

# Gauge mediation

# Messengers of SUSY breaking

We will first consider a model with  $N_f$  messengers  $\phi_i, \bar{\phi}_i$  Goldstino multiplet  $X$  with an expectation value:

$$\langle X \rangle = M + \theta^2 \mathcal{F}$$

$$W = X \bar{\phi}_i \phi_i$$

for gauge unification,  $\phi_i$  and  $\bar{\phi}_i$  should form complete GUT multiplets. The existence of the messengers shifts the coupling at the GUT scale

$$\delta\alpha_{\text{GUT}}^{-1} = -\frac{N_m}{2\pi} \ln\left(\frac{\mu_{\text{GUT}}}{M}\right)$$

where

$$N_m = \sum_{i=1}^{N_f} 2T(r_i)$$

For the unification to remain perturbative we need

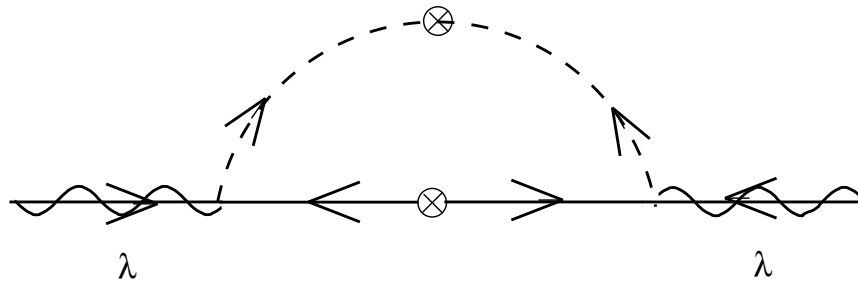
$$N_m < \frac{150}{\ln(\mu_{\text{GUT}}/M)}$$

# Soft Masses

$$\langle X \rangle = M + \theta^2 \mathcal{F}$$

$\Rightarrow$  messenger fermion mass =  $M$

$\Rightarrow$  messenger scalars mass<sup>2</sup> =  $M^2 \pm \mathcal{F}$



one-loop gaugino mass:

$$M_{\lambda i} \sim \frac{\alpha_i}{4\pi} N_m \frac{\mathcal{F}}{M}$$

# Scalar Soft Masses

two-loops squared masses for squarks and sleptons

$$M_s^2 \sim \sum_i \left( \frac{\alpha_i}{4\pi} \frac{\mathcal{F}}{M} \right)^2 \sim M_\lambda^2$$

inserting messenger loop corrections in the one-loop sfermion mass diagrams spoils the cancellation by destroying the relation between the couplings

# RG calculation of soft masses

effective Lagrangian below the messenger mass:

$$\mathcal{L}_G = -\frac{i}{16\pi} \int d^2\theta \tau(X, \mu) W^\alpha W_\alpha$$

Taylor expanding in the  $\mathcal{F}$

$$M_\lambda = \left. \frac{i}{2\tau} \frac{\partial \tau}{\partial X} \right|_{X=M} \mathcal{F} = \left. \frac{i}{2} \frac{\partial \ln \tau}{\partial \ln X} \right|_{X=M} \frac{\mathcal{F}}{M}$$

$$\tau(X, \mu) = \tau(\mu_0) + i \frac{b'}{2\pi} \ln \left( \frac{X}{\mu_0} \right) + i \frac{b}{2\pi} \ln \left( \frac{\mu}{X} \right)$$

$b'$  is the  $\beta$  function coefficient including the messengers

$b$  is the  $\beta$  function coefficient in the effective theory (i.e. the MSSM)

$$b' = b - N_m$$

So the gaugino mass is simply given by

$$M_\lambda = \frac{\alpha(\mu)}{4\pi} N_m \frac{\mathcal{F}}{M}$$

# Gaugino Masses

$$M_\lambda = \frac{\alpha(\mu)}{4\pi} N_m \frac{\mathcal{F}}{M}$$

the ratio of the gaugino mass to the gauge coupling is universal:

$$\frac{M_{\lambda_1}}{\alpha_1} = \frac{M_{\lambda_2}}{\alpha_2} = \frac{M_{\lambda_3}}{\alpha_3} = N_m \frac{\mathcal{F}}{M}$$

this was once thought to be a signature of gravity mediation models

# Sfermion Masses

consider wavefunction renormalization for the matter fields of the MSSM:

$$\mathcal{L} = \int d^4\theta Z(X, X^\dagger) Q'^\dagger Q' ,$$

$Z$  is real and the superscript ' indicates not yet canonically normalized  
Taylor expanding in the superspace coordinate  $\theta$

$$\mathcal{L} = \int d^4\theta \left( Z + \frac{\partial Z}{\partial X} \mathcal{F}\theta^2 + \frac{\partial Z}{\partial X^\dagger} \mathcal{F}^\dagger \bar{\theta}^2 + \frac{\partial^2 Z}{\partial X \partial X^\dagger} \mathcal{F}\theta^2 \mathcal{F}^\dagger \bar{\theta}^2 \right) \Big|_{X=M} Q'^\dagger Q'$$

Canonically normalizing:

$$Q = Z^{1/2} \left( 1 + \frac{\partial \ln Z}{\partial X} \mathcal{F}\theta^2 \right) \Big|_{X=M} Q'$$

so

$$\mathcal{L} = \int d^4\theta \left[ 1 - \left( \frac{\partial \ln Z}{\partial X} \frac{\partial \ln Z}{\partial X^\dagger} - \frac{1}{Z} \frac{\partial^2 Z}{\partial X \partial X^\dagger} \right) \mathcal{F}\theta^2 \mathcal{F}^\dagger \bar{\theta}^2 \right] \Big|_{X=M} Q^\dagger Q$$

# Sfermion Masses

$$m_Q^2 = - \frac{\partial^2 \ln Z}{\partial \ln X \partial \ln X^\dagger} \Big|_{X=M} \frac{\mathcal{F} \mathcal{F}^\dagger}{M M^\dagger}$$

Rescaling also introduces an  $A$  term in the effective potential from Taylor expanding the superpotential:

$$W(Q') = W \left( Q Z^{-1/2} \left( 1 - \frac{\partial \ln Z}{\partial X} \mathcal{F} \theta^2 \right) \Big|_{X=M} \right)$$

so the  $A$  term is

$$Z^{-1/2} \frac{\partial \ln Z}{\partial X} \Big|_{X=M} \mathcal{F} Q \frac{\partial W}{\partial (Z^{-1/2} Q)}$$

which is suppressed by a Yukawa coupling

# RG Calculation

calculate  $Z$  and replace  $M$  by  $\sqrt{XX^\dagger}$ , so that  $Z(X, X^\dagger)$  is invariant under  $X \rightarrow e^{i\beta} X$ . At  $l$  loops RG analysis gives

$$\ln Z = \alpha(\mu_0)^{l-1} f(\alpha(\mu_0)L_0, \alpha(\mu_0)L_X)$$

where

$$L_0 = \ln \left( \frac{\mu^2}{\mu_0^2} \right) , \quad L_X = \ln \left( \frac{\mu^2}{XX^\dagger} \right)$$

so

$$\frac{\partial^2 \ln Z}{\partial \ln X \partial \ln X^\dagger} = \alpha(\mu)^{l+1} h(\alpha(\mu)L_X)$$

two-loop scalar masses are determined by a one-loop RG equation

# RG Calculation

at one-loop

$$\frac{d \ln Z}{d \ln \mu} = \frac{C_2(r)}{\pi} \alpha(\mu)$$

$$Z(\mu) = Z_0 \left( \frac{\alpha(\mu_0)}{\alpha(X)} \right)^{2C_2(r)/b'} \left( \frac{\alpha(X)}{\alpha(\mu)} \right)^{2C_2(r)/b}$$

where

$$\alpha^{-1}(X) = \alpha^{-1}(\mu_0) + \frac{b'}{4\pi} \ln \left( \frac{XX^\dagger}{\mu_0^2} \right)$$
$$\alpha^{-1}(\mu) = \alpha^{-1}(X) + \frac{b}{4\pi} \ln \left( \frac{\mu^2}{XX^\dagger} \right)$$

So we obtain

$$m_Q^2 = 2C_2(r) \frac{\alpha(\mu)^2}{16\pi^2} N_m \left( \xi^2 + \frac{N_m}{b} (1 - \xi^2) \right) \left( \frac{\mathcal{F}}{M} \right)^2$$

where

$$\xi = \frac{1}{1 + \frac{b}{2\pi} \alpha(\mu) \ln(M/\mu)}$$

# Gauge mediation and the $\mu$ problem

electroweak sector in the MSSM needed two types of mass terms: a supersymmetric  $\mu$  term:

$$W = \mu H_u H_d$$

and a soft SUSY-breaking  $b$  term:

$$V = b H_u H_d$$

with a peculiar relation between them

$$b \sim \mu^2$$

In gauge mediated models we need

$$\mu \sim m_{soft} \sim \frac{1}{16\pi^2} \frac{\mathcal{F}}{M}$$

# Gauge mediation and the $\mu$ problem

If we introduce a coupling of the Higgses to the SUSY breaking field  $X$ ,

$$W = \lambda X H_u H_d$$

we get

$$\mu = \lambda M, \quad b = \lambda \mathcal{F} \sim 16\pi^2 \mu^2$$

so  $b$  is much too large A more indirect coupling

$$W = X(\lambda_1 \phi_1 \bar{\phi}_1 + \lambda_2 \phi_2 \bar{\phi}_2) + \lambda H_u \phi_1 \phi_2 + \bar{\lambda} H_d \bar{\phi}_1 \bar{\phi}_2$$

yields a one-loop correction to the effective Lagrangian:

$$\Delta \mathcal{L} = \int d^4\theta \frac{\lambda \bar{\lambda}}{16\pi^2} f(\lambda_1/\lambda_2) H_u H_d \frac{X}{X^\dagger}$$

This unfortunately still gives the same, nonviable, ratio for  $b/\mu^2$

# Gauge mediation and the $\mu$ problem

The correct ratio can be arranged with two additional singlet fields:

$$W = S(\lambda_1 H_u H_d + \lambda_2 N^2 + \lambda \phi \bar{\phi} - M_N^2) + X \phi \bar{\phi}$$

then

$$\mu = \lambda_1 \langle S \rangle, \quad b = \lambda_1 \mathcal{F}_S$$

A VEV for  $S$  is generated at one-loop

$$\langle S \rangle \sim \frac{1}{16\pi^2} \frac{\mathcal{F}_X^2}{M M_N^2}$$

but  $\mathcal{F}_S$  is only generated at two-loops:

$$\mathcal{F}_S \sim \frac{1}{(16\pi^2)^2} \frac{\mathcal{F}_X^2}{M^2} \sim \frac{1}{16\pi^2} \mu \frac{M_N^2}{M}$$

Thus,  $b \sim \mu^2$  provided that  $M_N^2 \sim \mathcal{F}_X$