SUSY breaking
and the MSSM
Spontaneous SUSY breaking at tree-level

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Spontaneous SUSY Breaking

\[ \langle 0 | H | 0 \rangle > 0 \]

implies that SUSY is broken.

\[ V = \mathcal{F}_i^* \mathcal{F}_i + \frac{g^2}{2} D^a D^a , \]

find models where \( \mathcal{F}_i = 0 \) or \( D^a = 0 \) cannot be simultaneously solved

then use this SUSY breaking sector to generate the soft SUSY breaking
O’Raifeartaigh model

have nonzero $\mathcal{F}$-terms

$$W_{O' R} = -k^2 \Phi_1 + m \Phi_2 \Phi_3 + \frac{y}{2} \Phi_1 \Phi_2^2.$$ 

scalar potential is

$$V = |\mathcal{F}_1|^2 + |\mathcal{F}_2|^2 + |\mathcal{F}_3|^2$$

$$= |k^2 - \frac{y}{2} \phi_3^*|^2 + |m \phi_3^*|^2 + |m \phi_2^* + y \phi_1^* \phi_3^*|^2.$$ 

no solution where both $\mathcal{F}_1 = 0$ and $\mathcal{F}_2 = 0$

For large $m$, minimum is at $\phi_2 = \phi_3 = 0$ with $\phi_1$ undetermined

vacuum energy density is

$$V = |\mathcal{F}_1|^2 = k^4.$$
O’Raifeartaigh model

Around $\phi_1 = 0$, the mass spectrum of scalars is

$$0, \ 0, \ m^2, \ m^2, \ m^2 - yk^2, \ m^2 + yk^2.$$

There are also three fermions with masses

$$0, \ m, \ m.$$

Note that these masses satisfy a sum rule for tree-level breaking

$$\text{Tr}[M^2_{\text{scalars}}] = 2\text{Tr}[M^2_{\text{fermions}}]$$
For $k^2 \neq 0$, loop corrections will give a mass to $\phi_1$

$yk^2$ insertions must appear with an even number in order to preserve the orientation of the arrows flowing into the vertices correction to the $\phi_1$ mass from the top three graphs vanishes by SUSY
O’Raifeartaigh model: One Loop

bottom two graphs give

\[-i m_1^2 = \int \frac{d^4 p}{2 \pi^4} \left( -i y^2 \right) \frac{i y^2 k^4}{(p^2-m^2)^3} + (iy m)^2 \frac{i}{p^2-m^2} \frac{iy^2 k^4}{(p^2-m^2)^3} ,\]
yields a finite, positive, result

\[m_1^2 = \frac{y^4 k^4}{48 \pi^2 m^2} = \frac{y^4}{48 \pi^2} \frac{|F_1|^2}{m^2}.\]

the classical flat direction is lifted by quantum corrections, the potential is stable around \( \phi_1 = 0 \)

the massless fermion \( \psi_1 \) stays massless since it is the Nambu–Goldstone particle for the broken SUSY generator, a \textit{goldstino}.

\( \psi_1 \) is the fermion in the multiplet with the nonzero \( F \) component.
Fayet–Iliopoulos mechanism

uses a nonzero \( D \)-term for a \( U(1) \) gauge group
add a term linear in the auxiliary field to the theory:

\[
\mathcal{L}_{\text{FI}} = \kappa^2 D ,
\]

where \( \kappa \) is a constant parameter with dimensions of mass scalar potential is

\[
V = \frac{1}{2} D^2 - \kappa^2 D + g D \sum_i q_i \phi_i^* \phi_i ,
\]

and the \( D \) equation of motion gives

\[
D = \kappa^2 - g \sum_i q_i \phi_i^* \phi_i .
\]

If the \( \phi_i \)s have large positive mass squared terms, \( \langle \phi \rangle = 0 \) and \( D = \kappa^2 \)
in the MSSM, however, squarks and sleptons cannot have superpotential mass terms
Problems

Fayet–Iliopoulos and O’Raifeartaigh models set the scale of SUSY breaking by hand. To get a SUSY breaking scale that is naturally small compared to the Planck scale, $M_{Pl}$, we need an asymptotically free gauge theory that gets strong through RG evolution at some much smaller scale

$$\Lambda \sim e^{-8\pi^{2}/(bg_{0}^{2})}M_{Pl},$$

and breaks SUSY nonperturbatively. Can’t use renormalizable tree-level couplings to transmit SUSY breaking, since SUSY does not allow scalar–gaugino–gaugino couplings. We expect that SUSY breaking occurs dynamically in a “hidden sector” and is communicated by non-renormalizable interactions or through loop effects. If the interactions that communicate SUSY breaking to the MSSM (“visible”) sector are flavor-blind it is possible to suppress FCNCs.
Gauge-Mediated Scenario

add “messenger” chiral supermultiplets where the fermions and bosons are split and which couple to the SM gauge groups
MSSM superpartners get masses through loops:

\[ m_{\text{soft}} \sim \frac{\alpha_i}{4\pi} \frac{\langle F \rangle}{M_{\text{mess}}} \]

If \( M_{\text{mess}} \sim \sqrt{\langle F \rangle} \), then the SUSY breaking scale can be as low as
\( \sqrt{\langle F \rangle} \sim 10^4–10^5 \) GeV.
Gravity-Mediated Scenario

interactions with the SUSY breaking sector are suppressed by powers of $M_{Pl}$ hidden sector field $X$ with a nonzero $\langle \mathcal{F}_X \rangle$, then MSSM soft terms of the order

$$m_{\text{soft}} \sim \frac{\langle \mathcal{F}_X \rangle}{M_{Pl}}.$$ 

To get $m_{\text{soft}}$ to come out around the weak scale we need $\sqrt{\langle \mathcal{F}_X \rangle} \sim 10^{10} - 10^{11}$ GeV. Alternatively, if SUSY is broken by a gaugino condensate $\langle 0 | \lambda^a \lambda^b | 0 \rangle = \delta^{ab} \Lambda^3 \neq 0$, then

$$m_{\text{soft}} \sim \frac{\Lambda^3}{M_{Pl}^2},$$

which requires $\Lambda \sim 10^{13}$ GeV. This can, of course, be rewritten as: $\langle \mathcal{F}_X \rangle = \Lambda^3 / M_{Pl}$. 

Effective Lagrangian

Below the $M_{Pl}$ is:

$$\mathcal{L}_{\text{eff}} = -\int d^4 \theta \frac{X^*}{M_{Pl}} b''^{ij} \psi_i \psi_j + \frac{XX^*}{M_{Pl}^2} \left( \hat{m}^i_j \psi_i \psi_j^* + \hat{b}^{ij} \psi_i \psi_j \right) + h.c.$$  

$$-\int d^2 \theta \frac{X}{2M_{Pl}} \left( \hat{M}_3 G^\alpha G_\alpha + \hat{M}_2 W^\alpha W_\alpha + \hat{M}_1 B^\alpha B_\alpha \right) + h.c.$$  

$$-\int d^2 \theta \frac{X}{M_{Pl}} \hat{a}^{ijk} \psi_i \psi_j \psi_k + h.c.$$  

where $G_\alpha$, $W_\alpha$, $B_\alpha$, and $\psi_i$ are the chiral superfields of the MSSM, and the hatted symbols are dimensionless.

If $\langle X \rangle = \langle F \rangle$ then

$$\mathcal{L}_{\text{eff}} = -\frac{\langle F_X \rangle}{2M_{Pl}} \left( \hat{M}_3 \tilde{G} \tilde{G} + \hat{M}_2 \tilde{W} \tilde{W} + \hat{M}_1 \tilde{B} \tilde{B} \right) + h.c.$$  

$$-\frac{\langle F_X \rangle \langle F_X^* \rangle}{M_{Pl}^2} \left( \hat{m}^i_j \tilde{\psi}_i \tilde{\psi}_j^* + \hat{b}^{ij} \tilde{\psi}_i \tilde{\psi}_j \right) + h.c.$$  

$$-\frac{\langle F_X \rangle}{M_{Pl}} \hat{a}^{ijk} \tilde{\psi}_i \tilde{\psi}_j \tilde{\psi}_k - \frac{\langle F_X^* \rangle}{M_{Pl}} \int d^2 \theta \hat{b}''^{ij} \tilde{\psi}_i \tilde{\psi}_j + h.c.$$
Assumptions

Assume \( \hat{M}_i = \hat{M} \), \( \hat{m}_j^i = \hat{m} \delta^i_j \)

we have generated a \( \mu \)-term with \( \mu^{ij} = \hat{b}' \delta^i_{H_u} \delta^j_{H_d} \langle \mathcal{F}_X \rangle / M_{Pl} \) assuming \( \hat{a}^{ijk} = \hat{a} Y^{ijk} \) and \( \hat{b}^{ij} = \hat{b} \delta^i_{H_u} \delta^j_{H_d} \), then soft parameters have a universal form (when renormalized at \( M_{Pl} \))

gaugino masses are equal

\[
M_i = m_{1/2} = \hat{M} \frac{\langle \mathcal{F}_X \rangle}{M_{Pl}},
\]

the scalar masses are universal

\[
\mathbf{m}^2_f = m^2_{H_u} = m^2_{H_d} = m^2_0 = \hat{m} \frac{|\langle \mathcal{F}_X \rangle|^2}{M_{Pl}^2},
\]

\( A \) and \( b \) terms are given by

\[
A_f = A \mathbf{Y}_f = \hat{a} \frac{\langle \mathcal{F}_X \rangle}{M_{Pl}} \mathbf{Y}_f, \quad b = B \mu = \frac{\hat{b}}{\hat{b}'} \frac{\langle \mathcal{F}_X \rangle}{M_{Pl}} \mu.
\]

\( \mu^2 \) and \( b \) are naturally of the same order of magnitude if \( \hat{b} \) and \( \hat{b}' \) are of the same order of magnitude
Justified Assumptions?

the assumptions avoid problems with FCNCs. Since gravity is flavor-blind, it might seem that this a natural result of gravity mediation. However, the equivalence principle does not guarantee these universal terms, since nothing forbids a Kähler function of the form

\[ K_{\text{bad}} = f(X^*, X)_{j}^{i} \psi_{j}^{\dagger} \psi_{i}, \]

which leads directly to off-diagonal terms in the matrix \( \hat{m}_{j}^{i} \).

Taking \( \mu \) and the four SUSY breaking parameters and running them down from the unification scale (rather than the Planck scale as one would expect) is referred to as the minimal supergravity scenario.
scalar mass $m^2$, gaugino mass $M$, $A = 0$

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The goldstino

Consider the fermions in a general SUSY gauge theory. Take a basis \( \Psi = (\lambda^a, \psi_i) \). The mass matrix is

\[
M_{\text{fermion}} = \begin{pmatrix}
0 & \sqrt{2} g_a (\langle \phi^* \rangle T^a)^i \\
\sqrt{2} g_a (\langle \phi^* \rangle T^a)^j & \langle W^{ij} \rangle
\end{pmatrix}
\]

eigenvector with eigenvalue zero:

\[
\begin{pmatrix}
\langle D^a \rangle / \sqrt{2} \\
\langle F_i \rangle
\end{pmatrix}
\]

eigenvector is only nontrivial if SUSY is broken. The corresponding canonically normalized massless fermion field is the goldstino:

\[
\Pi = \frac{1}{F_\Pi} \left( \frac{\langle D^a \rangle}{\sqrt{2}} \lambda^a + \langle F_i \rangle \psi_i \right)
\]

where

\[
F_{\Pi}^2 = \sum_a \frac{\langle D^a \rangle^2}{2} + \sum_i \langle F_i \rangle^2
\]
The Goldstino

masslessness of the goldstino follows from two facts. First the superpotential is gauge invariant,

\[(\phi^* T^a)^i W_i^* = -(\phi^* T^a)^i \mathcal{F}_i = 0\]

second, the first derivative of the scalar potential

\[\frac{\partial V}{\partial \phi_i} = -W_i^* \frac{\partial W^i}{\partial \phi_i} - g_a (\phi^* T^a)^j D^a\]

vanishes at its minimum

\[\langle \frac{\partial V}{\partial \phi_i} \rangle = \langle \mathcal{F}_i \rangle \langle W^{ij} \rangle - g_a \langle (\phi^* T^a)^j \rangle \langle D^a \rangle = 0\]
The Supercurrent

\[
J^\mu_\alpha = iF_\Pi (\sigma^\mu \bar{\Pi})_\alpha + (\sigma^\nu \bar{\sigma}^\mu \psi_i)_\alpha D_\nu \phi^{*i} - \frac{1}{2\sqrt{2}} (\sigma^\nu \bar{\sigma}^\rho \sigma^\mu \lambda^a)_\alpha F^a_{\nu\rho},
\]

\[\equiv iF_\Pi (\sigma^\mu \bar{\Pi})_\alpha + j^\mu_\alpha.\]

terms included in \(j^\mu_\alpha\) contain two or more fields.
supercurrent conservation:

\[
\partial_\mu J^\mu_\alpha = iF_\Pi (\sigma^\mu \partial_\mu \bar{\Pi})_\alpha + \partial_\mu j^\mu_\alpha = 0 \quad (\ast)
\]

effective Lagrangian for the goldstino

\[
L_{\text{goldstino}} = i\bar{\Pi} \bar{\sigma}^\mu \partial_\mu \Pi + \frac{1}{F_\Pi} (\Pi \partial_\mu j^\mu + h.c.).
\]

The EQOM for \(\Pi\) is just eqn (\ast)

goldstino–scalar–fermion and goldstino–gaugino–gauge boson interactions allow the heavier superpartner to decay interaction terms have two derivatives, coupling is proportional to the difference of mass squared
Eat the Goldstino

Nambu–Goldstone boson can be eaten by a gauge boson for gravity, Poincaré symmetry, and hence SUSY, must be a local SUSY spinor $\epsilon^\alpha \rightarrow \epsilon^\alpha(x)$: supergravity

spin-2 graviton has spin-3/2 fermionic superpartner, gravitino, $\tilde{\Psi}_\mu^\alpha$, which transforms inhomogeneously under local SUSY transformations:

$$\delta \tilde{\Psi}_\mu^\alpha = -\partial_\mu \epsilon^\alpha + \ldots .$$

gavitino is the “gauge” particle of local SUSY transformations

when SUSY is spontaneously broken, the gravitino acquires a mass by “eating” the goldstino: the other super Higgs mechanism

gavitino mass:

$$m_{3/2} \sim \frac{\langle \mathcal{F}_X \rangle}{M_{Pl}}$$
Gravitino Mass

In gravity-mediated SUSY breaking, the gravitino mass \( \sim m_{\text{soft}} \)
In gauge-mediated SUSY breaking the gravitino is much lighter than the MSSM sparticles if \( M_{\text{mess}} \ll M_{Pl} \), so the gravitino is the LSP.

For a superpartner of mass \( m_{\tilde{\psi}} \approx 100 \text{ GeV} \), and \( \sqrt{\langle F_X \rangle} < 10^6 \text{ GeV} \)

\[ m_{3/2} < 1 \text{ keV} \]

the decay \( \tilde{\psi} \rightarrow \psi \Pi \) can be observed inside a collider detector
The goldstino theorem

no matter how SUSY is spontaneously broken, even if it is dynamical, there is a goldstino. Using the SUSY algebra it follows

\[ \langle 0 | \{ Q_\alpha, J^\mu_{\alpha}^\dagger (y) \} | 0 \rangle = \sqrt{2} \sigma^\nu_{\alpha \dot \alpha} \langle 0 | T^\mu_\nu (y) | 0 \rangle = \sqrt{2} \sigma^\nu_{\alpha \dot \alpha} E \eta^\mu_\nu , \]

where \( E \) is the vacuum energy density. When \( E \neq 0 \), SUSY is spontaneously broken. Taking the location of the current to be at the origin, and writing out \( Q_\alpha \) as an integral over a dummy spatial variable

\[ \sqrt{2} \sigma^\mu_{\alpha \dot \alpha} E = \langle 0 | \{ \int d^3 x J^0_\alpha (x), J^\mu_{\alpha}^\dagger (0) \} | 0 \rangle \]

\[ = \sum_n \int d^3 x \left( \langle 0 | J^0_\alpha (x) | n \rangle \langle n | J^\mu_{\alpha}^\dagger (0) | 0 \rangle + \langle 0 | J^\mu_{\alpha}^\dagger (0) | n \rangle \langle n | J^0_\alpha (x) | 0 \rangle \right) \]

where we have inserted a sum over a complete set of states. Choosing \( x^0 = 0 \), use the generator of translations \( (P^\mu) \) to show that

\[ \langle 0 | J^0_\alpha (x) | n \rangle = \langle 0 | e^{i P \cdot x} J^0_\alpha (0) e^{-i P \cdot x} | n \rangle \]

\[ = \langle 0 | J^0_\alpha (0) e^{-i \vec{p}_n \cdot \vec{x}} | n \rangle \]
So we have

$$\sqrt{2}\sigma_{\alpha\dot{\alpha}}^\mu E = \sum_n (2\pi)^3 \delta(p_n^\prime) \left( \langle 0 | J^0_\alpha(0) | n \rangle \langle n | J^{\mu\dagger}_\dot{\alpha}(0) | 0 \rangle + \langle 0 | J^{\mu\dagger}_\dot{\alpha}(0) | n \rangle \langle n | J^0_\alpha(0) | 0 \rangle \right)$$

write the term in parenthesis as $f_n(E_n, \vec{p}_n)$ We can also write our anti-commutator as

$$\sqrt{2}\sigma_{\alpha\dot{\alpha}}^\mu E = \int d^4x \left( \langle 0 | J^0_\alpha(x) J^{\mu\dagger}_\dot{\alpha}(0) | 0 \rangle + \langle 0 | J^{\mu\dagger}_\dot{\alpha}(0) J^0_\alpha(x) | 0 \rangle \right) \delta(x^0)$$

$$= \int d^4x \partial_\rho \left( \langle 0 | J^\rho_\alpha(x) J^{\mu\dagger}_\dot{\alpha}(0) | 0 \rangle \Theta(x^0) - \langle 0 | J^{\mu\dagger}_\dot{\alpha}(0) J^\rho_\alpha(x) | 0 \rangle \Theta(-x^0) \right)$$

where $\Theta(x^0)$ is the step function

$E$ is related to the integral of a total divergence. Nonvanishing if there is a massless particle contributing to the two-point function.
The goldstino theorem

Inserting a sum over a complete set of states we have

\[
\sqrt{2}\sigma_{\alpha\dot{\alpha}}^\mu E = \sum_n \int d^4x \partial_\rho \left( \begin{array}{c}
\langle 0| J_\alpha^\rho (0) e^{-i\vec{p}_n \cdot \vec{x}} | n \rangle \langle n | J_\dot{\alpha}^{\mu\dagger} (0) | 0 \rangle \Theta (x^0) \\
- \langle 0| J_{\dot{\alpha}}^{\mu\dagger} (0) | n \rangle \langle n | e^{i\vec{p}_n \cdot \vec{x}} J_\alpha^\rho (0) | 0 \rangle \Theta (-x^0) \\
- i p_n \rho \left( e^{-i\vec{p}_n \cdot \vec{x}} \langle 0| J_\alpha^\rho (0) | n \rangle \langle n | J_{\dot{\alpha}}^{\mu\dagger} (0) | 0 \rangle \Theta (x^0) \\
+ e^{i\vec{p}_n \cdot \vec{x}} \langle 0| J_{\dot{\alpha}}^{\mu\dagger} (0) | n \rangle \langle n | J_\alpha^\rho (0) | 0 \rangle \Theta (-x^0) \\
+ \delta (x^0) \left( e^{-i\vec{p}_n \cdot \vec{x}} \langle 0| J_\alpha^\rho (0) | n \rangle \langle n | J_{\dot{\alpha}}^{\mu\dagger} (0) | 0 \rangle \\
+ e^{i\vec{p}_n \cdot \vec{x}} \langle 0| J_{\dot{\alpha}}^{\mu\dagger} (0) | n \rangle \langle n | J_\alpha^\rho (0) | 0 \rangle \right) \right)
\end{array} \right) \\
= \sum_n (2\pi)^3 \delta (\vec{p}_n) \left( f_n (E_n, \vec{p}_n) - i \int_0^\infty dx^0 \, e^{i\vec{E}_n \cdot \vec{x}} E_n f_n (E_n, \vec{p}_n) \right)
\]
The goldstino theorem

Comparing the two eqns we see that

$$\int_0^\infty dx^0 \, e^{iE_n x^0} E_n f_n(E_n, 0^+, 0^-) = 0$$

and if SUSY is spontaneously broken

$$f_n(E_n, 0^+, 0^-) \neq 0$$

The only possibility is that

$$f_n(E_n, 0^+, 0^-) \propto \delta(E_n)$$

so a state contributes to our two-point function with the quantum numbers of $J^0_\alpha$ (i.e. a fermion) with $\vec{p} = 0$ and $E = 0$. In other words there must be a goldstino!