta \varphi(t, \vec{x}) \]

\[ \Rightarrow \quad g_{\mu\nu}(t, \vec{x}) = g_{\mu\nu}^{(0)}(t) + \delta g_{\mu\nu}(t, \vec{x}). \]

★ Can induce fictitious metric fluctuations by performing small coordinate transformations: \( x^\mu \rightarrow x^\mu + \xi^\mu \).

★ Physical observables must be gauge invariant.

★ Introduce the curvature perturbation \( \phi = \delta g_{00} \):

\[ \zeta \cong -\phi - \frac{H}{\varphi_0} \delta \varphi \]

★ The basic observables are the correlators: \( \langle \zeta \zeta \cdots \zeta \rangle \).
Spectrum and Gaussianity

★ The spectrum (two-point function) is almost scale invariant on large scales $k \ll aH$:

$$\langle \zeta_k \zeta_{k'} \rangle = \frac{1}{2 \epsilon} \left( \frac{H}{M_p} \right)^2 \frac{1}{2k^3} \left( \frac{k}{aH} \right)^{n-1} \delta^3(k + k')$$

★ Spectral index: $n - 1 = 2\eta - 6 \epsilon \ll 1$.

★ To linear order $\zeta$ contains only one $a_k, a_k^\dagger$:

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = 0$$

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = \langle \zeta_{k_1} \zeta_{k_2} \rangle \langle \zeta_{k_3} \zeta_{k_4} \rangle + \text{perms}$$

★ Two-point correlator is the only independent statistics.
Nongaussianity

★ **Gaussian** fluctuations: the connected part of the $n$-point functions vanishes for $n \geq 3$.

★ At linear order in perturbation theory the fluctuations are exactly gaussian.

★ Nongaussianity is expected due to **nonlinearities** in the KG and gravity equations.

★ The three-point function (bispectrum) is the lowest order statistics which can discriminate between gaussianity and nongaussianity:

\[
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = (2\pi)^{-3/2} B(k_i) \delta^3(k_1 + k_2 + k_3)
\]
Nonlinearity Parameter

* Usually nongaussianity is parametrized in terms of the nonlinearity parameter $f_{NL}$ as

$$\zeta = \zeta_g - \frac{3}{5} f_{NL} (\zeta_g^2 - \langle \zeta_g^2 \rangle)$$

* Yields a nontrivial bispectrum:

$$B(k_i) \approx -\frac{6}{5} f_{NL} [P(k_1)P(k_2) + \text{perms}]$$

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = (2\pi)^{-3/2} B(k_i) \delta^3(k_1 + k_2 + k_3)$$

$$\langle \zeta_{k_1} \zeta_{k_2} \rangle = P(k_i) \delta^3(k_1 + k_2)$$

* WMAP analysis constrains $|f_{NL}| \lesssim 100$. $^a$

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Nonlinearity Parameter

- Various scenarios for generating $\zeta$ give distinct predictions for $f_{NL}$.
- Measurement of $f_{NL}$ can discriminate between different models.
- Expect $f_{NL} \sim n - 1$ for the simplest models, which is unlikely to ever be detectable.
- Can get observably large nongaussianity from:
  - Curvaton mechanism.$^a$
  - Single field models with small inflaton sound speed.$^b$
    (For example the D-celleration model.$^c$)
  - Preheating.$^d$
  - ...

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$^b$ Chen et al., arXiv:hep-th/0605045
Part 2: Hybrid Inflation and Tachyonic Preheating

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Hybrid Inflation: Potential

\[ V(\varphi, \sigma) = \frac{\lambda}{4} (\sigma^2 - v^2)^2 + \frac{g^2}{2} \varphi^2 \sigma^2 + \frac{m^2}{2} \varphi^2 \]

\[ = \frac{\lambda v^4}{4} + \frac{1}{2} \left( g^2 \varphi^2 - \lambda v^2 \right) \sigma^2 + \frac{\lambda}{4} \sigma^4 + \frac{m^2}{2} \varphi^2 = m_{\sigma}^2 \]

Herdeiro et al., JHEP 0112, 027 (2001).
The tachyon is trapped in the false vacuum:
\[ \langle \sigma \rangle \equiv \sigma_0 = 0. \]

Potential along the inflationary trajectory:
\[ V_{\text{inf}} = \frac{\lambda v^4}{4} + \frac{1}{2} m^2 \varphi^2 \approx \frac{\lambda v^4}{4} \]

Slow roll solutions:
\[ \langle \varphi \rangle \equiv \varphi_0(t) \approx \frac{\lambda^{1/2} v}{g} \left( \frac{a(t_c)}{a(t)} \right)^\eta \]
\[ 3H^2 \approx \frac{\lambda v^4}{(4M_p^2)} \]

Slow roll parameters never get large: \( \dot{\epsilon} < 0, \eta = \text{const.} \)
Tachyonic Preheating

★ Tachyon mass-squared:

\[ m_\sigma^2 = g^2 \phi_0^2 - \lambda v^2 \cong -2\lambda v^2 \eta H(t - t_c) \equiv -cH^2 N \]

★ Tachyonic preheating: transfer of energy from the false vacuum \( \lambda v^4/4 \) to the fluctuations \( \delta^{(1)} \sigma_k \).

Domain Walls

★ Symmetry breaking leads to **domain walls**.

★ At late times universe consists of many domains with $\sigma \sim \pm \nu$.

★ Even at late times the tachyon averages to zero $\langle \sigma \rangle = \sigma_0 = 0$ over many domains.

★ Domain walls will overclose the universe so one should add symmetry breaking terms or consider a complex tachyon which gives cosmic strings...

Tachyon Dynamics

★ The tachyon mass-squared varies linearly with the number of e-foldings:

\[ m_\sigma^2 \approx -cH^2 N \]

★ At early times \( m_\sigma^2 > 0 \):
  - Large scale tachyon fluctuations get damped as \( a^{-3/2} \) during any e-foldings where \( m_\sigma^2 > H^2 \).
  - Scale invariant large scale fluctuations for \( m_\sigma^2 < H^2 \).

★ At late times \( m_\sigma^2 < 0 \):
  - Large scale tachyon fluctuations are exponentially amplified.
  - Within a time \( t_* \) the energy from the false vacuum is transferred into the large scale fluctuations \( \delta \sigma_k \).
During both inflation and the (early) instability phase the tachyon mode functions obey:

\[
\frac{d^2}{dN^2} \delta^{(1)} \sigma_k + 3 \frac{d}{dN} \delta^{(1)} \sigma_k + \left[ \frac{k^2}{H^2} e^{-2N} - cN \right] \delta^{(1)} \sigma_k = 0
\]

where \( N = H(t - t_c) \), \( c = 2\eta\lambda v^2 / H^2 \).

In the far UV where the \( k^2 / a^2 \) term dominates \((k^2 H^{-2} e^{-2N} \gg c|N|)\) have Minkowski space modes.

In the far IR where the \( m_\sigma^2 \) term dominates \((k^2 H^{-2} e^{-2N} \ll c|N|)\) are exponentially damped if \( m_\sigma^2 > 0 \) or amplified if \( m_\sigma^2 < 0 \).

Match solutions at \( N = N_k \) defined by: \( \frac{k^2}{cH^2} e^{-2N_k} = |N_k| \).
Matching Conditions

- Modes which cross $|m_\sigma|$ while $m_\sigma^2 > 0$ are damped exponentially as $a^{-3/2}$ before the instability sets in.
- Modes which cross $|m_\sigma|$ when $m_\sigma^2 < 0$ were light throughout inflation and experience no damping.
Modes in the UV $N < N_k (k \gg a|m_\sigma|)$ feel only Minkowski space: $a \delta^{(1)} \sigma_k \sim e^{-ik\tau} / \sqrt{2k}$.

These are red-shifted into the IR region $N > N_k$ ($k \ll a|m_\sigma|$) where the mass term becomes important:

$$|\delta^{(1)} \sigma_k(N)| \sim |b_k| \exp \left[ -\frac{3}{2} N + \frac{9}{4c} \left( 1 + \frac{4}{9c} N \right)^{3/2} \right]$$
The End of Exponential Growth

★ Once the tachyon fluctuations become sufficiently large, the exponential growth is replaced by oscillations about the minima $\pm v$.

★ Our condition for the end of tachyonic growth:

$$\langle (\delta^{(1)} \sigma)^2 \rangle^{1/2} \bigg|_{N=N_*} = \frac{v}{2}$$

★ Numerical solutions of this equation agree with previous authors.$^a$

★ NOTE: For a very slowly rolling inflaton can have $N_* \gtrsim 1$.

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Part 3: Cosmological Perturbation Theory

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Cosmological Perturbations

★ Expand the metric in longitudinal gauge as:

\[
\begin{align*}
    g_{00} &= -a(\tau)^2 \left[ 1 + 2\phi^{(1)} + \phi^{(2)} \right] \\
    g_{0i} &= 0 \\
    g_{ij} &= a(\tau)^2 \left[ \left( 1 - 2\psi^{(1)} - \psi^{(2)} \right) \delta_{ij} \\
    &\quad + \frac{1}{2} \left( \partial_i \chi^{(2)}_{j} + \partial_j \chi^{(2)}_i + \chi^{(2)}_{ij} \right) \right]
\end{align*}
\]

★ Expand the matter fields as:

\[
\begin{align*}
    \varphi(\tau, \vec{x}) &= \varphi_0(\tau) + \delta^{(1)} \varphi(\tau, \vec{x}) + \frac{1}{2} \delta^{(2)} \varphi(\tau, \vec{x}) \\
    \sigma(\tau, \vec{x}) &= \delta^{(1)} \sigma(\tau, \vec{x}) + \frac{1}{2} \delta^{(2)} \sigma(\tau, \vec{x})
\end{align*}
\]

★ Neglect vectors, tensors at first order.

★ Recall that \( \langle \sigma \rangle \equiv \sigma_0 = 0 \).
At first order there are two independent scalar degrees of freedom: $\phi^{(1)}, \delta^{(1)}\sigma$.

Can write a master equation for $\phi^{(1)}$:

$$\phi''^{(1)} - \frac{2}{\tau}(\eta - \epsilon)\phi''^{(1)} + \left[ \frac{2}{\tau^2}(\eta - 2\epsilon) + k^2 \right] \phi^{(1)} = 0$$

Tachyon fluctuation does not couple to the metric fluctuations:

$$\delta^{(1)}\ddot{\sigma}_k + 3H\delta^{(1)}\dot{\sigma}_k + \left[ \frac{k^2}{a^2} + m^2_{\sigma} \right] \delta^{(1)}\sigma_k = 0$$

Constraint equations fix $\delta^{(1)}\varphi, \psi^{(1)}$. 
First Order Curvature Perturbation

- Physical quantity of interest is the curvature perturbation

\[ \zeta = \zeta^{(1)} + \frac{1}{2} \zeta^{(2)} \]

defined so that \( \langle \zeta \rangle = 0 \).

- For \( \sigma_0 = 0 \) the first order piece depends only on the inflaton:

\[ \zeta^{(1)} \cong -\phi^{(1)} - \frac{\mathcal{H}}{\varphi_0} \delta^{(1)} \phi \]

- First order curvature perturbation is conserved on large scales:

\[ \frac{\partial}{\partial \tau} \zeta_k^{(1)} \cong 0 \quad \text{for} \quad k \ll aH \]

- Have the usual scale invariant spectrum from \( \langle \zeta_{k_1} \zeta_{k_2} \rangle \).
Second Order Einstein Equations

★ Second order fluctuations are **sourced** by first order fluctuations.

★ **Two independent scalar fluctuations** at second order: \( \phi^{(2)}, \delta^{(2)} \sigma \).

★ Can write a **master equation** for \( \phi^{(2)} \):

\[
\phi''_{k}^{(2)} - \frac{2}{\tau} (\eta - \epsilon) \phi'_{k}^{(2)} + \left[ \frac{2}{\tau^{2}} (\eta - 2\epsilon) + k^{2} \right] \phi_{k}^{(2)} = J_{k}(\tau)
\]

where the **source** \( J \) is constructed from \( \delta^{(1)} \sigma, \delta^{(1)} \varphi, \phi^{(1)} \).

★ Can solve for the other second order fluctuations using constraints.

★ Curvature perturbation does not depend on \( \delta^{(2)} \sigma \) up to second order.
Second Order Curvature Perturbation

★ Split the curvature perturbation into inflaton and tachyon contributions:

\[ \zeta^{(2)} = \zeta_\varphi^{(2)} + \zeta_\sigma^{(2)} \]

★ The inflaton part has already been studied and yields negligible nongaussianity\(^a\)

\[ \zeta_\varphi^{(2)} \approx \frac{1}{4} (2\eta - 6\epsilon) \left( \zeta^{(1)} \right)^2 \approx \text{const} \quad \text{for} \quad k \ll aH \]

★ Non-adiabatic pressures at second order will amplify large scale \( \zeta_\sigma^{(2)} \) during the instability phase so that \( \zeta^{(2)} \approx \zeta_\sigma^{(2)} \) after preheating.

\(^a\)Maldacena, JHEP 0305, 013 (2003).
Calculation of $\zeta^{(2)}$

\[
\zeta^{(2)} \equiv -\frac{\phi'^{(2)}}{\epsilon H} - \left(\frac{1}{\epsilon} + 1\right)\phi^{(2)} + \frac{1}{3 - \epsilon}\frac{\partial^k \partial_k \phi^{(2)}}{\epsilon H^2}
\]
\[
+ \frac{1}{\epsilon H} \Delta^{-1} \gamma' + \Delta^{-1} \gamma - \frac{1}{3 - \epsilon}\frac{1}{\epsilon H^2} \gamma
\]
\[
+ \frac{1}{3 - \epsilon} \frac{1}{(\phi_0')^2} \left[ \left(\delta^{(1)} \sigma'\right)^2 + a^2 m_\sigma^2 \left(\delta^{(1)} \sigma\right)^2 \right]
\]
\[
+ \ldots
\]

\[
\phi_k^{(2)}(\tau) = \int d\tau' G_k(\tau, \tau')(-\tau')^{2(\epsilon - \eta)} J_k(\tau')
\]

\[
G_k(\tau, \tau') = \frac{\pi}{2} \Theta(\tau - \tau')(\tau \tau')^{1/2+\eta-\epsilon}
\]
\[
\times \left[ J_\nu(-k\tau)Y_\nu(-k\tau') - J_\nu(-k\tau')Y_\nu(-k\tau) \right]
\]
\[
\nu \cong 1/2 + 3\epsilon - \eta
\]
Calculation of $\zeta_\sigma^{(2)}$

\[ J(\tau, \vec{x}) = a^2 \kappa^2 m_\sigma^2 \left( \delta^{(1)} \sigma \right)^2 - 2\kappa^2 \left( \delta^{(1)} \sigma' \right)^2 \]

\[ + 2\kappa^2 \mathcal{H} (1 + \eta - \epsilon) \Delta^{-1} \partial_i \left( \delta^{(1)} \sigma' \partial^i \delta^{(1)} \sigma \right) \]

\[ + 4\kappa^2 \Delta^{-1} \partial_\tau \partial_i \left( \delta^{(1)} \sigma ' \partial^i \delta^{(1)} \sigma \right) \]

\[ - \mathcal{H} (1 + 2\epsilon - 2\eta) \Delta^{-1} \gamma' + \Delta^{-1} \gamma'' \]

\[ + \text{ inflaton contributions} \]

\[ \gamma = -3\kappa^2 \Delta^{-1} \partial_i \left( \partial^k \partial_k \delta^{(1)} \sigma \partial^i \delta^{(1)} \sigma \right) \]

\[ - \frac{\kappa^2}{2} \left( \partial_i \delta^{(1)} \sigma \partial^i \delta^{(1)} \sigma \right) + \cdots . \]
Large Scale $\zeta^{(2)}_{\sigma}$

★ The leading contribution to $\zeta^{(2)}_{\sigma}$ on large scales:

$$
\zeta^{(2)}_{\sigma} \equiv \frac{\kappa^2}{\epsilon} \int_{-1/a_i H}^{\tau} d\tau' \left[ \frac{\left( \delta^{(1)} \sigma' \right)^2}{\mathcal{H} (\tau')} \right]
- \frac{\mathcal{H} (\tau')^2}{\mathcal{H} (\tau)^3} \left( \left( \delta^{(1)} \sigma' \right)^2 - a^2 m_{\sigma}^2 \left( \delta^{(1)} \sigma \right)^2 \right)
$$

★ The result is manifestly local consistent with the results of other authors.\(^a\)

★ We explicitly identify the error in previous calculations which leads to a nonlocal result.

Part 4: Nongaussianity and Constraints

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Generating $\zeta$

★ The gauge invariant curvature perturbation:

$$\zeta = \zeta^{(1)} + \frac{1}{2}\zeta^{(2)} \sim \zeta^{(1)} + \frac{3}{5} f_{NL} (\zeta^{(1)})^2$$

★ The first order curvature perturbation $\zeta^{(1)}$ is the usual scale invariant and conserved quantity.

★ The second order curvature perturbation is split into

$$\zeta^{(2)} = \underbrace{\zeta^{(2)}_{\varphi}}_{\propto (2\eta - 6\epsilon)(\zeta^{(1)})^2} + \underbrace{\zeta^{(2)}_{\sigma}}_{\text{amplified by instability}}$$

★ After the symmetry breaking completes only one field is dynamical so $\zeta$ is conserved on large scales for $t > t_\star$. 

– p.34/52
Tachyon Bispectrum

★ The bispectrum is dominated by the tachyon part of $\zeta$:

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \approx \frac{1}{2^3} \langle \zeta^{(2)}_{\sigma,k_1} \zeta^{(2)}_{\sigma,k_2} \zeta^{(2)}_{\sigma,k_3} \rangle$$

$$\equiv (2\pi)^{-3/2} B(k_i) \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$

★ Should be compared to the usual inflationary spectrum:

$$\langle \zeta^{(1)}_{k_1} \zeta^{(1)}_{k_2} \rangle = P_\varphi(k_i) \delta^3(\vec{k}_1 + \vec{k}_2)$$

where $P_\varphi^{1/2} \sim (2\pi) 10^{-5} k^{-3/2}$.

★ Nonlinearity parameter $f_{NL}$:

$$B(k_i) \equiv -\frac{6}{5} f_{NL} \left[ P_\varphi(k_1) P_\varphi(k_2) + \text{perms} \right]$$

★ Demand that $|f_{NL}| < 100$. 

– p.35/52
The Linearity Parameter

★ The two-point function also gets contributions from the tachyon:

\[
\langle \zeta_{k_1} \zeta_{k_2} \rangle \approx \langle \zeta^{(1)}_{k_1} \zeta^{(1)}_{k_2} \rangle + \frac{1}{2^2} \langle \zeta^{(2)}_{\sigma,k_1} \zeta^{(2)}_{\sigma,k_2} \rangle
\]

★ Should compare the **second order tachyon spectrum** to the **first order inflaton spectrum**:

\[
\frac{1}{2^2} \langle \zeta^{(2)}_{\sigma,k_1} \zeta^{(2)}_{\sigma,k_2} \rangle \equiv S(k_i) \delta^3(\vec{k}_1 + \vec{k}_2)
\]

★ Define the **linearity parameter**:

\[
f_L \equiv \frac{S(k_i)}{P_\phi(k_i)}
\]

★ Demand that \(|f_L| < 1\) so that the spectrum is due to the inflaton.
If $m_\sigma^2$ varies slowly:

- Tachyon is almost massless throughout inflation.
- Instability sets in very slowly.
- Have $N_* \gg 1$.
- Tachyon fluctuations, bispectrum are scale invariant and can have $|f_{NL}| > 1$.

If $m_\sigma^2$ varies quickly:

- Tachyon curvature perturbation is blue ($n = 4$).
- $\zeta^{(2)}_\sigma$ gets contributions from all tachyon modes in the instability band.
- Preheating distorts the power spectrum on small scales: strongest constraint from $|f_L| < 1$. 
Hybrid Inflation: $\nu / M_p = 10^{-3}$

- Spectral index: $n - 1 \sim 1.84g$. 
Hybrid Inflation

- The size of the excluded region depends sensitively on $v/M_p$.

- Larger effect for smaller $v/M_p$ since the amplification goes like $v/H \sim M_p/v$. 

![Diagram showing excluded and invariant regions](image)
Inverted Hybrid Inflation

- Simple modification of hybrid inflation which gives spectral index $n < 1$.
- SUSY, string theory embeddings of hybrid inflation are more similar to inverted model.
- Inverted hybrid inflation potential:

$$V(\varphi, \sigma) = \frac{\lambda}{4}(\sigma^2 + v^2)^2 - \frac{g^2}{2} \varphi^2 \sigma^2 - \frac{m_\varphi^2}{2} \varphi^2$$

- Obtained from hybrid inflation by flipping the sign of $m_\varphi^2$, $v^2$, $g^2$.
- Potential is unbounded from below without the addition of a $\tilde{\lambda} \varphi^4$ term...
Inverted vs. non-Inverted Model

- Potential along inflationary trajectory:

\[ V_{\text{inf}} = \frac{\lambda v^4}{4} \pm \frac{1}{2} m_\varphi^2 \varphi^2 \]

Hybrid Inflation

- \( n > 1 \)
- Inflaton rolls towards the flat point \( \varphi = 0 \).
- Still have slow roll as \( \varphi \to \varphi_c \).
- Possible to have a light tachyon, scale invariant fluctuations.

Inverted Hybrid Inflation

- \( n < 1 \)
- Inflaton rolls away from the flat point \( \varphi = 0 \).
- As \( \varphi \to \varphi_c \) the inflaton need not be slowly rolling.
- Requires more tuning to keep the tachyon light.
Inverted Hybrid Inflation

* Constraints weakened: new allow regions correspond to fast roll through the instability point $\varphi = \varphi_c$. 
Part 5: Implications for Brane Inflation

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**Brane Inflation**

- **Brane inflation** is a particularly appealing embedding of hybrid inflation into string theory.
- Inflation is driven by potential between $D_3/D_\bar{3}$ which are parallel to our 3 large dimensions and separated in the extra dimensions.
- **Inter-brane separation**, $y$, plays the role of the inflation.
- Lightest stretched string mode between the branes becomes tachyonic at $y \sim l_s$ (open string tachyon).
- **Open string tachyon** plays the role of the waterfall field $\sigma$.
- Tachyon in the spectrum signals instability of the system to annihilate.
Flux Compactifications

- Realistic models of brane inflation are embedded in GKP-KKLT flux vacua.
- Complex structure moduli and dilaton are fixed by addition of fluxes of the NS-NS and R-R gauge fields.
- Kahler modulus fixed by nonperturbative effects (e.g., gaugino condensation).

Compactification has warped throat regions where exponentially large hierarchy can be generated from a small hierarchy in the ratio of fluxes.
In the throat the geometry is locally $\text{AdS}_5 \times S^5$:

$$ds^2 \simeq e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 + y^2 d\Omega_5^2.$$ 

Geometry is identical to Randall-Sundrum I.

Set-up: mobile D3 falling down the throat from the UV $(y = 0)$ end towards a fixed D3 in the IR $(y = y_i > 0)$ end.

Exchange of massless gauge fields gives rise to a Coulomb potential between the branes.

The large warping $a_i = e^{-ky_i} \ll 1$ flattens inter-brane potential.
The $\eta$-problem

- Coulomb potential between the branes:
  
  $$V = a_i^4 \tau_3 \left[ 1 - \frac{1}{N} \left( \frac{\varphi_0}{\varphi} \right)^4 \right]$$

  is extremely flat.

- Unfortunately consistent introduction of volume stabilization introduces an $\mathcal{O}(H)$ contribution to the inflaton mass.

- Can salvage inflation to adding some corrections to the superpotential which cancel the large inflaton mass coming from volume stabilization.

- Inflation is fine tuned in this scenario.\(^a\)

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\(^a\)Kachru et al., JCAP 0310, 013 (2003); Burgess et al., JHEP 0409, 033 (2004).
The End of Inflation

★ Lightest stretched string mode between the branes has mass:

\[ M_T^2 = \frac{M_s^2}{2} \left[ \frac{(M_s y)^2}{(2\pi)^2} - \frac{1}{2} \right] \]

which becomes tachyonic at \( y \lesssim l_s \).

★ Brane annihilation is described by the tachyon condensation:

- Field theory about the tachyon false vacuum \( T = 0 \) describes the coincident brane-antibrane system
- The tachyon rolls to \( |T| = \infty \) and field theory about this point describes the vacuum with no brane-antibrane

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Brane Inflation and Nongaussianity

★ Brane inflation is similar to inverted hybrid inflation:

\[ V_{\text{inf}} = V_0 \left[ 1 - \frac{1}{N} \left( \frac{\varphi_0}{\varphi} \right)^4 - \frac{\beta}{3} \left( \frac{\varphi}{M_p} \right)^2 \right] \]

★ Some differences:

- tachyon field is complex
- the tachyon potential is minimized at \( T = \pm \infty \):
  \[ V(T, y = 0) = \tau_3 e^{-|T|^2} \]
- tachyon DBI action: \(^a\)

\[ \mathcal{L}_{\text{tac}} = -V(T, y) \sqrt{1 + M_s^{-2} |\partial_\mu T|^2} \]

Excluded Regions

★ Dimensionally reduce the DBI action on $AdS_5$ and expand to quadratic order in fields.
★ Match reduced action to inverted hybrid inflation to estimate $g, \lambda, v$ in terms of stringy quantities $g_s, M_s, a_i$. 
Conclusions

★ Variation of the second order curvature perturbation from tachyonic preheating puts interesting constraints on hybrid inflation.

★ Strongest constraints for small symmetry breaking scale $v/M_p \ll 1$.

★ Constraints on inverted hybrid inflation are weaker since it is harder to keep the tachyon light during inflation.

★ Nontrivial constraints on KKLMMT.
Future Directions

- This model leads to domain walls which will overclose the universe.
- Generalization to D-term inflation, $D_3/D_7$, ...
- Interesting hint of excess power on small scales in the CBI and ACBAR CMB data...